The Role of Risk-Neutral Probabilities in Enhancing Market Price Forecasts and Investment Strategies

ABSTRACT

This study explores market-based risk-neutral probabilities, rooted in the 1978 Breeden-Litzenberger method, for forecasting future asset prices and market trends. We assess the predictive power of these probabilities and confirm their effectiveness through confidence intervals provided by the Federal Reserve. Employing Lasso regression, we achieve an R squared of 87.9% in return prediction and 79.4% for volatility on training sets, with a KNN classifier reaching 78.4% in-sample accuracy for reversal detection. Based on these probabilities and historical pricing, our strategies yield backtest results with 37% and 40% excess returns over passive strategies, indicating that risk-neutral probabilities can be effectively exploited for market predictions despite market and financial constraints.

Keywords: *Market probability distribution, Lasso, Arbitrage, Efficient Market*

1. INTRODUCTION

Investors often tap into the insights embedded in financial asset prices as a cornerstone for making informed investment decisions. In this context, derivative markets provide them with a rich source of information for gauging market sentiment; due to their forward-looking nature, futures and options prices efficiently encapsulate market perceptions about underlying asset prices in the future[1]. Building on this foundation, an essential concept that investors encounter within these markets is the notion of market-based probabilities.

Theoretically, market-based probabilities allow investors to view the expected values of assets through a lens that discounts the varying attitudes toward risk. Since 1978, when Doug Breeden and Bob Litzenberger[2] proposed using traded options to infer those future probabilities, risk-neutral probabilities have been widely used in market pricing to find intrinsic value. This research aims to uncover and exploit mispricing in the market, so excess returns might be fair compensation for bearing hard-to-edge risks that are not included in the pricing model. The mispricing may reflect the effects of market constraints, such as short sale restrictions and financing issues[3].

This paper focuses on how the distribution of expected underlying asset returns, as outlined by the Minneapolis Fed's data, can provide a more accurate and actionable framework for investment decision-making. To demonstrate the effectiveness of our data, we conducted experiments using the S&P 500 Index as a case study. Firstly, by constructing confidence intervals, we evaluate the potency of risk-neutral probabilities in forecasting future market returns and volatility—a facet crucial for investors and policymakers alike. Upon confirming their predictive capability, we employ these projections to anticipate market prices and trends through Lasso regression analysis. Subsequently, we devise and retrospectively evaluate a price correction trading strategy that exploits these probabilities, aiming to exploit and profit from the data inherent in option prices.

2. THE EFFECTIVENESS OF MPD

With the Breeden-Litzenberger (1978) method, the Federal Reserve Bank of

Minneapolis utilized the option price to compute the market probability distribution of the underlying asset. Subsequently, the Fed calculated market probability density functions (MPDs) for various asset classes by adapting Shimko's (1993)[4] approach. This methodology is detailed in the Federal Reserve Bank of Minneapolis's documentation for current and historical market-based probabilities, published in 2024[5]. However, these foundational articles did not evaluate the predictive accuracy of these estimates, nor did they assess the effectiveness of the MPD. In this context, we introduce methods and approaches to evaluate their predictive efficacy and conduct empirical tests in Section 6.1

The Minneapolis Fed estimated the distribution of expected returns for the underlying assets over the next six months based on option prices. Our analysis involves hypothesis testing to examine the accuracy of this distribution prediction. Concurrently, we assess the precision of significant price movement predictions for the underlying assets using metrics and methods such as Brier Score, ROC, and AUC.

2.1 Hypothesis Testing for Prediction Accuracy

The Minneapolis Fed provides weekly forecasts of the probability distribution of log returns for assets over the subsequent six months. Generally, verifying whether a distribution accurately describes data charac-teristics is equivalent to exploring how data conforms to a specific distribution. Statistical methods addressing this issue require a sample set to test their derivation from the presumed distribution. However, for our target data, each distribution forecast corresponds to a single verifiable real data point, precluding traditional distribution verification statistical methods.

