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OUTLINES

- ❖ Block Diagrams Algebra
 - Space Shuttle pitch control
 - Hybrid electrical vehicle
 - UFSS pitch control



CLOSED-LOOP TRANSFER FUNCTION

From the block diagram

$$C(s) = G(s)E(s)$$

$$\begin{aligned} E(s) &= R(s) - C(s)H(s) \\ &= R(s) - G(s)H(s)E(s) \end{aligned}$$

Therefore

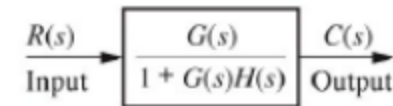
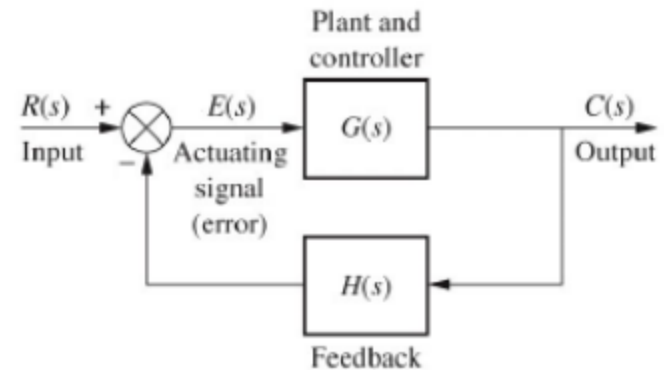
$$E(s) = \frac{1}{1 + G(s)H(s)} R(s)$$

And

$$C(s) = \frac{G}{1 + G(s)H(s)} R(s)$$

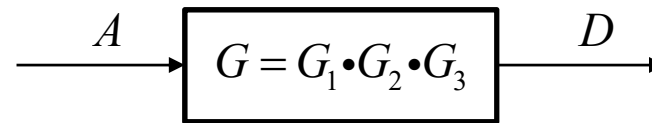
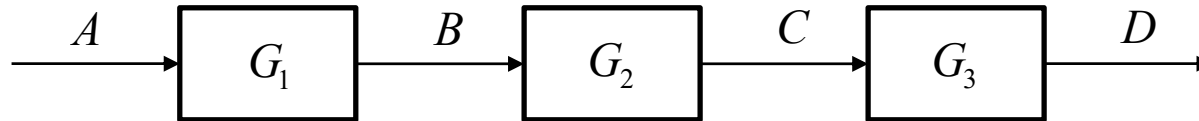
▪ Transfer function

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$





SERIES

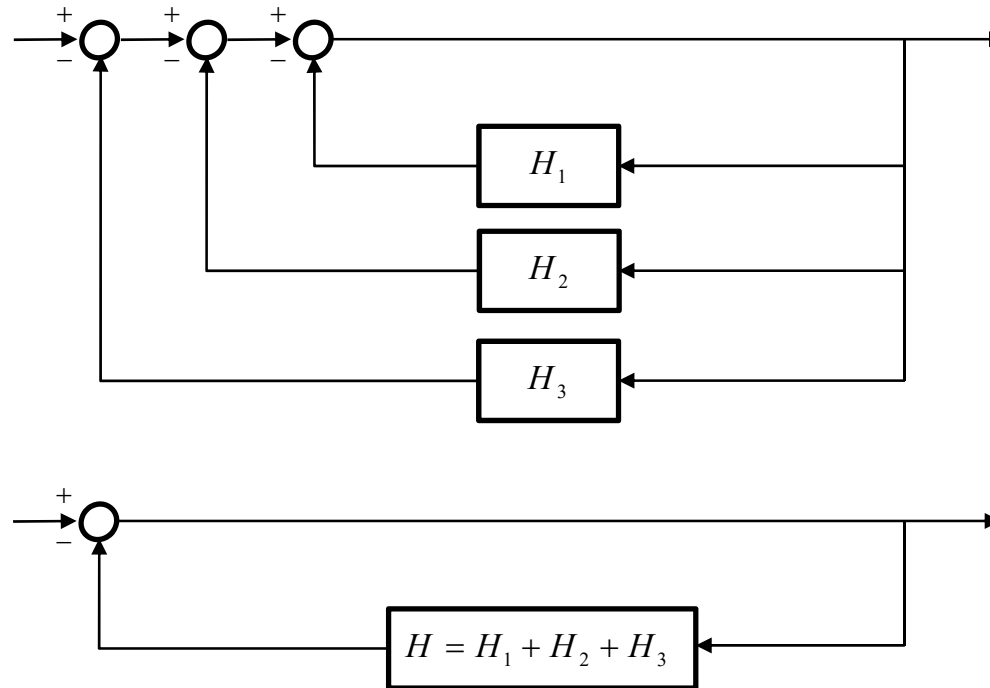


$$\frac{D}{A} = \frac{D}{C} \cdot \frac{C}{B} \cdot \frac{B}{A} \quad G = G_3 \cdot G_2 \cdot G_1$$



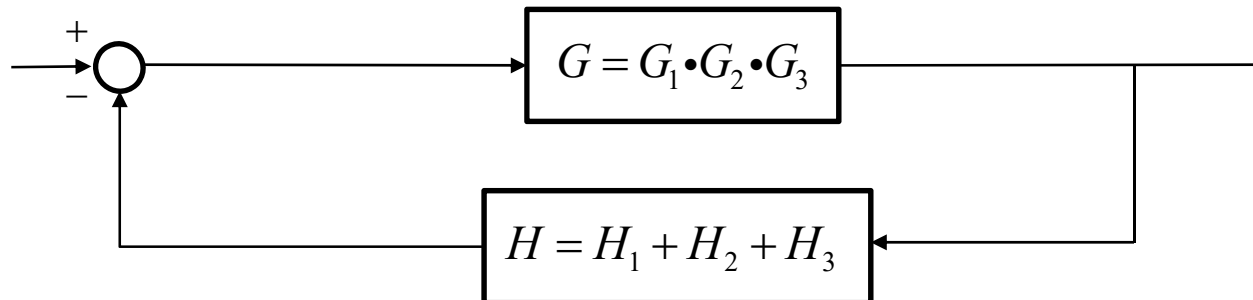
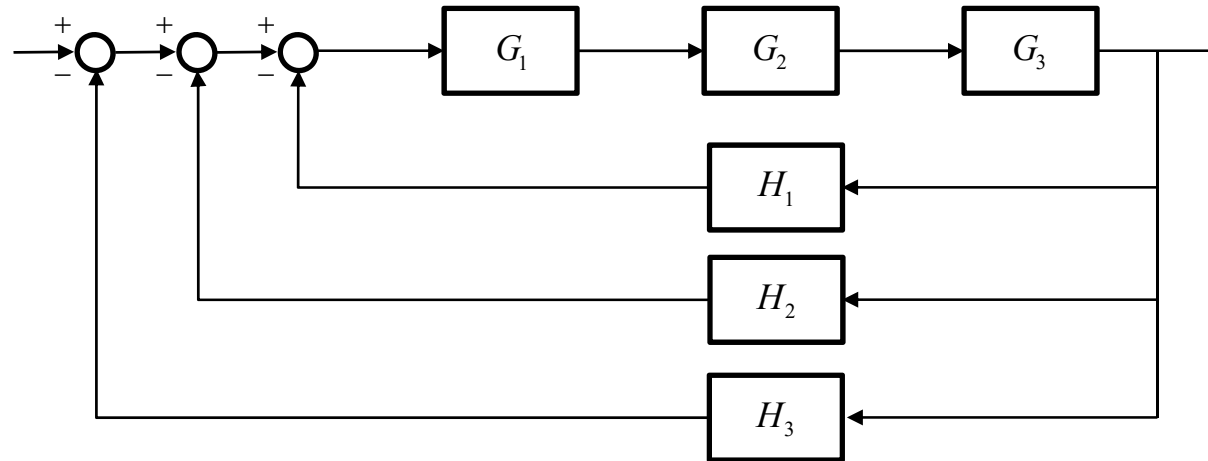


PARALLEL





CLOSED-LOOP TRANSFER FUNCTION



$$T = \frac{G}{1 + G \cdot H} = \frac{G_1 \cdot G_2 \cdot G_3}{1 + (G_1 \cdot G_2 \cdot G_3)(H_1 + H_2 + H_3)}$$

