CSC420 Assignmnet 1

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October 30, 2020

Part1: Theory

Q1

Since the factor $d = 4$, let us construct a form G:

G(0)	G(1)	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)	G(9)	G(10)	G(11)	G(12)	G(13)	G(14)	G(15)	G(16)
F(0)				F(1)				F(2)				F(3)				F(4)

Let the function in the diagram be f, and $F(0) = f(-2) = 4$, $F(1) = f(-1) = 1$, $F(2) = f(0) = 5$, $F(3) = f(1) = 1$, $F(4) = f(2) = 4$.

For $G(0)$, $\frac{i}{d}$ is integer,$G(0) = F(0) = 4$

For $G(1)$, $x = \frac{1}{4}$, $x_1 = \lfloor \frac{1}{4} \rfloor = 0$, $x_2 = \lceil \frac{1}{4} \rceil = 1$.

$$G(1) = \frac{1-\frac{1}{4}}{1} F(0) + \frac{\frac{1}{4}-0}{1} F(1) = \frac{13}{4}$$

For $G(2)$, $x = \frac{2}{4}$, $x_1 = \lfloor \frac{2}{4} \rfloor = 0$, $x_2 = \lceil \frac{2}{4} \rceil = 1$.

$$G(2) = \frac{1-\frac{2}{4}}{1} F(0) + \frac{\frac{2}{4}-0}{1} F(1) = \frac{5}{2}$$

For $G(3)$, $x = \frac{3}{4}$, $x_1 = \lfloor \frac{3}{4} \rfloor = 0$, $x_2 = \lceil \frac{3}{4} \rceil = 1$.

$$G(3) = \frac{1-\frac{3}{4}}{1} F(0) + \frac{\frac{3}{4}-0}{1} F(1) = \frac{7}{4}$$

For $G(4)$, $\frac{i}{d}$ is integer,$G(4) = F(1) = 1$

For $G(5)$, $x = \frac{5}{4}$, $x_1 = \lfloor \frac{5}{4} \rfloor = 1$, $x_2 = \lceil \frac{5}{4} \rceil = 2$.

$$G(5) = \frac{2-\frac{5}{4}}{1} F(1) + \frac{\frac{5}{4}-1}{1} F(2) = 2$$

For $G(6)$, $x = \frac{6}{4}$, $x_1 = \lfloor \frac{6}{4} \rfloor = 1$, $x_2 = \lceil \frac{6}{4} \rceil = 2$.

$$G(6) = \frac{2-\frac{6}{4}}{1} F(1) + \frac{\frac{6}{4}-1}{1} F(2) = 3$$

For $G(7)$, $x = \frac{7}{4}$, $x_1 = \lfloor \frac{7}{4} \rfloor = 1$, $x_2 = \lceil \frac{7}{4} \rceil = 2$.

$$G(7) = \frac{2-\frac{7}{4}}{1} F(1) + \frac{\frac{7}{4}-1}{1} F(2) = 4$$

For $G(8)$, $\frac{i}{d}$ is integer,$G(8) = F(2) = 5$

For $G(9)$, $x = \frac{9}{4}$, $x_1 = \lfloor \frac{9}{4} \rfloor = 2$, $x_2 = \lceil \frac{9}{4} \rceil = 3$.

$$G(9) = \frac{3-\frac{9}{4}}{1} F(2) + \frac{\frac{9}{4}-2}{1} F(3) = 4$$

For $G(10)$, $x = \frac{10}{4}$, $x_1 = \lfloor \frac{10}{4} \rfloor = 2$, $x_2 = \lceil \frac{10}{4} \rceil = 3$.

$$G(10) = \frac{3-\frac{10}{4}}{1} F(2) + \frac{\frac{10}{4}-2}{1} F(3) = 3$$

For $G(11)$, $x = \frac{11}{4}$, $x_1 = \lfloor \frac{11}{4} \rfloor = 2$, $x_2 = \lceil \frac{11}{4} \rceil = 3$.

