Financial Risk Management And Regulation

Group Project

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1.Model Documentation

1.1 Model Abstract

The risk calculation system is designed to help users analyze the market risk by computing 3 VaRs, including parametric VaR, Monte Carlo VaR and historical VaR. The system runs in 2 different ways that users can choose in the main program: (1)Users input their own model parameters to the system and calculate the 3 VaRs based on these given parameters. (2) Users input the historical data without giving specific model parameters. The system will back test the results based on 5 scenarios which definitely include all possible portfolio structures that users might face. These arbitrary portfolios can consist of assets including stocks and options, positions including long and short.

1.2 Risk Model Description

VaR is a percentile of the loss distribution from the present to a future horizon date. In other words, the p level VaR is X if p of the time our losses are less than or equal to X.

Suppose the cumulative distribution function (CDF) of the portfolio losses is given by

$G(X)=P[V_0 - V_T \leq X]= E^P[1_{V_0 - V_T \leq X}]$

where P is the probability distribution of the risk factors, V is the random variable giving the t future value of the portfolio at time t.

The pth percentile VaR for a horizon time T is:

VaR(V, T, p) = $G^{-1}(p)$

Often, it can also be given as

VaR(V, T, p) = inf{I|P[V0-VT>I]≤1-P}

1.2.1 Parametric VaR

The parametric method VaR is also known as Variance/Covariance VaR. The biggest assumption that parametric VaR makes is that the returns from the portfolios are normally distributed. This assumption makes it possible to yield a formula for the VaR based on approximate mean and variance calculations. In our system, we assume the whole portfolio follows GBM. In addition, the returns are assumed to be serially independent in that no prior return should influence the current return. For option value, we use Black-Scholes formula to calculate the result.
First, we express the portfolio as a sum of stock and option positions,and then we assume the whole portfolio follows the GBM as below.

Vt= stock position + option position.

Then we calculate the drift and volatility terms using the rolling windows. The parametric VaR for a given position S_0 is calculated as:

$$Parametric_{VaR(t,p)} = S_0 - S_0 \times e^{\left(\sigma\sqrt{t}\phi^{-1}(1-p)+\left(\mu-\frac{\sigma^2}{2}\right)t\right)}$$

The Expected Shortfall is calculated as:

$$X = S_0 - Parametric_{VaR(t,p)}$$

$$d_1 = \frac{(\log\left(\frac{S_0}{X}\right) + \left(\mu + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$ES(t,p) = S_0 - S_0 \frac{e^{\mu t}(1-\phi(d_1))}{1-p}$$

1.2.2 Monte Carlo VaR

Monte Carlo VaR is fitting the model to risk factors and calculate VaR by simulation. It uses a computer program to genereate a series of random numbers to predict scenarios (or market conditions) . Based on these predicted values we can see what would happen to the assets within a portfolio under certain conditions. The value of the portfolio being assessed is then calculated for each set of market conditions generated.

Basically, we use Monte Carlo simulation to calculate VaR. Since we have multiple stocks (n) in our portfolio and we assume each stocks follows Geometric Brownian Motion, we first generate independent identical distributed random samples from multiple multivariate gaussian distribution with mean equals to 0 and correlation matrix calculated from historical covariance of the risk factor changes. The correlation between each two stocks could be calculated easily using the formula below:

$$\begin{aligned}
\bar{\mu}_i &= \mathrm{mean}[\{R_{i,k}\}] \\
\bar{\sigma}_i &= \mathrm{stddev}[\{R_{i,k}\}] \\
\sigma_i &\approx \bar{\sigma}_i\sqrt{252} \\
\mu_i &\approx 252\bar{\mu}_i + \frac{\sigma_i^2}{2} \\
\rho_i &\approx 252\frac{\mathrm{cov}(\{R_{1,k}\},\{R_{2,k}\})}{\sigma_1\sigma_2}
\end{aligned}$$

Then we need to simulate each stocks using the generated random sample to calculate their values and combine to get our samples of P.

After calculating the results of the Monte Carlo VaR and ES for the portfolio based on fitting GBM to each underlying stock returns, we sort the results of the scenarios for each date, from lowest value to highest value.

At last, in order to get the VaR, we just need the (1-VaRp)th scenario and also calculate the mean of the first (1-ESp)th values to get Expected Shortfall.

1.2.3 Historical VaR

Historical value at risk (VaR), also known as historical simulation or the historical method, refers to a particular way of calculating VaR. In this approach, we calculate VaR directly from past returns. The historical method simply reorganizes actual historical returns, putting them in order from worst to best. It then assumes that history will repeat itself, from a risk perspective.

For example, suppose we want to calculate the 1-day 95% VaR for an equity using 100 days of data. The 95th percentile corresponds to the least worst of the worst 5% of returns. In this case, because we are using 100 days of data, the VaR simply corresponds to the 5th worst day.

In the system, we calculate historical VaR by assuming that risk factors follow the actual historical distribution instead of a GBM. And we also assume today's distribution of market changes equals the historical distribution of market changes. In other word, the future market can replicate the historical distribution.

1.3 Model Usage

The VaR Model System is designed in 2 different ways for future users to manipulate this system. (1)The users can first choose to input their individual parameters of the model into the system to calculate the 3 VaRs(i.e. Parametric VaR, Monter Carlo VaR, Historical VaR). This option will appear in the main program. (2) The users can choose not to input specific model parameters, and just use the existing historical data as the input. And then the system will calibrate the model and calculate 3 VaRs.

1.4 Back-testing

Back-testing against historical data will automaticly run in our system to validate our calculation process. The paremters could be change in back test. Right now, we use 5-day holding period calibration based on 5 year windows which corresponding to the parameters we used in calculating VaR. We then compare the calculated VaRs to the actual profit and loss of the porfolio. Each day we assume a 10,000 position in P.The days that actual portfolio P&L exceeds 99% VaR are considered to be exceptions. We would compute max of numbers of exceptions over last 252 days and compare the results with the expected number of exceptions.

1.5 Model Limits

The VaR Model System provides users who are not programming professional with an easy and clear way to compute 3 different VaRs (i.e. parametric VaR, Monter Carlo VaR, historical VaR). However, there exist some limitations in the VaR Model System which are originated from the VaR calculating principle and the system itself. First, there exist different limits in these 3 VaR calculation principles because of their different assumptions that they use in their calculating process.

- For parametric VaR, the biggest weakness of this method is the assumption of normality. Without actually plotting your data on a histogram to ensure such an assumption, you are exposing yourself to an enormous underestimate of possible standard deviation moves away from your historical mean.
- For Monte Carlo VaR, the outcome of the simulation is dependent on how the simulation is built. Besides, the operation time largely depends on the size of our portfolio.Because of the difficulty to calculate the VaR of even a small portfolio, the computational requirements and the need to establish parameters together mean that Monte Carlo simulations have limitations as to the size of portfolio they can cope with.
- For historical VaR, one disadvantage of the historical simulation approach is that it can be very slow to react to changing market environments. The hybrid approach tries to address this problem with the addition of a decay factor.

We still have some limits in our calculating system. When calculating parametric VaR and historical VaR, we modeled the whole portfolio instead of individual stocks. When building the models, we didn't take into consideration of volatility surface and term structure. For the distribution of the portfolio, we assume it follows Geometric Brownian Motion while there are other stochastic models might be a better fit. Besides, our assumption of fixed holding might limit the efficiency of this system. Future work could be done to improve the aspects mentioned above.

2.Software Design Documentation

2.1 Introduction

This document is designed to demonstrate the implementation of the risk calculation system as well as the instructions of how to use this system.

Our risk calculation system is designed to calculate different types of Value-at-Risk for portfolio with arbitrage stocks and options using parametric, historical and Monte Carlo methods. The system can both calibrate to historical data and take parameters as input.

This document consists of two parts. The first part is Design Overview which describes the software structure and explains some key codes. The second part is User Instruction which shows users how to use the system as well as the requirements of data input format. Examples will be presented as well. It is important to notice that this document does not cover model description or test result analysis. Please refer to relevant documents for further information.

2.2 Design Overview

2.2.1 System Platform

This system is implemented on R. Users should have both R and RStudio downloaded.

2.2.2 System Component

The system has 10 component files listed as below:

- **main.R**: the main execution file that takes input data, performs calculations and outputs results
- **DataPrepare.R**: a file that is designed to prepare data, e.g. calculate option value, portfolio value and log returns, for further use
- **BS.formula.R**: function file that returns European call/put option price
- **winEst.R** and **winEst2.R**: function files that perform calibration using rolling window methods and return drift and volatility terms for a given date
- **ParametricVaR.R** and **ParametricES.R**: function files that perform Parametric VaR and ES calculation for a given date respectively
- **HistoricalVaR.R** and **HistoricalES.R**: function files that perform Historical VaR and ES calculation for a given date respectively
- **MC.VaR.R**: function files that perform Monte Carlo VaR and ES calculation respectively
- **Calculation.R**: function that run the codes above when the data entered to the model to calcultate different types VaR and ES, actual loss and number of dates that actural loss exceed VaR.

2.3 System Structure and Code Explanation

2.3.1 Data Input

Users need to input the necessary data into the system. The required data consist of two parts: equity data and option data. Then the system will read the input data and convert them into variables in R.

In the read.csv function, users need to type the name of the equity/option file as the input parameter, then the function will convert the input csv files into two dataframes containing equity data and option data separately.

```
# Input the data
# Equity data, type t read.csv(file, header = TRUE, sep = ",", c
data.stock<-read.csv( TRUE, comment.char = "", ...)
# Option data, type the name of the option file
data.option<-read.csv("test1_option.csv",as.is=T)
```

2.3.2 Calculate the Parametric VaR and ES

The system will calculate the Parametric VaR and ES by assuming the entire portfolio follows the Geometric Brownian Motion.

The system will first calculate the value of options, value of the portfolio as well as log returns in the DataPrepare.R file.