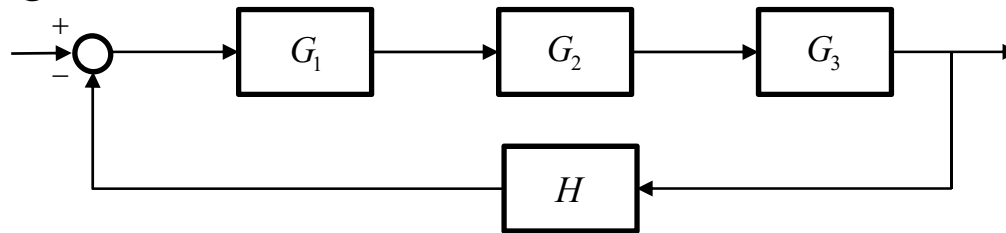


REDUCTION TECHNIQUES

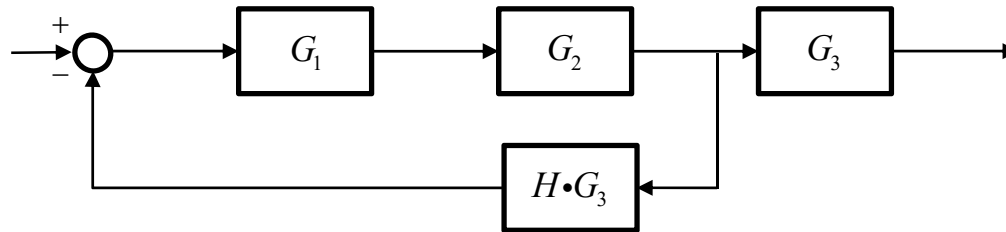


Summing

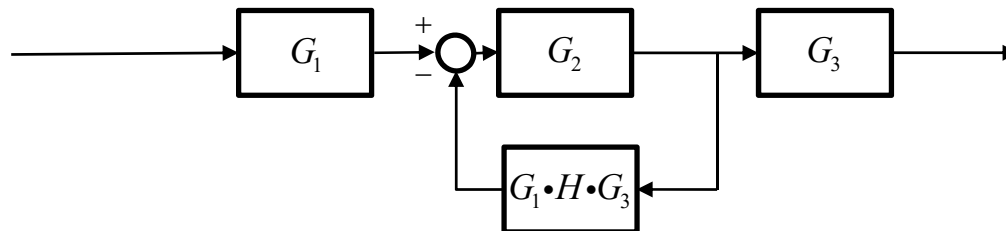
Pickoff



$$T = \frac{G_1 G_2 G_3}{1 + G_1 G_2 G_3 H}$$



$$T = \frac{G_1 G_2}{1 + G_1 G_2 \cdot H G_3} \cdot G_3$$



$$T = G_1 \cdot \frac{G_2}{1 + G_2 \cdot H G_1 G_3} \cdot G_3$$

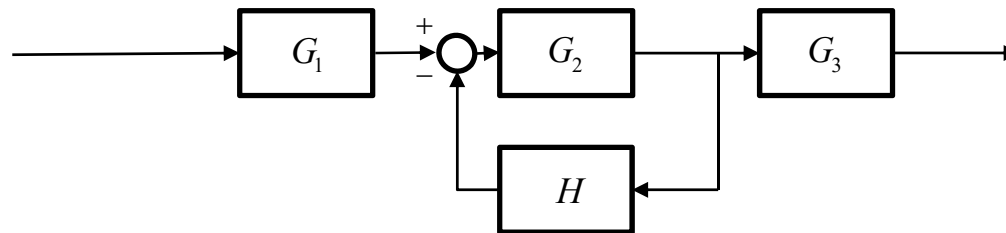


REDUCTION TECHNIQUES

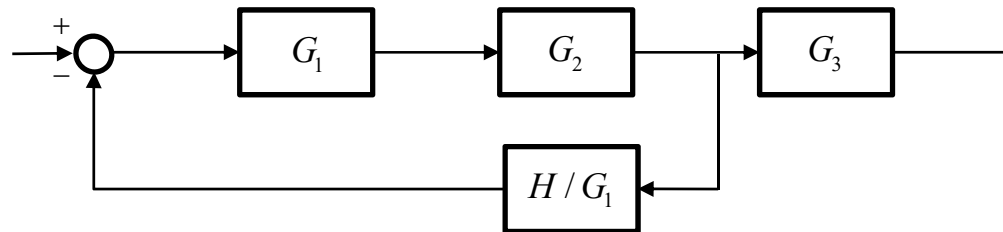


Summing

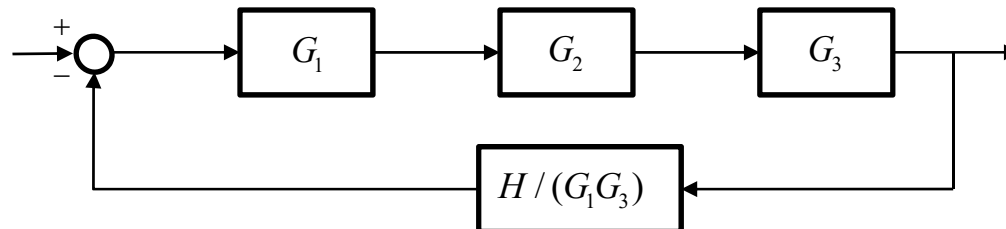
Pickoff



$$T = G_1 \cdot \frac{G_2}{1 + G_2 H} \cdot G_3$$



$$T = \frac{G_1 G_2}{1 + G_1 G_2 H / G_1} \cdot G_3$$



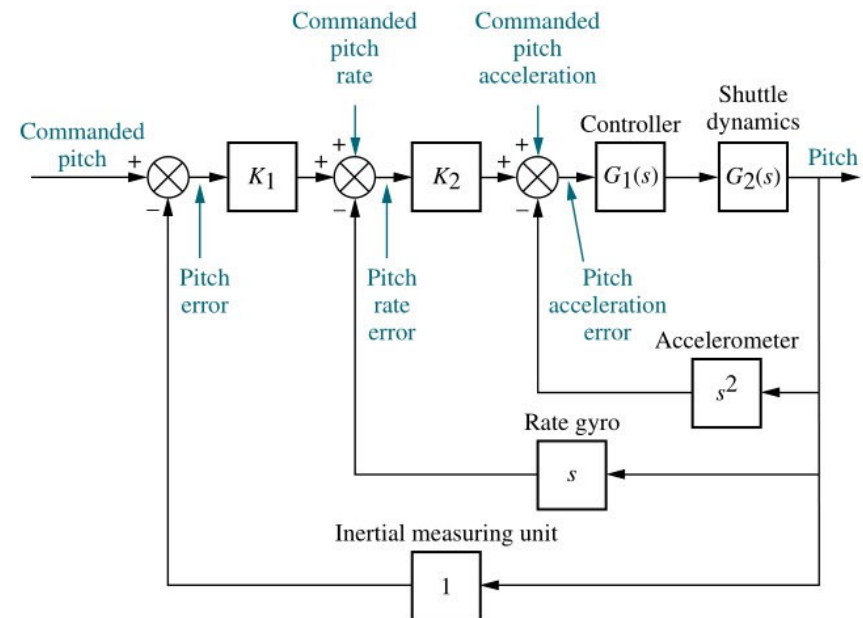
$$T = \frac{G_1 G_2 G_3}{1 + G_1 G_2 G_3 H / (G_1 G_3)}$$



SPACE SHUTTLE PITCH CONTROL

- Verify that the closed-loop transfer function from the commanded pitch input to actual pitch output is given by (assume all other inputs are zeros)

