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ch07 xlin11

7.1

(a)monomial basis: $p_2(t) = x_1 + x_2 t + x_3 t^2$

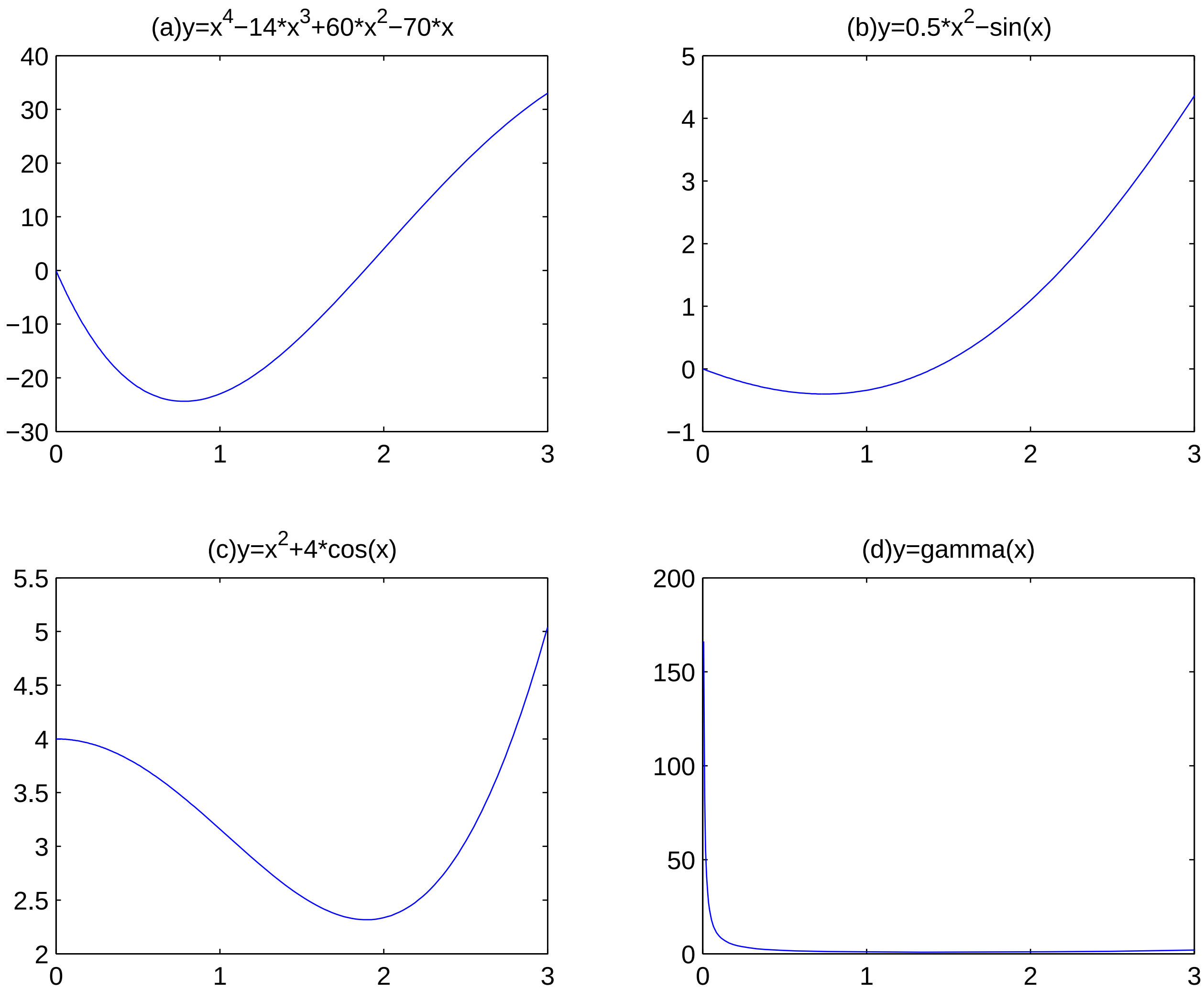
$$\boldsymbol{Ax} = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$p_2(t) = t^2$

