440 Assignment 3

Xiaowei Zhang, 204007085
Xinyuan Chen, 197001668

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Problem 1

Game tree see Figure 1. We don't care about the "?" values because when we use the MINIMAX strategy, at state $(3, 2)$, it's B's turn, so we apply MIN, then the path to state $(2, 4)$ will be pruned. Also, at state $(2, 3)$, it's A's turn, so we apply MAX, then the path to state $(1, 4)$ will be pruned.

Problem 2

a) $P(A, B, C, D, E)$
$= P(A) \cdot P(B) \cdot P(C) \cdot P(D|A, B) \cdot P(E|B, C)$
$= 0.2 * 0.5 * 0.8 * 0.1 * 0.3$
$= 0.0024$

b) $P(\neg A, \neg B, \neg C, \neg D, \neg E)$
$= P(\neg A) \cdot P(\neg B) \cdot P(\neg C) \cdot P(\neg D|\neg A, \neg B) \cdot P(\neg E|\neg B, \neg C)$
$= 0.8 * 0.5 * 0.2 * 0.1 * 0.8$
$= 0.0064$

c) $P(\neg A|B, C, D, E)$
$= \alpha \cdot P(\neg A, B, C, D, E)$
$= \alpha \cdot (0.8 * 0.5 * 0.8 * 0.6 * 0.3)$
$= \alpha \cdot 0.0576$
$\alpha = \frac{1}{P(A,B,C,D,E)+P(\neg A,B,C,D,E)}$
$= \frac{1}{(0.2*0.5*0.8*0.1*0.3)+(0.8*0.5*0.8*0.6*0.3)}$
$= \frac{1}{0.0024+0.0576}$
$= \frac{50}{3}$
Thus $P(\neg A|B, C, D, E)$
$= \frac{50}{3} * 0.0576$
$= 0.96$

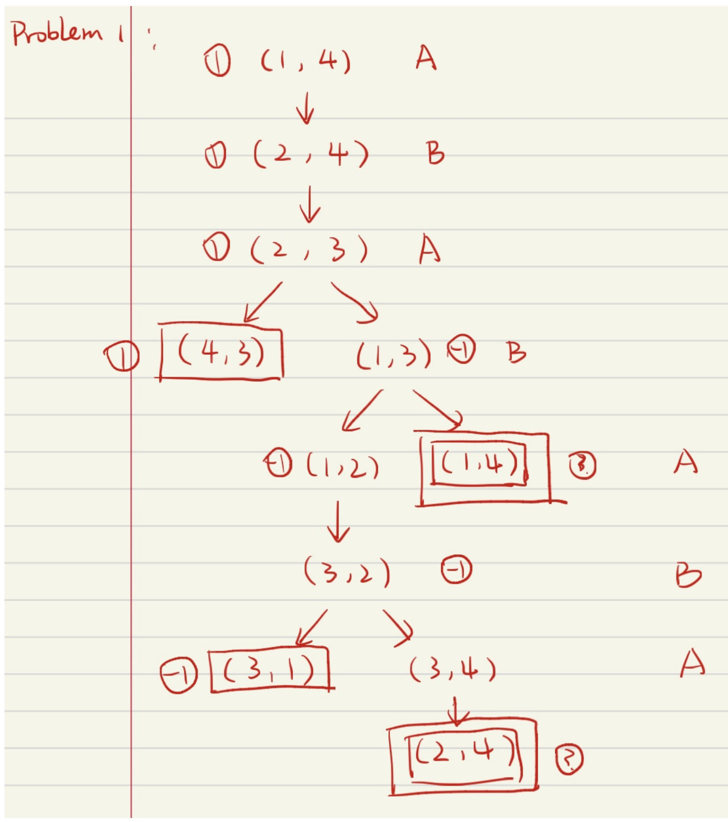

Figure 1: Game Tree

Problem 3

a) $P(B|j,m)$
$= \alpha P(B,j,m)$
$= \alpha \sum_e \sum_a P(B,j,m,e,a)$
For $B = true$:
$P(b|j,m)$
$= \alpha \sum_e \sum_a P(b)P(e)P(a|b,e)P(j|a)P(m|a)$
$= \alpha P(b) \sum_e P(e) \sum_a P(a|b,e)P(j|a)P(m|a)$
$= \alpha[P(b)P(e)P(a|b,e)P(j|a)P(m|a) + P(b)P(\neg e)P(a|b,\neg e)P(j|a)P(m|a) +$
$P(b)P(e)P(\neg a|b,e)P(j|\neg a)P(m|\neg a) + P(b)P(\neg e)P(\neg a|b,\neg e)P(j|\neg a)P(m|\neg a)]$
$= \alpha[(0.001 * 0.002 * 0.95 * 0.9 * 0.7) + (0.001 * 0.998 * 0.94 * 0.9 * 0.7) +$
$(0.001 * 0.002 * 0.05 * 0.05 * 0.01) + (0.001 * 0.998 * 0.06 * 0.05 * 0.01)]$
$= \alpha \cdot 0.00059224$
For $B = false$:
$P(\neg b|j,m)$
$= \alpha \sum_e \sum_a P(\neg b)P(e)P(a|\neg b,e)P(j|a)P(m|a)$
$= \alpha P(\neg b) \sum_e P(e) \sum_a P(a|\neg b,e)P(j|a)P(m|a)$
$= \alpha[P(\neg b)P(e)P(a|\neg b,e)P(j|a)P(m|a) + P(\neg b)P(\neg e)P(a|\neg b,\neg e)P(j|a)P(m|a) +$
$P(\neg b)P(e)P(\neg a|\neg b,e)P(j|\neg a)P(m|\neg a) + P(\neg b)P(\neg e)P(\neg a|\neg b,\neg e)P(j|\neg a)P(m|\neg a)]$
$= \alpha[(0.999 * 0.002 * 0.29 * 0.9 * 0.7) + (0.999 * 0.998 * 0.001 * 0.9 * 0.7) +$
$(0.999 * 0.002 * 0.71 * 0.05 * 0.01) + (0.999 * 0.998 * 0.999 * 0.05 * 0.01)]$
$= \alpha \cdot 0.00149194$
$\alpha = \frac{1}{P(b,j,m)+P(\neg b,j,m)}$
Thus $P(B|j,m) = \alpha < 0.00059224, 0.00149194 > \approx < 0.284, 0.716 >$