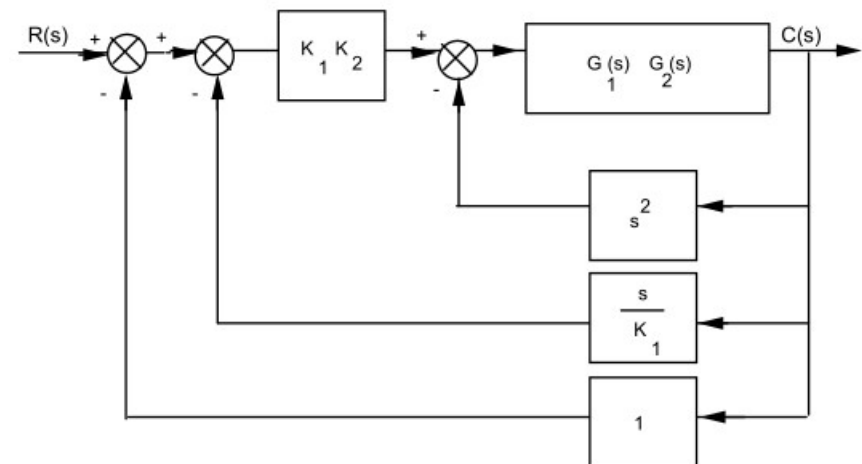
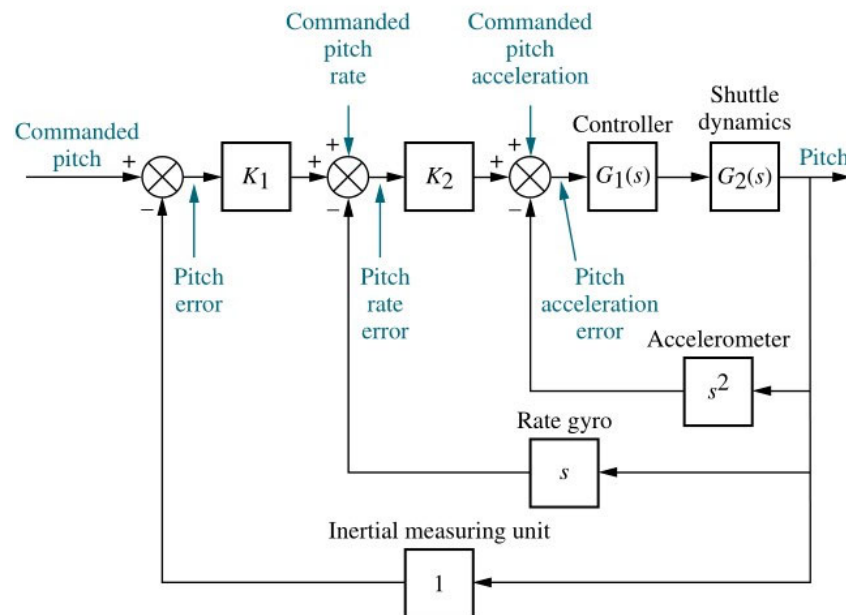
$$T(s) = \frac{K_1 K_2 G_1(s) G_2(s)}{1 + K_1 K_2 G_1(s) G_2(s) \left[1 + \frac{s}{K_1} + \frac{s^2}{K_1 K_2} \right]}$$





SPACE SHUTTLE PITCH CONTROL

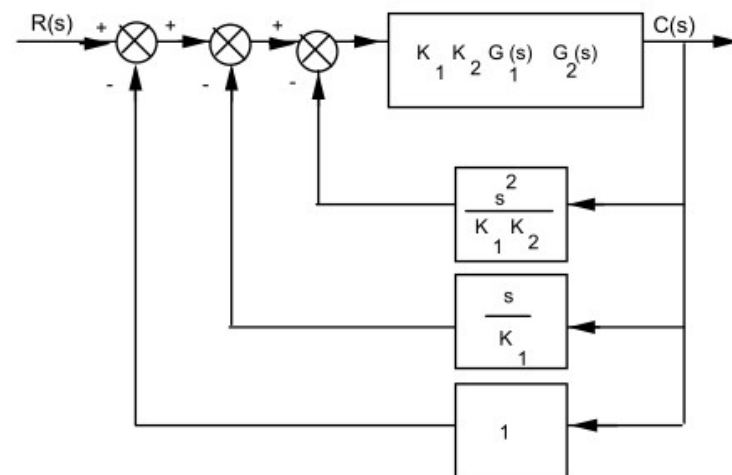
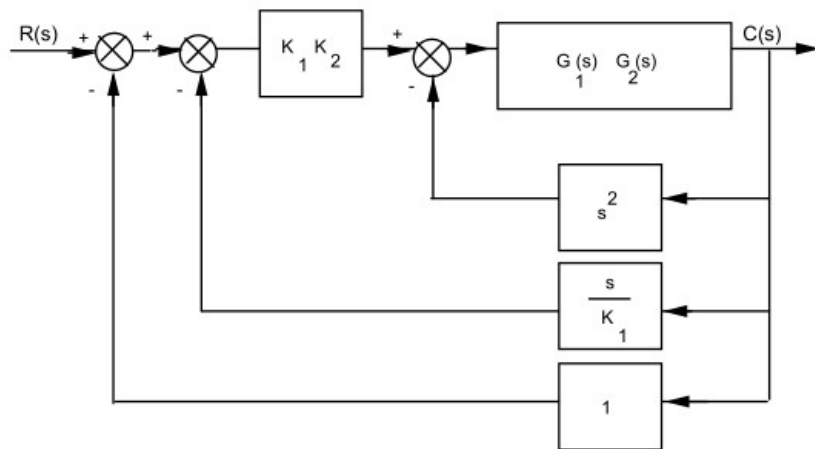
- a) combine G_1 and G_2 . Then push K_1 to the right past the summing junction





SPACE SHUTTLE PITCH CONTROL

- b) Push $K_1 K_2$ to the right past the summing junction

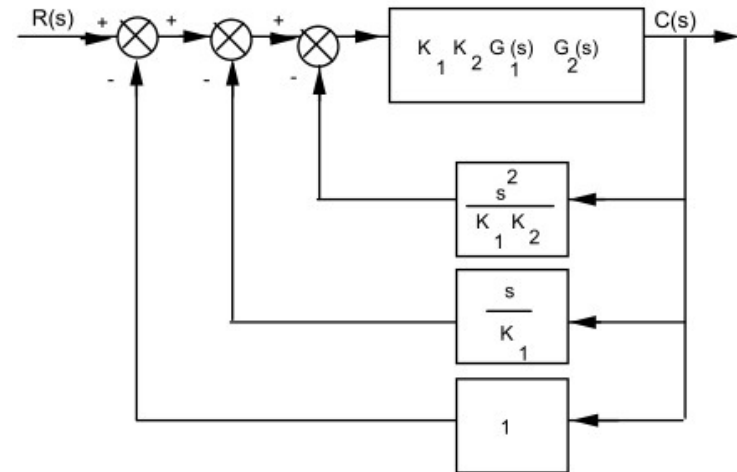




SPACE SHUTTLE PITCH CONTROL

- c) Write the transfer function

$$T(s) = \frac{K_1 K_2 G_1(s) G_2(s)}{1 + K_1 K_2 G_1(s) G_2(s) \left[1 + \frac{s}{K_1} + \frac{s^2}{K_1 K_2} \right]}$$

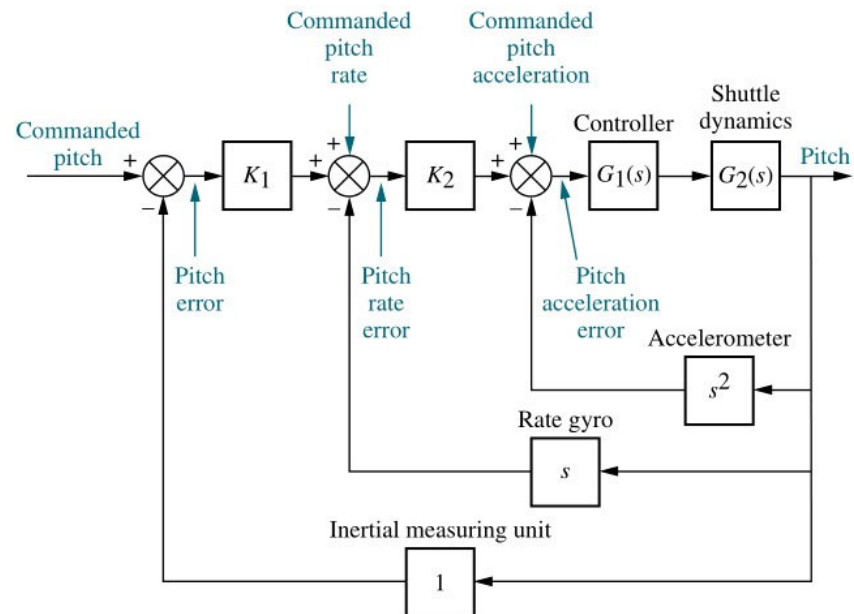




SPACE SHUTTLE PITCH CONTROL

- Verify that the closed-loop transfer function from the **commanded pitch rate input** to **actual pitch rate** is given by (assume all other inputs are zeros)

