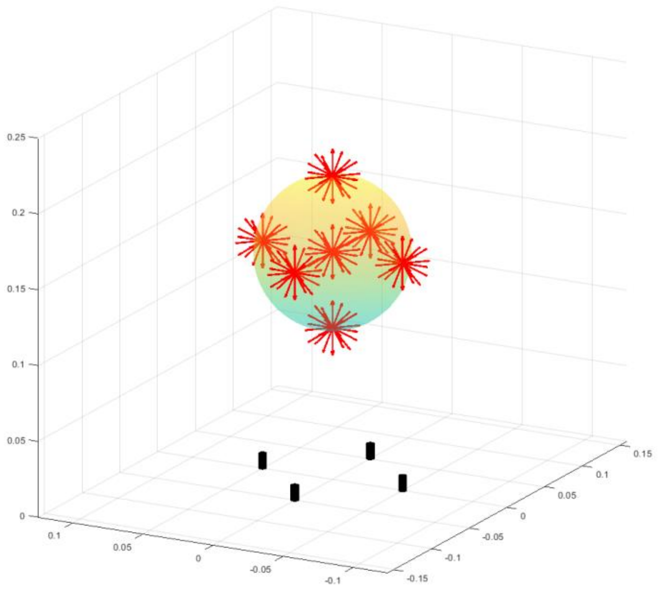
Hypothesis and test configuration

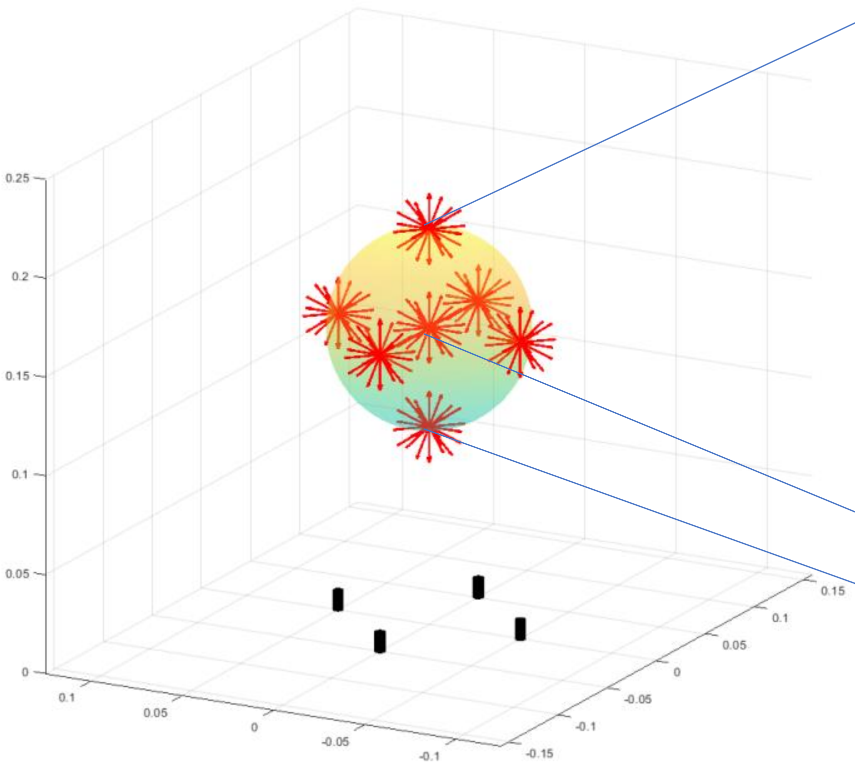
Hypothesis: If the singular value is higher, the same motion will generate a bigger signal compared to the noise level. The algorithm will converge closer to the ground truth when noise presents.

Test configuration: the red arrows are all the magnet configurations where we record the magnetic field. The 4 black cylinders are an example of the sensor put on a 5cm circle.

To ensure convergence, the algorithm starts the initial guess at the nominal configuration (close enough to the solution).



Radius = 5cm



$p = [0; 0; 0.20m], Mean(\sigma_{mean}) = 1.15e-5, Mean(e_p) = 7.41mm, Mean(e_R) = 4.52^{\circ}$

$p = [0; -0.05m; 0.15m], Mean(\sigma_{mean}) = 2.78e-5, Mean(e_p) = 4.56mm, Mean(e_R) = 3.91^{\circ}$

$p = [-0.05m; 0; 0.15m], Mean(\sigma_{mean}) = 2.77e-5, Mean(e_p) = 6.11mm, Mean(e_R) = 3.83^{\circ}$

$p = [0; 0.05; 0.15m], Mean(\sigma_{mean}) = 2.78e-5, Mean(e_p) = 5.68mm, Mean(e_R) = 4.36^{\circ}$