(b)Lagrange basis: $p_2(t) = 1 \times \frac{(t-0)(t-1)}{(-1-0)(-1-1)} + 0 + 1 \times \frac{(t+1)(t-0)}{(1+1)(1-0)} \Rightarrow p_2(t) = t^2$

(c)Newton basis: $$\boldsymbol{Ax} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & t_2 - t_1 & 0 \\ 1 & t_3 - t_1 & (t_3 - t_1)(t_3 - t_2) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow p_2(t) = t^2$$

```
function cp06_03
syms x;
x1=0;
x2=3;
%(a)
subplot(2,2,1);
disp('(a)');
f1=@(x)x^4-14*x^3+60*x^2-70*x;
min1=fminbnd(f1,x1,x2);
fplot(f1,[x1 x2]);
title('(a)y=x^4-14*x^3+60*x^2-70*x');
disp('Minimum of function in (a) is ');
disp('         x          y');
disp([min1 subs(f1, x, min1)]);
%(b)
subplot(2,2,2);
disp('(b)');
f2=@(x)0.5*x^2-sin(x);
min2=fminbnd(f2,x1,x2);
fplot(f2,[x1 x2]);
title('(b)y=0.5*x^2-sin(x)');
disp('Minimum of function in (b) is ');
disp('         x          y');
disp([min2 subs(f2, x, min2)]);
%(c)
subplot(2,2,3);
disp('(c)');
f3=@(x)x^2+4*cos(x);
min3=fminbnd(f3,x1,x2);
fplot(f3,[x1 x2]);
title('(c)y=x^2+4*cos(x)');
disp('Minimum of function in (c) is ');
disp('         x          y');
disp([min3 subs(f3, x, min3)]);
%(d)
subplot(2,2,4);
disp('(d)');
f4=@(x)gamma(x);
min4=fminbnd(f4,x1,x2);
fplot(f4,[x1 x2]);
title('(d)y=gamma(x)');
disp('Minimum of function in (d) is ');
disp('         x          y');
disp([min4 subs(f4, x, min4)]);
```


(a)y=x^4−14*x^3+60*x^2−70*x
40
30
20
10
0
−10
−20
−30
0
1
2
3
(b)y=0.5*x^2−sin(x)
5
4
3
2
1
0
−1
0
1
2
3
(c)y=x^2+4*cos(x)
5.5
5
4.5
4
3.5
3
2.5
2
0
1
2
3
(d)y=gamma(x)
200
150
100
50
0
0
1
2
3

```
>> cp06_03
(a)
Minimum of function in (a) is
         x         y
    0.7809  -24.3696

(b)
Minimum of function in (b) is
         x         y
    0.7391   -0.4005

(c)
Minimum of function in (c) is
         x         y
    1.8955    2.3168

(d)
Minimum of function in (d) is
         x         y
    1.4616    0.8856

>>
```

```
function cp06_08
syms x y;
f=2*x^2-1.05*x^4+x^6/6+x*y+y^2;
fx=diff(f,x);
fy=diff(f,y);
[xcr,ycr]=solve(fx,fy);
cr=vpa([xcr,ycr],6);

disp('Critical points:');
disp('         x         y');
disp(cr);

disp('Corresponding Hessian matrixes and eigenvalues are:');
HessM=hessian(f);
for k = 1:5
    Hess=subs(HessM, [x,y], [xcr(k), ycr(k)]);
    disp(vpa(Hess,6));
    disp('Eigenvalue:');
    disp(vpa(eig(Hess), 6));
end
disp('Non of them is singular.');

disp('If Hessian Matrix is positive definite, then it is local minimum point of f.');
disp('So points');
disp('         x         y');
disp(vpa(cr([1 2 4],1:2),6));
disp('are local minimum of f.');
disp('If Hessian Matrix is indefinite, then it is local minimum point of f.');
disp('And points');
disp('         x         y');
disp(vpa(cr([3 5],1:2),6));
disp('are saddle points of f.');
```

```
>> cp06_08
Critical points:
         x         y
[        0,         0]
[ -1.74755,  0.873776]
[ -1.07054,  0.535271]
[  1.74755, -0.873776]
[  1.07054, -0.535271]

Corresponding Hessian matrixes and eigenvalues are:
[ 4.0, 1.0]
[ 1.0, 2.0]

Eigenvalue:
 1.58579
 4.41421

[ 12.1531, 1.0]
[     1.0, 2.0]

Eigenvalue:
 1.90245
 12.2506

[ -3.87309, 1.0]
[      1.0, 2.0]

Eigenvalue:
 -4.03869
   2.1656

[ 12.1531, 1.0]
[     1.0, 2.0]

Eigenvalue:
 1.90245
 12.2506

[ -3.87309, 1.0]
[      1.0, 2.0]

Eigenvalue:
 -4.03869
   2.1656

Non of them is singular.
If Hessian Matrix is positive definite, then it is local minimum point of f.
So points
         x         y
[        0,         0]
[ -1.74755,  0.873776]
```

```
[  1.74755, -0.873776]

are local minimum of f.
If Hessian Matrix is indefinite, then it is local minimum point of f.
And points
         x         y
[ -1.07054,  0.535271]
[  1.07054, -0.535271]

are saddle points of f.
>>
```

