Studio 10: Root Locus Design-Gain Adjustment and PI and Lag Compensators

1. Gain Adjustment via Root Locus

For the unity feedback control system shown in Figure 1, a proportional control gain is used. The control gain K is multiplied by the error signal and is fed to the plant. Changing the value of the control gain changes the closed-loop poles of the system and hence changes the transient response characteristics and stability.

Recall that the closed-loop transfer function is given by

$$T(s) = \frac{C(s)}{R(s)} = \frac{KG(s)}{1 + KG(s)}$$

The characteristic equation is given by

$$1 + KG(s) = 0.$$

Values of s that satisfy the characteristic equation are poles of the closed-loop system. Changing the value of K changes the closed-loop poles and causes them to move along the root locus. If closed-loop poles can be found on the root locus that meet the time domain specifications, the control design problem is a simple one and only involves finding a control gain value that will place the closed-loop poles at the desired locations. One can verify that the time domain specifications are met by simulating the closed-loop system as was done in previous studios. If the desired closed-loop pole locations are not on the system root locus, a compensator is needed to modify the shape of the root locus in such a way that the desired closed-loop poles are on the modified root locus. Adding a compensator in effect adds zero(s) and/or pole(s) to the system and thus modifies the shape of the root locus. Choosing the location of the compensator zero(s) and/or pole(s) and gain carefully results in a closed-loop system that meets the desired specifications. This approach will be demonstrated in detail in this studio.

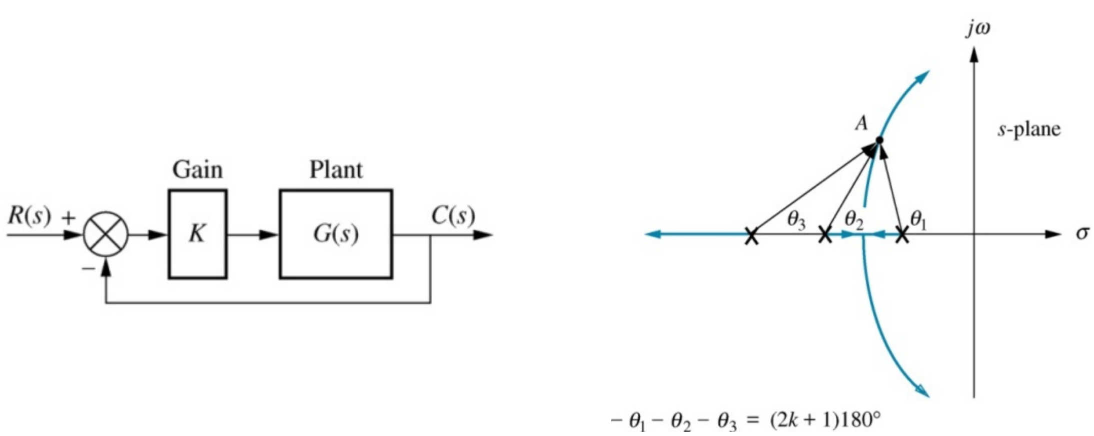
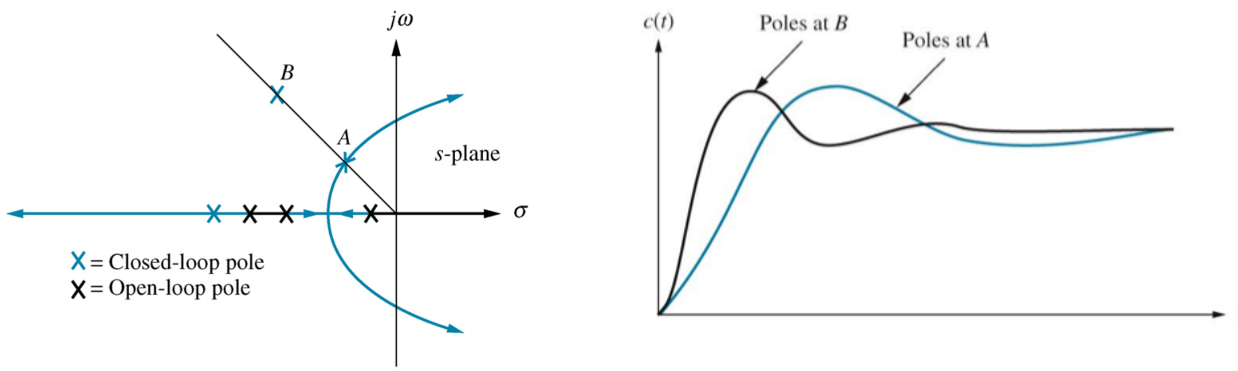