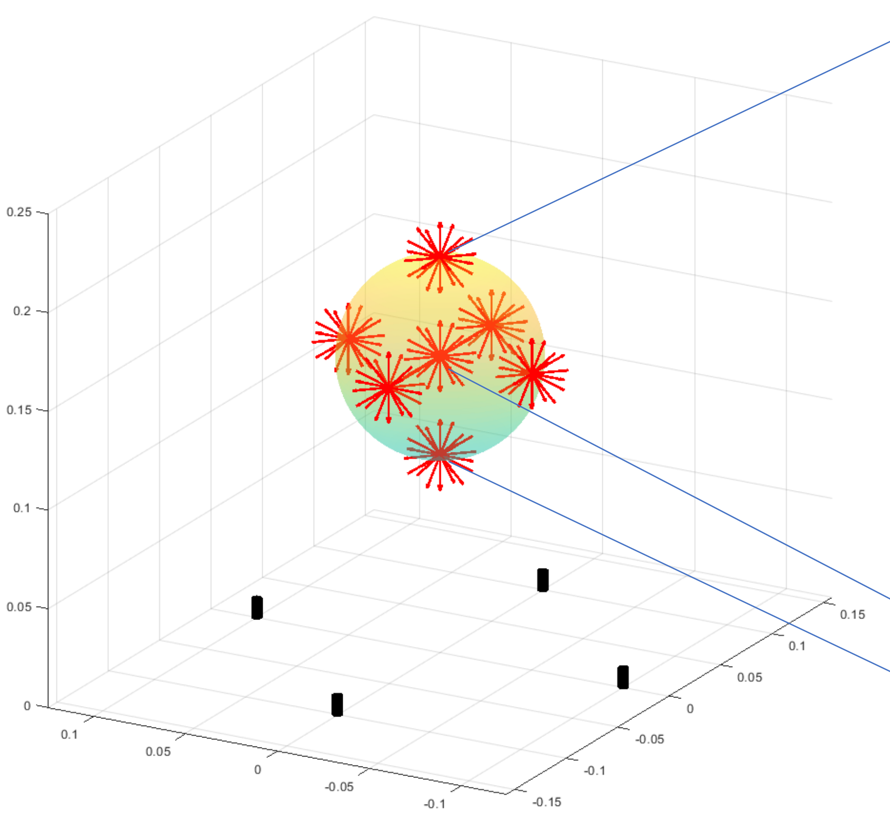
$p = [0.05m; 0; 0.15m], Mean(\sigma_{mean}) = 2.77e-5, Mean(e_p) = 5.11mm, Mean(e_R) = 3.55^{\circ}$

$p = [0; 0; 0.15m], Mean(\sigma_{mean}) = 3.11e-5, Mean(e_p) = 5.19mm, Mean(e_R) = 3.20^{\circ}$

$p = [0; 0; 0.1m], Mean(\sigma_{mean}) = 1.20e-4, Mean(e_p) = 5.09mm, Mean(e_R) = 2.24^{\circ}$

Overall: $Mean(\sigma_{mean}) = 3.90e-5, Mean(e_p) = 5.59mm, Mean(e_R) = 3.66^{\circ}$

Radius = 10cm



$p = [0; 0; 0.20m], Mean(\sigma_{mean}) = 1.15e-5, Mean(e_p) = 5.08mm, Mean(e_R) = 2.76^{\circ}$

$p = [0; -0.05m; 0.15m], Mean(\sigma_{mean}) = 2.78e-5, Mean(e_p) = 3.94mm, Mean(e_R) = 2.15^{\circ}$

$p = [-0.05m; 0; 0.15m], Mean(\sigma_{mean}) = 2.77e-5, Mean(e_p) = 4.63mm, Mean(e_R) = 1.79^{\circ}$

$p = [0; 0.05; 0.15m], Mean(\sigma_{mean}) = 2.78e-5, Mean(e_p) = 4.11mm, Mean(e_R) = 2.66^{\circ}$

$p = [0.05m; 0; 0.15m], Mean(\sigma_{mean}) = 2.77e-5, Mean(e_p) = 3.44mm, Mean(e_R) = 1.80^{\circ}$

$p = [0; 0; 0.15m], Mean(\sigma_{mean}) = 3.11e-5, Mean(e_p) = 4.26mm, Mean(e_R) = 2.21^{\circ}$

$p = [0; 0; 0.1m], Mean(\sigma_{mean}) = 1.20e-4, Mean(e_p) = 4.30mm, Mean(e_R) = 1.77^{\circ}$

Overall: $Mean(\sigma_{mean}) = 2.44e-5, Mean(e_p) = 4.25mm, Mean(e_R) = 2.16^{\circ}$
Even though σ is smaller compared to the 5cm circle, $3.90e-5$, the error is smaller.

