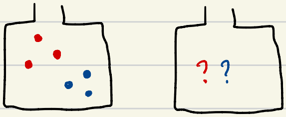
Lecture 12



if the indexing variable of variable space is a finite dimensional vector, it's parametric.

6 balls, draw 2 at random

prior on all urn compositions

$$p(3\text{ red}, 3\text{ blue}) \propto p(\text{draw } 1\text{ red}, 1\text{ blue} \mid 3,3) \times p(3,3)$$

Inductive reasoning

→ Fisher's significance testing

$X \sim P,\ p \in \mathcal{P}$

H_0: $p \in \mathcal{P}_0$ some subset of $\mathcal{P}$ (Single hypothesis)

$X \sim \text{Bin}(500, \phi)$, $\phi \in [0,1]$

H_0: $\phi \le 0.5$, $T(x) = x$

Some statistic $T = T(X)$ such that T is unlikely to be large for any $p \in \mathcal{P}_0$

Fix the statistic and observe data $X = x_{obs}$

Calculate: $t_{obs} = T(x_{obs})$

Report p-value $p = \max_{p \in \mathcal{P}_0} \mathbb{P}(T \geq t_{obs})$

$$X = 270, \quad p = \max_{0 \le \phi \le 0.5} \mathbb{P}(T \geq 270 \mid \phi) = \mathbb{P}(X \geq 270 \mid \phi = 0.5) \approx 4\%$$

Interpretation: for $p = 4\%$, either H_0 is false OR something unlikely (only 4% chance) has happened.

If instead obs: $X = 300$ then $p = P(X \geq 300 \mid \phi = 0.5) \approx 4 \times 10^{-6}$

Either H_0 is false, or something extremely rare (only 4 in a million) has happened.
"$\phi \le 0.5$"

↓ get a bad sample

→ Why Baysian prior has issues.

Binomials → $\{\text{Bin}(500, \phi) : 0 \le \phi \le 1\}$

$= \{\text{Bin}(500, e^{\eta}/(1+e^{\eta})) : -\infty < \eta < +\infty\}$

$\phi \leftrightarrow \eta = \log \frac{\phi}{1-\phi}$

No rejection or acceptance.
$p \downarrow$, evidence $\uparrow$ that the hypothesis is false.

p-value $\neq \mathbb{P}(H_0 \mid X = x_{obs})$ ← This is Bayesian

But prob of data given H_0.

Fisher creates an illusion of inductive reasoning w/o priors. But if you don't do that, there's no way doing that. It's flawed since it cannot say something $\mathbb{P}(H_0 | X = x_{obs})$.
It cannot perform what it's pretending to do.

Inductive behavior

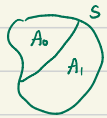
→ Neyman - Pearson

Forget inductive reasoning, embrace "inductive behavior".

$H_0: p \in \mathcal{P}_0$ vs. $H_1: p \in \mathcal{P}_1$, where $\mathcal{P} = \mathcal{P}_0 \cup \mathcal{P}_1$

statistical practice

= applying some well-defined rule of inference on observed data



$$\delta(x) = \begin{cases} \text{acc } H_0, & \text{if } x \in A_0 \\ \text{acc } H_1, & \text{if } x \in A_1 \end{cases}$$

Rule. $\delta: x \mapsto \{\text{accept } H_0, \text{accept } H_1\}$

Inductive behavior

→ focus on guarantees or "error probs" of a choosen rule.

	acc H_0	acc H_1
H_0	V	Type I
H_1	Type II	V

$H_0: p = p_0$, $H_1: p = p_1$ (simple)

$\alpha = P_0(\delta(X) = \text{acc } H_1)$ WT small α α, β quantifies how good the

$\beta = P_1(\delta(X) = \text{acc } H_0)$ small β.   rule is.

$X_1, \dots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, $\sigma = 0.5$, $\mu \in \mathbb{R}$, $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

$H_0: \mu = 2$ vs. $H_1: \mu = 3$

$\mathbb{P}(X_1 > 4 | \mu = 3) = 1 - \Phi(2) = 2.5\%$

Think of a Rule:

$$\delta(x) = \begin{cases} \text{acc } H_0 \text{ if } \bar{x} < c \\ \text{acc } H_1 \text{ if } \bar{x} \geq c. \end{cases}$$

target $\alpha = 1\%$, $\beta = 1\%$

Then,

$$0.01 = \alpha = \mathbb{P}(\bar{X} \geq c | \mu = 2) = 1 - \Phi\left(\frac{c-2}{\sigma/\sqrt{n}}\right)$$

$$0.99 = \Phi\left(\frac{c-2}{\sigma/\sqrt{n}}\right) \Rightarrow c = 2 + 2.33 \cdot \sigma/\sqrt{n}$$

$$0.01 \geq \beta = \mathbb{P}(\bar{X} < c | \mu = 3) = \Phi\left(\frac{c-3}{\sigma/\sqrt{n}}\right)$$

$$\frac{c-3}{\sigma/\sqrt{n}} \leq -2.33 \Rightarrow c \leq 3 - 2.33\frac{\sigma}{\sqrt{n}} \Rightarrow 2 + 2.33\frac{\sigma}{\sqrt{n}} \leq 3 - 2.33\frac{\sigma}{\sqrt{n}}$$

$$\Rightarrow n \geq 6 \Rightarrow c \leq 2.5$$