Our adopted methodology involves analy-zing a sequence of time points from t_0 to $t_n = T$, where at each time point $t_i, i = 0,1,\dots,n$, there exists a forecast for the distribution of log returns of the underlying asset at $(t_i + 0.5)^1$, denoted as f_i , for $i = 0,1,\dots,n$. The actual log return of the underlying asset at time t_i is denoted as x_i. For each forecasted distribution f_i, we identify its 10th and 90th percentiles, denoted as $P_{10,i}$ and $P_{90,i}$, respectively. This allows us to construct an 80% confidence interval $[P_{10,i}, P_{90,i}]$. We then compare whether the actual value x_i falls within this 80% confidence interval. If x_i falls within the interval, we mark it as 1; otherwise, we mark it as 0. If the forecast distribution is accurate, the proportion of actual values that fall within the 80% confidence interval should approximate 80%, indicating:

$$Accuracy = \frac{\sum_i^n 1_{\{x_i \in [P_{10,i}, P_{90,i}]\}}}{n} \approx 0.8 \quad (1)$$

where $1_{\{\}}$ is an indicator function that takes the value of 1 if x_i falls within $[P_{10,i}, P_{90,i}]$, and 0 otherwise.

By examining the proportion of points that fall within the confidence interval, we can assess the rationality of the distribution forecasts.

2.2 Assessing Predictive Accuracy of Large Price Fluctuations

The Federal Reserve Bank of Minneapolis provides forecasts that include the probability of significant[2] upward or downward movements in the returns of underlying assets. To validate the accuracy of these probability predictions, we calculated the actual returns of the underlying assets to determine if they meet the criteria for substantial fluctuations. At time t_i , if the actual return x_i reaches or exceeds an upward threshold, we assign $I_i = 1$; otherwise, $I_i = 0$. This process creates a categorical variable used to indicate whether the actual returns have experienced a significant increase. The formulation is as follows:

$$I_i = \begin{cases} 1 & if\ x_i \geq upward\ threshold \\ 0 & otherwise \end{cases} \quad (2)$$

Similarly, we can construct a categorical variable D_i to indicate whether the returns have

[1] This paper assumes T = 1 as one cycle, meaning T = 0.5 represents half a year.

[2] An increase or decrease of 20% within six months is considered a significant price movement.

significantly decreased, with the following formula:

$$D_i = \begin{cases} 1 & if\ x_i \geq downward\ threshold \\ 0 & otherwise \end{cases} \quad (3)$$

The data analysis then focuses on the probability of these events occurring and the categorical variables indicating whether the events have occurred. To assess the accuracy of these predictions, we employ commonly used metrics to evaluate the performance of classification models. These include the Brier Score[6], ROC[7] (Receiver Operating Charac-teristic) Curve, and AUC[Error! Bookmark not defined.] (Area Under the Curve).

The Brier Score is a measure of the accuracy of probabilistic predictions. It is used when predictions must assign probabilities to a set of mutually exclusive outcomes. The score is calculated as the mean squared difference between the predicted probabilities and the actual out-comes, where the outcomes are coded as 1 for the event happening and 0 for it not happening. The formula for the Brier Score is:

$$BS = \frac{1}{n}\sum_{i=1}^{n}(F_i - R_i)^2 \quad (4)$$

Here, n is the number of forecasting instances, F_i is the forecasted probability for the event at instance t_i, and R_i is the actual outcome of the event at instance t_i. The Brier Score should range between [0, 1], with a lower Brier Score indicating a better model fit.[8]

Typically, the ROC curve and the AUC are performance metrics widely adopted in machine learning for evaluating binary classifiers [9]. The ROC curve is created by plotting the True Positive Rate (TPR)[3] against the False Positive Rate (FPR)[4] at various threshold settings.

To construct an ROC curve, the decision threshold of the classifier is varied, with both the TPR and FPR calculated at each threshold level. These rates are subsequently plotted on a graph, positioning the FPR on the x-axis and the TPR on the y-axis.

AUC serves as a comprehensive measure for assessing the overall performance of the classifier. An AUC value of 1 indicates an ideal classifier with perfect accuracy, whereas an AUC value of 0.5 denotes a classifier lacking any discriminative power, equivalent to random guessing, and thus unable to differentiate between positive and negative classes effectively.

3 PREDICTIONS BASED ON MPD

Following the validation of the MPD effectiveness, our objective is to predict the returns, volatilities, and reversals of the underlying assets using MPD. We selected the Lasso model to achieve this goal. Section 3.1 provides a brief overview of the principles behind Lasso, while Section 3.2 details how we specifically construct models for different regression targets.