Then two functions----Parametric_VaR and Parametric_ES will be implemented to calculate the Parametric VaR and ES on a single given date.

```
##### Calculate the Parametric VaR on a single date
source("winEst.R")
Parametric_VaR<-function(data,current_date,p=0.99,t=5/252,S0=10000){

  # data: A dataframe contains the Date, Portfolio_Value, log_return and square_log_return
  # current_date: The specific date when the Parametric VaR is calculated
  # p: Confidence Level, Default = 99%
  # t: VaR Time Horizon, Default = 5 days
  # S0: Assume a fixed position in the portfolio
```

```
##### Calculate the Parametric ES on a single date
source("winEst.R")
source("ParametricVaR.R")

Parametric_ES<-function(data,current_date,p=0.975,t=5/252,S0=10000){

  # data: A dataframe contains the Date, Portfolio_Value, log_return and square_log_return
  # current_date: The specific date when the Parametric ES is calculated
  # p: Confidence Level, Default = 97.5%
  # t: VaR Time Horizon, Default = 5 days
  # S0: Assume a fixed position in the portfolio
```

Take the Parametric_VaR function as an example, the function will calculate the Parametric VaR using the Parametric VaR function as defined in the lecture.

```
  para<-winEst(data,current_date,l=5)

  # Long Portflio
  if(data[current_date,"Portfolio_Value"]>0){
    ans<-S0-S0*exp(para[2]*sqrt(t)*qnorm(1-p)+(para[1]-para[2]^2/2)*t)
  }
```

Finally, the system will utilize for loop to calculate the Parametric VaR and ES for the past 20 years (default).

```
for(i in 1:VaR_length){
  ParaVaR[i]<-Parametric_VaR(data.portfolio,i,p.VaR,t,S0)
}

for(i in 1:VaR_length){
  ParaES[i]<-Parametric_ES(data.portfolio,i,p.ES,t,S0)
}
```

2.3.3 Calculate the Historical VaR and ES

The system will calculate the Historical VaR and ES by assuming the entire portfolio follows the historical return pattern.

Similar to the calculation of Parametric VaR and ES, two functions----Historical_VaR and Historical_ES will be implemented to calculate the Historical VaR and ES on a single given date.

```
# Calculate the historical VaR on a single date
historical_VaR<-function(data,return.vector,current_date,p=0.99,S0){

  # data: A dataframe contains the Date, Portfolio_Value, log_return and square_log_return
  # return.vector: A vector contains the log returns over a given time horizon
  # current_date: The specific date when the Historical VaR is calculated
  # p: Confidence Level, Default = 99%
  # S0: Assume a fixed position in the portfolio
```

```
# Calculate the historical ES on a single date
historical_ES<-function(data,return.vector,current_date,p=0.975,S0)
{

  # data: A dataframe contains the Date, Portfolio_Value, log_return and square_log_return
  # return.vector: A vector contains the log returns over a given time horizon
  # current_date: The specific date when the Historical ES is calculated
  # p: Confidence Level, Default = 97.5%
  # S0: Assume a fixed position in the portfolio
```

Take the Historical_VaR function as an example, the function will calculate the Historical VaR by selecting the 100*(1-p) percentile of the historical return.

```
  cutoff<-quantile(return.vector,1-p,na.rm=T)

  ans<-S0-S0*exp(cutoff)
```

Finally, the system will utilize for loop to calculate the Historical VaR and ES for the past 20 years (default).

```
for(i in 1:VaR_length){
  HistVaR[i]<-historical_VaR(data.portfolio,portfolio.return[i:(i+num.return-1)],i,p.VaR,S0)
}

for(i in 1:VaR_length){
  HistES[i]<-historical_ES(data.portfolio,portfolio.return[i:(i+num.return-1)],i,p.ES,S0)
}
```

2.3.4 Calculate the Monte Carlo VaR and ES

The system will calculate the Monte Carlo VaR and ES by assuming the underlying stocks follow the Geometric Brownian Motion. We assume the underlying stocks follow the Geometric Brownian Motion instead of assuming the entire portfolio follows the Geometric Brownian Motion because MC VaR with the portfolio following GBM ties out with the Parametric VaR as we mentioned above.

First, we calculate the volatility and drift terms of the stocks and get correlation matrix. Note that if the input stocks are more than one stock, we will store all the results in matrix format instead of vector and we will get p times p matrix if we input p different stocks.

```r
# portfolio contains multiply stocks

# Initialize matrix
covriance <- matrix(nrow = ncol(rtn), ncol = ncol(rtn))
simga <- matrix(nrow = ncol(rtn), ncol = ncol(rtn))
rho <- matrix(nrow = ncol(rtn), ncol = ncol(rtn))

# get the volitility and drift
vol <- unlist(winEst2(rtn, current_date, l,flag)[2])
mu <- unlist(winEst2(rtn, current_date, l,flag)[1])
# Caculate stocks' covariance
endtemp <- min(length(rtn[,1]),l*252+current_date)
covmatrix <- cov(logrtn[current_date:endtemp,])
# caculate correlated matrix
volB <-  matrix(rep(vol,length(vol)),nrow = length(vol), byrow = TRUE)
volA <- matrix(rep(vol,length(vol)), nrow = length(vol))
volmatrix <- volA*volB
rho <- 252*(covmatrix/volmatrix)
diag(rho) <- rep(1,nrow(rho))
```

Secondly, we generate multivariate standard normal random samples from the correlation matrix, then use these random samples to generate Brownian motion and Geometric Brownian motion sample set. For each simulation, we calculate the stocks' value using the estimated GBM path.

```r
# generate random numbers
randomsample<-mvrnorm(npaths,rep(0,ncol(stock)),rho)
# caculate the value of portfolio
volm <- matrix(rep(vol,npaths),ncol = ncol(stock), byrow = TRUE)
mum <- matrix(rep(mu,npaths),ncol = ncol(stock), byrow = TRUE)
S0 <- sum(stock[current_date,]*stockpositiontemp)
portfoliotemp <- (exp(randomsample*volm*sqrt(dt)+ (mum - volm^2/2)*dt)) * matrix(rep(stock[current_date,]*stockpositiontemp,npaths),ncol = ncol(stock), byrow = TRUE)
St <- rowSums(portfoliotemp)
```

Third, we calculate corresponding option price and add it to the portfolio. We calculate VaR by selecting the p*100 loss of the portfolio and calculate ES by calculating the mean of top p*100 loss of the portfolio.

```
} else {
  # portfoio has option
  tag <- ifelse(Type=="p",2,1)
  strike <- S0
  put0 <- unlist(blackScholes(S0,strike,iv, r, mat, 0)[,tag])
  putt <- unlist(blackScholes(St,strike,iv, r, mat-dt, 0)[,tag])

  Vt.option <- optionshare * putt
  V0.option <- optionshare * put0

  portfolio <- Vt.stock + Vt.option
  if(sum(stockposition < 0) < length(stockposition)){
    # long postition
    loss <- V0.option+investment-portfolio
    # sort Portfolio (decreasing)
    loss <- sort(loss,decreasing = F)
    portsort <- sort(portfolio,decreasing = F)

    VaR <- quantile(loss, p.VaR,na.rm = TRUE)
    ES <-  V0.option+investment-mean(portsort[1:ceiling((1-p.ES)*npaths)])
  } else {
    # short position
    loss <- -V0.option-investment+portfolio
    # sort Portfolio (decreasing)
    loss <- sort(loss,decreasing = F)
    portsort <- sort(portfolio,decreasing = F)

    VaR <- quantile(loss, p.VaR,na.rm = TRUE)
    ES <- -V0.option-investment+mean(portsort[ceiling((p.ES)*npaths):npaths])
  }
```

2.3.5 Backtest

The system will back test all above VaR by counting the number of times the VaR on each date is exceeded by the subsequence D ("hist.t" in code) day changes in N ("bktesting.length" in code) year window. We back test from 1+ D day.

First, we calculate the real loss of the portfolio.

```
# Caculate real loss
length <- length(data.portfolio[,1])
realloss <- (1-data.portfolio[1:(length-hist.t),2]/data.portfolio[(hist.t+1):length,2])*10000
```

Secondly, count the number of dates that the real loss excesses the estimated VaR separately.

```r
# Caculated how many times that real loss exceed VaR in h year
count.ParaVaR<-c()
count.HistVaR<-c()
count.MC.VaR<-c()
count.ParaES<-c()
count.HistES<-c()
count.MC.ES<-c()

for (i in 1:VaR_length) {
  count.ParaVaR[i] <- sum(realloss[i:(i+bktesting.length*252-1)]>rev(ParaVaR)[(i+5):(i+bktesting.length*252+4)])
  count.HistVaR[i] <- sum(realloss[i:(i+bktesting.length*252-1)]>rev(HistVaR)[(i+5):(i+bktesting.length*252+4)])
  count.MC.VaR[i] <- sum(realloss[i:(i+bktesting.length*252-1)]>rev(ParaES)[(i+5):(i+bktesting.length*252+4)])
  count.ParaES[i] <- sum(realloss[i:(i+bktesting.length*252-1)]>rev(HistES)[(i+5):(i+bktesting.length*252+4)])
  count.HistES[i] <- sum(realloss[i:(i+bktesting.length*252-1)]>rev(MC.VaR)[(i+5):(i+bktesting.length*252+4)])
  count.MC.ES[i] <- sum(realloss[i:(i+bktesting.length*252-1)]>rev(MC.ES)[(i+5):(i+bktesting.length*252+4)])
}
```

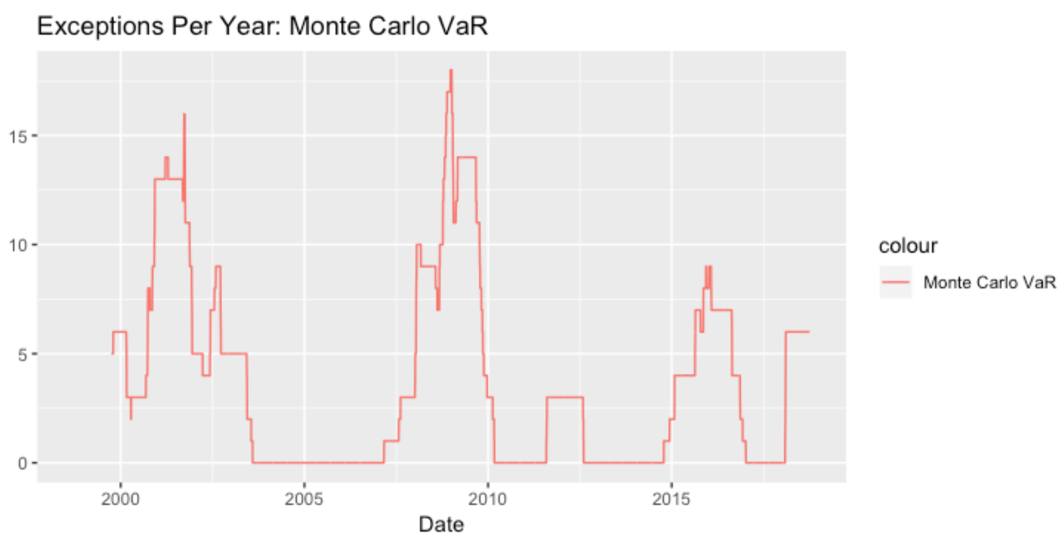
2.4 Output Explanation

The system will generate the following outputs:

- Plot the estimated VaR and show the trend of Historical VaR, Monte Carlo VaR and Parametric VaR.