Figure 1 A closed-loop control system with proportional control gain and a typical root locus plot.

2. Root Locus Design: Compensators to Reduce Steady-State Error

In case the system's response cannot be improved to meet the specification with simple gain adjustment (i.e., migrating on the root locus) or the steady-state error does not meet design requirements using a proportional gain only, the system needs to be "compensated" via adding poles and/or zeros to (i) increase the system type (PI control) or static error constant (lag compensation) in order to improve the steady-state errors, and/or (ii) re-shape the root locus so that the desired closed-loop poles are located on the re-shaped root locus.



(a) (b)

Figure 2 (a) An example root locus. Desired CL poles at B are not on RL. (b) placing CL poles at A does not meet T_s requirement.

2.1. Proportional-Integral (PI) Compensators

A PI compensator improves the steady-state response via increasing the system type, without distorting the shape of the original system's root locus.

A PI compensator has two components, a proportional component and an integral component and takes the form

$$G_c(s) = K_p + \frac{K_I}{s} = \frac{K_p s + K_I}{s} = \frac{K_p\left(s + \frac{K_I}{K_p}\right)}{s}$$

A PI compensator adds a pole at the origin and a zero close to the origin to prevent the additional pole from significantly distorting the shape of the original system's root locus. A block diagram of a unity feedback system with a PI compensator is shown in Figure 3.

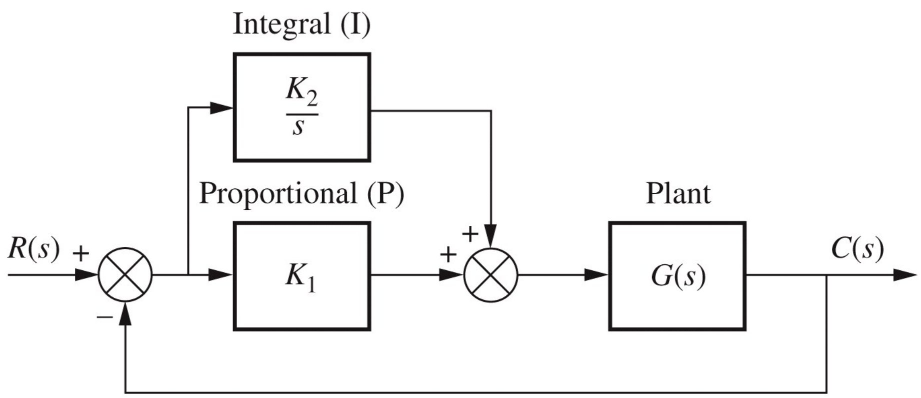


Figure 3 A closed-loop control system with a PI compensator.

2.2 Integral Compensators

An integral compensator adds a pole at the origin, thus increases the system type by 1. An integral compensator takes the form

$$G_c(s) = \frac{K}{s}$$

This type of compensation can be used to reduce the steady-state error. However, adding a pole to the system significantly changes the shape of the root locus, see Figure 4.