3.1 Method in predictions: lasso

Lasso[10] (The Least Absolute Shrinkage and Selection Operator) is an efficient variable selection technique across various application scenarios. Fundamentally, it incorporates L1 regularization as a penalty term to linear regression. During model training, Lasso applies this penalty to reduce the coefficients of independent variables with minor influence on the dependent variable to zero. This process effectively conducts variable selection within the fitting procedure. The general formula for Lasso is presented as follows:

$$\hat{\beta}^{lasso} = arg\,\min_{\beta}(\frac{1}{2n}||Y - X\beta||_2^2 + \lambda||\beta||_1) \quad (5)$$

Where Y represents the dependent variable. X represents the matrix of independent variables. β is the vector of coefficients. n is the number of observations.

This formulation allows Lasso to perform coefficient estimation and model complexity re-

[3] True Positive Rate (TPR): It measures the proportion of actual positives that are correctly identified by the classifier. It is calculated as:

$$TPR = \frac{TP}{TP + FN}$$

where TP is the number of true positives, and FN is the number of false negatives.

[4] False Positive Rate (FPR): It measures the proportion of actual negatives that are incorrectly classified as positives. It is calculated as:

$$FPR = \frac{FP}{FP + TN}$$

Where FP is the number of false positives, and TN is the number of true negatives.

duction simultaneously, making it a valuable tool for achieving sparse models in high-dimensional data settings.

3.2 Predicting Returns, Volatilities, and Reversals

These estimators reflect market expectations for the future. Due to the characteristics of time series data, there is generally a higher correlation between data points closer in time. Therefore, we use the target asset's estimators and lagged data for prediction purposes. Moreover, our focus is forecasting the asset's value changes one week ahead.

In predicting returns, we designate the complete set of estimates data as matrix $E_{n\times m}$, where i and j index the rows and columns, respectively, denoting the observation at the i^{th} time point for the j^{th} feature. Similarly, the data related to the target asset are represented by matrix $A_{p\times q}$. It is important to note that n and p may not necessarily be equal, owing to differences in the frequency of data generation, which results in varying amounts of data. Unless specified otherwise, the data structure within this paper is organized such that each row corresponds to observations of different features at a given time point.

Following the construction methodology typical of time series analysis, we apply lags to all variables intended to serve as independent predictors, incorporating these lagged variables into our predictive model. The dependent variable, or the target of our prediction, is constructed accordingly to facilitate a Lasso regression model. Specifically, only those estimands related to the target asset are selected as predictors. For instance, in predicting the returns of the S&P 500, we utilize the forecast data from options on the S&P 500 that expire in 6 months and 12 months as our independent variables.

To align the frequency of the independent variables, we first adjust the dimensions of $A_{p\times q}$ to match those of $\mathrm{E}_{\mathrm{n\times m}}$, resulting in a modified matrix $A'_{n\times q}$. This adjustment ensures that all independent variables are based on the same time intervals, facilitating a coherent analysis. Subsequently, we concatenate $\mathrm{E}_{\mathrm{n\times m}}$ and $A'_{n\times q}$ to form a new matrix, denoted as $[E_{n\times m}, A'_{n\times q}]$. This matrix is then subjected to lag operations to incorporate historical data into the predictive model.

The final matrix of independent variables, X, is composed of the original matrices and their lagged versions, symbolically represented as

$$\mathrm{X} = [E_{n\times m}, A'_{n\times q}, \mathrm{lag}_1, \ldots, \mathrm{lag}_\ell] \quad (6)$$

where lag_ℓ denotes the operation of applying ℓlags to the matrix $[E_{n\times m}, A'_{n\times q}]$. Symbolically represented as

$$lag_\ell[E_{n\times m}, A'_{n\times q}] \quad (7)$$

This structured approach captures the current and historical information deemed relevant for prediction.

Our Lasso regression model forecasts returns and volatilities, differentiating solely by the dependent variable Y while maintaining uniformity in all other modeling aspects. Lasso's regularization aids in selecting pivotal variables, bolstering predictive accuracy and reducing overfitting.

On the other hand, we employed the MACD indicator to identify momentum and signal potential price reversals. By analyzing the logarithmic mean price of the S&P 500 over bi-weekly and monthly intervals, we use the crossover of these moving averages post-divergence to anticipate price direction changes.

