## **Mutable Functions** const first = head(k); let j = i - 1; d reverse const sec = head(tail(k)); Map while $(i \ge 0 \&\& A[i] > A[i + 1])$ { function d\_reverse(xs) { if (first > sec) { function map\_array(f, arr) { swap(A, j, j + 1);if (is\_null(xs)) { set\_head(tail(k), first); const len = array\_length(arr); $j = j - 1; \} \}$ return xs; set\_head(k, sec); function iter(i) { } else if (is\_null(tail(xs))) { } else {} if (i < len) { **Merge Sort** $k = tail(k); }$ arr[i] = f(arr[i]); function merge\_sort(A) { return xs; } else { return L; } iter(i + 1);merge\_sort\_helper(A, 0, array\_length(A) -1); let temp = d reverse(tail(xs)); } else {}} set\_tail(tail(xs), xs); **Array Processing** iter(0);function merge\_sort\_helper(A, low, high) { set\_tail(xs, null); Flatten Array return temp; }} function flatten array(arr) { **Binary Search** if (low < high) { let output = $\Pi$ : function binary search loop(A, v) { const mid = math floor((low + high) / 2); d append function append\_to\_output(arr) { let low = 0: merge\_sort\_helper(A, low, mid); function d\_append(xs, ys) { for (let k = 0; $k < array_length(arr)$ ; k = k + 1) merge\_sort\_helper(A, mid + 1, high); let high = array\_length(A) - 1; if (is\_null(xs)) { while (low <= high) { merge(A, low, mid, high); return ys; if (is\_array(arr[k])){ const mid = math floor((low + high) / 2): } else{ }} append\_to\_output(arr[k]); } else { if (v === A[mid]) { set\_tail(xs, d\_append(tail(xs), ys)); function merge(A, low, mid, high) { } else { break; return xs; }} output[array length(output)] = arr[k]; $else if (v < A[mid]) {$ const B = []; // temporary array }} high = mid - 1;let left = low: d\_filter return output;} let right = mid + 1; } else { function d filter(pred, xs) { return append to output(arr); } $low = mid + 1; \}$ let Bidx = 0: if (is\_null(xs)) { return(low <= high);} return xs; Append while (left <= mid && right <= high) { function append\_array(lhs, rhs) { } else if (pred(head(xs))) { Selection Sort if (A[left] <= A[right]) { function find\_min\_pos(A, low, high) { set\_tail(xs, d\_filter(pred, tail(xs))); const result = []; B[Bidx] = A[left];left = left + 1: return xs: for (let i = 0; $i < array_length(lhs)$ ; i = i + 1) { let min pos = low: } else { result[array\_length(result)] = lhs[i]; } for (let j = low + 1; j <= high; j = j + 1) { } else{ return d\_filter(pred, tail(xs)); }} for (let i = 0; $i < array_length(rhs)$ ; i = i + 1) { $if(A[j] < A[min_pos])$ B[Bidx] = A[right];result[array\_length(result)] = rhs[i]; } $min_pos = j$ ; right = right + 1;return result:} } else{} } d map function d\_map(f, xs) { Bidx = Bidx + 1;return min\_pos; } if (!is null(xs)) { Reverse set\_head(xs, f(head(xs))); function swap(A, i, j) { function selection sort(A) { while (left <= mid) { let temp = A[i]; const len = array\_length(A); B[Bidx] = A[left];d\_map(f, tail(xs)); } else {}} A[i] = A[j];for (let i = 0; i < len - 1; i = i + 1) { Bidx = Bidx + 1;A[i] = temp;let min\_pos = find\_min\_pos(A, i, len-1); left = left + 1;**Bubble Sort (List)** swap(A, i, min\_pos);}} function bubblesort\_list(L) { function reverse\_array(A) { while (right <= high) { const len = length(L); const len = array\_length(A); B[Bidx] = A[right];**Insertion Sort** for (let i = len - 1; i >= 1; i = i - 1) { const half\_len = math\_floor(len / 2); function insertion\_sort(A) { Bidx = Bidx + 1: let k = L: for (let i = 0; $i < half_len$ ; i = i + 1) { const len = array\_length(A); right = right + 1; } for (let i = 1; i < len; i = i + 1) { for (let j = 0; j < i; j = j + 1) { swap(A, i, len - 1 - i); }}

```
for (let k = 0; k < high -low + 1; k = k + 1) {
                                                               ? pair(head(s),() => stream_filter(p,
    A[low + k] = B[k];
                                                     stream tail(s)))
                                                               : stream_filter(p, stream_tail(s));}
function mergeC(xs, xs_len, ys, ys_len) {
 let result = [];
                                                     Append
 let result_len = xs_len + ys_len;
                                                     function stream_append_pickle(xs, ys) {
 let xi = 0:
                                                       return is null(xs)
 let yi = 0;
                                                          ? ys()
                                                          : pair(head(xs), () =>
  for (let i = 0; i < result_len; i = i + 1) {
                                                     stream_append_pickle(stream_tail(xs), ys));}
         if (xi === xs_len) {
                   result[i] = vs[vi];
                                                     Add Streams
                   vi = vi + 1;
                                                     function add streams(s1, s2) {
         } else if ( yi === ys_len) {
                                                       if (is_null(s1)) {
      result[i] = xs[xi];
                                                          return s2:
      xi = xi + 1;
                                                       } else if (is_null(s2)) {
         else if (xs[xi] \le ys[yi]) {
                                                          return s1:
      result[i] = xs[xi];
                                                       } else {
      xi = xi + 1;
                                                          return pair(head(s1) + head(s2),
                                                                () => add streams(stream tail(s1),
    } else {
      result[i] = ys[yi];
                                                     stream_tail(s2))); }}
      yi = yi + 1; \}
                                                     Multiply Streams
return result; }
                                                     function mul streams(s1, s2) {
Stream Processing
                                                       if (is_null(s1)) {
                                                          return s2:
Map
function stream_map(f, s) {
                                                       } else if (is_null(s2)) {
 return is_null(s)
                                                          return s1:
    ? null
                                                       } else {
         : pair(f(head(s)),() => stream_map(f,
                                                          return pair(head(s1) * head(s2),
stream_tail(s)));}
                                                                () => mul_streams(stream_tail(s1),
                                                     stream_tail(s2))); }}
function stream_map_optimized(f, s) {
  return is null(s)
                                                     Scale Streams
                                                     function scale_stream(factor, stream) {
    ? null
         : pair(f(head(s)),
                                                        return stream_map(x => x * factor, stream); }
       memo_fun(() =>
stream_map_optimized(f, stream_tail(s))));}
                                                     merge list of streams(list(s1,s2,s3)) returns
                                                     the stream 1, 2, 3, 11, 22, 33,...
Filter
                                                     function merge_n(ss) {
function stream_filter(p, s) {
```