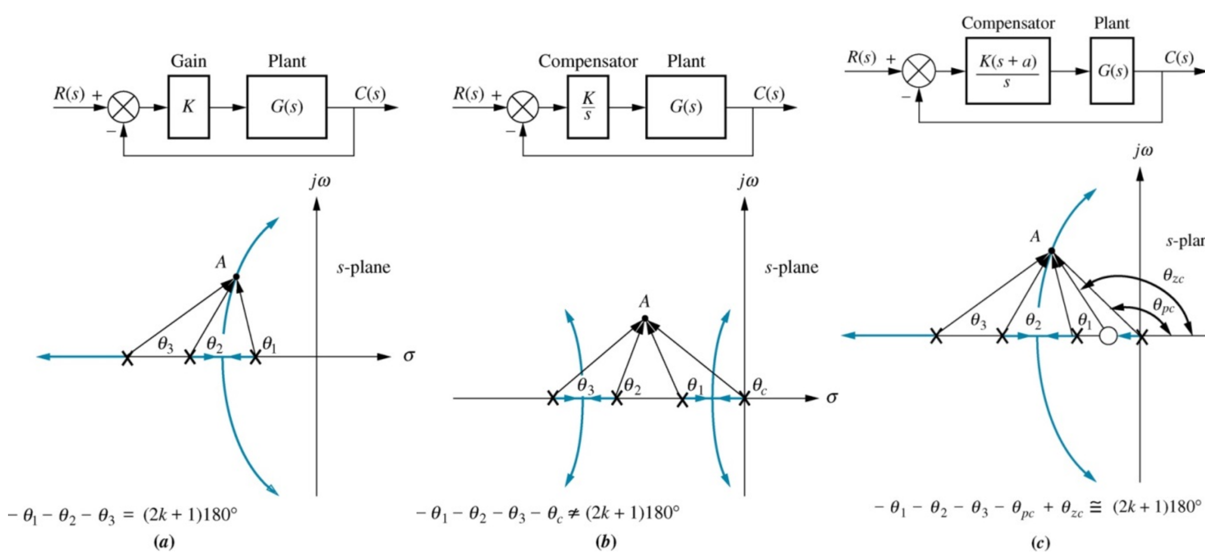


Figure 4 (a) Proportional gain, (b) Integral compensation; reduces SS error but modifies the shape of the RL, and (c) PI compensation; can be used to reduce SS error without modifying the shape of the root locus.

2.3 Lag Compensators

Improve the steady-state response via increasing the static error constants, without distorting the shape of the original system's root locus. Additional poles are put at a location close to the origin, and additional zeros are put close to (but farther from the origin) the additional poles to prevent the additional poles from distorting the shape of the root locus (see Figure 5 & Figure 6). This type of compensation is used when transient response system characteristics are met with proportional gain but the steady state error is not satisfactory.