$$G(11) = \frac{3-\frac{11}{4}}{1} F(2) + \frac{\frac{11}{4}-2}{1} F(3) = 2$$

For $G(12)$, $\frac{i}{d}$ is integer,$G(12) = F(3) = 1$
For $G(13)$, $x = \frac{13}{4}$, $x_1 = \lfloor \frac{13}{4} \rfloor = 3$, $x_2 = \lceil \frac{13}{4} \rceil = 4$.

$$G(13) = \frac{4-\frac{13}{4}}{1}F(3) + \frac{\frac{13}{4}-3}{1}F(4) = \frac{7}{4}$$

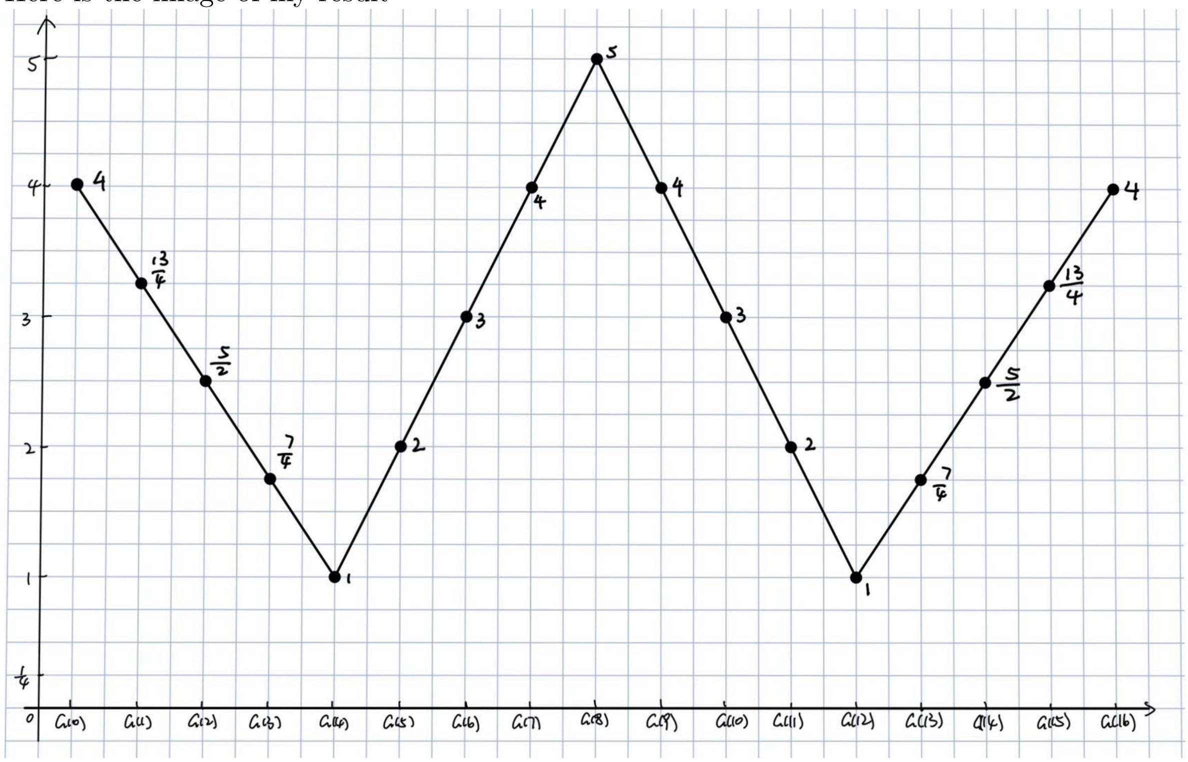
For $G(14)$, $x = \frac{14}{4}$, $x_1 = \lfloor \frac{14}{4} \rfloor = 3$, $x_2 = \lceil \frac{14}{4} \rceil = 4$.

$$G(14) = \frac{4-\frac{14}{4}}{1}F(3) + \frac{\frac{14}{4}-3}{1}F(4) = \frac{5}{2}$$

