HW0 Part 1. Basic Algebra

(no complex or imaginary number is considered below)

C 1. For any two nonzero values a and b, the expression $\frac{1}{a}+\frac{1}{b}=?$

(a) $\frac{2}{a+b}$

(b) $\frac{1}{ab}$

(c) $\frac{ab}{a+b}$

(d) $\frac{a+b}{ab}$

(e) None of the above

d 2. For any value x, the expression $\sqrt{x^2}=?$

(a) x

$x>0$ or $x<0$

(b) $-x$

(c) $\pm x$

(d) $|x|$

(e) None of the above

e 3. For any two values x and y, the expression $\sqrt{x^2+y^2}=?$

(a) $x+y$

(b) $\pm(x+y)$

(c) $|x+y|$

(d) $|x|+|y|$

(e) None of the above

$p-2\le 7$ $p\le 9$

C 4. Solve for p: $|p-2|\le 7$

(a) 9

$2-p\le 7$

(b) -5 and 9 are both solutions

(c) $[-5,9]$

$-5\le p$

(d) $(-\infty,-5]\cup[9,\infty)$

(e) None of the above

$p-2\le -7$ $p\le -5$ X

e 5. Solve for p: $|p-2|\le -7$

$|p-2|>0$

(a) 9

$2-p\le -7$ X

(b) -5 and 9 are both solutions

(c) $[-5,9]$

-5 $9\le p$ X

(d) $(-\infty,-5]\cup[9,\infty)$

(e) None of the above

$\ln(e^z) = \ln(e^x + e^y)$

b 6. Solve for z: $e^z = e^x + e^y$

(a) $z = x + y$

(b) $z = \ln(e^x + e^y)$

(c) $z = \ln(x + y)$

(d) $z = \ln(x) + \ln(y)$

(e) None of the above

a 7. For any value x, the expression $\sqrt[3]{-x^3} = ?$

(a) $-x$

(b) $-|x|$

(c) $|x|$

(d) Does not exist

(e) None of the above

$-x$

b 8. Which of the following statements about logarithms is (are) TRUE?

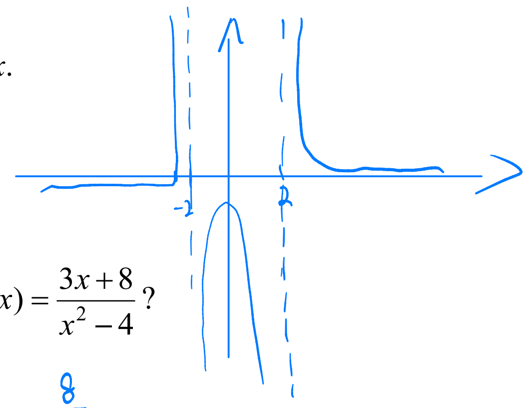
(a) $\log_b a = -\log_a b$

(b) $\ln(x^r) = r\ln(x)$ for any value of r and positive value x.

(c) $\ln(x^r) = (\ln(x))^r$ for any value of r and positive value x.

(d) Both (b) and (c)

(e) None of the above



d 9. Which of the following statements is (are) TRUE about $f(x) = \dfrac{3x+8}{x^2-4}$?

(a) The graph of $f(x)$ passes through the origin.

(b) $f(x)$ has vertical asymptotes at $x = \pm 2$.

(c) The X-axis is a horizontal asymptote for $f(x)$.

(d) Both (b) and (c)

(e) None of the above

$\frac{8}{-4}$

$x = 0$ $\quad \frac{8}{4} = 2$

c 10. Which of the following statements is (are) TRUE about $g(x) = \dfrac{3x+8}{x^2+4}$? >0

(a) The graph of $g(x)$ passes through the origin.

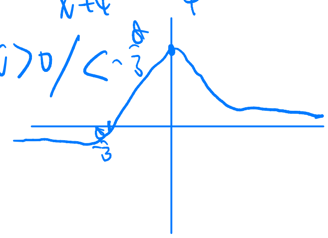
(b) $g(x)$ has vertical asymptotes at $x = \pm 2$.