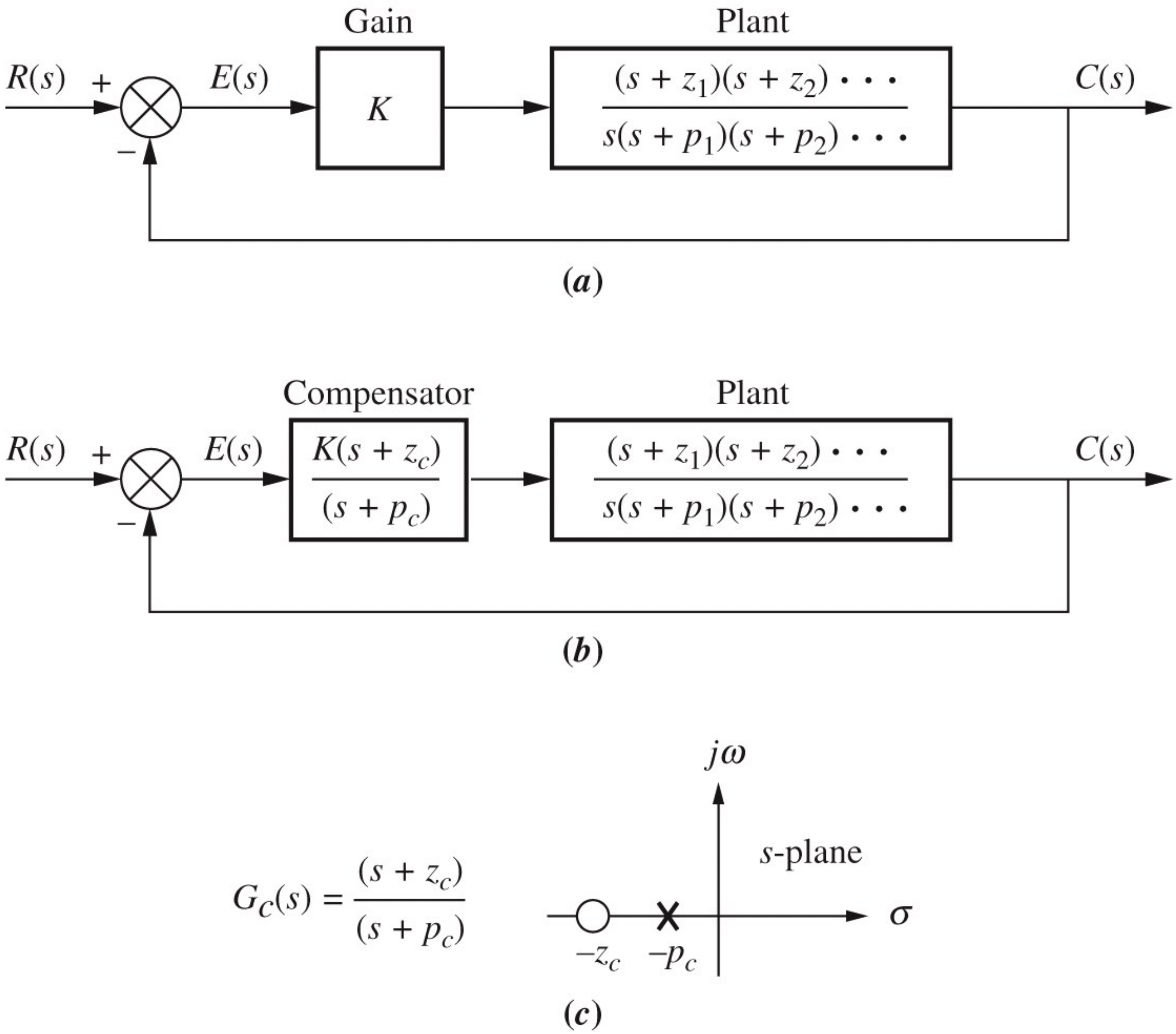


Figure 5 (a) Uncompensated system, (b) Lag compensated system, and (c) Lag compensator pole/zero configuration.

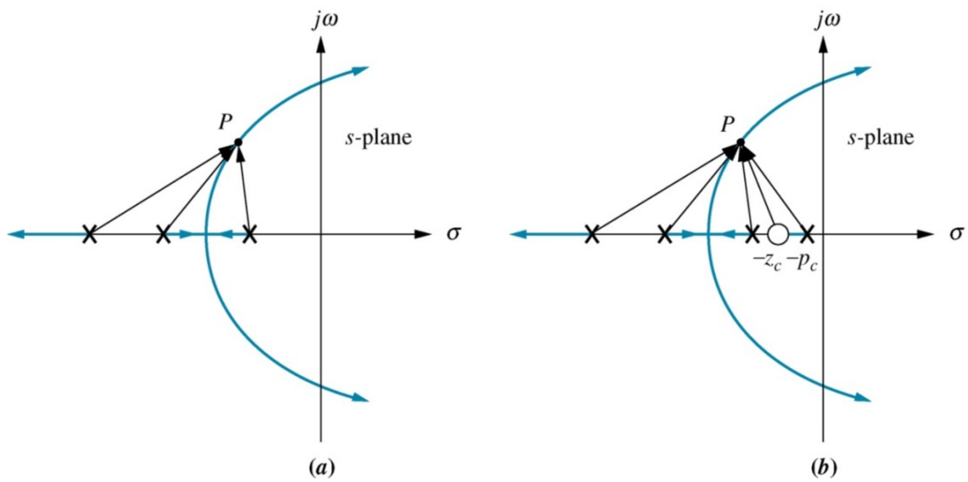


Figure 6 (a) An example root locus with proportional gain, (b) RL for the same system with lag compensation (the RL is approximately the same but the SS error can be reduced).