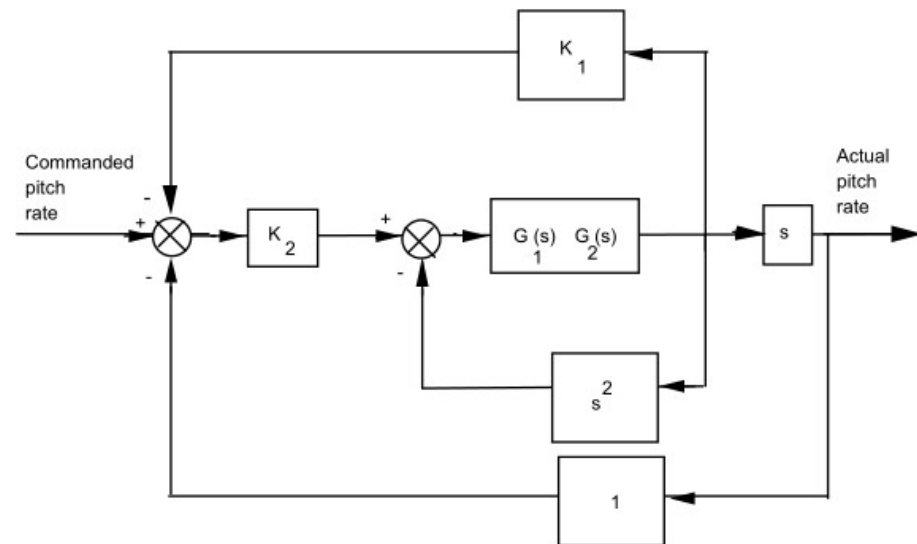
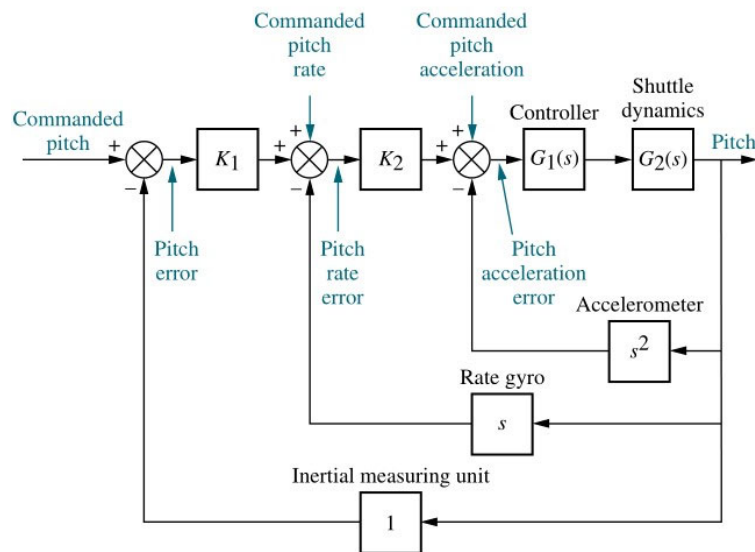
$$T(s) = \frac{K_2 s G_1(s) G_2(s)}{1 + G_1(s) G_2(s) [s^2 + K_2 s + K_1 K_2]}$$





SPACE SHUTTLE PITCH CONTROL

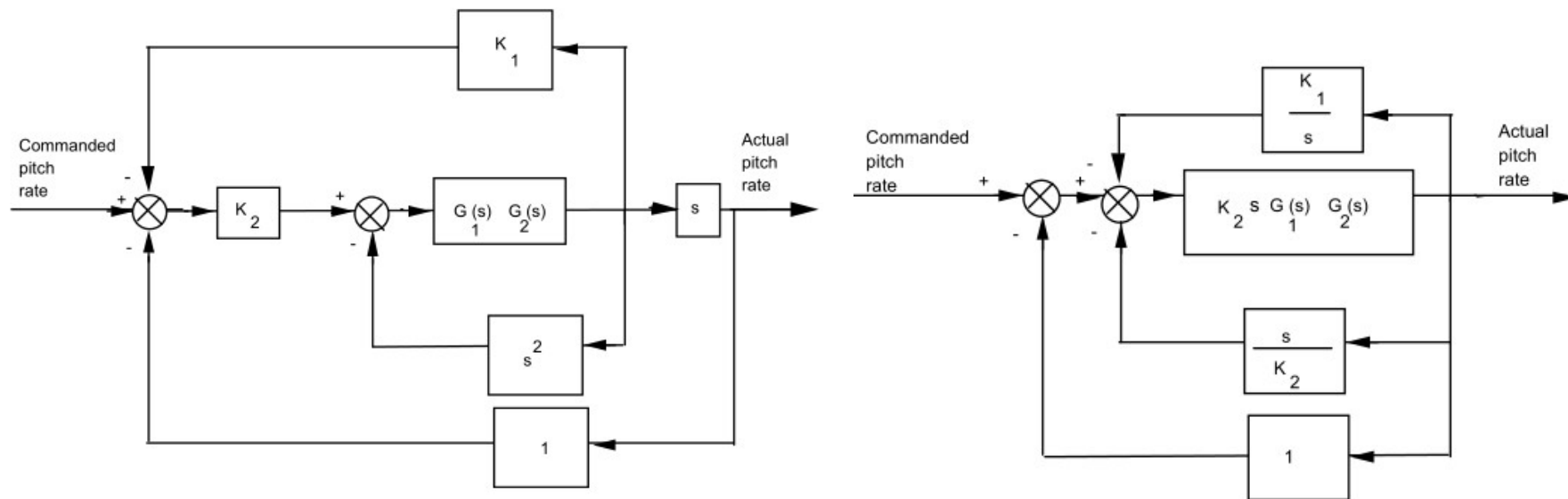
- Rearranging the block diagram to show commanded pitch rate as the input and actual pitch rate as the output:





SPACE SHUTTLE PITCH CONTROL

- Push K_2 to the right past the summing junction; and push s to the left past the pick-off point yields,

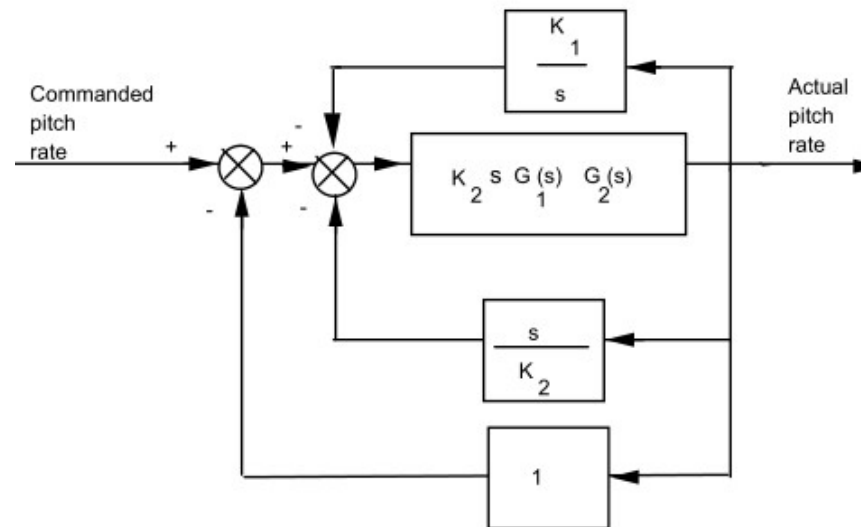




SPACE SHUTTLE PITCH CONTROL

- Then the closed-loop transfer function:

$$T(s) = \frac{K_2 s G_1(s) G_2(s)}{1 + K_2 s G_1(s) G_2(s) \left(1 + \frac{s}{K_2} + \frac{K_1}{s} \right)} = \frac{K_2 s G_1(s) G_2(s)}{1 + G_1(s) G_2(s) (s^2 + K_2 s + K_1 K_2)}$$