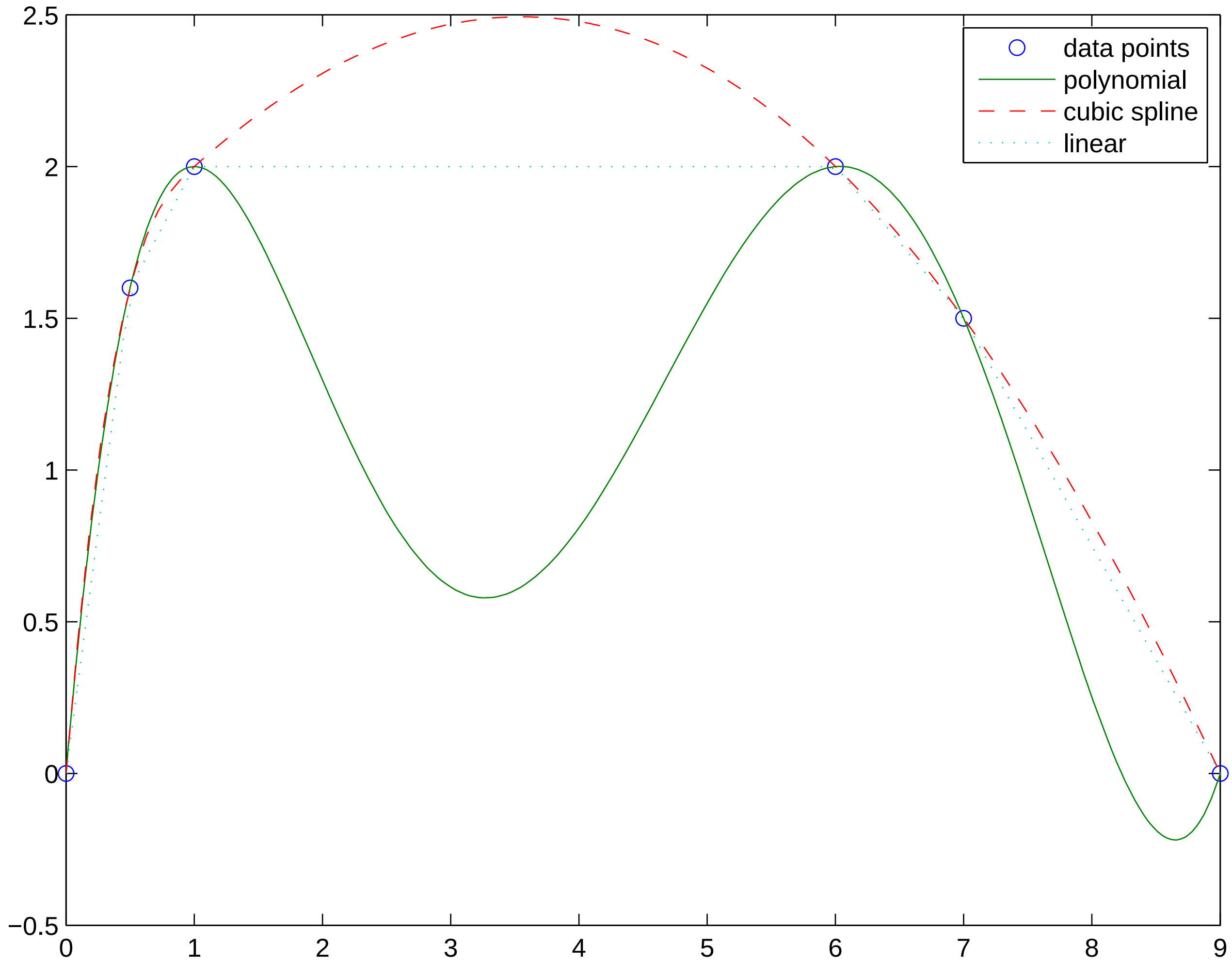
```
function cp07_05
syms x;
X=[0 0.5 1 6 7 9];
Y=[0 1.6 2 2 1.5 0];

%(a)

disp('(a)Monomial basis mathod:');
V = vander(X);
c=V\Y';
f1=poly2sym(c);
disp(f1);
disp('It is about ');
disp(vpa(f1,5));
F=@(x)[x^5 x^4 x^3 x^2 x^1 x^0]*c;
plot(X,Y,'o');
hold all
fplot(F, [0 9]);
hold all
%(b)

disp('(b)');
cs=spline(X, Y);
disp('Coeffeicients of the cubic spline method:');
disp(cs.coefs);
xx=linspace(0,9,101);
plot(xx,ppval(cs,xx),'--');
hold all
%(c)
disp('(c)The cubic spline.');
disp('Polynomial methods have n-1 critical points, where produce the "wiggles", ...which
may bear no relation to data to be fit. And it may oscillate wildly between data
points.');
disp('Piecewise interpolation eliminates excessive oscillation and nonconvergence.The
physical spline minimizes potential energy subject to the interpolation constraints. The
corresponding cubic spline must have a continuous second derivative and satisfy the same
interpolation constraints.');

%(d)
disp('(d)');
yy = interp1(X,Y,xx);
plot(xx,yy,':');
legend('data points','polynomial','cubic spline','linear');
disp('Yes. It avoids the problem of wiggles and non convergence of polynomial method. And
there is no enough evidence provided by the data to say that the y value of x in (1,6)
should increase or decrease, while piecewise linear interpolation provides simplisity.');
```