return is null(s)

? null: p(head(s))

return pair(head(head(ss)),

() => merge\_n(append(tail(ss),

list(stream\_tail(head(ss))))); }

```
More
function more(a, b) {
 return (a > b)
    ? more(1, 1 + b)
    : pair(a, () => more(a + 1, b)); }
eval_stream(more(1,2), 15);// [1, [1, [2, [1, [2, [3,
[1, [2, [3, [4, [1, [2, [3, [4, [5, []]]]]]]]]]]]]
Partial Sums
function partial_sums(s) {
 return pair(head(s),
        () => add streams(stream tail(s),
partial sums(s)));}
Memoized Streams
function memo_fun(fun) {
 let already_run = false;
 let result = undefined:
  function mfun() {
    if (!already run) {
      result = fun();
      already_run = true;
      return result:
    } else {
      return result; }}
  return mfun;}
function ms(m, s) {
  display(m);
  return s;}
function m_integers_from(n) {
  return pair(n, memo_fun(() => ms("M: " +
stringify(n), m integers from(n + 1)));
Memoized Coin Change
const mem = \Pi;
function read(n, k) {
  return (mem[n] === undefined)?
    undefined : mem[n][k];}
function write(n, k, value) {
 if (mem[n] === undefined) {
    mem[n] = [];
```

```
} else {}
  mem[n][k] = value;
function mcc(n, k) {
 if (n === 0){
    return 1;
 else if (n < 0 | k === 0)
    return 0;
  } else if (read(n,k) !== undefined){
    return read(n,k);
  } else {
    const value = mcc(n, k-1) + mcc(n -
first denomination(k),k);
    write(n,k,value);
    return value;}}
Rotate Matrix
```

```
function rotate matrix(M) {
  const n = array_length(M);
  function swap(r1, c1, r2, c2) {
    const temp = M[r1][c1];
    M[r1][c1] = M[r2][c2];
    M[r2][c2] = temp; 
// Transpose Matrix
  for (let r = 0; r < n; r = r + 1) {
    for (let c = r + 1; c < n; c = c + 1) {
      swap(r, c, c, r); }}
// Then reverse each row
  const half_n = math_floor(n / 2);
  for (let r = 0; r < n; r = r + 1) {
    for (let c = 0; c < half_n; c = c + 1) {
      swap(r, c, r, n - c - 1); \}\}
```

## Order of Growth

(1) Loop  $\rightarrow$  if WD inside the loop is constant,

for(let i = 0; i < N; i = i + 1)  $\rightarrow \Theta$ n

- ② Nested Loop  $\rightarrow$   $\Theta$ n<sup>2</sup>
- ③ Sequential → add up & give the highest pow
- (4) If Else → only give the block with the highest pow

Wu Xiaoyun NUS AY20/21 Sem 1