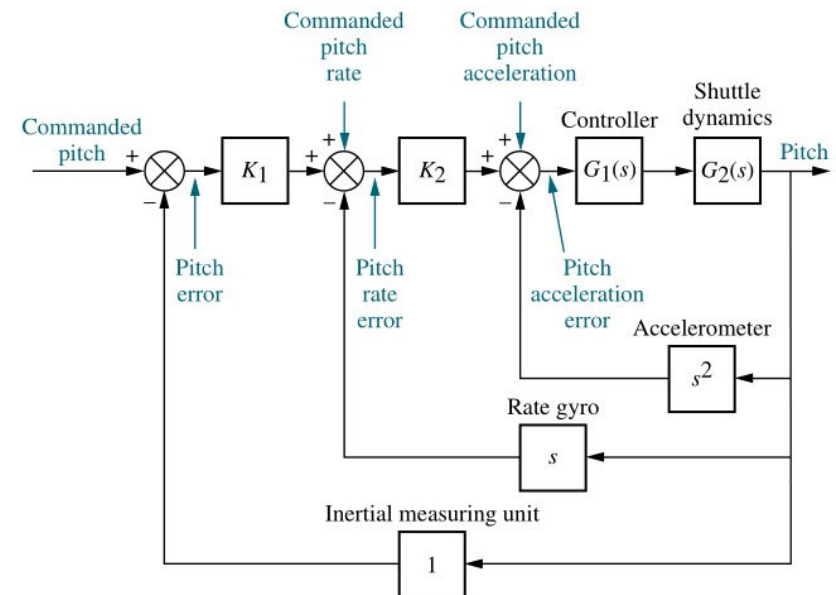




SPACE SHUTTLE PITCH CONTROL

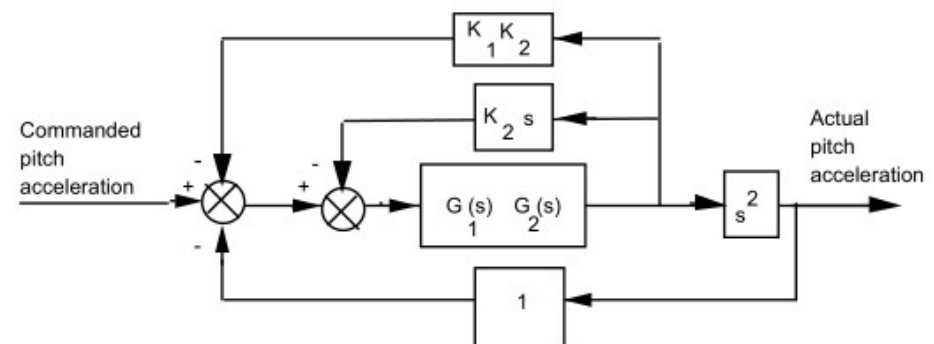
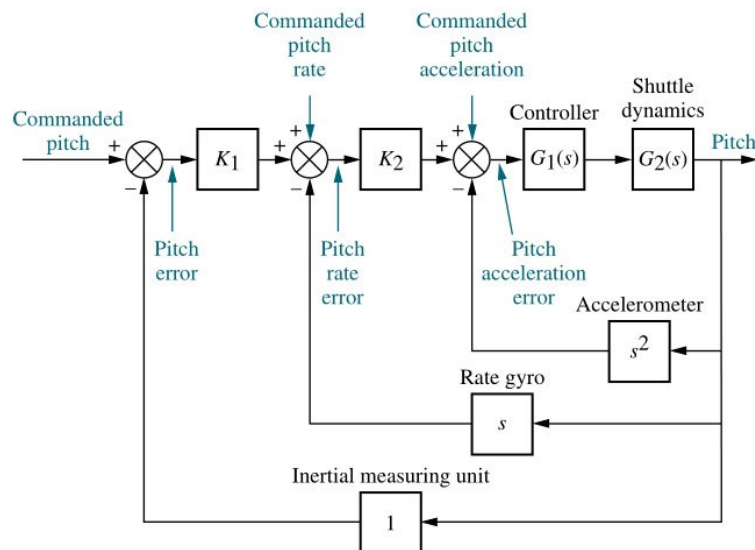
- Verify that the closed-loop transfer function from the *commanded pitch acceleration input* to *actual pitch acceleration* is given by (assume all other inputs are zeros)

$$T(s) = \frac{s^2 G_1(s) G_2(s)}{1 + G_1(s) G_2(s) [s^2 + K_2 s + K_1 K_2]}$$



SPACE SHUTTLE PITCH CONTROL

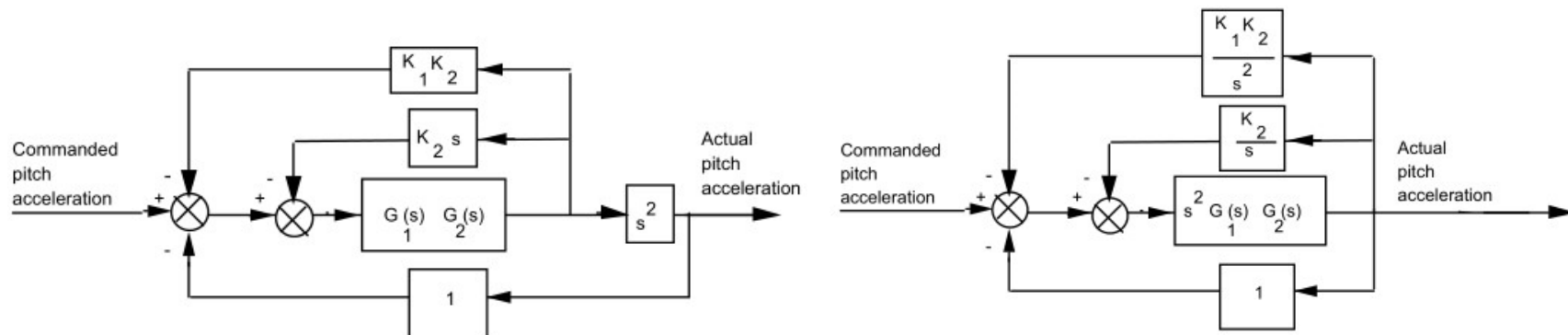
- Rearranging the block diagram to show commanded pitch acceleration as the input and actual pitch acceleration as the output:





SPACE SHUTTLE PITCH CONTROL

- Push s^2 to the left past the pick-off point yields,

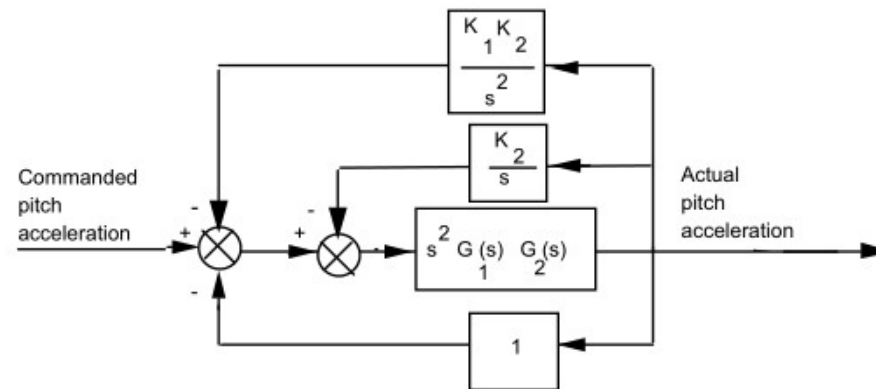




SPACE SHUTTLE PITCH CONTROL

- Then the closed-loop transfer function:

$$T(s) = \frac{s^2 G_1(s) G_2(s)}{1 + s^2 G_1(s) G_2(s) \left(1 + \frac{K_1 K_2}{s^2} + \frac{K_2}{s} \right)} = \frac{s^2 G_1(s) G_2(s)}{1 + G_1(s) G_2(s) (s^2 + K_2 s + K_1 K_2)}$$