- Plot estimated Expected Shortfall and show the trend of Historical ES, Monte Carlo ES and Parametric ES.

- Plot back test results for each VaR. For each VaR, there is one plot comparing the real loss with estimated VaR and one plot showing the number of exceptions in N (default as one year) window year.



2.5 User Instruction

There are specific requirements for data input file format as demonstrated below.

2.5.1 Equity File

Below is an example of the proper file format of equity file.

Dates	XOM	XOM_Position	INTC	INTC_Position
10/3/2018	86.15	156	48.76	200
10/2/2018	86.46	156	48.1	200
10/1/2018	85.81	156	46.45	200
9/28/2018	85.02	156	47.29	200
9/27/2018	85.77	156	45.88	200
9/26/2018	85.78	156	45.7	200
9/25/2018	86.5	156	45.91	200
9/24/2018	86.6	156	46.91	200
9/21/2018	85.17	156	46.66	200
9/20/2018	84.82	156	47.2	200
9/19/2018	84.63	156	46.15	200
9/18/2018	83.63	156	46.1	200
9/17/2018	83.41	156	45.42	200
9/14/2018	82.92	156	45.54	200

The equity file format should follow the following requirements:

- All data are listed by date from newest to oldest.

- The first column is always the date.
- Starting from the second column, every two columns represent one stock. The first column is the historical price and the second column is the position of that stock. Positive position means long while negative position means short.
- The data of each stock should begin and end with the same date. i.e. The number of rows should be the same.

2.5.2 Option File

Below is an example of the proper file format of option file corresponding to the equity file above. As we can see, the option file contains no data, which means we have a stock only portfolio. However, it is important to notice that even we do not have options in our portfolio, we still need to create and input the option file into our system.

Date	Risk_free	Vol1	Call1_Position	Call1_Strike	Call1_Maturity	Put1_Position	Put1_Strike	Put1_Maturity	Vol2	Call2_Position	Call2_Strike	Call2_Maturity	Put2_Position	Put2_Strike	Put2_Maturity
10/3/2018															
10/2/2018															
10/1/2018															
9/28/2018															
9/27/2018															
9/26/2018															
9/25/2018															
9/24/2018															
9/21/2018															
9/20/2018															

The option file format should follow the following requirements:

- All data are listed by date from newest to oldest.
- The first column is always the date and the second column is always the risk-free rate.
- Starting from the third column, every seven columns represent one option. The first column is the volatility corresponding to this option. The second to forth column is the call position, call strike and call maturity. The fifth to seventh column is the put position, put strike and put maturity. Once again, no matter whether you have options in your portfolio or not, you still need to organize your option file in this way and input it into your system.
- The data of each option should begin and end with the same date. i.e. The number of rows should be the same.
- The column order of the option file must correspond to the equity file. For instance, in the equity file, the first stock is XOM and the second stock is INTC. Therefore, in the option file,

the first option should be XOM and the second option should be INTC.

- Option file should have the same number of rows as that of equity file, namely, both files should begin and end with the same date.

3.Test Plan

3.1 Introduction

For the VaR Model System, it is intended for the programming professional or unprofessional users in hedge funds, investment banks or other financial companies. They are intended to do some risk management or want to use the risk model system to analyze market risk. Our model system and test plan provides a clear and easy way for them to calculate 3 basic VaRs (i.e. parametric VaR, Monter Carlo VaR and historical VaR). The system is the fundamental for market risk management.

The objective of the test plan is to guarantee our VaR Model System can work normally in real-life environment with any arbitrary portfolios consisting of stocks and options with any positions in long or short.

The data recourses come from both Bloomberg and Yahoo. We use the portfolio consisting of stocks and options of XOM, INTC, AAPL, MSFT and GE to test our system. We mainly calculate 3 types of VaRs and Expected Shortfall(i.e. parametric, Monter Carlo and historical VaR), the exceptions compared with the actual profit and loss of the tested portfolios. And we finally plot them to analyze.

3.2 Test Data Description

Test Data	Data Description	Objective
XOM (Long) + INTC (Long)	Using the stock price of XOM and INTC from the homework and assuming long position	This is the simplest case. Test whether the results match the homework solutions to decide the correctness of the VaR calculations in the software.
XOM (Short) + INTC (Long)	Using the stock price of XOM and INTC from the homework and assuming short position	Test whether the system applies to the short position as well

XOM (Long) + XOM call option (Long)	Using the stock price of XOM from homework and XOM Call Option from Bloomberg and assuming long stock and long call option position	Test whether the system applies to portfolio consisting of long stock and long option position
XOM (Short) + XOM call option (Short)	Using the stock price of XOM from homework and XOM Call Option from Bloomberg and assuming short stock and short call option position	Test whether the system applies to portfolio consisting of short stock and short option position
XOM (Long) + INTC (Long) + AAPL (Long) + MSFT (Long) + GE (Short) + XOM call option (Long)	Using the stock price of XOM, INTC, AAPL, MSFT and GE and XOM Call Option, and assuming a portfolio consisting of long and short positions of stocks and long position of call options	Test whether the system applies to portfolio consisting of arbitrage stocks and options in long and short position.

4. Test Analysis

4.1 Expected Outcome

In the test, we are going to calculate the protfolios' 99% 5-day VaR and 97.5% 5-day Expected Shortfall using 5 year windows for each day over the last 20 years. The first portfolio consists of XOM and INTC long positions. The second portfolio consist of XOM and INTC short positions. The third portfolio consist of XOM and XOM call option in long positions. The forth portfolio consist of XOM and XOM call option in short positions. The last one is the arbitrary portfolio which consists of both stocks and options in long and short positions.

We expect that we have the following outcomes.

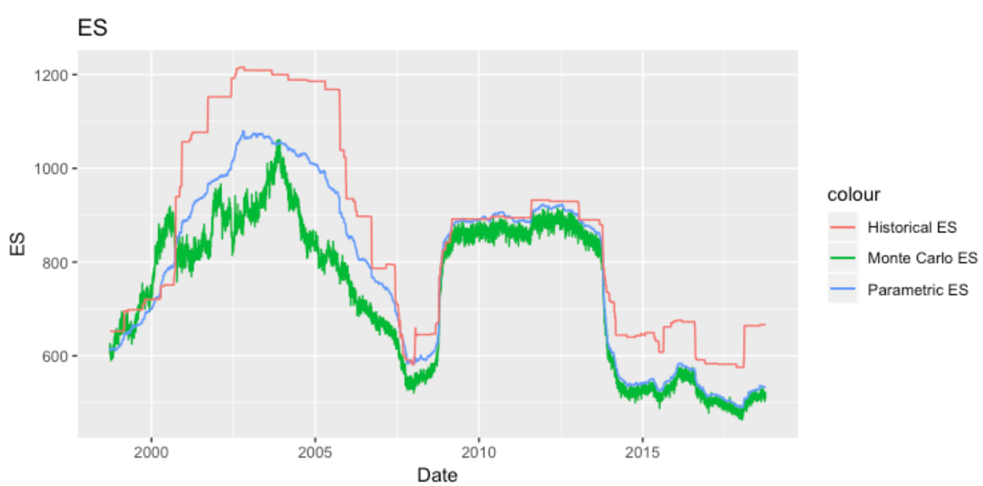
- First of all, because we use the same data frame as the homework uses, the output from the software should match perfectly with the homework solutions.
- When the positions in the portfolios go from long to short, the VaR of the portfolios should go up. This is because we put the portfolios into more risk exposure like margin recall risk,

liquidity risk and so on. But the whole trend of the VaR should be in the same pattern no matter what positions we hold.

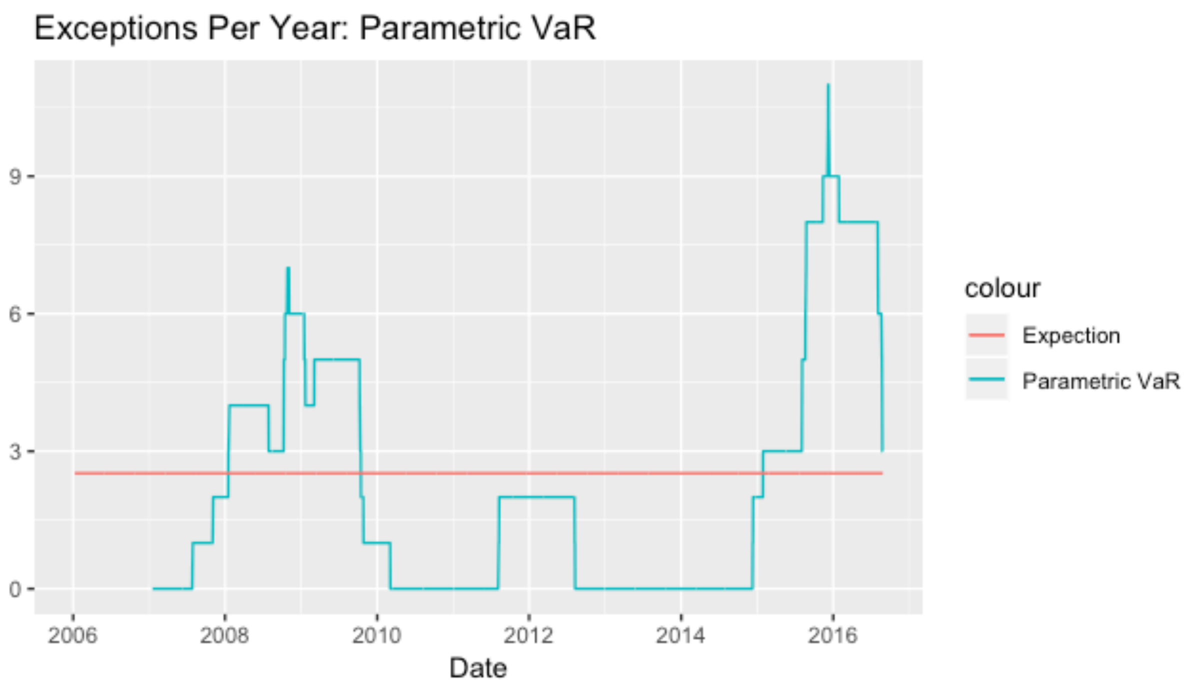
- Cause we have already set up the exit criteria of p=0.99. If the number of dates in one year that actual losses exceed VaRs is greater than exception, we need to exit and do some research for the abnormal phenomenon.