4 STRATEGIES

4.1 Price Correction Strategy

The Lasso regression model is designed to forecast the log price of the S&P 500 at a specific time T. It operates under the hypothesis that the model's predictive capacity is sufficiently robust, such that the actual log price of S&P 500 at time T should conform to a projected threshold, expressed as a percentage change. Should the accurate log price of S&P 500 deviate from this calculated range, it may be interpreted as a mispricing—anomalies expected to correct over time, thereby aligning with the model's predicted value.

The mechanisms for triggering investment actions within our strategy are articulated through the following well-defined criteria:

Long Position: Initiate or maintain a long position for the next trading week (T+1) if the percentage of change between predicted time T log return and the actual time T log return exceeds the threshold of positive 0.05%.

LONG:

$$\frac{Predicted\ Price - True\ Price_t}{True\ Price_t} \times\ 100\% \ \geq 0.05\% \quad (8)$$

Short Position: Initiate or maintain a short position if the aforementioned normalized difference falls below a threshold of negative 2%.

SHORT:

$$\frac{Predicted\ Price - True\ Price_t}{True\ Price_t} \times\ 100\% \ \leq -0.05\% \quad (9)$$

Hold Position: The strategy recommends maintaining the current portfolio stance without initiating new trades in scenarios not meeting the criteria for long or short positions.

HOLD:

$$Otherwise$$

The strategy articulates the robustness of the underlying predictive model. Leveraging the high degree of accuracy afforded by the Lasso regression, the model has demonstrated a compelling performance profile. The efficacy of the trading strategy is illustrated below, where it is evident that the model's predictive power has been adeptly harnessed to realize substantial returns.

4.2 Long-Short Strategy

The regression architecture provides predictive capability, particularly concerning the expected log price of the S&P 500 in the forthcoming week. Therefore, a long-short trading strategy is constructed. The criteria for generating these signals are delineated in the same way as those in Price Correction Strategy.

5 DATA

We first introduce the data utilized in our study. The estimands under evaluation are derived from the Federal Reserve Bank of Minneapolis [5], which provides Statistics for Market-Based Probability Densities (MPDs) calculated from fifty-two different options or forward prices across six asset categories[6]. The original dataset comprises 12,249 observations and 14 independent variables.

The Federal Reserve Bank of Minneapolis provides forecasts weekly or bi-weekly. The data coverage period varies across different markets. We selected markets with complete data available from January 10, 2013, to February 7, 2024, and subsequently removed any data before January 10, 2013. Before September 4, 2014, the Federal Reserve Bank of Minneapolis issued data forecasts bi-weekly. To ensure consistency in data interval frequency, we used forward filling to address missing data, resulting in a weekly data frequency after the adjustment.

Our training data consists of known risk-neutral density statistics from the MPD data, including mean, standard deviation, skewness, and kurtosis for the S&P 500 index. Using the Lasso model for regression, we have also incorporated past actual S&P 500 index prices[7] into our time series for regression analysis. We have taken lags from 1 to 6 for our training data, and in each window, data from different lags are included in the model for training. Our training set comprises the time series's first 75% (427 rows), leaving the latter 25% (143 rows) for testing.

6 EMPIRICAL ANALYSIS

Building on the previously outlined data processing and methodological framework, we now provide a detailed examination of the performance of the above-mentioned methods on actual data sets.

[5] The data originate from : https://www.minneapolisfed.org/banking/current-and-historical-market--based-probabilities

[6] The six asset categories are: Rates, Inflation, Equity, Currency, Commodity, Bank.

[7] The data originate from : https://finance.yahoo.com/quote/%5EGSPC

Given that the Federal Reserve Bank of Minneapolis has conducted MPD calculations using options on the S&P 500 index with maturities of 6 months and 12 months. The data have been adjusted to reflect the distribution of expected values six months after each observation date, and our analysis will incorporate both the 6-month (sp6m) and 12-month (sp12m) options data.

6.1 Validating the Effectiveness of MPDs through S&P 500 Data

In line with the approach described in Section 2.1, there are 578 observations each for sp6m and sp12m over the entire period. Given that both datasets pertain to the same underlying asset, we analyze the data from both markets. Our calculations reveal that the within interval accuracy for sp6m predictions is 0.8616, while for sp12m predictions, it is 0.9446. It indicates that the projections for sp6m are relatively close to the 80% mark, though there are reasons for the discrepancies observed. Firstly, the limited number of available data points for validation (578) does not fully attest to the accuracy of the predictions, suggesting potential data biases; moreover, the projections appear to be conservatively biased, as more actual data fell within the predicted intervals than expected, thus reducing the probability of Type I errors while increasing the probability of Type II errors. Overall, these estimates are usable, albeit with a possibly underestimated variance. Figure 1 illustrates that the major data points fall within the 80% confidence interval.

