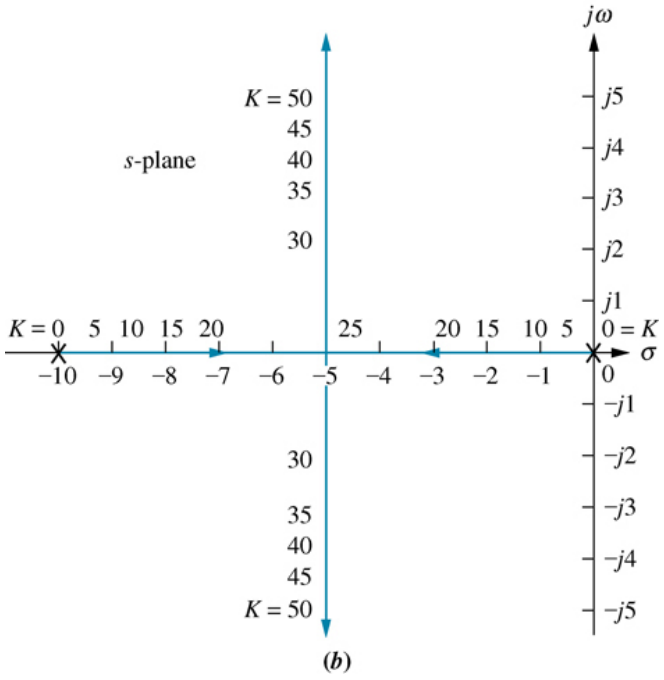
Studio #08: Root Locus Basics

The objective of this studio is to reinforce recently learnt concepts in the class on Root Locus method.

Properties of Root Locus/Sketching Rules

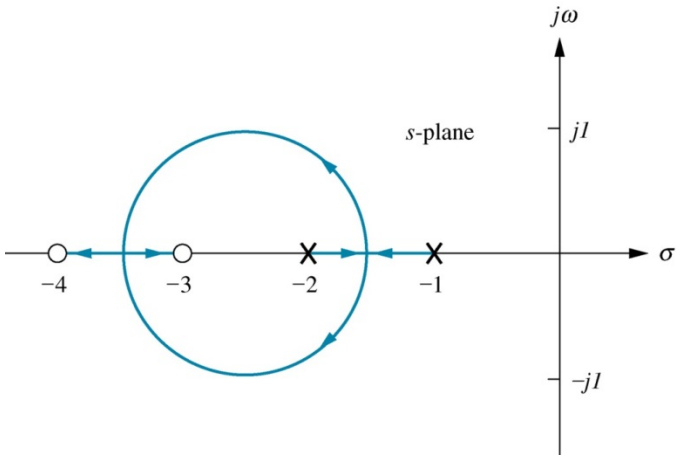
1. Number of branches is the same as the number of closed-loop poles.

2. Root locus is symmetric about the real axis.



3. Real-axis part of the root locus is to the left of an odd number of open-loop poles/zeros.

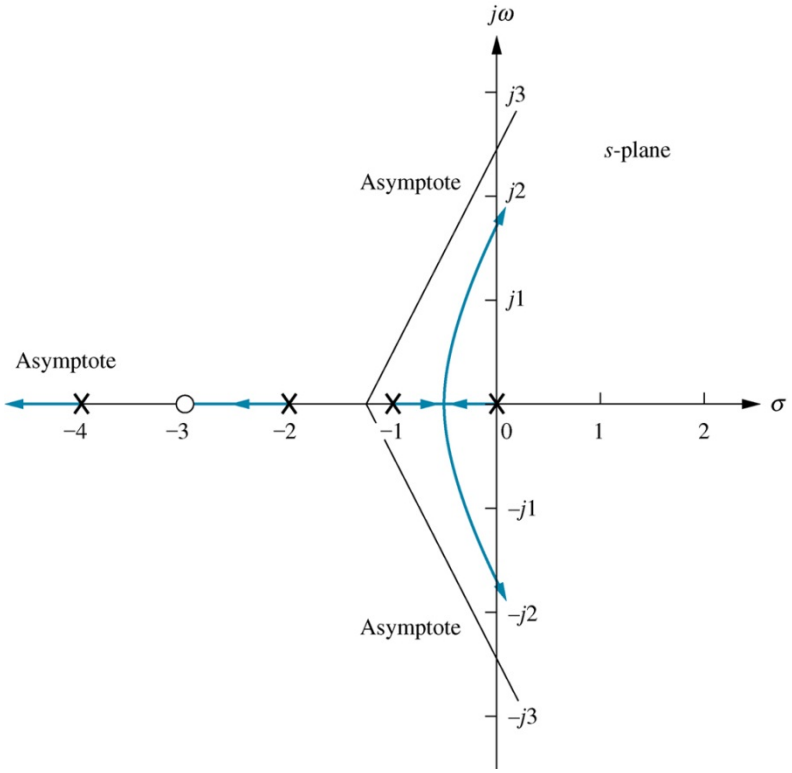
4. The root locus starts from open-loop poles and ends at open-loop zeros.