For $G(15)$, $x = \frac{15}{4}$, $x_1 = \lfloor \frac{15}{4} \rfloor = 3$, $x_2 = \lceil \frac{15}{4} \rceil = 4$.

$$G(15) = \frac{4-\frac{15}{4}}{1}F(3) + \frac{\frac{15}{4}-3}{1}F(4) = \frac{13}{4}$$

For $G(16)$, $\frac{i}{d}$ is integer,$G(16) = F(4) = 4$
Here is the image of my result



Q2

From the formula $cos2\alpha = 1 - 2sin^2\alpha$, we can have $sin^2\alpha = \frac{1-cos2\alpha}{2}$

So $x(t) = \frac{1-cos(4F_a\pi t)}{2} + \frac{1}{7}cos(F_b\pi t) + \frac{1}{5}sin(5F_c\pi t)$

$\quad = \frac{1}{2} - \frac{1}{2}cos(4F_a\pi t) + \frac{1}{7}cos(F_b\pi t) + \frac{1}{5}sin(5F_c\pi t)$

The period of $cos(4F_a\pi t)$, $cos(F_b\pi t)$, $sin(5F_c\pi t)$ are $\frac{1}{2F_a}$, $\frac{1}{F_b}$, $\frac{2}{5F_c}$ respectively. So their frequencies are $2F_a$, F_b, $\frac{5F_c}{2}$

Thus, the frequency for $x(t)$ is $max(2F_a, F_b, \frac{5F_c}{2})$.

By Nyquist-Shannon sampling theorem, the sample-rate is at least twice of the frequency. So the sufficient sample-rate for $x(t)$ is $2max(2F_a, F_b, \frac{5F_c}{2})$

Q3

1.

$M = \sum_x \sum_y w(x,y) \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix}$

So trace(M) $= I_x^2 + I_y^2$.

At the same time, by the definition of T, we can know trace(M) $= \lambda_0 + \lambda_1$. We have already known $\lambda_0 = 0$. Thus, $\lambda_1 = I_x^2 + I_y^2$

2.

Since N is positive semi-definite, for every vector v, we can have $v^T N v \geq 0$.

Since windows function must non-negative, $w(x,y) \geq 0$. So we can have $w(x,y)v^T N v = v^T w(x,y) N v \geq 0$.

To prove M is positive semi-definite, we want to show that $v^T M v \geq 0$ for any vector v.

$$v^T M v = v^T (\sum_x \sum_y w(x,y) \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix})v$$
$$= v^T (\sum_x \sum_y w(x,y) N)v$$
$$= \sum_x \sum_y (v^T w(x,y) N v)$$

We have proven that $v^T w(x,y) N v \geq 0$, so $\sum_x \sum_y (v^T w(x,y) N v) \geq 0$, which means that $v^T M v \geq 0$.

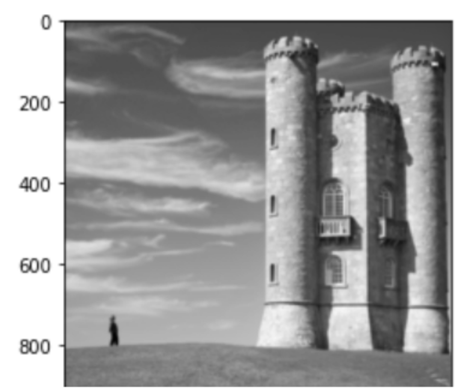
Thus, M is positive semi-definite.

Part2: Image Resizing with Seam Carving.

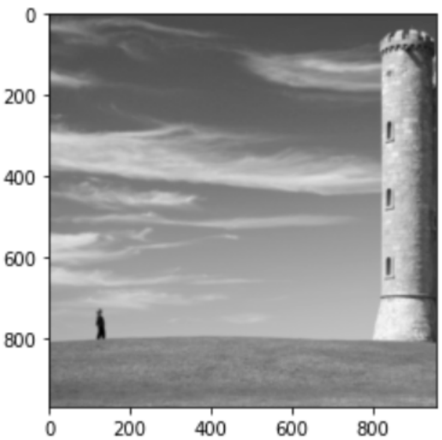
For the code, please see Part2.py.