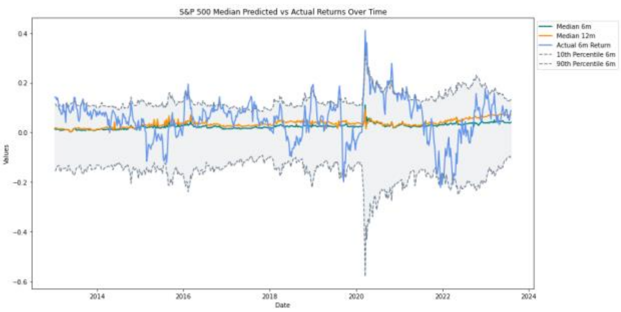

Figure 1 S&P 500 Median Predicted vs Actual Returns Over Time

Next, we analyze the predictions of significant price movements in the S&P 500 from 2013 to 2023. Initially, we calculate the actual price changes within six months and then determine whether these changes meet or exceed an upward or downward threshold, which we have set at 20%. If met, we mark it as 1; if not, we mark it as 0. Subsequently, we calculate the Brier Score and AUC for sp6m and sp12m predictions. The results are presented in Table 1.

Table 1. Brier Score and AUC for Significant Price Movement Predictions by sp6m and sp12m

	Brier Score	AUC
sp6m Increase	0.0124	0.9888
sp6m Decrease	0.0138	0.6867
sp12m Increase	0.0192	0.9560
sp12m Decrease	0.0240	0.6841

The data reveal that sp6m and sp12m perform exceptionally well in predicting significant upward movements, with sp6m yielding slightly better results, achieving an AUC of 0.9888. This superiority can be attributed to the Breeden-Litzenberger approach, which forecasts the expiration day prices for futures, options, and forwards. The Federal Reserve Bank of Minneapolis builds on this by calculating predicted distributions of returns and adjusting all data to an expected value distribution within a six months following each observation date. Since sp6m directly predicts the returns after six months, its predictions tend to be more accurate.

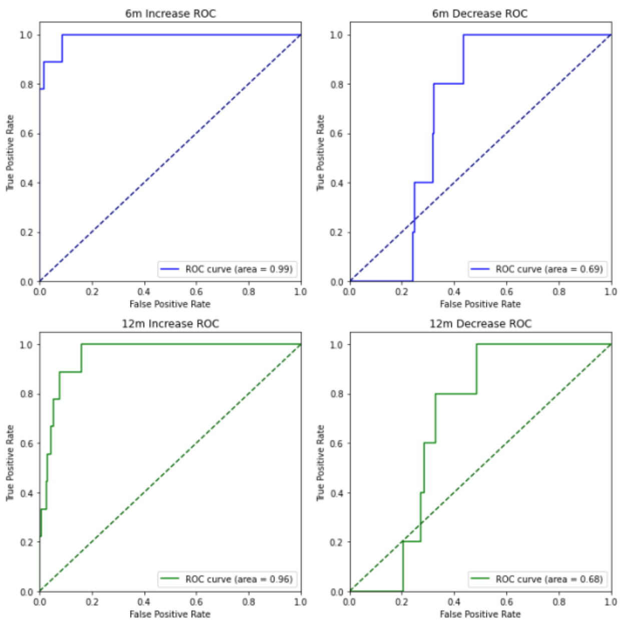

Figure 2 ROC Curve for Large Rise and Fall Predictions

6.2 Predicting with MPD

In predicting future returns with our Lasso model, we achieved an R squared of 99.323% on the training set and 87.946% on the test set. On the test set, our error RMSE is 0.02442, and MAE is 0.01839. The Lasso model identifies 17 significant parameters.

Besides, we use the VIX index[8] to represent market volatility and predict future volatility, resulting in an R squared of 79.446% on the training set and 63.972% on the test set. Our error RMSE is 0.03182 on the test set, and MAE is 0.02228. The Lasso model identifies 18 significant parameters this time.

