Lecture 13

$X_1, \dots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, $\sigma = 0.5$, $\mu \in \mathbb{R}$

$H_0: \mu = 2$, v.s. $H_1: \mu = 3$

Rule based on $(T = \bar{X}, c)$

$$\text{rule} = \begin{cases} \text{acc } H_1 & \text{if } \bar{x} \geq c \\ \text{acc } H_0 & \text{if } \bar{x} < c \end{cases}$$

$\alpha = p(\text{type I}) = P_0(\bar{X} \geq c)$

$\beta = p(\text{type II}) = P_1(\bar{x} < c)$

Target: $\alpha = 1\%$, $\beta \leq 1\%$

$\downarrow$

$C = 2 + \frac{\sigma}{\sqrt{n}} \cdot 2.33$

$n \geq 6$.

→ $H_0: \mu = 2$, v.s. $H_1: \underbrace{\mu > 2}_{\text{composite}}$

Any time you have a composite hypothesis which get the null hypothesis on its boundary, it's always that $\beta = 1 - \alpha$. (can't control it by n).

$\alpha = 1\%$, $T = \bar{X}$

$C = 2 + \frac{\sigma}{\sqrt{n}} \times 2.33$

Consider $\beta(\mu) = P(\bar{x} < c \mid \mu) = \phi\left(\frac{c - \mu}{\sigma / \sqrt{n}}\right)$, for $\mu > 2$.

$\max_{\mu > 2} \beta(\mu) = \beta(2) = 1 - \alpha = 99\%$

Min effect size $\Delta = 1$, $\bar{\beta} = \max_{\mu > 3} \beta(\mu) = \beta(3)$ $\begin{cases} \bar{\beta} \leq 1\% \\ \text{with } n \geq 6 \end{cases}$

→ $X \sim \text{Bin}(500, \phi)$, $\phi \in [0,1]$

$H_0: \phi = 0.5$ vs $H_1: \phi \geq 0.55$

δ: reject H_0 if $\underbrace{x \geq 269}$

$\alpha = 5\%$, $\bar{\beta} = \max_{\phi \geq 0.55} \beta(\phi) = 28\%$

① obs $x = 270$ reject H_0 with $\alpha = 5\%$, $\beta = 28\%$

② obs $x = 300$ reject H_0 with $\alpha = 5\%$, $\beta = 28\%$

p-value $= 4 \times 10^{-6}$

reject H_0 at $p < 0.0001$ ****

Look at all tests based on X and different possible thresholds for which H_0 is rejected, and choose the one w/ the smallest α value.

"reject H_0 if $X \geq 300$" : δ^*

$\alpha = P(X \geq 300 \mid \phi = 0.5) = 4 \times 10^{-6}$

Roving-α. reject H_0 with $\alpha = 4 \times 10^{-6}$ and $\beta = 97\%$

$\text{Bin}(500, 0.55) \approx N(275, \underbrace{500 \times 0.55 \times 0.45}_{\|^2})$

$P(X < 300 \mid \phi = 0.55)$

$\underbrace{\alpha, \beta}$

ideal $\Downarrow$ realistic

Design

Interval estimation

population feature

$X \sim P$, $P \in \mathcal{P}$, Qty of interest $\xi = \xi(P)$

$P = \text{Bin}(500, \phi)$,
$\xi = \phi$

$\gamma : x \longrightarrow [\underline{\gamma}(x), \overline{\gamma}(x)]$

Error : $\xi(P) \notin [\underline{\gamma}(x), \overline{\gamma}(x)]$

coverage $(\gamma; P) = 1 - \mathbb{P}[\text{exclusion error}] = 1 - \mathbb{P}[\xi(P) \notin [\underline{\gamma}(x), \overline{\gamma}(x)]])$

are all the same.

$X \sim \text{Bin}(500, \phi)$, $\phi \in [0,1]$, est. ϕ

$\gamma(x) = [\frac{x}{n} - \frac{1}{\sqrt{n}}, \frac{x}{n} + \frac{1}{\sqrt{n}}]$

$\phi = 0.5$

Coverage$(\gamma; \phi = 0.5) = \mathbb{P}(0.5 \in [\frac{x}{n} \pm \frac{1}{\sqrt{n}} \mid \phi = 0.5)$

Conf coef = min coverage $(\gamma; P)$ over $P \in \mathcal{P}$

conf = 95.6%

[$n \geq 100$, conf coef $\geq 94.3\%$]

regardless of ϕ.