5. If the number of finite poles is different from the number of finite zeros, some branches of the root locus converge to the infinite poles or zeros, and the asymptotes are given by the real-axis intercept σ_a and the angles θ_a:

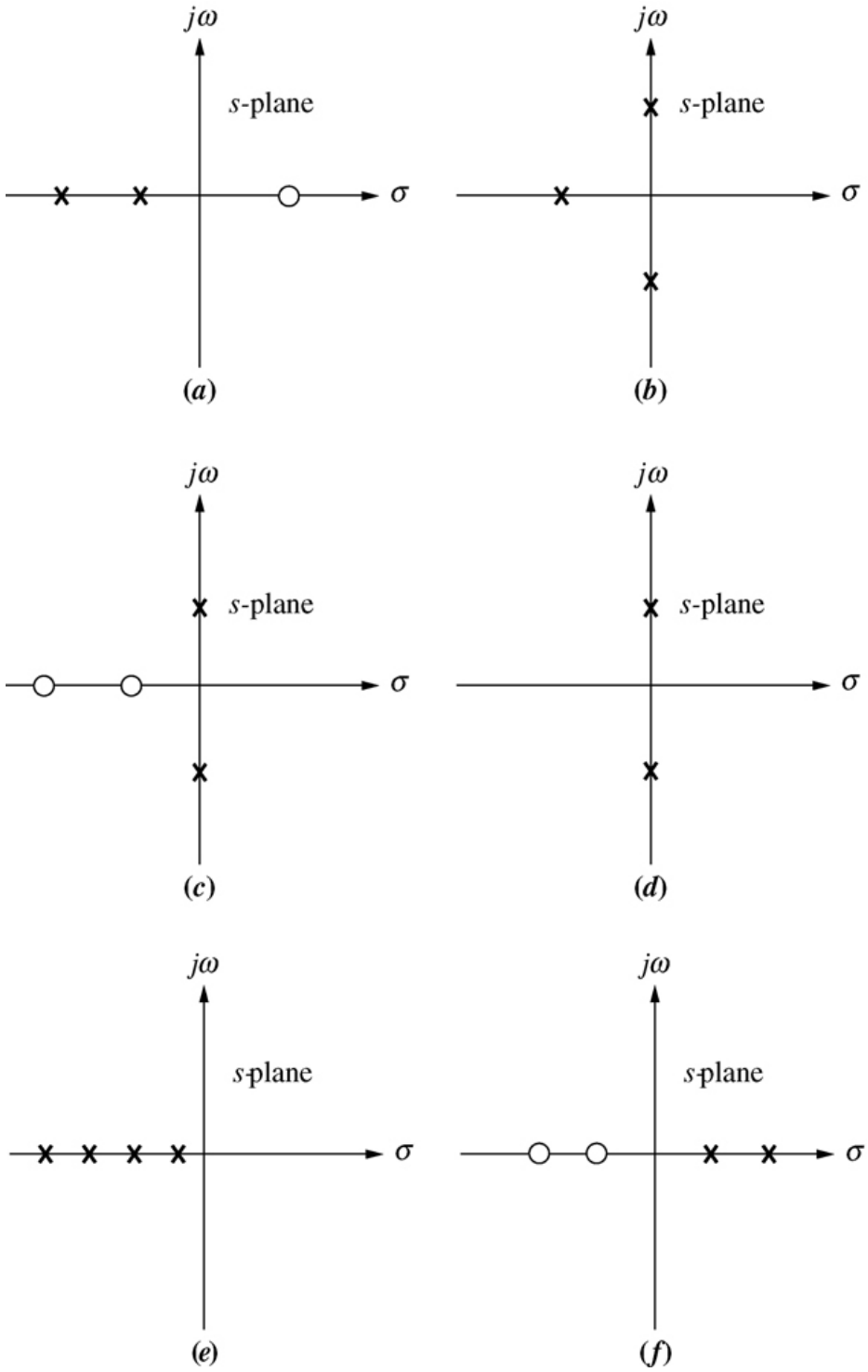
$$\sigma_a = \frac{\sum finite\ poles - \sum finite\ zeros}{\#finite\ poles - \#finite\ zeros}$$

$$\theta_a = \frac{(2k+1)\pi}{\#finite\ poles - \#finite\ zeros},\ k = 0, \pm 1, \pm 2, \cdots$$