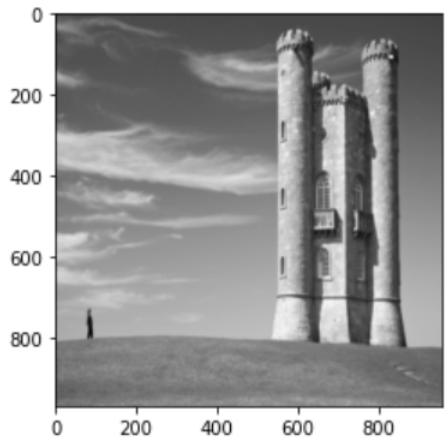
For ex1:

Here is the image produced by seam carving with size 968 $\times$ 957:

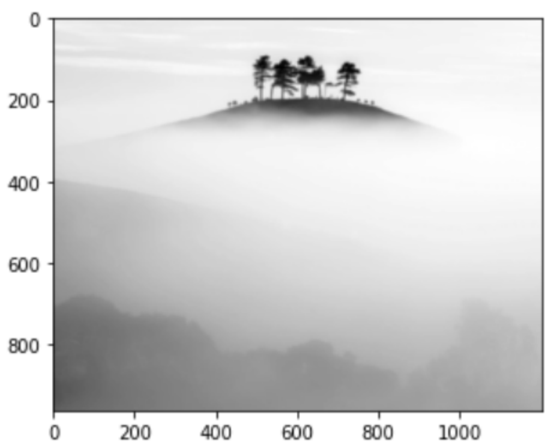


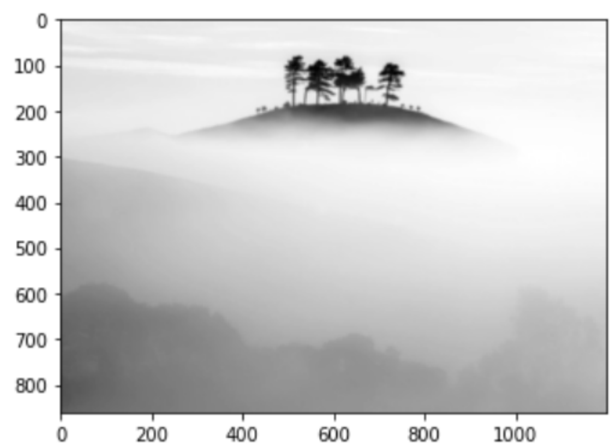
The images produced by cropping and scaling functions:



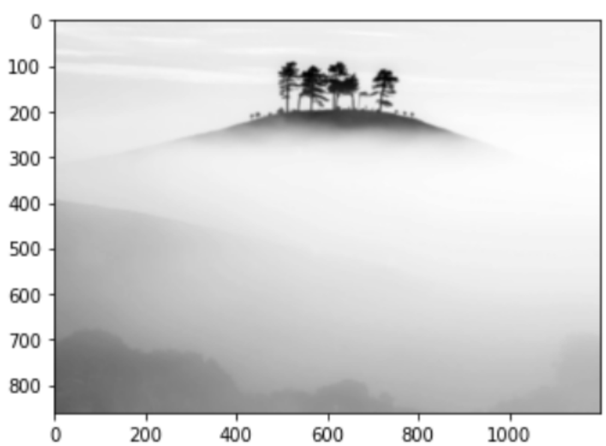


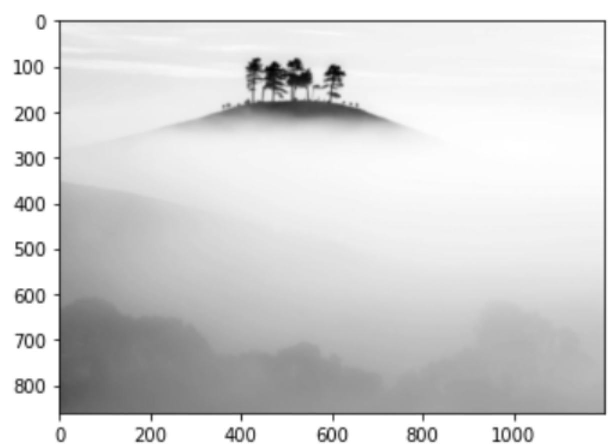
For ex2:
Here are images produced by seam carving with size 961 $\times$ 1200 and 861 x 1200:





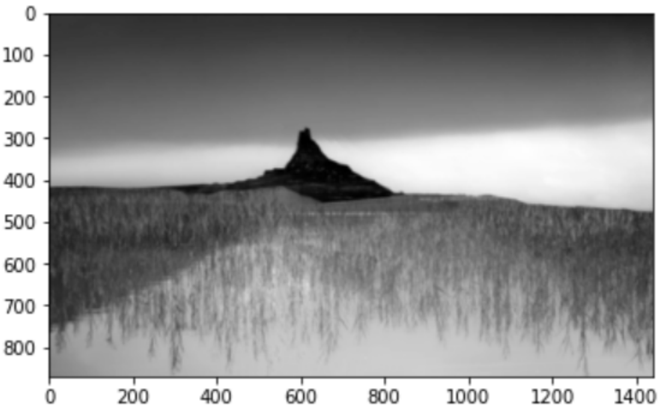
The images produced by cropping and scaling functions:

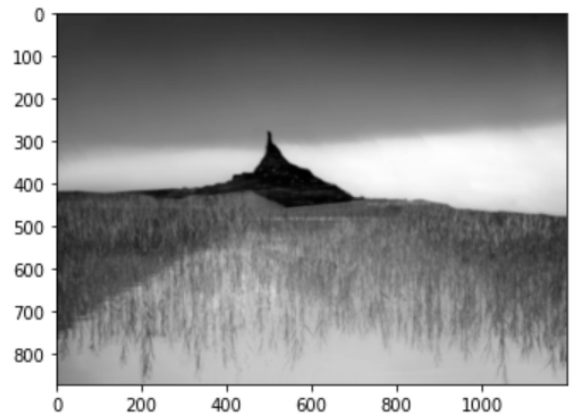




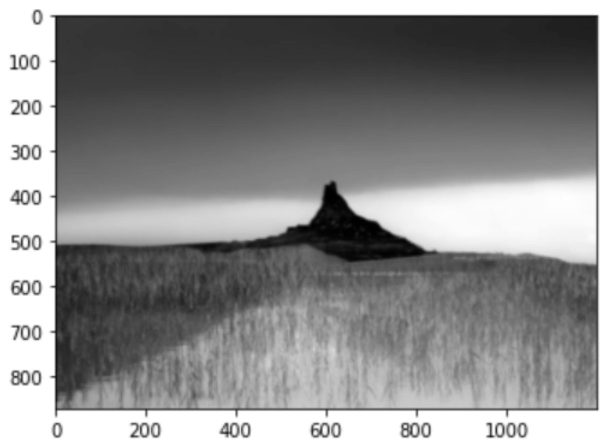
For ex3:

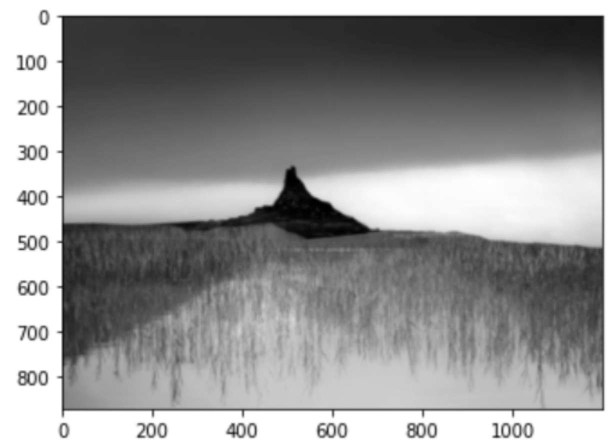
Here are images produced by seam carving with size 870 x 1440 and 870 x 1200:





The images produced by crooping and scalling functions:



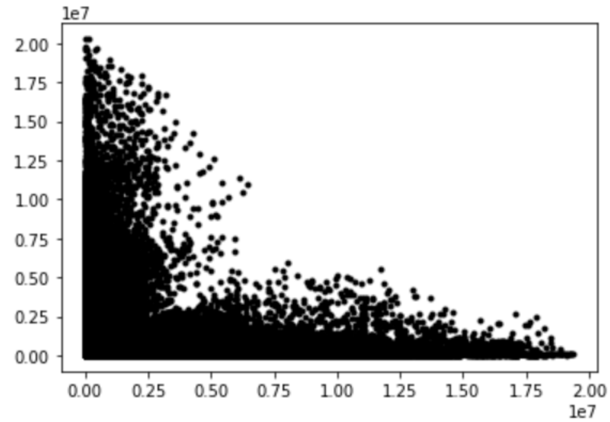


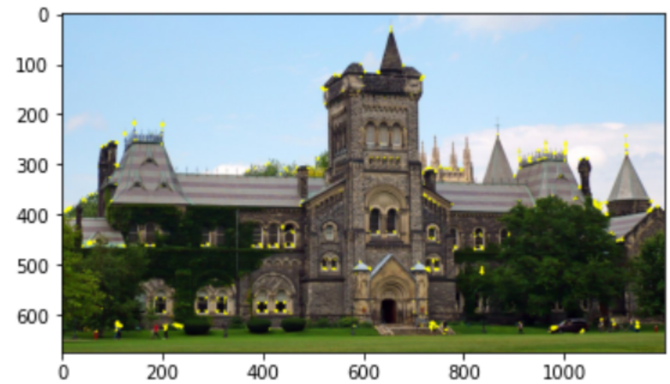
By the comparison of images, we can see that compare to the cropping and scaling, seam carving can save more features of the picture and delete some repeat element, such as the sky in ex1. However, if the size we want to compute have only a little change compared with the original image, all those three methods have no big differences. If the size changed is huge, seam carving's image seems more reasonable, where it includes enough important information and keep the scale. In contrast, cropping lose a lot of information and scaling looks really strange.

Part3:

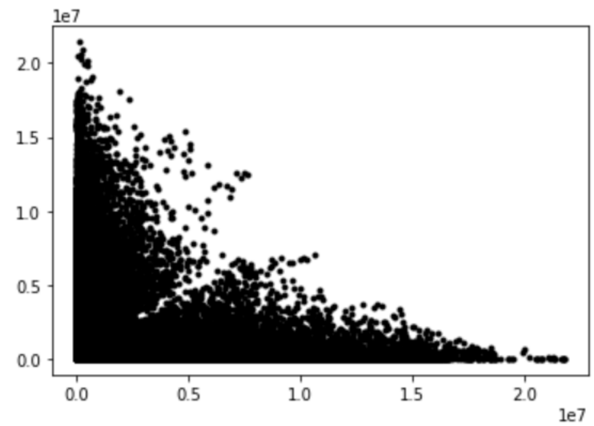
Firstly, I use window function as $\begin{pmatrix} 0.25 & 0.25 \\ 0.25 & 0.25 \end{pmatrix}$.

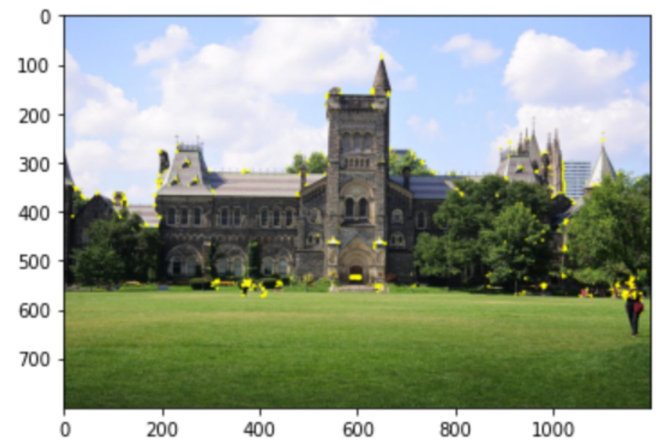
Here are the scatterplot of λ_1 and λ_2, and the image for the corners for Image1.jpg:





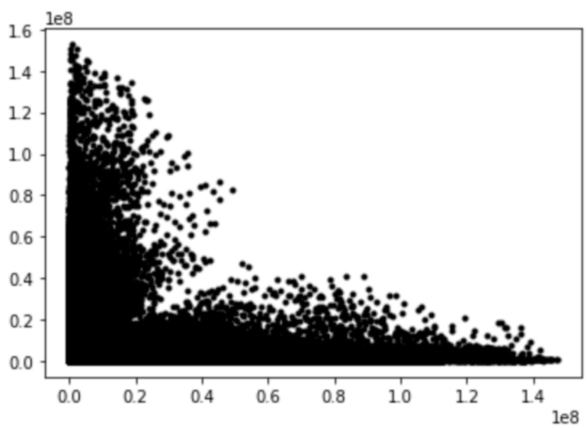
Here are the images for Image2.jpg:

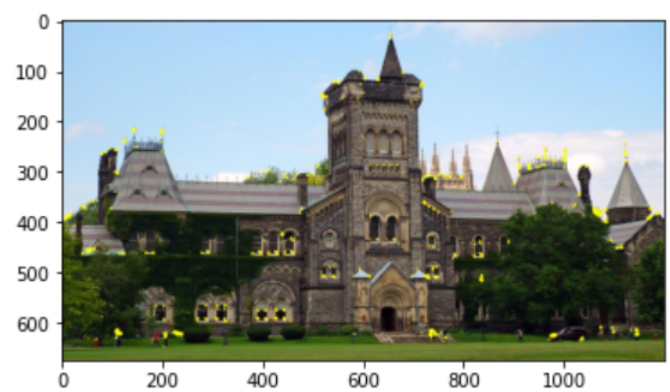




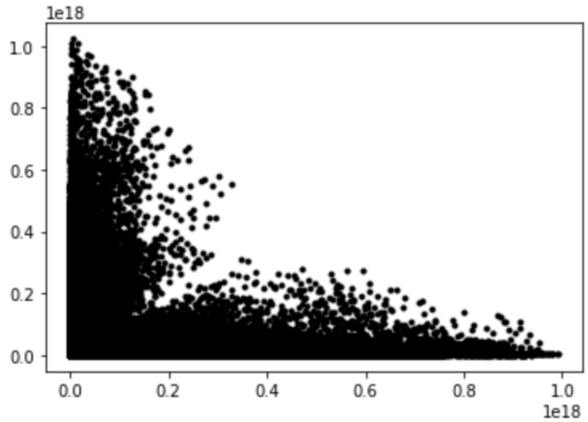
Using Gaussian kernel on Image1:

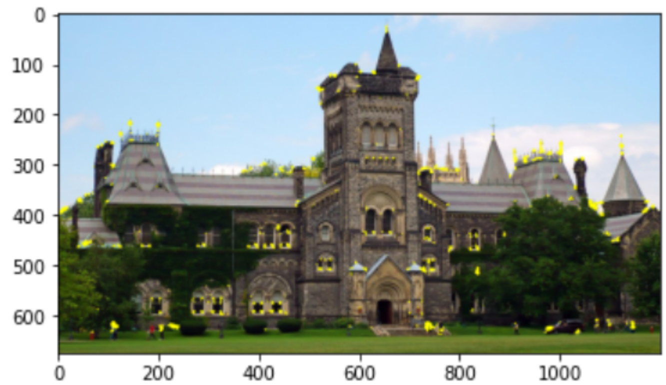
Using Gaussian kernel with $\sigma = 1$:



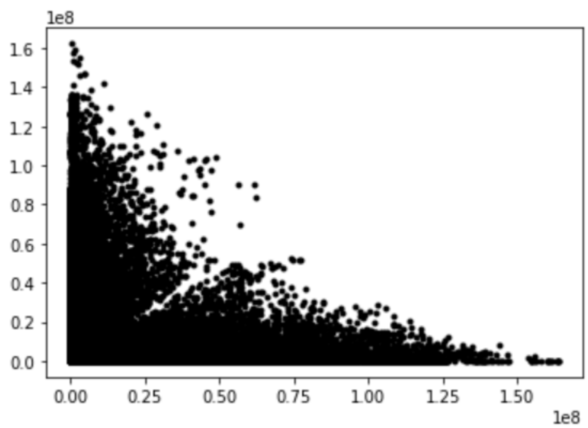


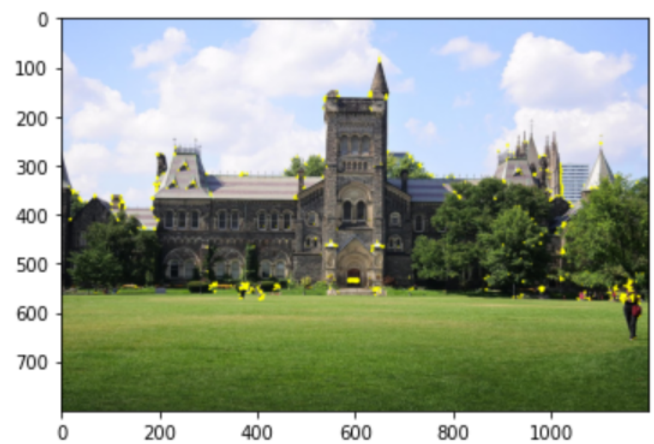
Using Gaussian kernel with $\sigma = 100000$:



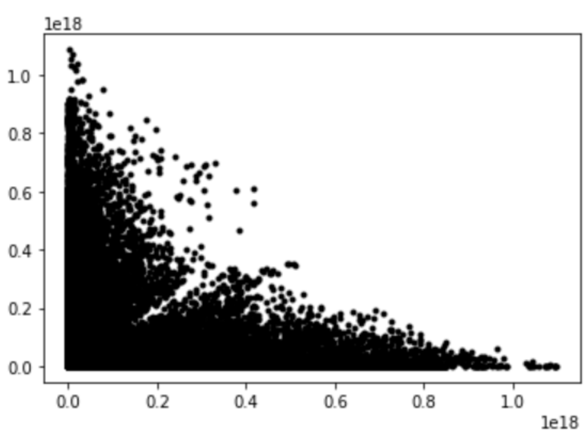


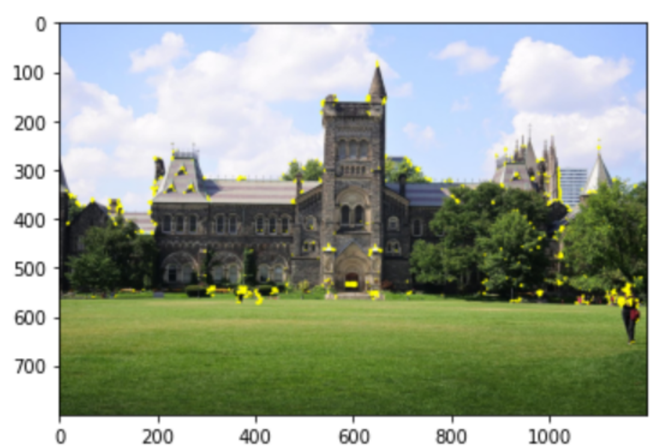
Using Gaussian kernel on Image2:
Using Gaussian kernel with $\sigma = 1$:





Using Gaussian kernel with $\sigma = 100000$:





Explanation: Actually, from picture, we can see that σ does not influence too much in corner detecting. Even the λ increases 100000 times, the images do not change a lot. However, we can still find that the image with large σ find more corners. So, when the threshold does fit well, increasing σ could help us to find more corners instead of losing data. But if the threshold is fit well for the small σ, larger σ may lead to overfit. There may be much more corners than we need.