Exercise 3.28(2)

Xiaozhu Zhang

```
library(TeachingDemos)
library(ggplot2)
```

Since the posterior distribution is

$$\pi(\theta|x) \propto \exp\left\{-\frac{1}{2}(x-\theta)^2\right\}\frac{1}{1+\theta^2},$$

where $x = 6$, we first should compute the denominator Z:

```
f <- function(x){
  exp(-0.5 * (x - 6)^2) / (1 + x^2)
}
Z <- integrate(f, -Inf, Inf)$value
Z
```

```
## [1] 0.07384742
```

Then we shell sample enough values from the posterior distribution, using the accept-reject method.

Step 1: the proposal distribution.

We can use the Gaussian distribution $N(5.5, 2^2)$ as the proposal distribution. Set $c = 2$. Such a choice is justified by the following test and graph.

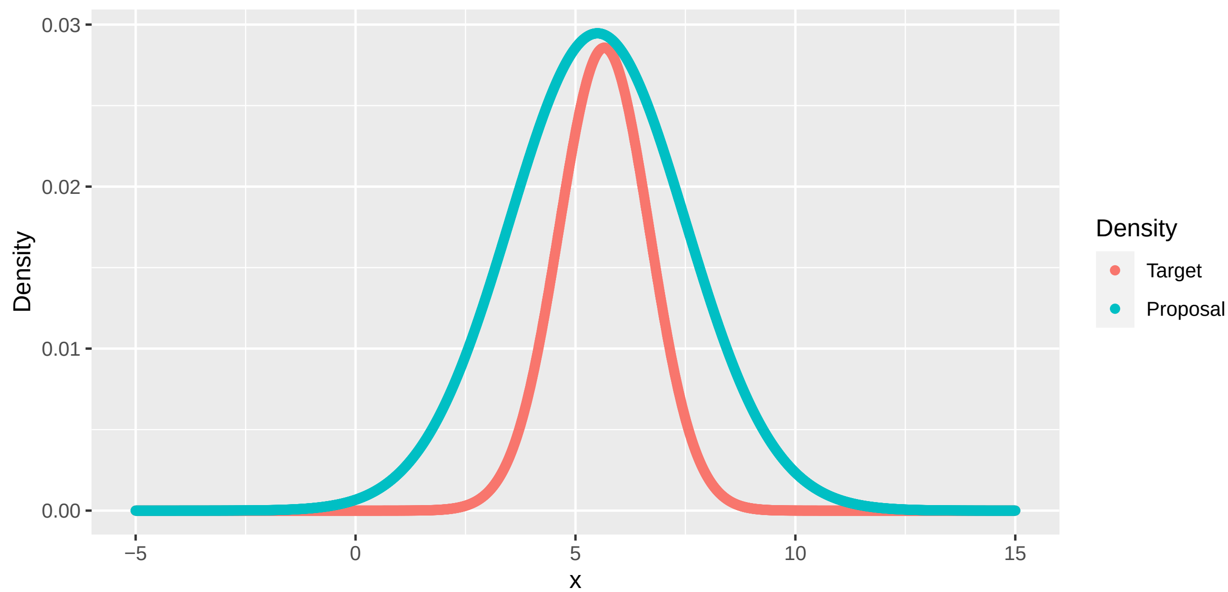
```
set.seed(54321)
# Test the validity of proposal distribution
seq <- sapply(seq(-5, 15, 0.01), f)
gam <- sapply(seq(-5, 15, 0.01), function(x){
  dnorm(x, mean = 5.5, sd = 2)})
gam.n <- gam * Z * 2

# Number of invalid points
sum(gam.n <= seq)
```

```
## [1] 0
```

```
# Visualization
gp <- c(rep(0,length(seq)), rep(1, length(gam.n)))
df <- data.frame(x = c(seq(-5, 15, 0.01), seq(-5, 15, 0.01)),
                 y = c(seq, gam.n),
                 gp = as.factor(gp))
```

```r
ggplot(data = df, aes(x = x, y = y, color = gp)) +
  geom_point() +
  scale_color_discrete(name = "Density", labels = c("Target", "Proposal")) +
  ylab("Density")
```



In a nutshell, there is no invalid points, and the graph above shows that the proposal distribution is completely above the target distribution.

Step 2: the accept-reject procedure.

```r
c = 2
n = 1e6
# Sample from the proposal distribution
x.star <- rnorm(n, mean = 5.5, sd = 2)

# Compute the accept-reject threshold
ratio <- sapply(x.star, f) /
  (c * Z * sapply(x.star, function(x){dnorm(x, mean = 5.5, sd = 2)}))

# Decide to accept or not
u <- runif(n)
ind <- u <= ratio

# Obtain the sample values
x.final <- x.star[ind]
length(x.final)
```

```
## [1] 500126
```

With the sampled values `x.final`, we can finaly find the 95% HPD of θ:

```r
emp.hpd(x.final)
```

```
## [1] 3.629926 7.680003
```