Example: Liquid Level Control

Figure 7 shows a two-tank system. The liquid inflow to the upper tank can be controlled using a valve and is represented by F_0. The upper tank's outflow equals the lower tank's inflow and is represented by F_1. The outflow of the lower tank is F_2. The control design objective is to control the liquid level in the lower tank $y(t)$.

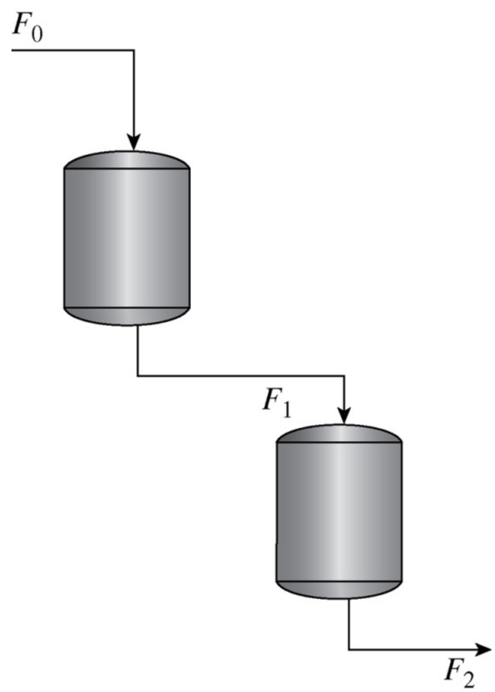


Figure 7 A two-tank system.

The open-loop transfer function from the flow input to the liquid level is given by:

$$G(s) \coloneqq \frac{Y(s)}{F_0(s)} = \frac{a_2 a_3}{s^2 + (a_1 + a_4)s + a_1 a_4}$$

where a_1=0.04, a_2=0.0187, a_3=1, a_4=0.227, and Y(s) is the liquid level at the lower tank. The system is in a unity-feedback configuration. The liquid level will be measured and compared to a set point. The resulting error will be fed to a controller, which in turn will open or close the valve feeding the upper tank.

(i) Consider the system in a unity feedback configuration. Plot the root locus and determine the steady state error.
(ii) Design a lag compensator to yield a 10% steady-state error to step inputs.
(iii) Plot the root locus of the lag-compensated system.
(iv) Plot the step response for each of the system in (i) and (ii) on the same plot.

Exercise

Each student shall complete the exercise below and get their work checked off by the studio instructor/TA.

For remote students and students who do not finish within the studio session, compile your answers and outputs of Exercises 1 along with the MATLAB code you used to answer the questions and screenshots of MATLAB outputs in a word file and name it LastNameFirstNameStudio10.docx and upload your file to Canvas under the Studio10 link.

Exercise 1. Consider the unity feedback system with

$$G(s) = \frac{K}{(s+1)(s+3)(s+10)}$$

Assume for the remainder of this problem that the system operates with a damping factor of 0.4.

(i) Plot the root locus for the system as the gain K is varied.

(ii) Determine the value of K that will yield a damping factor of 0.4 and determine the corresponding percent overshoot.

(iii) Determine the steady state error due to a step input for the value of K you found in (ii).

(iv) Design a PI compensator to drive the step response error to zero while maintaining operation with damping ratio of 0.4 for the following two cases
   a) Compensator zero at $s = -0.1$
   b) Compensator zero at $s = -0.7$

(v) Compare the specifications of the uncompensated system and each one of the compensated systems (settling time, percent overshoot, steady state error).

(vi) Simulate each one of the systems using MATLAB and plot the step response for the uncompensated system and the two compensated systems.

(vii) Design a Lag compensator that will reduce the steady-state error in (iii) by at least a factor of 10 and plot the root locus of the lag-compensated system.

(viii) Plot the step response for the uncompensated system and the lag-compensated systems on the same plot.