CSC420 Assignmnet 4

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Q1

First of all, let us define and calculate some variables:
$s_1 = w_1 * x_1 + w_2 * x_2 = 0.75 * 0.9 + (-0.63) * (-1.1) = 1.368$
$l_1 = \sigma(s_1) = \frac{1}{1+e^{-1.368}} = 0.797$
$s_2 = w_3 * x_3 + w_4 * x_4 = (-0.3) * (0.24) + 0.8 * (-1.7) = -1.432$
$l_2 = \sigma(s_2) = \frac{1}{1+e^{1.432}} = 0.193$
$s_3 = w_5 * l_1 + w_6 * l_2 = 0.8 * 0.797 + (-0.2) * 0.193 = 0.599$
Thus, at the end, $\hat{y} = \sigma(s_3) = \frac{1}{1+e^{0.599}} = 0.645$
By using back-propagation algorithm, we can have $\frac{\partial L}{\partial w_3} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial s_3} \frac{\partial s_3}{\partial l_2} \frac{\partial l_2}{\partial s_2} \frac{\partial s_2}{\partial w_3}$
$\frac{\partial L}{\partial \hat{y}} = 2||y - \hat{y}|| = 2 * |0.5 - 0.645| = 0.29$
$\frac{\partial \hat{y}}{\partial s_3} = \frac{\partial \sigma(s_3)}{\partial s_3} = \frac{e^{-s3}}{(1+e^{-s3})^2} = 0.229$
$\frac{\partial s_3}{\partial l_2} = \frac{\partial w_5 * l_1 + w_6 * l_2}{\partial l_2} = w_6 = -0.2$
$\frac{\partial l_2}{\partial s_2} = \frac{\partial \sigma(s_2)}{\partial s_2} = \frac{e^{-s2}}{(1+e^{-s2})^2} = 0.156$
$\frac{\partial s_2}{\partial w_3} = \frac{\partial w_3 * x_3 + w_4 * x_4}{\partial w_3} = w_3 = -0.3$
Thus, we can have $\frac{\partial L}{\partial w_3} = 0.29 * 0.229 * (-0.2) * 0.156 * (-0.3) = 0.00062$

Q2

1.

The vanishing point of line L is the point that the line finally converges at. Thus, when the coordinates of the line L in the image is $\vec{p} = \begin{bmatrix} \omega x \\ \omega y \\ \omega \end{bmatrix} = K\vec{P} = K \begin{bmatrix} X_0 + td_x \\ Y_0 + td_y \\ Z_0 + td_z \end{bmatrix}$
$= \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_0 + td_x \\ Y_0 + td_y \\ Z_0 + td_z \end{bmatrix} = \begin{bmatrix} fX_0 + ftd_x + p_x Z_0 + tp_x d_z \\ fY_0 + ftd_y + p_y Z_0 + tp_y d_z \\ Z_0 + td_z \end{bmatrix}$, we can get the coordinates of its vanishing points by taking $t -> \infty$:
$x_v = \lim_{t->\infty} \frac{fX_0 + ftd_x + p_x Z_0 + tp_x d_z}{Z_0 + td_z} = \frac{fd_x + p_x d_z}{d_z}$
$y_v = \lim_{t->\infty} \frac{fY_0 + ftd_y + p_y Z_0 + tp_y d_z}{Z_0 + td_z} = \frac{fd_y + p_y d_z}{d_z}$
Thus, the pixel coordinates of the vanishing point is $(\frac{fd_x + p_x d_z}{d_z}, \frac{fd_y + p_y d_z}{d_z})$.

2.

Assume the normal vector of the plane is $\vec{n} = (n_x, n_y, n_z)$.
Since all the lines $\vec{d}$ on the plane are perpendicular to $\vec{n}$, $n_x d_x + n_y d_y + n_z d_z = 0$
Thus, we can have $d_x = -\frac{n_y d_y + n_z d_z}{n_x}$

From last part, we can have the coordinates of vanishing points is: $x_v = \frac{fd_x + p_x d_z}{d_z}$, $y_v = \frac{fd_y + p_y d_z}{d_z}$.