b) The complexity of computing $P(X_1|X_n = true)$ using enumeration is $O(2^n)$, and the complexity using variable elimination is $O(n)$.

Problem 4

OC : card holder owns a computer or smart phone.
Fraud : current transaction is fraudulent.
Trav : card holder is currently travelling.
FP : current transaction is a foreign purchase.
IP : current purchase is an internet purchase.
CRP : a computer related purchase was made in the past week.
$P(Frau|Trav) = 0.01$, $P(Frau|\neg Trav) = 0.004$
$P(Trav) = 0.05$
$P(FP|\neg Trav, Fraud) = 0.1$, $P(FP|\neg Trav, \neg Fraud) = 0.01$
$P(FP|Trav) = 0.9$
$P(OC) = 0.75$
$P(IP|OC, \neg Fraud) = 0.01$, $P(IP|OC, Fraud) = 0.02$
$P(IP|\neg OC, \neg Fraud) = 0.001$, $P(IP|\neg OC, Fraud) = 0.011$
$P(CRP|OC) = 0.1$, $P(CRP|\neg OC) = 0.001$

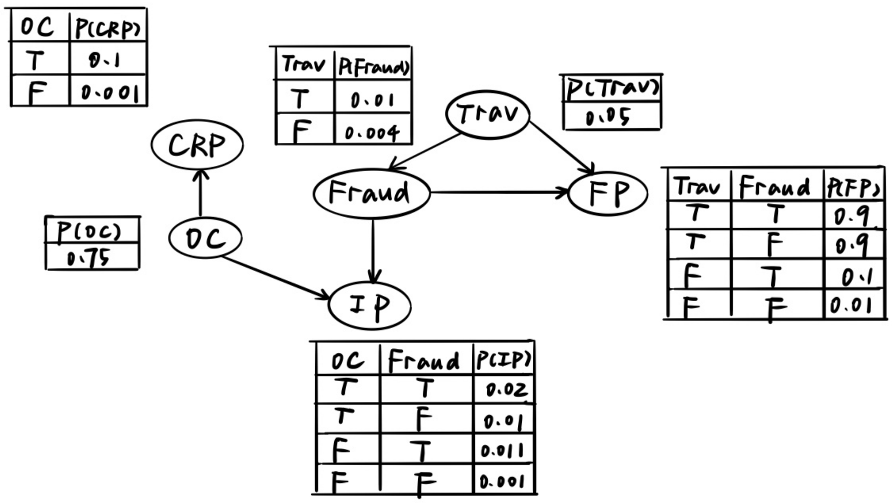

Figure 2: Bayes Network

a) Bayes Network see Figure 2.

b) $P(Fraud)$
$= P(Fraud|Trav)P(Trav) + P(Fraud|\neg Trav)P(\neg Trav)$
$= (0.01 * 0.05) + (0.004 * 0.95)$
$= 0.0043$
$P(Fraud|FP, \neg IP, CRP)$
$= \frac{P(Fraud, FP, \neg IP, CRP)}{P(FP, \neg IP, CRP)}$
$P(Fraud, FP, \neg IP, CRP)$
$= \sum_{OC} \sum_{Trav} P(Trav)P(FP|Trav, Fraud)P(\neg IP|OC, Fraud)P(CRP|OC)P(Fraud|Trav)$
$= 0.05 * 0.9 * 0.98 * 0.1 * 0.01 + 0.95 * 0.1 * 0.98 * 0.1 * 0.004 + 0.05 * 0.9 *$
$0.989 * 0.001 * 0.01 + 0.95 * 0.1 * 0.989 * 0.001 * 0.004$
$= 0.00008216087$
$P(\neg Fraud, FP, \neg IP, CRP)$
$= \sum_{OC} \sum_{Trav} P(Trav)P(FP|Trav, \neg Fraud)P(\neg IP|OC, \neg Fraud)P(CRP|OC)P(\neg Fraud|Trav)$
$= 0.05 * 0.9 * 0.99 * 0.1 * 0.99 + 0.95 * 0.01 * 0.99 * 0.1 * 0.996 + 0.05 * 0.9 *$
$0.999 * 0.001 * 0.99 + 0.95 * 0.01 * 0.999 * 0.001 * 0.996$
$= 0.005445651438$
$P(Fraud|FP, \neg IP, CRP)$
$= \frac{0.00008216087}{0.00008216087 + 0.005445651438}$
$= 0.01486318012$