

### Lecture 13

$X_1, \dots, X_n \stackrel{iid}{\sim} N(\mu, \sigma^2)$ ,  $\sigma = 0.5$ ,  $\mu \in \mathbb{R}$

$H_0: \mu = 2$ , v.s.  $H_1: \mu = 3$

Rule based on  $(T = \bar{X}, c)$

Target:  $\alpha = 1\%$ ,  $\beta \leq 1\%$

$$\text{rule} = \begin{cases} \text{acc } H_1 & \text{if } \bar{x} \geq c \\ \text{acc } H_0 & \text{if } \bar{x} < c \end{cases}$$

$$\downarrow$$

$$c = 2 + \frac{\sigma}{\sqrt{n}} \cdot 2.33$$

$$n \geq 6$$

$$\alpha = P(\text{type I}) = P_0(\bar{X} \geq c)$$

$$\beta = P(\text{type II}) = P_1(\bar{X} < c)$$

→  $H_0: \mu = 2$ , v.s.  $H_1: \mu > 2$   
composite

Any time you have a composite hypothesis which get the null hypothesis on its boundary, it's always that  $\beta = 1 - \alpha$ . (can't control it by  $n$ ).

$$\alpha = 1\%, T = \bar{X}$$

$$c = 2 + \frac{\sigma}{\sqrt{n}} \times 2.33$$

Consider  $\beta(\mu) = P(\bar{X} < c | \mu) = \Phi\left(\frac{c - \mu}{\sigma/\sqrt{n}}\right)$ , for  $\mu > 2$ .

$$\max_{\mu > 2} \beta(\mu) = \beta(2) = 1 - \alpha = 99\%$$

Min effect size  $\Delta = 1$ ,  $\bar{\beta} = \max_{\mu > 3} \beta(\mu) = \beta(3) \begin{cases} \bar{\beta} \leq 1\% \\ \text{with } n \geq 6 \end{cases}$

→  $X \sim \text{Bin}(500, \phi)$ ,  $\phi \in [0, 1]$

$H_0: \phi = 0.5$  vs  $H_1: \phi \geq 0.55$

$\delta$ : reject  $H_0$  if  $\underline{x \geq 269}$

$$\alpha = 5\%, \bar{\beta} = \max_{\phi \geq 0.55} \beta(\phi) = 28\%$$

① obs  $x = 270$  reject  $H_0$  with  $\alpha = 5\%$ ,  $\beta = 28\%$

② obs  $x = 300$  reject  $H_0$  with  $\alpha = 5\%$ ,  $\beta = 28\%$

$$p\text{-value} = 4 \times 10^{-6}$$

reject  $H_0$  at  $p < 0.00001$  \*\*\*

"reject  $H_0$  if  $X \geq 300$ ":  $\delta^*$

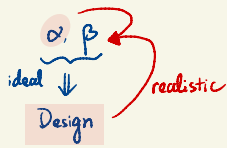
$$\alpha = P(X \geq 300 | \phi = 0.5) = 4 \times 10^{-6}$$

Roving- $\alpha$ . reject  $H_0$  with  $\alpha = 4 \times 10^{-6}$  and  $\beta = 97\%$

$$\text{Bin}(500, 0.55) \approx N(275, \underbrace{500 \times 0.55 \times 0.45}_{112})$$

$$P(X < 300 | \phi = 0.55)$$

Look at all tests based on  $X$  and different possible thresholds for which  $H_0$  is rejected, and choose the one w/ the smallest  $\alpha$  value.



→ Interval estimation

population feature

$X \sim P$ ,  $P \in \mathcal{P}$ , Qty of interest  $\vartheta = \vartheta(P)$

$$P = \text{Bin}(500, \phi), \\ \xi = \phi$$

$$\gamma : X \rightarrow [\underline{\gamma}(x), \bar{\gamma}(x)]$$

$$\text{Error: } \vartheta(P) \notin [\underline{\gamma}(x), \bar{\gamma}(x)]$$

$$\text{coverage}(\gamma; P) = 1 - \mathbb{P}[\text{exclusion error}] = 1 - \mathbb{P}[\vartheta(P) \notin [\underline{\gamma}(x), \bar{\gamma}(x)]]$$

● are all the same.

$$X \sim \text{Bin}(500, \phi), \phi \in [0, 1], \text{ est. } \phi$$

$$\gamma(x) = \left[ \frac{x}{n} - \frac{1}{\sqrt{n}}, \frac{x}{n} + \frac{1}{\sqrt{n}} \right]$$

$$\phi = 0.5$$

$$\text{coverage}(\gamma; \phi = 0.5) = \mathbb{P}(0.5 \in \left[ \frac{x}{n} \pm \frac{1}{\sqrt{n}} \mid \phi = 0.5 \right])$$

Conf coef = min coverage  $(\gamma; P)$  over  $P \in \mathcal{P}$

conf = 95.6%

[  $n \geq 100$ , conf coef  $\geq 94.3\%$  ]  
regardless of  $\phi$ .