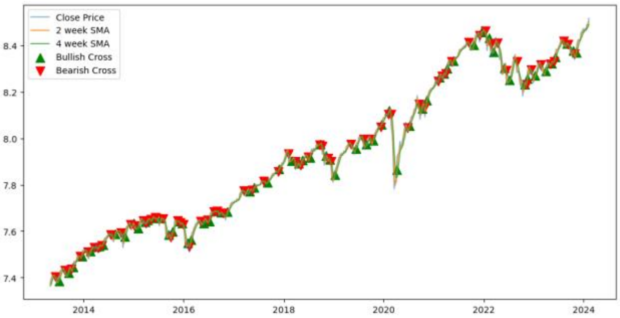

Figure 3 Stock Price Trend Analysis with Moving Average Crossovers

In addressing the classification problem, we have opted not to employ the previously discussed Lasso model to discern whether information can predict reversals. Instead, we have implemented a K-Nearest Neighbors (KNN) classifier and got an in-sample accuracy of 78.35% and an out-of-sample accuracy of 65.49%.

From the reslts presented above, estimates of risk-neutral probabilities indeed play a significant role in predicting future returns, volatility, and reversals. For the predictions concerning future returns and volatility, kurtosis occupies the most significant proportion among the coefficients selected by the Lasso method because it measures the "tailedness" of the probability distribution of a real-valued random variable. High kurtosis means the distribution has heavy tails or outliers, indicating extreme values far from the mean are more likely than in a normal distribution.

In financial markets, these extreme values can correspond to periods of high volatility or rapid changes in asset prices, which are critical for predicting future returns and market behavior. A distribution with high kurtosis is more likely to experience sudden, unexpected fluctuations, making it a valuable predictor for volatility and potential reversals in asset prices.

The confusion matrix from the KNN model shows that predictions for reversals are significantly more accurate when forecasting upward movements, and it aligns with our earlier analysis, which demonstrated a higher accuracy for predicting upward trends. Therefore, the estimators contain information that helps predict future returns, volatility, and reversals.

6.3 Strategy Construction Predictive

In the preceding discussion, we introduced using Lasso regression with simple training and testing data partitions. To augment the prediction robustness from comprehensive historical asset prices in constructing trading strategegis, we integrated a rolling Lasso regression approach, enhancing the predictive model by continuously expanding the training dataset with the latest available market data. This iterative process enables the model to dynamically update and refine its predictions in alignment with the most current market information, ensuring that the strategy remains responsive to evolving market conditions. A critical observation regarding the Federal Reserve's release schedule is that the MPD data for time T is published at T+1, causing direct application of T period MPD data for T+1 predictive analysis to be infeasible. Moreover, the imperative of executing a weekly frequency trading strategy necessitates the formulation of trading decisions before the closure of markets at time T. Consequently, our rolling regression model leverages all available MPD data up to T-1 to forecast the trading signal for T+1 while utilizing the S&P 500 log price data available by time T at the same time.

[8] The data originate from : https://finance.yahoo.com/quote/%5EVIX?.tsrc=fin-srch

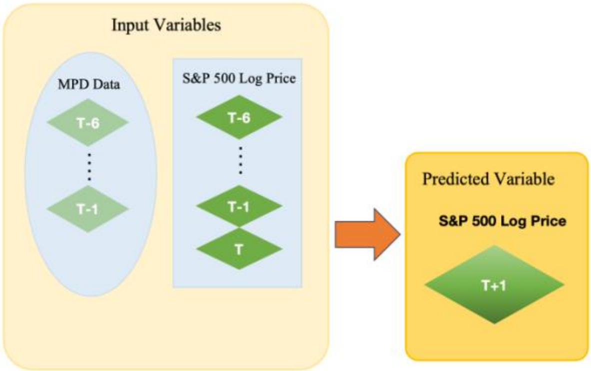

Figure 4: Data processing flow

6.3.1 Price Correction Strategy

Over three years, the active trading strategy realized a respectable cumulative return of 57%, with an alpha of more than 25% when contrasted against a passive S&P 500 buy-and-hold benchmark. The strategy's discerning approach, called the Price Correction Strategy, predominantly maintained a long position akin to a passive investment stance. Yet, it demonstrated strategic acuity by capitalizing on select windows for initiating short positions - most notably during the marked downturn between June 8th and June 14th, 2022, when the S&P 500 experienced a steep decline of approximately 10%. Additionally, the strategy identified and exploited transient shorting opportunities in February and March of 2023.