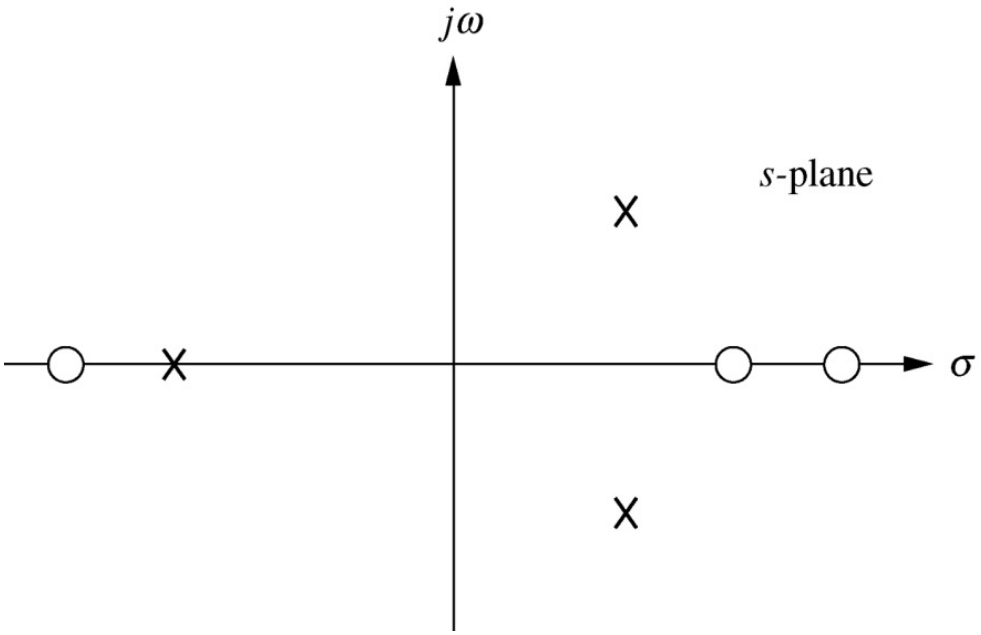


Example 1. Sketching the Root Locus

In each of the following plots, you are given the pole-zero diagram of the open-loop system. Sketch the root locus.



Example 2. Sketch the root locus for the following open-loop pole-zero configurations.



Example 3. For a negative unity-feedback system with a forward transfer function $G(s) = \frac{K}{s(s+6)(s+9)}$, plot the root locus and find breakaway points, asymptotes, and $j\omega$-axis crossing points.

Exercise

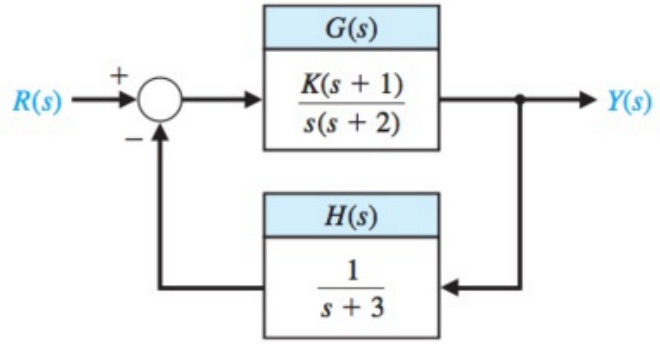
Each student shall complete the exercise below and get their work checked off by the studio Instructor.

For remote students and students and Students who do not finish within studio session:

Compile your answers and outputs of Exercises 1 in a word file and name it LastNameFirstNameStudio8.docx. Your file shall include the solution to parts (a) through (f) as well as the the Matlab code you used to answer the questions. Upload your file to Canvas under the Studio 8 link.

Exercise 1. Root Locus (RL) Analysis by hand and using Matlab

Given the feedback system shown below.



(a) Determine the characteristic equation for the closed loop system

(b) Determine the number of asymptotes and their angles

(c) Sketch the RL

(d) Determine the break-away point (if any)

(e) Determine the range of K for which the system is stable.

(f) Write a Matlab script to plot the root locus and compare your root locus sketch to the Matlab output. Use the Matlab plot to determine the break-away point and compare what you get to your answer in part (d).

Hint: use the function `rlocus(G*H)`