The Vicsek model:
Simulating particles' flocking behaviour

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The Vicsek models are a series of mathematical models used to describe the behaviours of flocks of birds or swarms of fishes, where the most famous is the model introduced by Tamás Vicsek et al. in 1995. In this article, we will simulate particles' motion by applying the earliest formulas in 1995, including extracting data of Vicsek order parameter. Additionally, after that we add a predator to our system where a particle that changes its orientation of motion towards the position of 'preys' within a particular radius. Conversely, the other 'preys' near enough this particle will gain an extra velocity that turns them away from the position of the predator.

I. THE VICSEK MODEL: THE CORE ELEMENTS

Given a flock of birds, there is one subject of particular interest: how do they fly coherently so that all the birds gather together and no one would be separated ultimately? One reasonable explanation is given by Vicsek Model[1], describing the birds' motions as a rule of alignment in discrete time steps. It interprets the flying mode for a particular bird as reorienting itself according to the mean direction vector calculated from neighbouring birds at each step. We simulate the birds' flocking behaviour with python based on the following mathematical setup for a small system.

Suppose that there exists a $L \times L$ container in $\mathbb{R}^2$ space, where L^2 many birds are put into this container, with uniformly random initial positions and angles. Denote the position and angle for particle i at discrete time step t as $\vec{r}_i(t)$ and $\theta_i(t)$ respectively. The state of particles at any time is decided by these two factors. In order to demonstrate the birds' motion in the single container, the positions of the particles in this container is subject to *periodic boundary conditions* which means that if a particle crosses over one edge, it will reappear at the opposite edge while retaining its direction.

The core formulas are two equations corresponding to the update of particles' positions and angles respectively.

$$\vec{r}_i(t+1) = \vec{r}_i(t) + \hat{n}_i(t)v_0 \tag{1}$$

$$\theta_i(t+1) = angle[\sum_{j=1}^{z_i} \hat{n}_i(t)] + \eta_i \tag{2}$$

where $\hat{n}_i(t) = (cos(\theta_i(t)), sin(\theta_i(t))$ and η_i is the normally distributed noise, with mean 0 and variance σ^2. Note that the direction $\hat{n}_i(t)$ in equation (1) is at time step t, which means that for every particle, it will first changes its position and then reorient by the previous location information.

II. EXTRACTION OF DATA

In order to draw the phase transition diagram of the Vicsek order parameter, we run simulations for 10000 steps with evenly spaced 20 sigmas between 0.05 and 1. To process the data as a whole, we save the output traces in a CSV file for each run. Here, we need to develop an appropriate algorithm to judge the time for a system to reach steady with varying sigmas, then drop the data before that time in each case.

Since the system is small, we would expect that it will reach steady state in hundreds of steps. Seeing that the first few hundreds of steps only make up a small proportion of the data we obtained, it makes little difference to the average of the whole data. As a consequence, we approximate the mean of the Vicsek order parameter for systems in equilibrium by the average of the whole data in our algorithm.

$$n(\sigma) = \lim_{T\to\infty} \frac{1}{T} \sum_{k=1}^{T} n(\sigma, k) \approx \frac{1}{10000} \sum_{k=1}^{10000} n(\sigma, k)$$

We refer to the difference between the means of data to develop our criteria of judgment.

We loop from the first time step. In a single iteration, the current time is denoted as t. Considering the fluctuation of the data, define the average of the data from this point on within 25 steps as the running average at time t.

$$running\_average(t) = \frac{1}{25} \sum_{k=t}^{t+24} n(\sigma, k)$$

if the difference between the running average at time t and the average of the whole data is less than a threshold(we take as 0.005).

$$|running\_average(t) - n(\sigma)| \approx |\frac{1}{25} \sum_{k=t}^{t+24} n(\sigma, k) - \frac{1}{10000} \sum_{k=1}^{10000} n(\sigma, k)| < 0.005$$

We define that the system reaches steady state by time step $t + 12$(the midpoint of the running time interval $[t, t + 24]$). Otherwise, t is incremented by 1 and the above judging process is repeated in the next iteration.

III. PREDATOR–SWARM INTERACTIONS

In nature, some species have the talent that 'confuses' the predator so that the predator is unlikely to catch anyone[2]. Meanwhile, there exists some species which have low cooperation abilities. They will escape directly once finding a predator. Now add a predator to our system. Denote the predator's position and angle as $\vec{p}(t)$ and $\alpha(t)$ respectively. The predator's location information is completely determined by preys whose distance with the predator is less than R_{pred}, the field of view of predator:

$$\vec{p}(t+1) = \vec{p}(t) + \hat{m}(t)v_{pred} \tag{3}$$

$$\hat{m}(t) = \sum_{i \in V(t)} \hat{n}_i(t) + \eta_i \tag{4}$$

Here η_i is defined as before and $V(t)$ is the visible space of predator.

$$V(t) = \{i \;\; | \;\; \|\vec{r}_i(t) - \vec{p}(t)\| < R_{pred}\}$$

Preys' motion will be determined by either the predator or nearby birds. If the distance between *bird* i and the predator is greater than R_{prey}, the field of view of preys, it will move as before. But if the distance is less than R_{prey}, then the bird will not consider nearby birds and gain an extra terms about its angle in terms of the distance.

$$\theta_i(t+1) = \begin{cases} angle[\sum_{j=1}^{z_i} \hat{n}_i(t)] + \eta_i & \text{if } \; i \notin V \\ \frac{\theta(t) - \theta_{escape}}{R_{prey}} \|\vec{r}_i(t) - \vec{p}(t)\| + \theta_{escape} & \text{if } \; i \in V \end{cases} \tag{5}$$

where

$$\theta_{escape} = angle[\vec{r}_i(t) - \vec{p}(t)]$$

which means that the bird will run directly in the opposite direction to the predator. Illustratively, in this model the predator computes the average direction of preys which surrounds it at each time step, then goes in this direction at the next step. And predator's influence on prey will change linearly with distance between them, which explains why we create the linear function of preys' angle in terms of the distance.

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Thanks for Yulin's extraordinary idea about optimizing the computation of periodic boundary conditions.

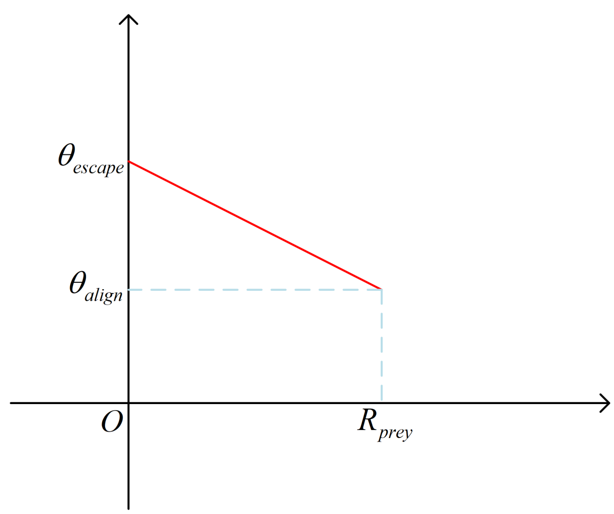

FIG. 1. How the predator–swarm interactions works for preys.

[1] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, Novel type of phase transition in a system of self-driven particles, Physical review letters **75**, 1226 (1995).

[2] Y. Chen and T. Kolokolnikov, A minimal model of predator–swarm interactions, Journal of The Royal Society Interface **11**, 20131208 (2014).