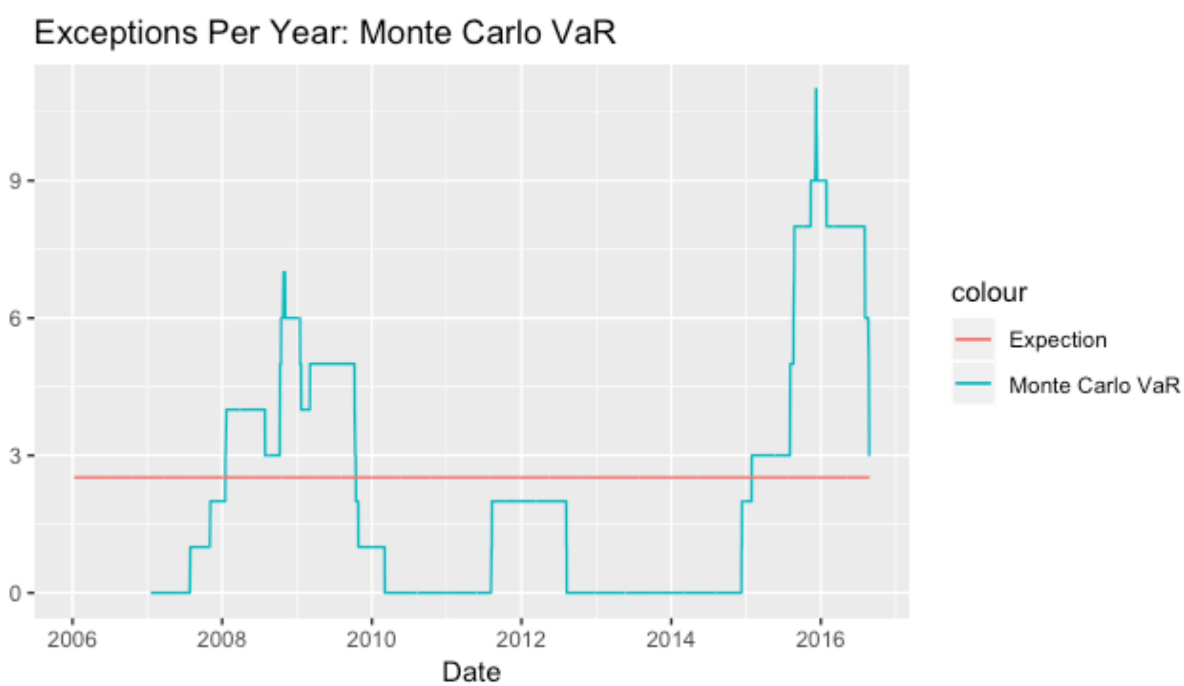
4.2 Performance

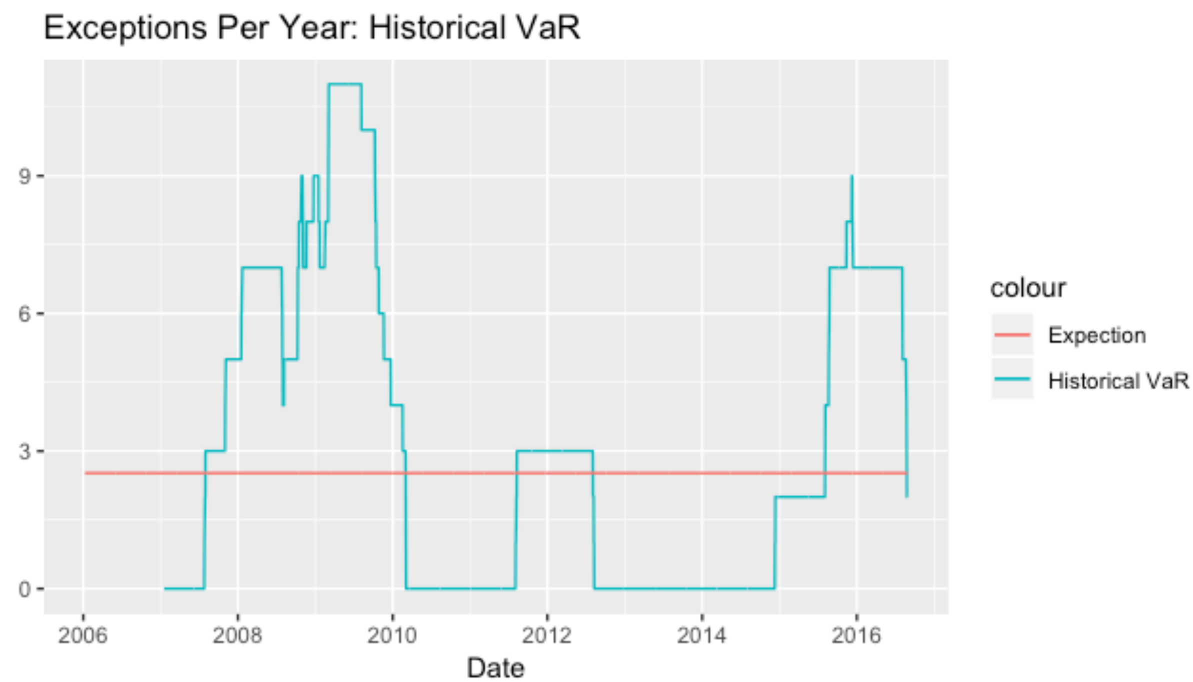
4.2.1 XOM (Long) and INTC (Long)



The trends for the 3 types of ES show the similar pattern. The historical VaR is similar to the parametric VaRs, but they are substantially higher. The reason for that is using GBM for the calculation will underestimate the risk. The Monte Carlo VaR is also similar to parametric VaR and sometimes lower than parametric VaR while sometime exceed it. The graph matched exactly with the homework solution indicate VsR calculating for the portfolios that only has the long positions in stocks works well in our system.







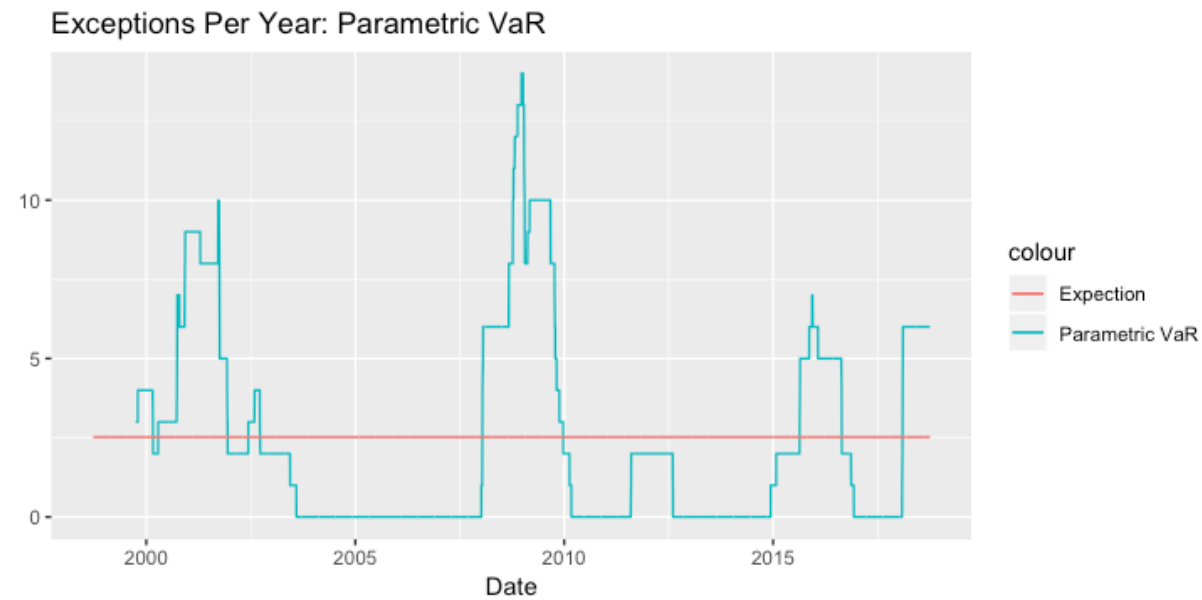
From the plots, the Monter Carlo VaR and parametric VaR have similar pattern in exception of VaR. It shows that at most of dates, the number of dates that real losses do not exceed VaR is under our expection. However, it is clear that VaR exceptions cluster, especially in three periods of time, year 2008-2010, year around 2012 and year around 2016. During 2008-2010, the number of exceptions reaches its peak. This may related to the famous financial crisis start from 2008 and the recovery of economics from 2010.

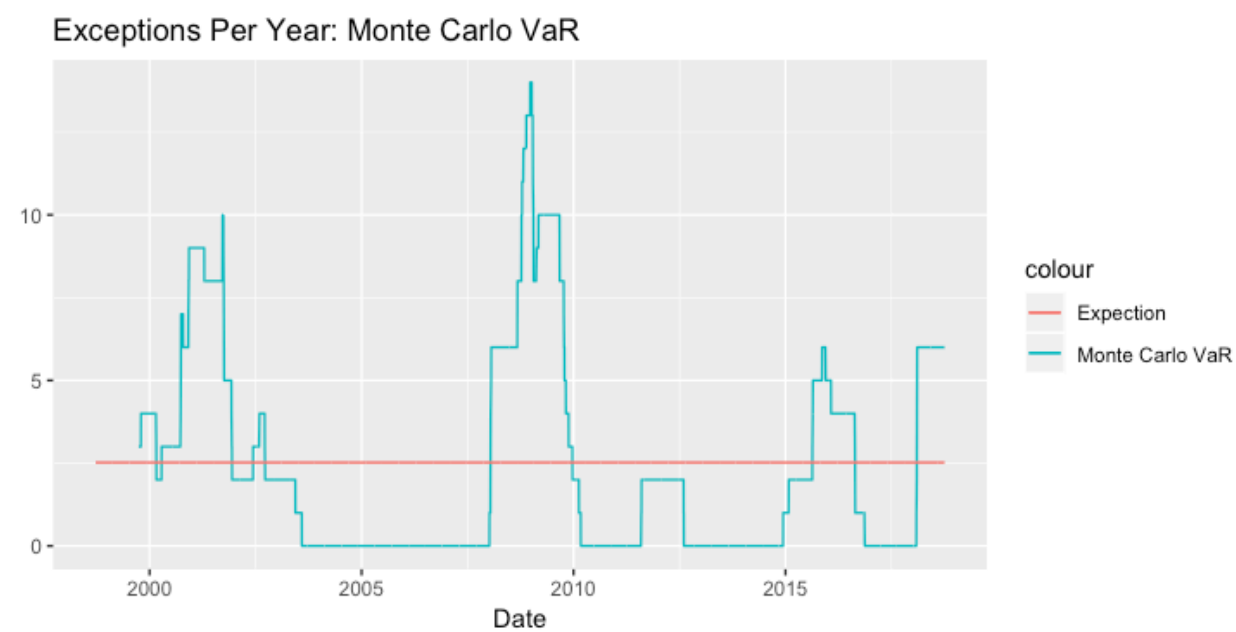
In the graph above, the green line represents the calculated VaR value by our system, the red line represents the actual loss of the portfolio. The VaR values match with the homework solution and the results match with our previous analysis.

