CSC165H1 Problem Set 1

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1. Pets

   (a) $\forall p \in P,\ Attacks(p, p) \Rightarrow \neg House(p)$

   (b) $\forall p_1 \in P,\ \exists p_2 \in P,\ p_2 \neq p_1 \land Attacks(p_1, p_2) \Rightarrow \neg Behaved(p_1)$

   (c) There is at least one dog who do not attack any cats.

   (d) $\forall p_1, p_2 \in P,\ p_1 \neq p_2 \land \Big(Attacks(p_1, p_2) \land Attacks(p_2, p_1)\Big) \Rightarrow \Big(\neg House(p_1) \lor \neg House(p_2)\Big)$

2. Working with strings

   (a) HasCSC(s): $\exists i \in \mathbb{N},\ 0 \leq i < (|s| - 2) \land s[i] = C \land s[i+1] = S \land s[i+2] = C$

   (b) Substring(s_1, s_2): $\Bigg(|s_1| \leq |s_2| \land \Big(\exists x \in \mathbb{N} \land 0 \leq x \leq |s_2| - |s_1|, (\forall i \in \mathbb{N}, 0 \leq i < |s_1| \Rightarrow s_1[i] = s_2[i + x])\Big)\Bigg) \lor |s_1| = 0$

   (c) Palindrome(s): $\forall i \in \mathbb{N}, 0 \leq i < |s| \Rightarrow s[i] = s[|s| - i - 1]$

   (d) False.
   Even $|s_1| \leq |s_2|$, s_1 can be not the substring of s_2 because their characters can be different. And if s_1 is not substring of s_2, the length of s_1 can be bigger, smaller or equal to the length of s_2.

3. Properties of functions

   (a) $\exists x, y \in \mathbb{R},\ x < y \land f_1(x) \geq f_2(y)$

   (b) $\Big(\exists x, y \in \mathbb{R}, x < y \land f_2(x) \geq f_2(y)\Big) \land \Big(\exists x, y \in \mathbb{R}, x < y \land f_2(x) \leq f_2(y)\Big)$

   (c) $\forall x \in \mathbb{R},\ \exists a, b \in \mathbb{R},\ a \neq b \land f_3(a) \geq f_3(x) \land f_3(b) \geq f_3(x)$

   (d) $\forall f : \mathbb{R} \to \mathbb{R},\ \Big(\forall x, y \in \mathbb{R}, x < y \Rightarrow f(x) < f(y)\Big) \Rightarrow \Big(\forall a \in \mathbb{R}, \exists x \in \mathbb{R}, f(a) < f(x)\Big)$

4. Choosing a universe and predicates

(a)

- If the statement is true: $P(n,m) : n > m$, where $n, m \in \mathbb{N}$. Because for natural number x, if $x > 165$, then $x > 3$.
- If the statement is false: $P(n,m) : n < m$, where $n, m \in \mathbb{N}$. Because for natural number x, if$x < 165$, then x might smaller or greater than 3, so the statement is false.

(b)

- If the statement is true:
Let $\mathbb{U} = \{0, 1, 2, 3, 4, 5, 6\}$
P(x):$x < 9$, $x \in \mathbb{U}$
Q(x):$x < 8$, $x \in \mathbb{U}$
R(x):$x < 7$, $x \in \mathbb{U}$
Explanation: Since all the items in U are smaller than 9, 8 and 7, so P(x), Q(x), R(x) are always True. Thus the statement "$\forall x \in \mathbb{U}, \Big(\big(P(x) \Rightarrow Q(x)\big) \Rightarrow R(x)\Big) \Leftrightarrow \Big(P(x) \Rightarrow \big(Q(x) \Rightarrow R(x)\big)\Big)$ is always true
- If the statement is false:
Let $\mathbb{U} = \{0, 1, 2, 3, 4, 5, 6\}$
P(x):$x < 5$, $x \in \mathbb{U}$
Q(x):$x < 4$, $x \in \mathbb{U}$
R(x):$x < 3$, $x \in \mathbb{U}$
Explanation: When $x = 4$, P(x) is True, Q(x) is False and R(x) is False. Thus $\big(P(x) \Rightarrow Q(x)\big) \Rightarrow R(x)$ is False and $P(x) \Rightarrow \big(Q(x) \Rightarrow R(x)\big)$ is True. Thus $\Big(\big(P(x) \Rightarrow Q(x)\big) \Rightarrow R(x)\Big) \Leftrightarrow \Big(P(x) \Rightarrow \big(Q(x) \Rightarrow R(x)\big)\Big)$ is True. Which means that not all x in U can make the statement True. So, the whole statement is False.