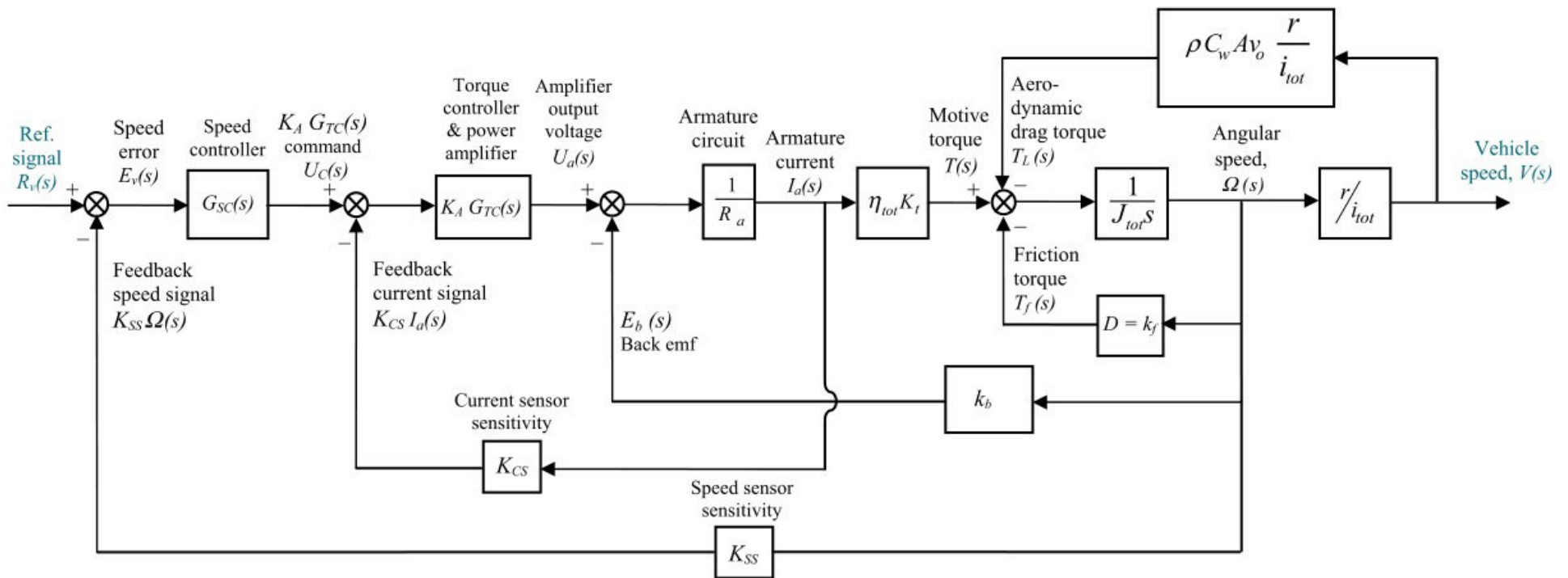


HYBRID ELECTRICAL VEHICLE

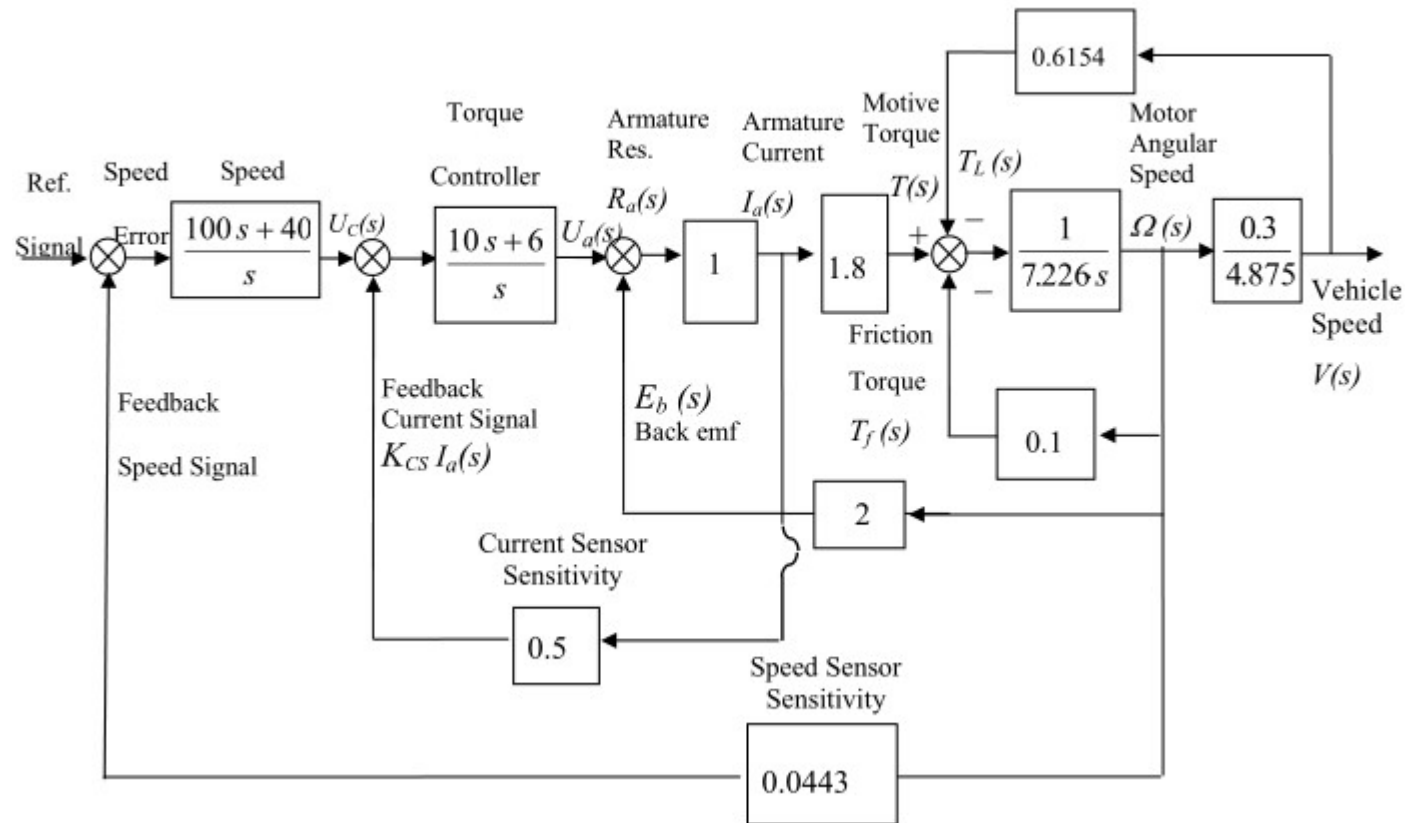
- The figure below shows the block diagram for a Hybrid Electrical Vehicle driven by a DC motor. Let the speed controller $G_{sc} = 100 + \frac{40}{s}$ the torque controller and the power amp $K_A G_{TC} = 10 + \frac{6}{s}$, the current sensor sensitivity $K_{SS} = 0.0433$.

$$\frac{1}{R_a} = 1; \eta_{tot} K_t = 1.8; k_b = 2; D = k_f = 0.1; \frac{1}{J_{tot}} = \frac{1}{7.226}; \frac{r}{i_{tot}} = 0.0615; \rho C_w A v_0 \frac{r}{i_{tot}} = 0.6154$$

HYBRID ELECTRICAL VEHICLE



HYBRID ELECTRICAL VEHICLE

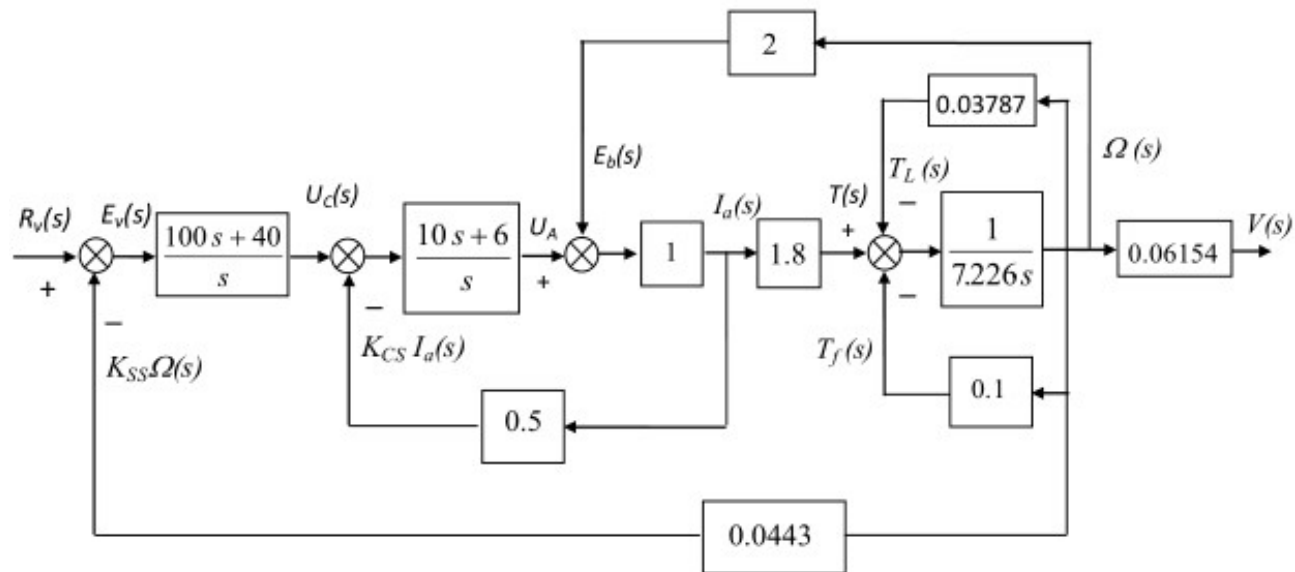




HYBRID ELECTRICAL VEHICLE

- Moving the last pick-off point to the left past the $\frac{r}{i_{tot}} = \frac{0.3}{4.875} = 0.06154$