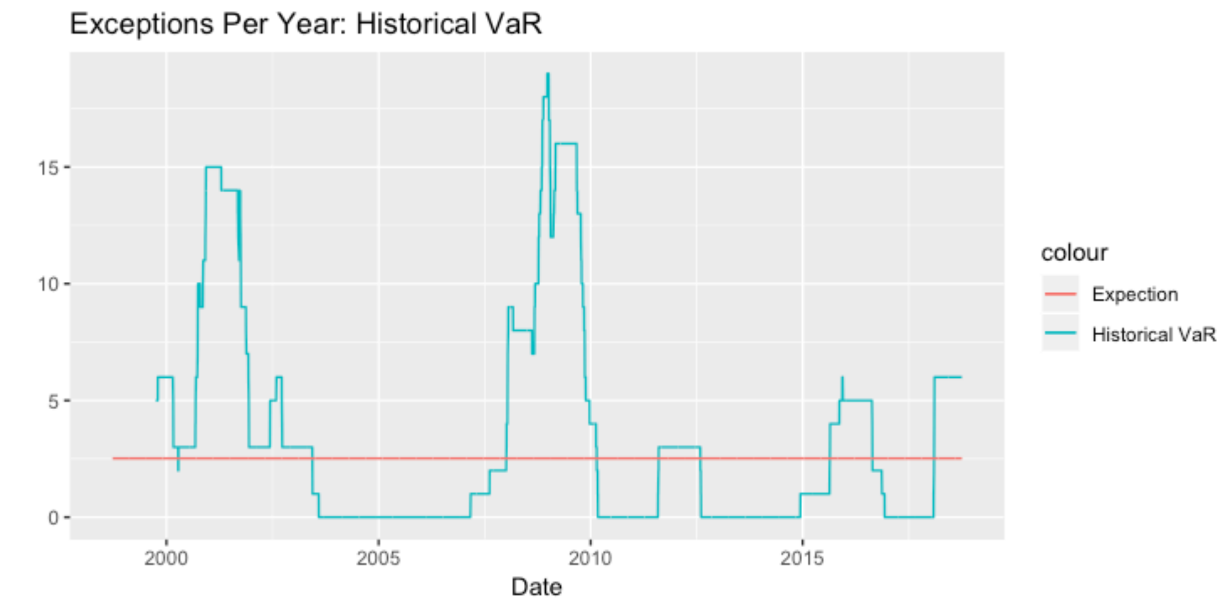
4.2.2 XOM (Short) and INTC (Short)

Compared with test one which has long positions in the same stocks, the VaR and ES are larger this time, which means the exceptions of VaR has increased. It matches the expectation that the risk of the portfolio in short position is larger than the portfolio in long position.

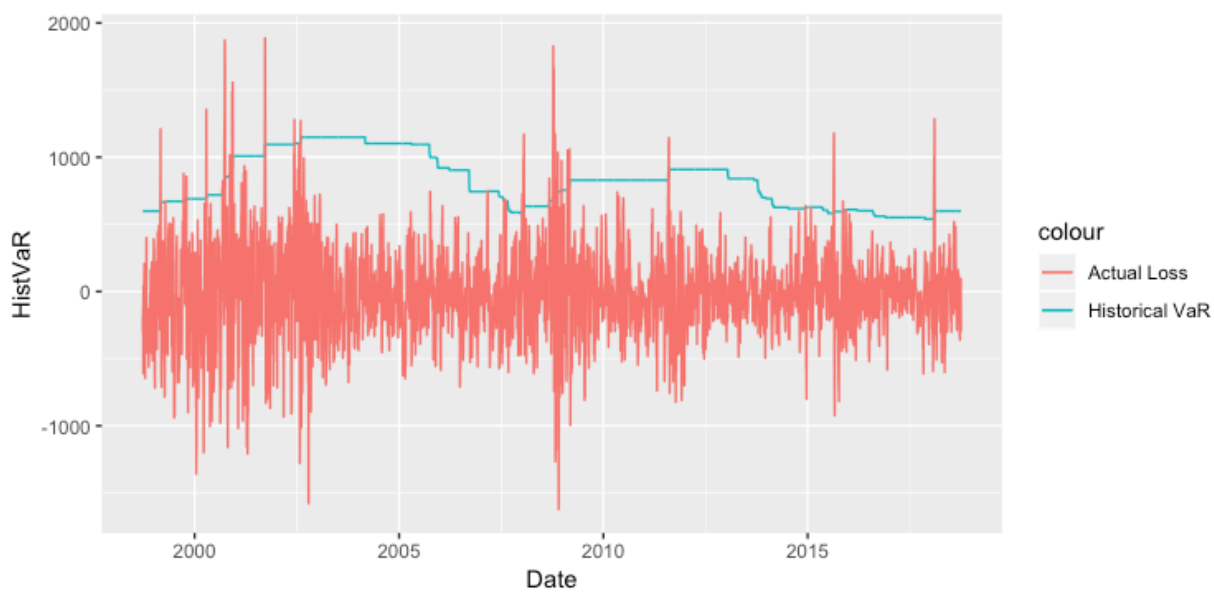
The trends for the 3 types of ES show the similar pattern. The historical VaR is similar to the parametric VaRs, but also have periods where they are substantially higher. The Monte Carlo VaR is also similar to parametric VaR and sometimes lower than parametric VaR while sometime exceed it. The graph matched exactly with the homework solution indicate VsR calculating for the portfolios that only has the short positions in stocks works well in our system.







From the plots, the Monter Carlo VaR and parametric VaR have similar pattern in exception of VaR as test one. It shows that at most of dates, the number of dates that real losses exceed VaR is above our expectation. This is different with test results one indicating the high risk user face when they only have short positions. It is clear that VaR exceptions cluster, especially in three periods of time, year 2000-2004, year 2008-2010, year around 2012 and year around 2016. During 2008-2010, the number of exceptions reaches its peak. This may related to the famous financial crisis start from 2008 and the recovery of economics from 2010.



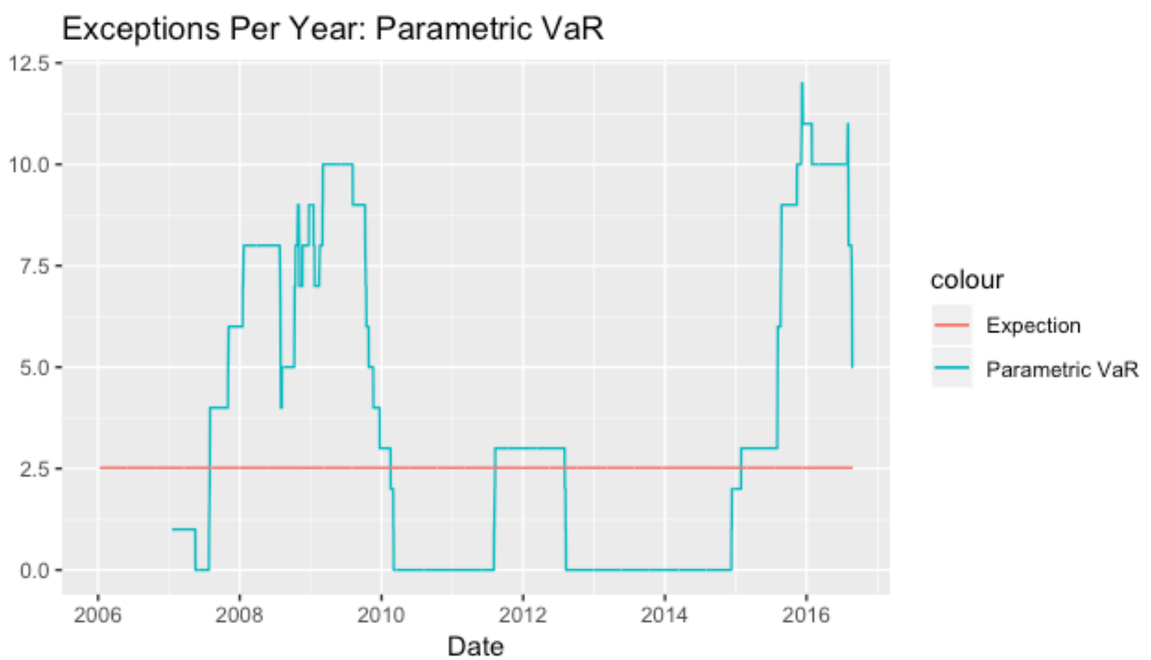
In the graph above, the green line represents the calculated VaR value by our system, the red line represents the actual loss of the portfolio. The VaR values match with the homework solution and the results match with our previous analysis.