(c) The X-axis is a horizontal asymptote for $g(x)$. $x > 0 \,/ < -\frac{8}{3}$

(d) Both (b) and (c)

(e) None of the above

$-\frac{8}{3}$ $\quad \frac{3x+8}{x^2+4} = 0$ $\quad \frac{8}{4}$



HW0 Part 2. Calculus (Integration)

1. $\int_0^1 (x^3 - 2x^2 + 4)dx$ (linearity rule and power rule)
2. $\int_1^2 \frac{1}{x+1} dx$ (power rule and by substitution)
3. $\int_1^2 \frac{x}{\sqrt{x^2+1}} dx$ (power rule and by sub: $u = x^2 + 1$)
4. $\int_0^1 3e^{2x} dx$ (exponential rule and by sub)
5. $\int_0^{+\infty} e^{-x} dx$ (improper integral and by sub)
6. $\int_0^{+\infty} xe^{-x} dx$ (improper integral and by parts)
7. $\int_0^{+\infty} xe^{-x^2} dx$ (improper integral and by sub)
8. $\int_0^{+\infty} e^{-\frac{x^2}{2}} dx$ (optional, very challenging! related to the standard normal distribution)
9. $\int_0^2 f(x)dx$, where $f(x) = \begin{cases} x-1, & 0 \le x \le 1 \\ 2x, & 1 < x \le 2 \end{cases}$ (piecewise integral)

(1) $\int_0^1 (x^3-2x^2+4)dx = \int_0^1 x^3 dx - 2\int_0^1 x^2 dx + 4\int_0^1 dx$

$= [\frac{1}{4}x^4]_0^1 - 2[\frac{1}{3}x^3]_0^1 + 4[x]_0^1 = \frac{1}{4} - (2\times\frac{1}{3}) + 4 = \frac{1}{4} - \frac{2}{3} + 4$

$= \frac{45}{12} = \frac{9}{4}$

(2) $\int_1^2 \frac{1}{x+1}dx = [\ln(x+1)]_1^2 = \ln(2+1) - \ln(1+1) = \ln 3 - \ln 2 = \ln\frac{3}{2}$

(3) let $u = x^2+1$, $du = d(x^2+1) = 2x dx$, $\frac{du}{2} = x dx$

$\int_1^2 \frac{x}{\sqrt{x^2+1}}dx = \int_2^5 \frac{du}{2\sqrt{u}} = \frac{1}{2}\int_2^5 u^{-\frac{1}{2}}du = \frac{1}{2}[2u^{\frac{1}{2}}]_2^5 = \cancel{\frac{1}{2}}\cdot\cancel{2}\cdot 5^{\frac{1}{2}} - \cancel{\frac{1}{2}}\cdot\cancel{2}\cdot 2^{\frac{1}{2}}$

$= \sqrt{5} - \sqrt{2}$

(4) let $u = 2x$, $x = \frac{1}{2}u$, $du = d(2x) = 2dx$, $dx = \frac{1}{2}du$

$\int_0^1 3e^{2x}dx = \int_0^2 3e^u \cdot \frac{1}{2}du = \frac{3}{2}\int_0^2 e^u du$

$= \frac{3}{2}[e^u]_0^2 = \frac{3}{2}e^2 - \frac{3}{2} = \frac{3}{2}(e^2-1)$

(5) $\int_0^{+\infty} e^{-x}dx = [-e^{-x}]_0^{+\infty} = 0 - (-1) = 1$

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6. $\int_0^{+\infty} xe^{-x} dx$ (improper integral and by parts)
7. $\int_0^{+\infty} xe^{-x^2} dx$ (improper integral and by sub)
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9. $\int_0^2 f(x)dx$, where $f(x) = \begin{cases} x-1, & 0 \le x \le 1 \\ 2x, & 1 < x \le 2 \end{cases}$ (piecewise integral)

(9)

for $0 \le x \le 1$:

$\int_0^2 f(x)\,dx = \int_0^1 (x-1)\,dx$

$= [\frac{1}{2}x^2 - x]_0^1 = (\frac{1}{2} - 1) - 0 = -\frac{1}{2}$

for $1 < x \le 2$:

$\int_0^2 f(x)\,dx = \int_1^2 (2x)\,dx = [x^2]_1^2$

$= 4 - 1 = 3$

$\int_0^2 f(x)\,dx = \int_0^1 f(x)dx + \int_1^2 f(x)\,dx$

$= -\frac{1}{2} + 3 = \frac{5}{2}$

(6) $\int u\,dv = uv - \int v\,du$, let $u = x$, $dv = e^{-x}dx$

$du = dx$, $v = -e^{-x}$

$\int_0^{+\infty} xe^{-x}dx = [-xe^{-x}]_0^{\infty} + \int_0^{\infty} e^{-x}dx = [-xe^{x}]_0^{\infty} + [-e^{-x}]_0^{\infty}$

$= 0 + 1 = 1$

(7) $\int_0^{+\infty} xe^{-x^2}dx$, let $u = x^2$, $du = d(x^2) = 2x\,dx$, $\frac{du}{2} = x\,dx$

$\int_0^{+\infty} xe^{-x^2}dx = \int_0^{+\infty} e^{-x^2} x\,dx = \frac{1}{2}\int_0^{+\infty} e^{-u}\,du = \frac{1}{2}[-e^{-u}]_0^{+\infty}$

$= \frac{1}{2}[0 - (-1)] = \frac{1}{2}$

(8)