$$0.06154 \times \frac{0.3}{4.875} = 0.03787$$

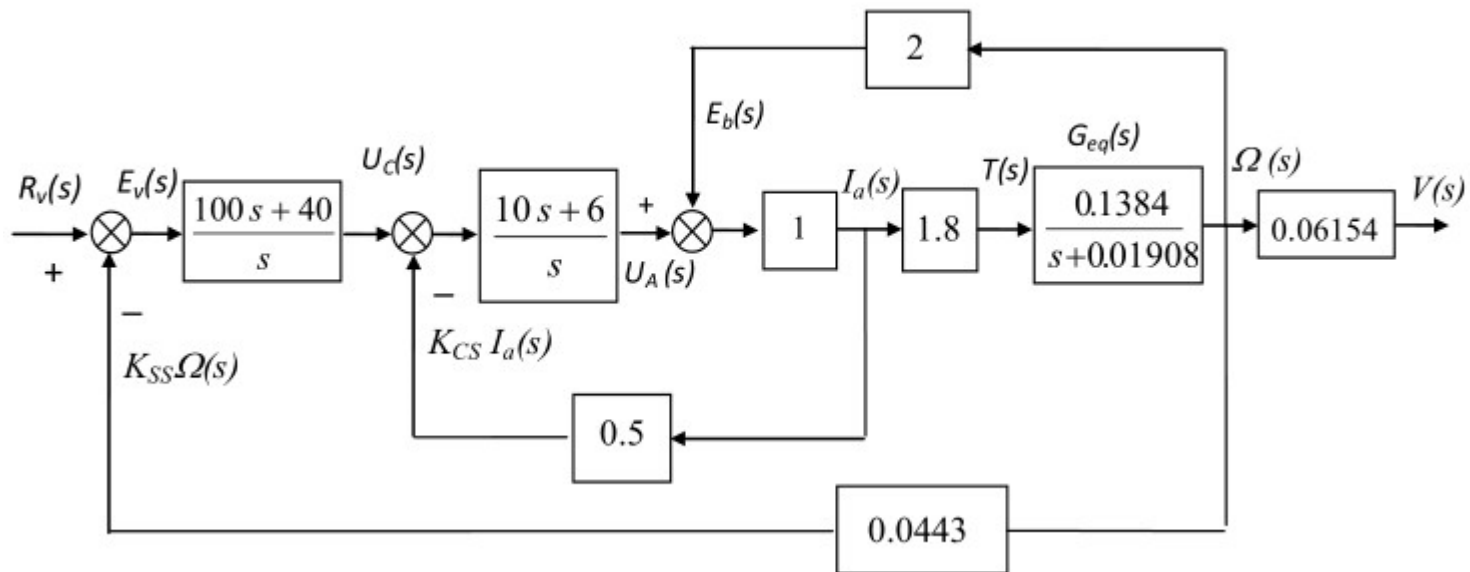




HYBRID ELECTRICAL VEHICLE

Transfer function

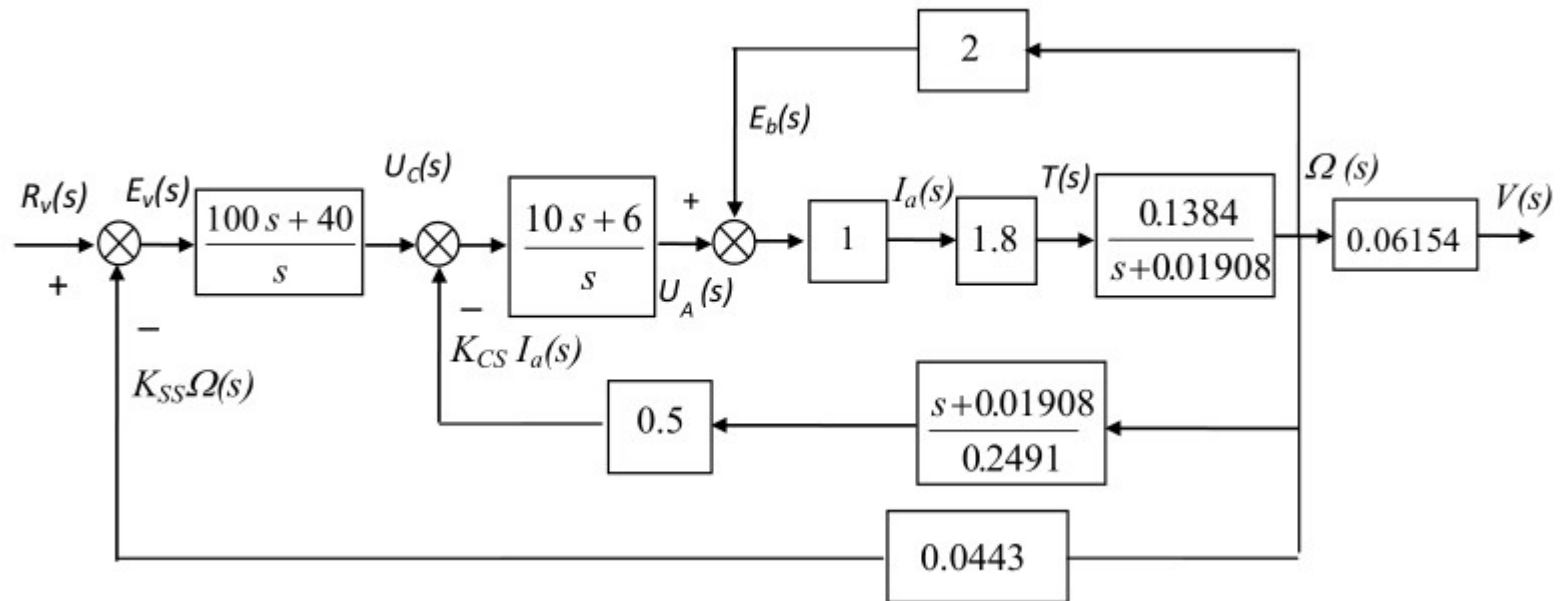
$$G_{eq}(s) = \frac{\Omega(s)}{T(s)} = \frac{1}{1 + \frac{7.226s}{0.13787}} = \frac{0.1384}{s + 0.01908}$$





HYBRID ELECTRICAL VEHICLE

$$\frac{s + 0.01908}{1.8 \times 0.2491} = \frac{s + 0.01908}{0.2491}$$

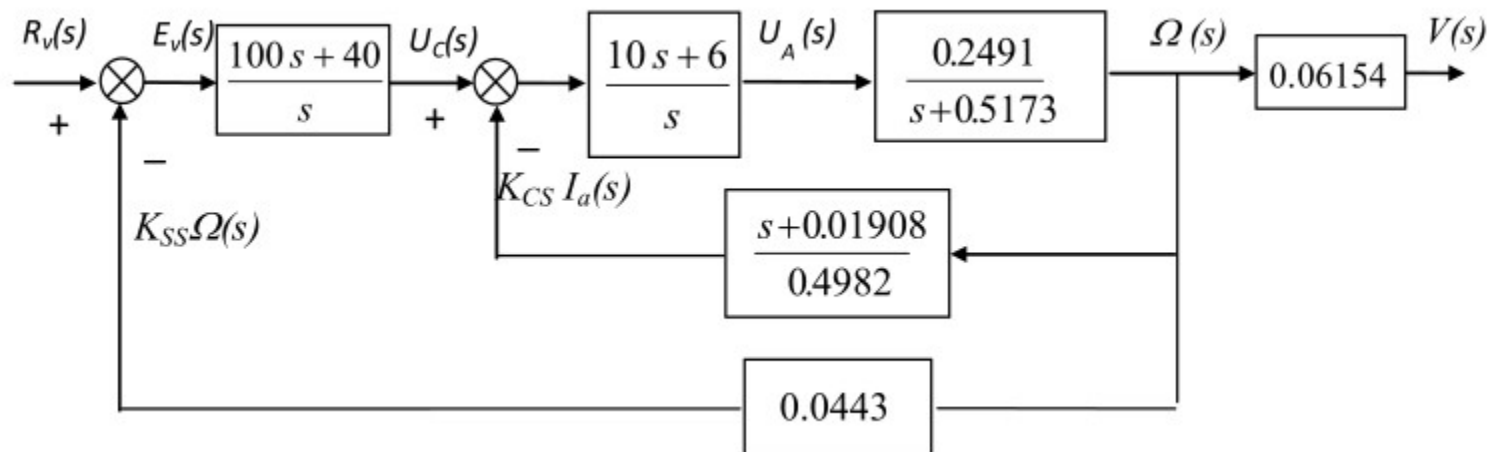




HYBRID ELECTRICAL VEHICLE

- Transfer function for torque controller

$$\frac{\Omega(s)}{U_A(s)} = \frac{\frac{0.2491}{s+0.01908}}{1 + \frac{0.2491}{s+0.01908} \times 2} = \frac{0.2491}{s+0.5173}$$