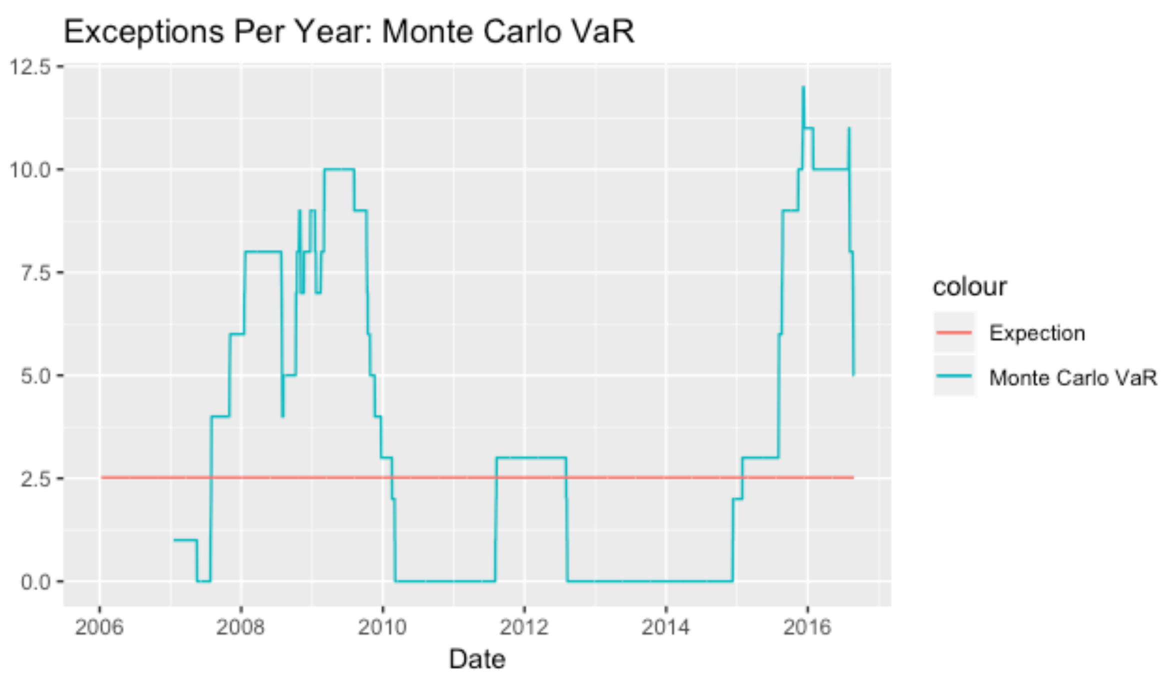
4.2.3 XOM (Long) and XOM Call Option (Long)

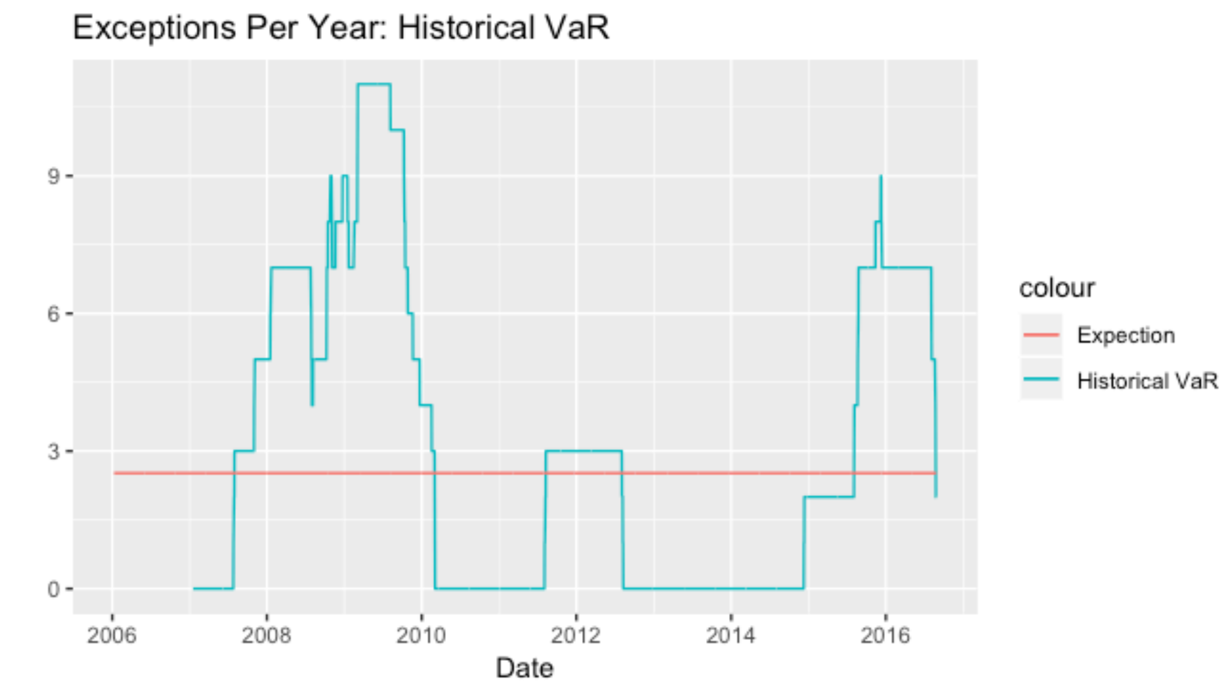
The ES of Monter Carlo and Parametric have similar patterns. Most of the time, the historical ES is lower than Monte Carlo and parametric ES. It lower the risk compared to the homework that we only calculate XOM VaR.

Compared with previous test case, where we have long positons in both XOM and INTC, the VaR value in this test case is much smaller than the values in previous case. This proves the hedging effect of the option, and helps us to verify the correctness of our functions when the portfolio consists of options and stocks.

In all the VaR calculation methods, VaR value drops from the peak in the year around 2012. Monte Carlo VaR seems to be close to parametric VaR. The results are perfectly matched with homework result, which proves that our model system is appropriate in this scenario.



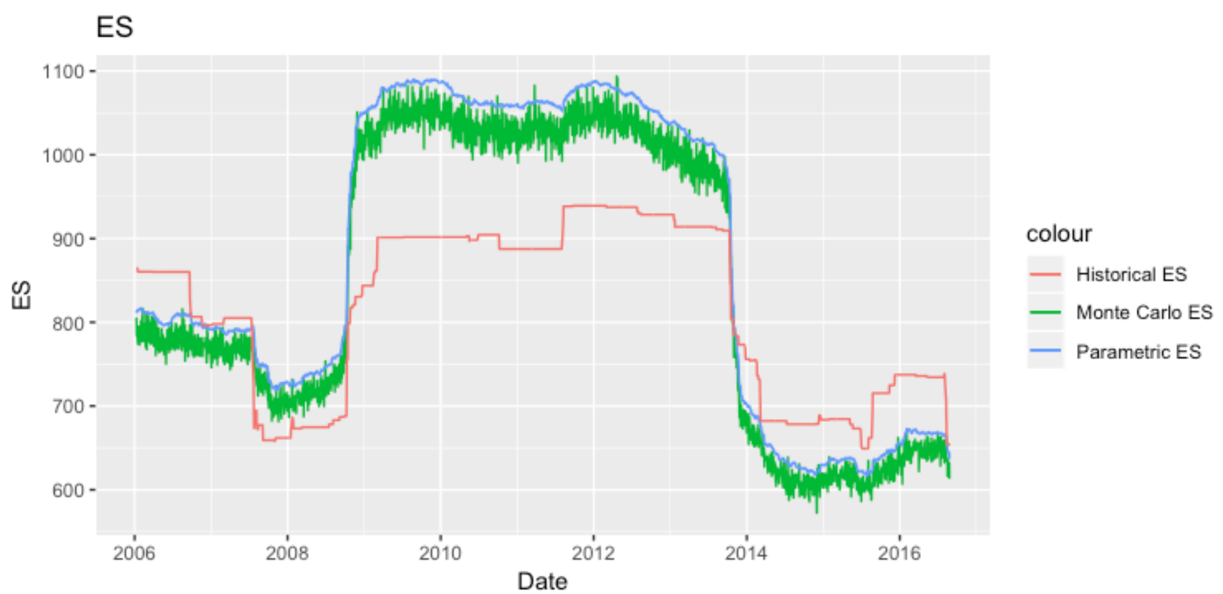




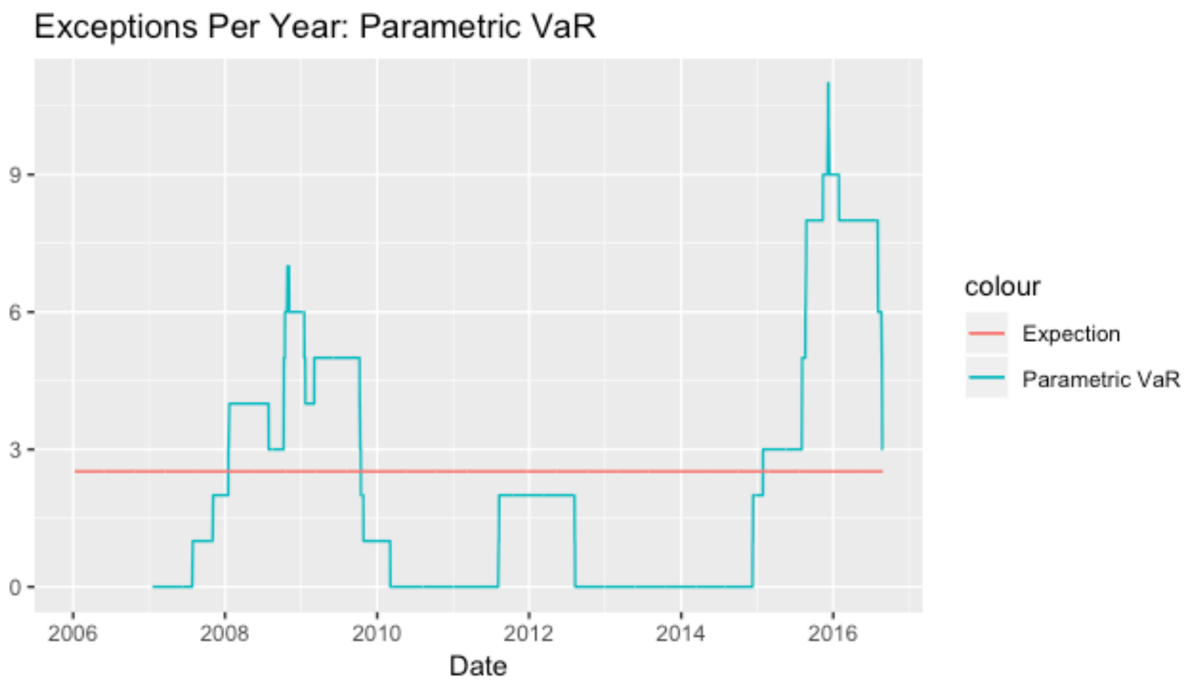
From the plot, it is clear that VaR exceptions cluster in three periods of time, year 2008-2010, year 2016. In the period of year 2009, the number of exceptions reaches its peak. In other periods of time, number of exceptions is below our expectation.