$x_v = \frac{fd_x + p_x d_z}{d_z}$

$= \frac{fd_x}{d_Z} + p_x$

$= \frac{f(-\frac{n_y d_y + n_z d_z}{n_x})}{d_Z} + p_x$

$= \frac{-fn_y d_y - fn_z d_z}{n_x d_z} + p_x$

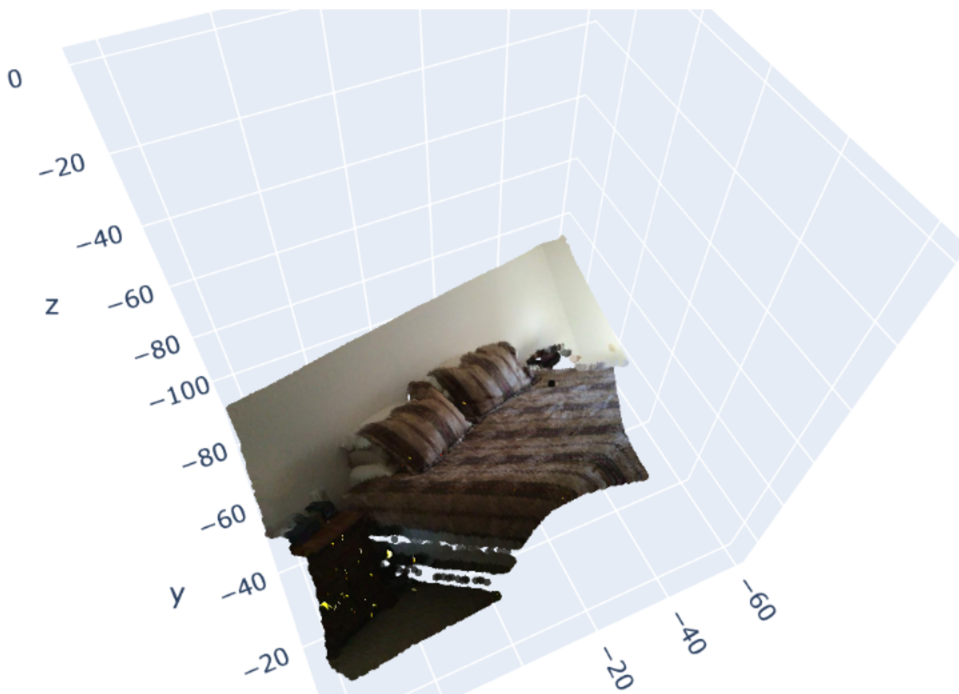
$= \frac{-n_y y_v d_z + n_y p_y d_z - fn_z d_z}{n_x d_z} + p_x$ (Since $y_v = \frac{fd_y + p_y d_z}{d_z}$, $d_y = \frac{y_v d_z - p_y d_z}{f}$.)

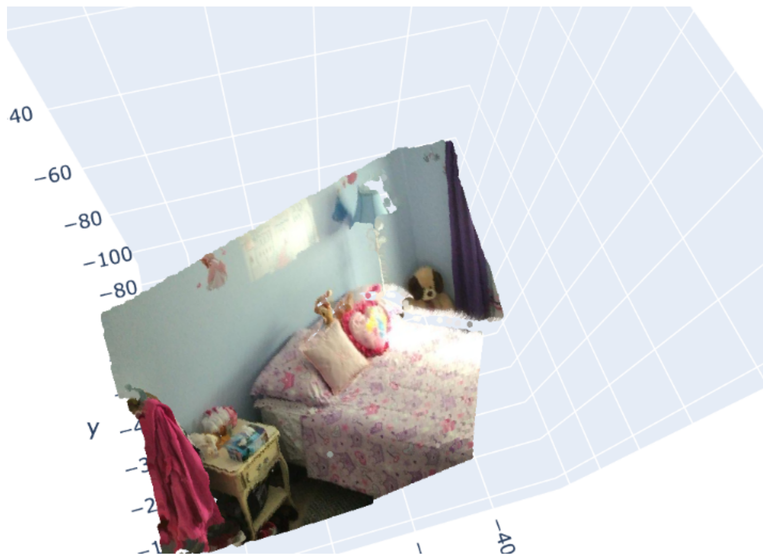
$= \frac{-n_y y_v + n_y p_y - fn_z}{n_x} + p_x$

Thus, we can see that v_x and v_y has a linear relationship, which is independent of lines' direction $\vec{d}$. Thus, the vanishing line of all lines lying on the plane form a line, regardless of the direction. And we can write the line as: $y_x = \frac{n_x}{n_y} x_v + \frac{p_x n_x + p_y n_y - fn_z}{n_y}$

Q3 a

Here are the results that I got:

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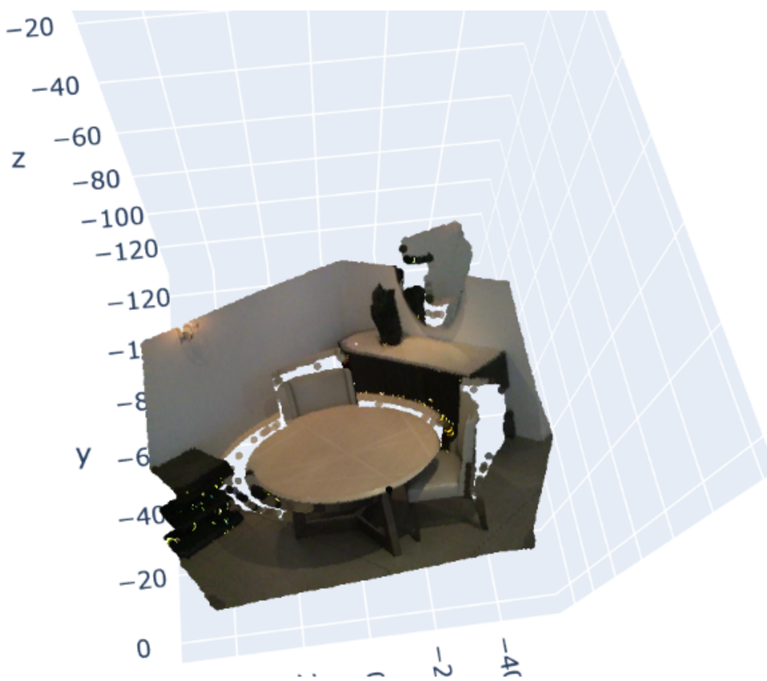
−60
−80
−100
−80
y
−40


−20
−40
z
−60
−80
−100
−120
−120
y
−20
0