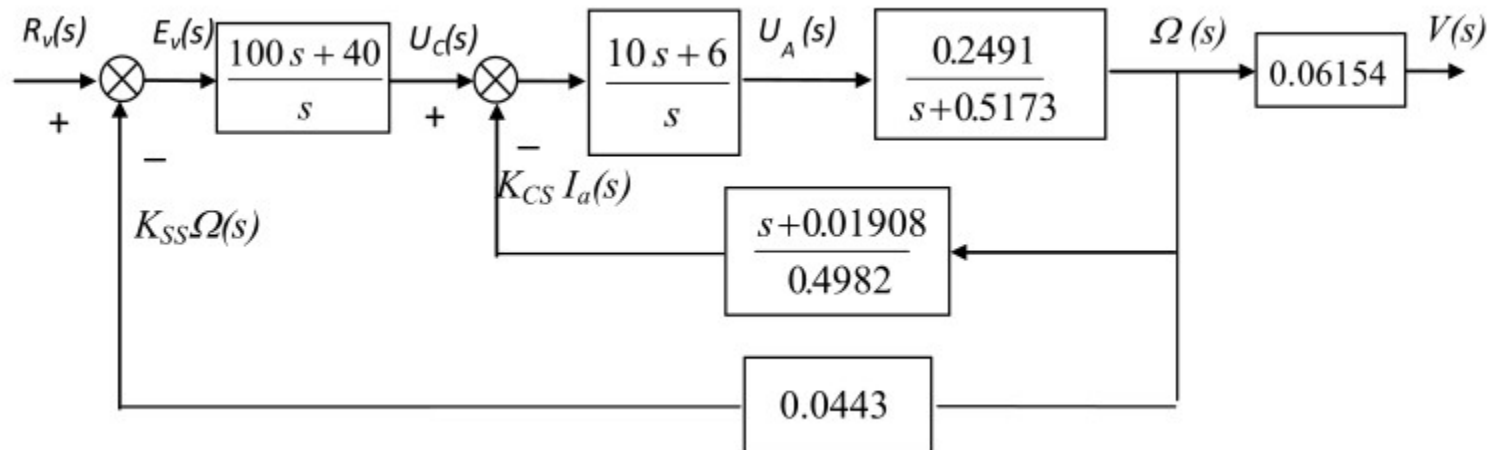




HYBRID ELECTRICAL VEHICLE

■ Thus

$$\frac{\Omega(s)}{U_c(s)} = \frac{\left(\frac{10s+6}{s}\right)\left(\frac{0.2491}{s+0.5173}\right)}{1 + \left(\frac{10s+6}{s}\right)\left(\frac{0.2491}{s+0.5173}\right)\left(\frac{s+0.01908}{0.4982}\right)} = \frac{0.2491(10s+6)}{s(s+0.5173) + 0.5(10s+6)(s+0.01908)}$$





HYBRID ELECTRICAL VEHICLE

■ Thus

$$\frac{\Omega(s)}{R_v(s)} = \frac{\left(\frac{100s+40}{s}\right) \left(\frac{0.2491(10s+6)}{s(s+0.5173)+0.5(10s+6)(s+0.01908)} \right)}{1+0.0443 \left(\frac{100s+40}{s}\right) \left(\frac{0.2491(10s+6)}{s(s+0.5173)+0.5(10s+6)(s+0.01908)} \right)}$$

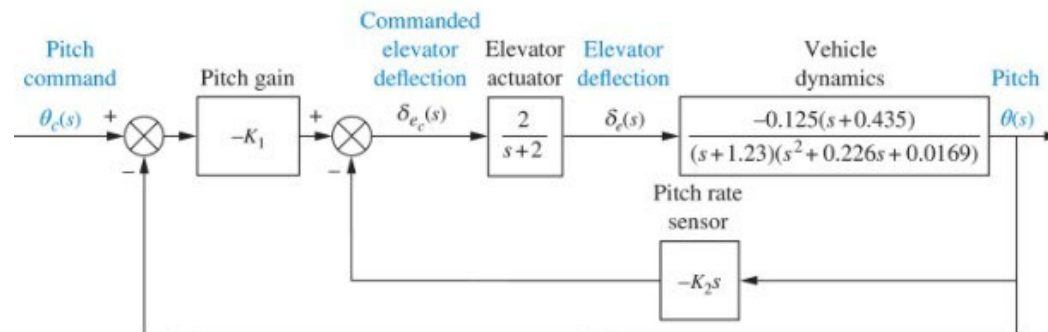
$$\frac{\Omega(s)}{R_v(s)} = \frac{2491(s+0.4)(s+0.6)}{6s^3 + 14.644s^2 + 11.09s + 2.65}$$

$$\frac{V(s)}{R_v(s)} = 0.06154 \frac{\Omega(s)}{R_v(s)} = \frac{15.33(s+0.4)(s+0.6)}{6s^3 + 14.644s^2 + 11.09s + 2.65}$$



UFSS

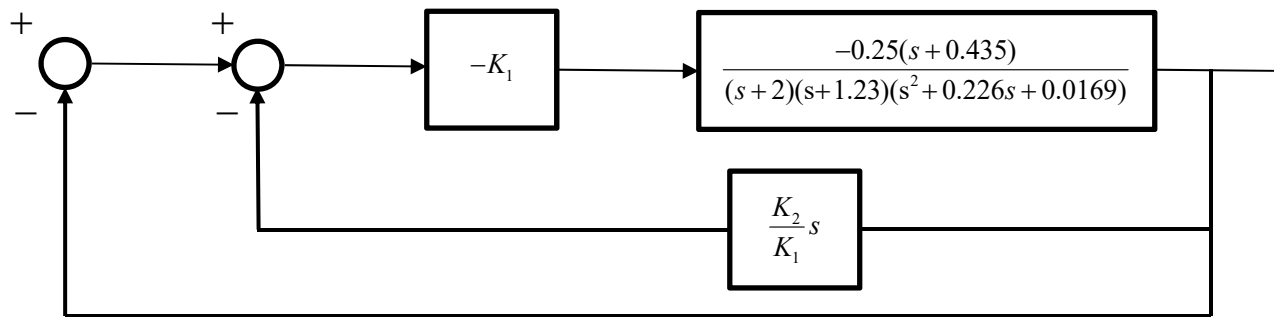
- Transfer function $\theta(s)/\theta_c(s)$ using block diagram reduction rules.





UFSS

- Transfer function $\theta(s)/\theta_c$ using block diagram reduction rules.

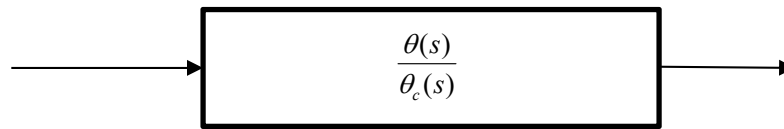


$$\frac{\theta(s)}{\theta_c(s)} = \frac{\frac{0.25K_1(s+0.435)}{(s+2)(s+1.23)(s^2+0.226s+0.169)}}{1 + \frac{0.25K_1(s+0.435)}{(s+2)(s+1.23)(s^2+0.226s+0.169)} \left(\frac{K_2}{K_1}s + 1 \right)}$$



UFSS

- Transfer function $\theta(s)/\theta_c(s)$ using block diagram reduction rules.



$$\frac{\theta(s)}{\theta_c(s)} = \frac{0.25K_1(s + 0.435)}{(s + 2)(s + 1.23)(s^2 + 0.226s + 0.169) + 0.25K_1(s + 0.435)(\frac{K_2}{K_1}s + 1)}$$

$$\frac{\theta(s)}{\theta_c(s)} = \frac{0.25K_1(s + 0.435)}{s^4 + 3.456s^3 + (3.359 + 0.25K_2)s^2 + (1.102 + 0.25K_1 + 0.109K_2)s + (0.416 + 0.109K_1)}$$