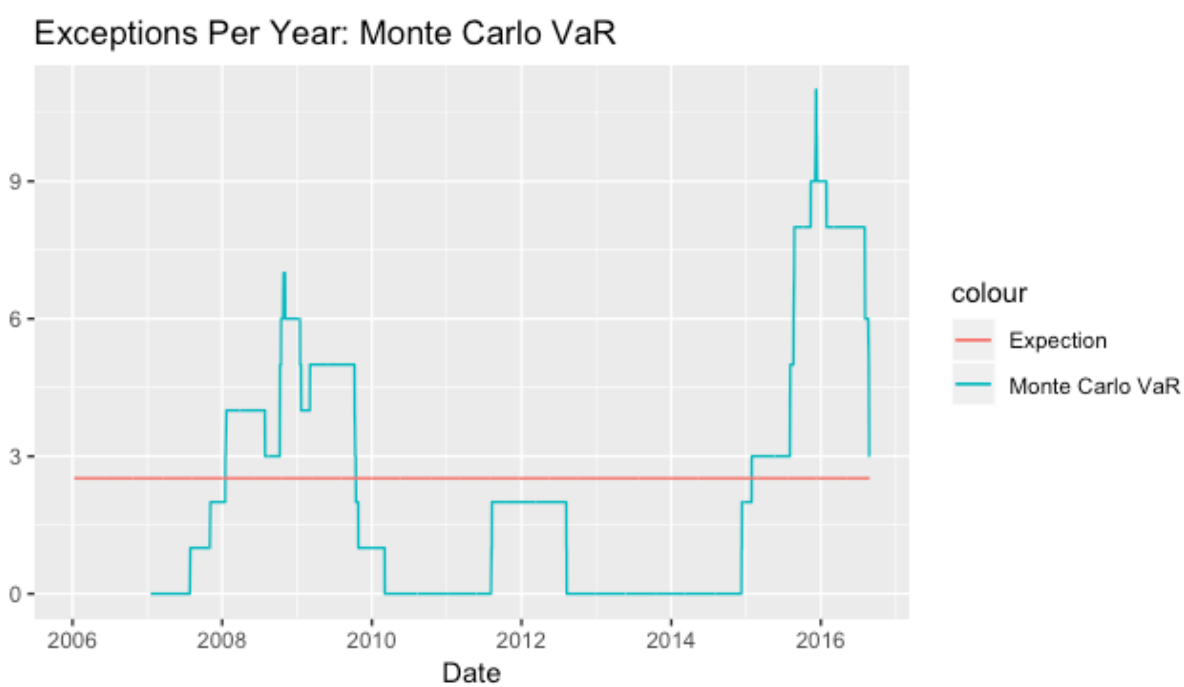
In parametric VaR and Monter Carlo VaR, the VaR value is steady around $1,000 from year 2009 to year 2013, and then has sharp decrease in year 2014. Historical VaR shows no such pattern. The number of exceptions has the largest value in year 2009.

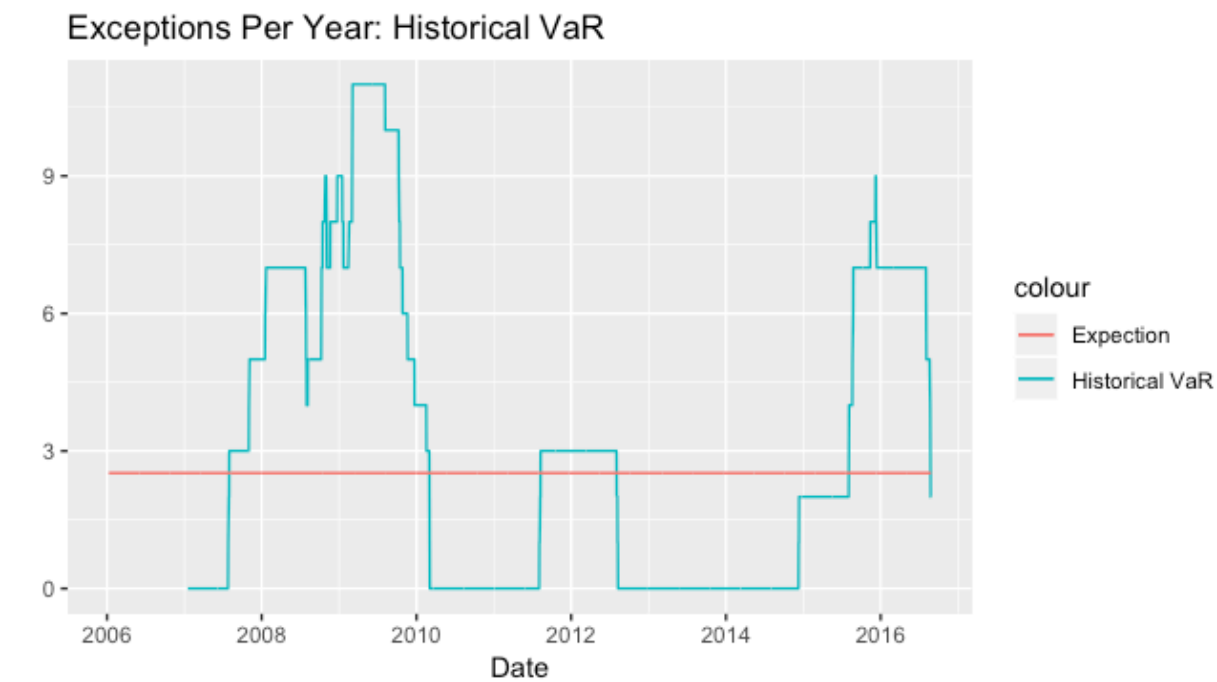
4.2.4 XOM (Short) and XOM Call Option (Short)



The ES and VaR are very large when modeling whole portfolio, compared with previous result. The results are perfectly matched the expectation that short posion is riskier than long position. The short positions have substantially larger downside, and hence larger VaRs. This is because the short positions expose the portfolios into more risk like margin recalling, liquidity risk and so on. Compared with previous test case, where we have short positons in both XOM and INTC, the VaR value in this test case is much smaller than the values in previous case. This proves the hedging effect of the option, and helps us to verify the correctness of our functions when the portfolio consists of options and stocks.



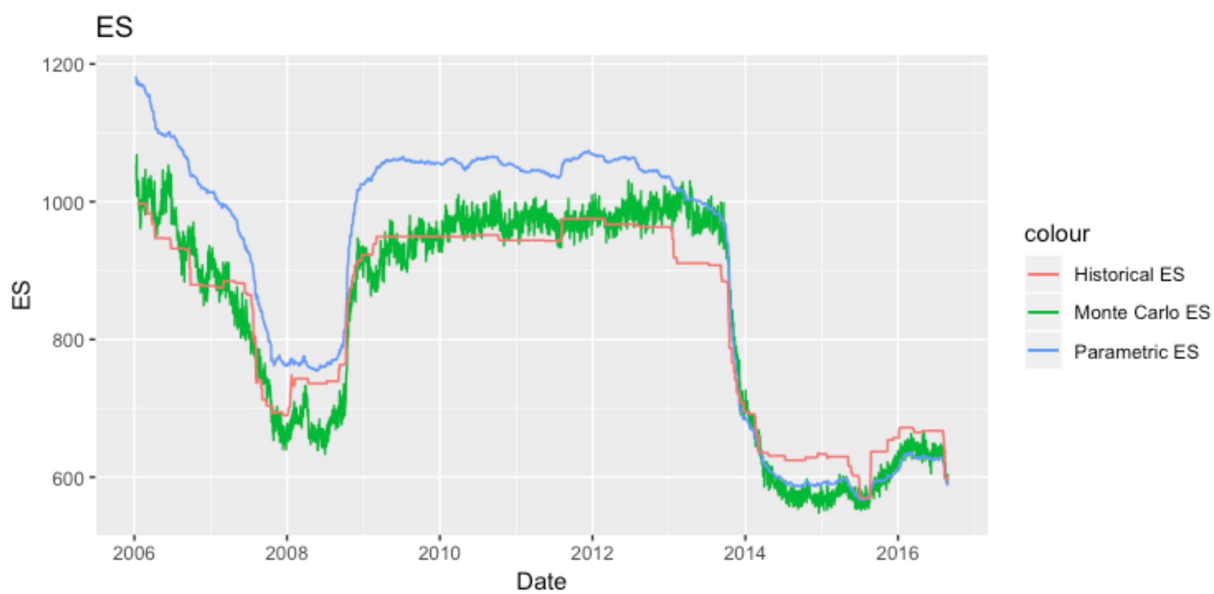




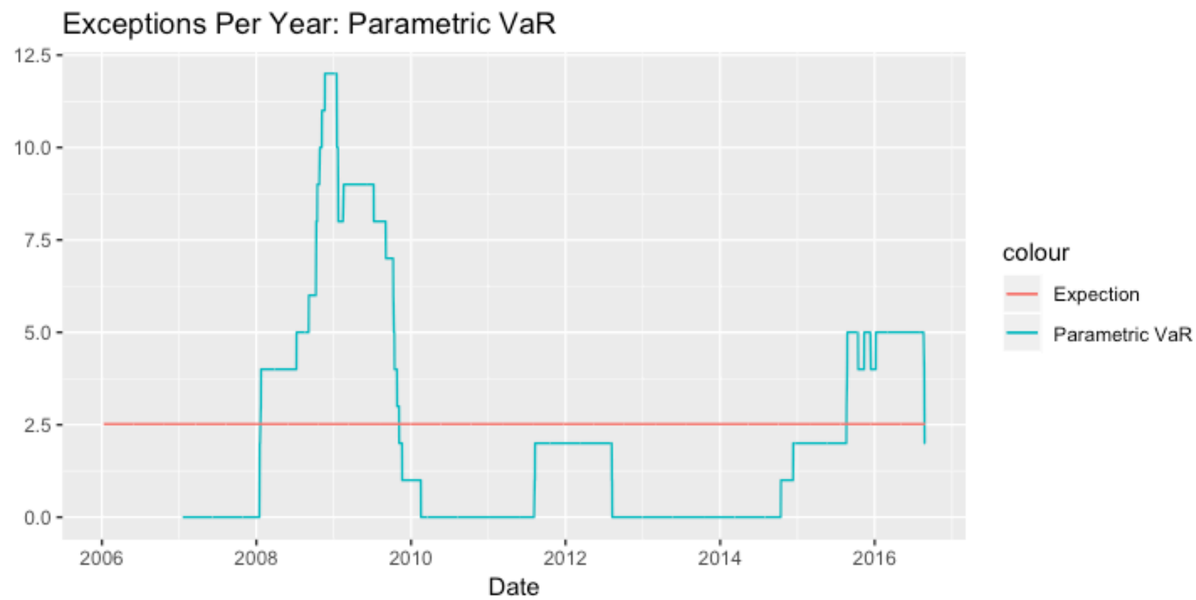
From the plot, it is clear that VaR exceptions cluster in three periods of time, year 2008-2010, year 2002, and year 2006. In the period of year 2008-2009, the number of exceptions reaches its peak. In other periods of time, number of exceptions is below our expectation.