The strategy's robustness is further underscored by its ability to sustain long positions through periods of heightened volatility. It is exemplified by the period from late April to early June 2022, where the strategy's adherence to long positions allowed it to benefit from the market's rebound off critical support levels. These strategic decisions are a testament to the sophisticated risk management and market timing embedded within the Price Correction Strategy, which aims to maximize returns while mitigating downside risk during turbulent market phases.

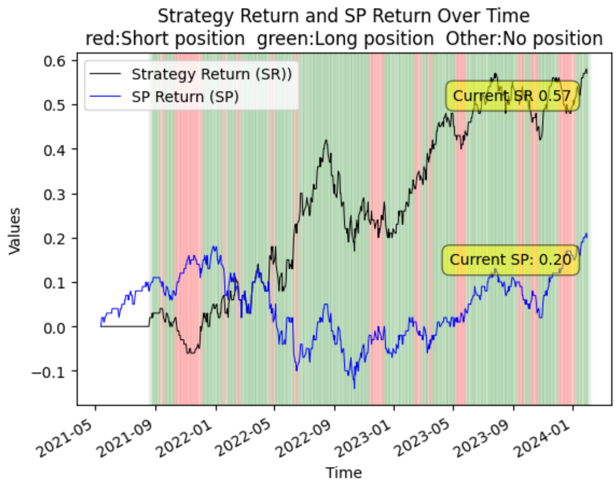

Figure 5: Price Correction Strategy Return vs. S&P 500 Return over Time

6.3.2 Long-Short Strategy

The diminished efficacy of recent short positions could indicate the U.S. stock market's recovery from the long-term impacts of the COVID-19 pandemic and the resolution of geopolitical tensions among nations. Using a Lasso regression model, calibrated with data from the past ten years, might have resulted in an over-representation of market volatility within its forecasting framework, thus raising concerns about overfitting. Furthermore, the model's training dataset predominantly encompasses a period before the Federal Reserve's interest rate hikes, implying that it may not have been adequately conditioned to account for the economic dynamics associated with increased interest rates. With the Federal Reserve maintaining interest rates at a 22-year peak[9], refining the model to accommodate the influence of such monetary policy shifts is essential. Hence, recalibrating the model to de-emphasize historical volatility could be instrumental in optimi-zing its future performance in a market that is entering a phase of stabilization.

[9] https://www.investopedia.com/federal-reserve-holds-interest-rate-at-22-year-high-8385747

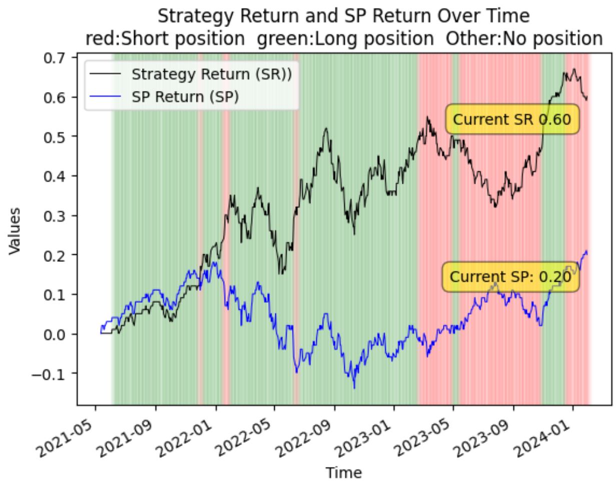

Figure 6: Long Short Strategy Return vs. S&P 500 Return over Time

6.3.3 Strategy Evaluation

The evaluation of performance metrics for the Price Correction and Long-Short strategies reveals congruent outcomes, attributable to their shared foundation in the Lasso predictive model. As illustrated in the Evaluation table, the analysis indicates a favorable trade success rate, with over half of the executed trades classified as winners. Furthermore, the average profit from successful trades is approximately double the average loss incurred from unsuccessful ones.

In addition, a Sharpe ratio nearing the 1.0 mark also denotes that both strategies deliver satisfactory risk-adjusted returns. However, it is crucial to note that a volatility measure of 17% for both strategies signifies a relatively aggressive risk profile. It is further evidenced by the substantial maximum drawdowns highlighting periods of significant decline from peak values, underscoring the potential for high returns alongside notable risks.