Comparison

	$Mean(\sigma_{mean})$ **5cm**	$Mean(e_p)$	$Mean(e_R)$	$Mean(\sigma_{mean})$ **10cm**	$Mean(e_p)$	$Mean(e_R)$
Overall	$3.90e-5$	$5.59mm$	3.66°	$2.44e-5$	$4.25mm$	2.16°
[0;0;0.1]	$1.20e-4$	$5.09mm$	2.24°	$5.40e-5$	$4.30mm$	1.77°
[0;0;0.15]	$3.11e-5$	$5.19mm$	3.20°	$2.11e-5$	$4.26mm$	2.21°
[0;0;0.20]	$1.15e-5$	$7.41mm$	4.52°	$9.31e-6$	$5.08mm$	2.76°
[0.05;0;0.15]	$2.77e-5$	$5.11mm$	3.55°	$2.16e-5$	$3.44mm$	1.80°
[0;0.05;0.15]	$2.78e-5$	$5.68mm$	4.36°	$2.16e-5$	$4.11mm$	2.66
[-0.05;0;0.15]	$2.77e-5$	$6.11mm$	3.83°	$2.16e-5$	$4.63mm$	1.79°
[0;-0.05;0.15]	$2.78e-5$	$3.94mm$	2.15°	$2.16e-5$	$3.94mm$	2.15°

For all the configurations, the singular value is higher for the 5cm circle than that for the 10cm circle, the hypothesis suggests the error after convergence would be smaller for a 5cm circle. But the experiment shows that the error is higher for a 5cm circle.

“Convergence ball” test

Hypothesis: starting from different initial guess, the configuration with higher singular value will converge to a smaller “sphere” that is closer to the ground truth.

But

Observation: As long as the algorithm converges, it converges to the same optima no matter where the initial guess is.

Hypothesis changes: given multiple noisy measurement, starting the initial guess at the ground truth configuration to ensure convergence, the error after convergence is smaller for the config that has a higher singular value.

10 noisy measurements are taken for each configuration. The 10 errors are averaged for each configuration and the errors for all configurations are averaged to get the overall performance.

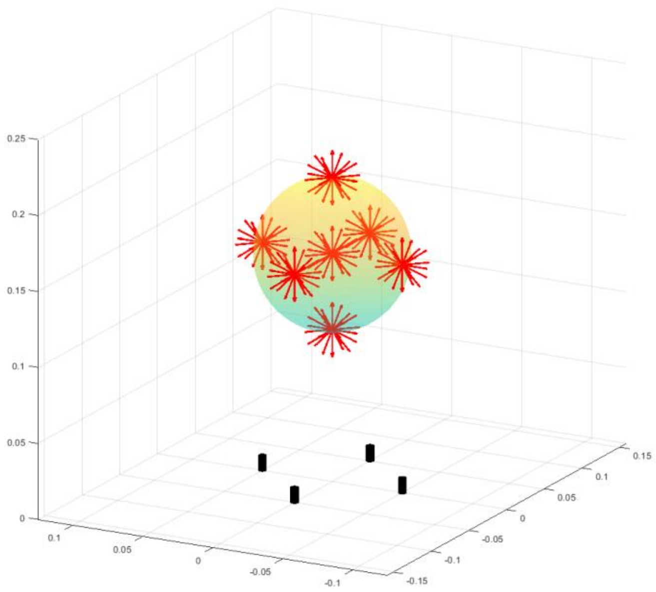
“Convergence ball” test

	$Mean(\sigma_{mean})$ **5cm**	$Mean(e_p)$	$Mean(e_R)$	$Mean(\sigma_{mean})$ **10cm**	$Mean(e_p)$	$Mean(e_R)$
Overall	$3.90e-5$	$5.60mm$	3.67°	$2.44e-5$	$4.26mm$	2.18°

The result is very close to the one on page 4. In face, running the algorithm for 10 different noisy measurements is essentially equivalent to taking the average of 10 noisy measurements and then run the algorithm. Still, the configuration with larger singular value has larger error.

Simulation

To further confirm the phenomenon, simulated experiment is done. For the same magnet configuration, add different level of random noise to the simulated sensor reading. Run the algorithm for 10 times. Compute the mean errors after convergence. Turns out that 10cm of a circle is better than not only 5cm, but also 15cm and 20cm.



$0.5\mu T$ noise	5cm	10cm	15cm	20cm
$Mean(e_p)$	$7.27mm$	$6.14mm$	$8.58mm$	$14.3mm$
$Mean(e_R)$	4.07°	3.31°	4.09°	5.93°

$0.1\mu T$ noise	5cm	10cm	15cm	20cm
$Mean(e_p)$	$1.43mm$	$1.23mm$	$1.70mm$	$2.82mm$
$Mean(e_R)$	0.80°	0.67°	0.81°	1.17°

$0.05\mu T$ noise	5cm	10cm	15cm	20cm
$Mean(e_p)$	$0.72mm$	$0.61mm$	$0.85mm$	$1.41mm$
$Mean(e_R)$	0.40°	0.33°	0.41°	0.59°