data points
polynomial
cubic spline
linear
2.5
2
1.5
1
0.5
0
−0.5
0
1
2
3
4
5
6
7
8
9

```
>> cp07_05
(a)Monomial basis mathod:
(6557559582006309*x^5)/1152921504606846976 - (607027170788621*x^4)/4503599627370496 +
(2523721185674223*x^3)/2251799813685248 - (8682767057338267*x^2)/2251799813685248 +
(5476675675435123*x)/1125899906842624

It is about
0.0056878*x^5 - 0.13479*x^4 + 1.1208*x^3 - 3.8559*x^2 + 4.8643*x

(b)
Coeffeicients of the cubic spline method:
    1.5544   -4.7315    5.1772         0
    1.5544   -2.4000    1.6114    1.6000
   -0.0014   -0.0685    0.3772    2.0000
    0.0015   -0.0894   -0.4121    2.0000
    0.0015   -0.0849   -0.5864    1.5000

(c)The cubic spline.
Polynomial methods have n-1 critical points, where produce the "wiggles", ...which may
bear no relation to data to be fit. And it may oscillate wildly between data points.
Piecewise interpolation eliminates excessive oscillation and nonconvergence.The physical
spline minimizes potential energy subject to the interpolation constraints. The
corresponding cubic spline must have a continuous second derivative and satisfy the same
interpolation constraints.
(d)
Yes. It avoids the problem of wiggles and non convergence of polynomial method. And there
is no enough evidence provided by the data to say that the y value of x in (1,6) should
increase or decrease, while piecewise linear interpolation provides simplisity.
>>
```