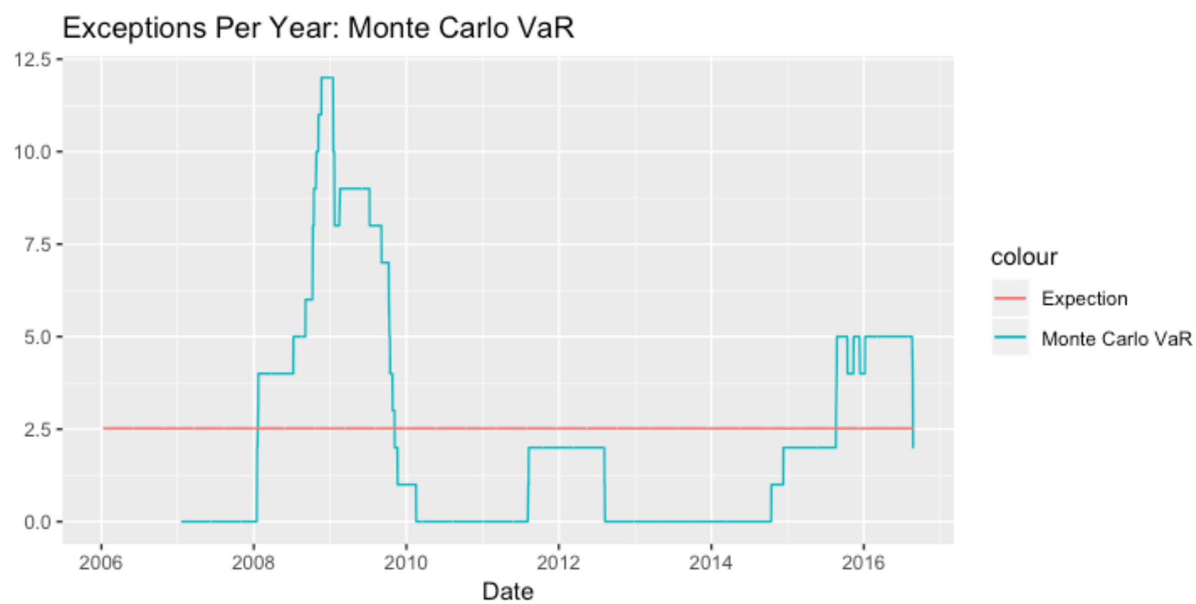
Concluded from the plot above, the Monte Carlo VaR has a virtually identical moving pattern with parametric VaR values. The results are perfectly matched with homework result, which proves that our model system is appropriate in this scenario.

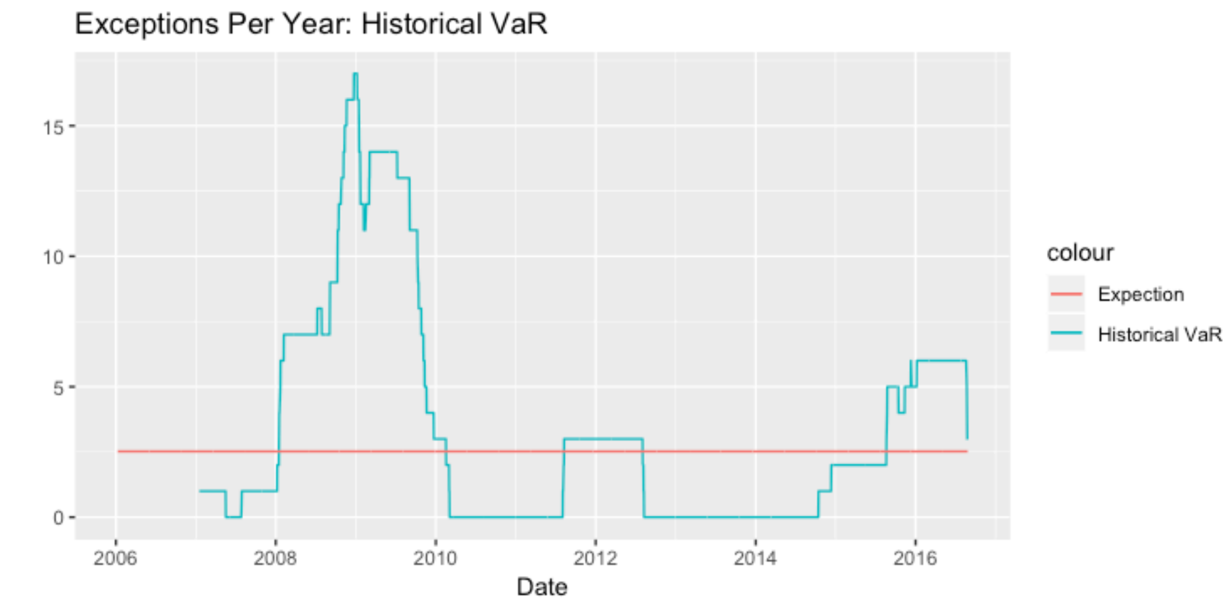
4.2.5 Arbitrary Portfolio



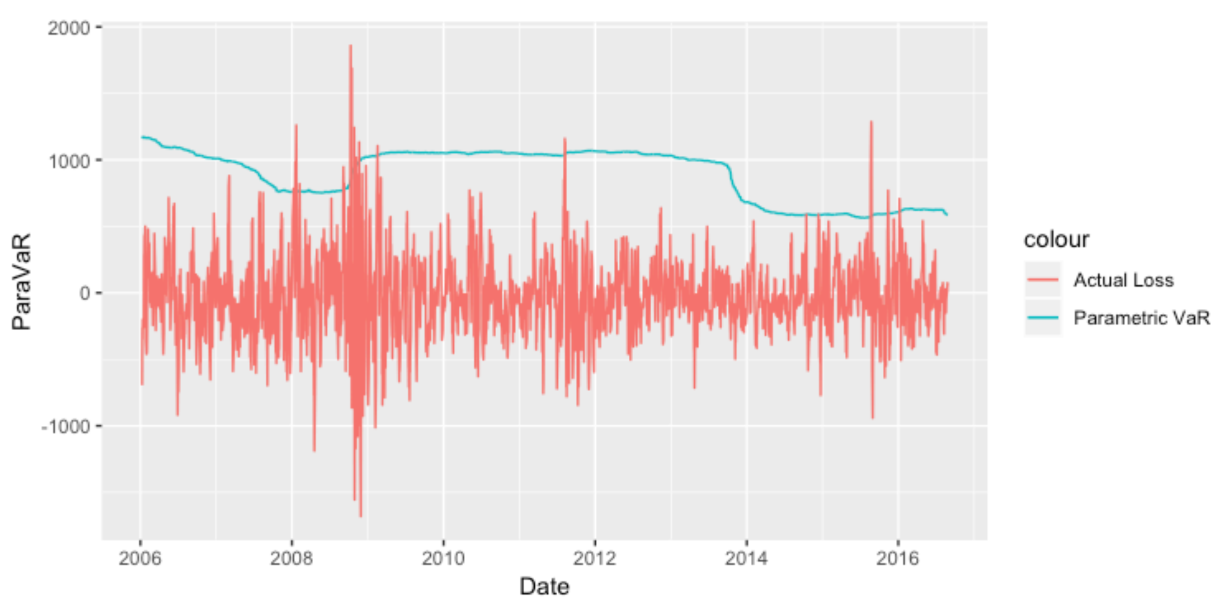
In this test case, we test the software with an arbitrary portfolio, which consists of a long position of 4 stocks, namely XOM, INTC, AAPL and MSFT, a short position of GE, and a long position of XOM AMT call option. The test results are demonstrated in the graphs.


Exceptions Per Year: Parametric VaR
12.5
10.0
7.5
5.0
2.5
0.0
2006
2008
2010
2012
2014
2016
Date
colour
Expection
Parametric VaR


Exceptions Per Year: Monte Carlo VaR
12.5
10.0
7.5
5.0
2.5
0.0
2006
2008
2010
2012
2014
2016
Date
colour
Expection
Monte Carlo VaR



From the plots, it is clear that VaR exceptions cluster in two periods of time, year 2008-2009, year 2016. In the period of year 2008-2009, the number of exceptions reaches its peak. In other periods of time, number of exceptions is below our expectation. The results are perfectly matched with homework result, which proves that our model system is appropriate in this scenario. The reason for that is because there was a financial crisis in the world as an abnormal scenario.



5 Summary and Conclusions

5.1 Demonstrated Capability

- This system could calculate parametric VaR, Monte Carlo VaR and historical VaR for an arbitrary portfolio.

- The system is able to VaRs of a portfolio that contains N stocks and one option, where N >=0. We only use one option to hedge the risk because it is difficult to measure the correlation within different options and also the correlation between different stocks and options. In real situation, we could hedge risk by adding position in options instead of buying multiple options
- The system is able to VaRs of a portfolio that has mixed positions in stocks and options.
- The system provides calculating assume that the whole portfolio follows GBM when we calculate Parametric VaR and historical VaR and calculating assume the underlying assets following GBM separately when we calculate Monte Carlo VaR. This is because the Monte Carlo VaRs that assume the whole portfolio follows GBM will be identical to the Parametric VaR in our system only be noisier. Besides, the reason of why we did not calculate Parametric VaR assuming the underlying assets following GBM separately is quite the same. It will be very similar to Monte Carlo VaR in our system and it is much easier to calculate the VaR under the assumption that the underlying assets following GBM separately using Monte Carlo methods instead of formula.
- The system is able to calculate VaR based on user supplied parameters or historical data.

5.2 Conclusions

- The perfectly matches between our portfolio with homework solution in the first two tests proves the correctness of our functions for VaR calculation.
- The short positions show similar trend with long positions but have larger VaRs and loss. This situation matches our expectation that short positions are risker than long position and proves the correctness of our systems when we add long and short options in our portfolio.
- The moving pattern of parametric VaR， historical VaR， Monte Carlo VaR is very similar prove the rightness of our system.

5.3 Recommended Improvements

- When we are calculating parametric VaR and historical VaR, we assume the whole portfolio follow GBM instead of the underlying individual stocks follow GBM. We can improve the system by adding calculating Parametric VaRs by adding the correlation of individual stocks.

- We only have one option positions in the system. We can improve the complexity of the system by adding more options to make the system more flexible.
- We assume the stocks follows Geometric Brownian Motion while it may be able to be more fit using other stochastic models. We could improve our model by adding more advanced stochastic model.
- We assume that user has a fix position during the whole period. We could improve our system by being able to adjusted to different positions during the calculating period.