The backtesting timeframe encapsulates the 2022 global stock market downturn, characterized by heightened uncertainties. Such conditions within the testing period provide context for the observed drawdowns, rendering them a plausible and understandable aspect of the strategies' performance during a notably turbulent phase in the markets. This consideration is essential for comprehensively interpreting the strategies' risk and return profiles.

Table 2. Strategy Evaluation Table

Metric	Price Correction	Long-Short
Strategy Return	57%	60%
Strategy Annualized Return	18.1%	18.9%
SP500 Return	20%	20%
SP500 Annualized Return	6.9%	6.9%
Excess Return	37%	40%
Win Rate	65.4%	81.8%
Profit/Loss Ratio	1.53	1.676
Sharpe Ratio	0.886	0.919
Volatility	17%	17.3%
Max Drawdown	-17.6%	-17.8%

7 ADDITIONAL DATA AND IMPROVED STRATEGY

To assess stock trend reversals more accurately, we need to construct a more stable reversal factor. Traditional reversal factors assume that individual stocks will regress toward the average level of the market:

$$Reverse_i = mean(Ret_{20}) - Ret_{i,20} \quad (10)$$

However, taking the average return of the entire market's stocks as a uniform benchmark for all stocks is overly simplistic and crude, as it neglects the specific qualitative information among different industries and companies. Therefore, finding a suitable expected return benchmark for each stock is crucial to determining the performance of the reversal factor. Tsinaslanidis and Kugiumtzis[2014][1] posit that similar background conditions underpin similar stock patterns, and by identifying the future performance of stocks with similar historical patterns, one can predict the future returns of stock[Error! Bookmark not defined.]. . In this context, we require a more extensive set of stock price data and provide a unified framework for cons-tructing reversal factors：

$$Reverse_i = \sum_{j=1}^{N} w_{ij} Ret_{j,20} - Ret_{i,20} \quad (11)$$

$$\sum_{j=1}^{N} w_{ij} = 1 \quad (12)$$

where N represents the total number of stocks in the market, and w_{ij} is the correlation function regarding stock i and stock j, representing the weighted weight of stock j.

This general form implies that the weighted return of similar stocks is the benchmark for their mean reversion. We need to find the most similar stocks for each individual stock and assign their respective weighted weights. Taking Bank of America as an example, to construct a reversal factor indicator, we can select the 10 stocks with the highest price correlation from the entire banking sector (JPMorgan Chase&Co., Wells Farfo&Co. and so on) and construct the reversal factor according to the previous formula. By constructing reversal factors, investors can discover the true intrinsic value of stocks for reasonable investment strategies and avoid downside risks when the market is overheated to achieve better risk management.

8 CONCLUSIONS AND DISCUSSIONS

The findings from this research underscore the critical role of market-based probabilities in enhancing market price forecasts and investment strategies. By constructing confidence intervals, we observed that the market-based probabilities possess a significant capability to estimate future market returns—over 80% of predicted returns lie in the confidence interval we constructed. It enhances our confidence in the utility of risk-neutral measurements for market forecasting.

To discover what information helps predict future returns, volatilities, and reversals, we adapted the Lasso Regression. We found some significant estimators with lag elements to capture the nuances of market movements. Furthermore, by constructing a MACD indicator and employing KNN classification, we validated the use of MPD statistics for predicting reversals. In the end, we achieve an R squared of 87.9% in return prediction and 79.4% for volatility on training sets, with a KNN classifier reaching 78.4% in-sample accuracy for reversal detection.

Based on all the information we have especially for the predicted price by lasso model, we constructed two trading strategies with 37% and 40% excess returns separately. It not only showcased the practical applications of these probabilities but also demonstrated robust theoretical excess returns, highlighting the strategies' potential profitability and efficacy in real world market scenarios.

This research contributes to the existing body of knowledge by providing empirical evidence on the effectiveness of risk-neutral probabilities in forecasting and strategy development. However, the potential inaccuracies arising from market constraints and financing issues prompt a discussion on further refinement of the models and strategies employed. Additionally, the study highlights the importance of considering market dynamics and external factors that influence the predictive accuracy of risk-neutral probabilities.

Future research could explore integrating more sophisticated models or including broader market indicators to enhance predictive accuracy. Moreover, examining the application of risk-neutral probabilities across different market conditions and asset classes could offer deeper insights into their versatility and effectiveness as a tool for investors.

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