



ENME 462 STUDIO 5

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OUTLINES

- ❖ Block Diagrams Algebra
- Space Shuttle pitch control
- Hybrid electrical vehicle
- UFSS pitch control

CLOSED-LOOP TRANSFER FUNCTION

From the block diagram

$$C(s) = G(s)E(s)$$

$$\begin{aligned} E(s) &= R(s) - C(s)H(s) \\ &= R(s) - G(s)H(s)E(s) \end{aligned}$$

Therefore

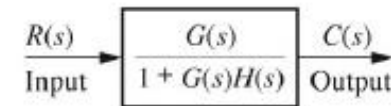
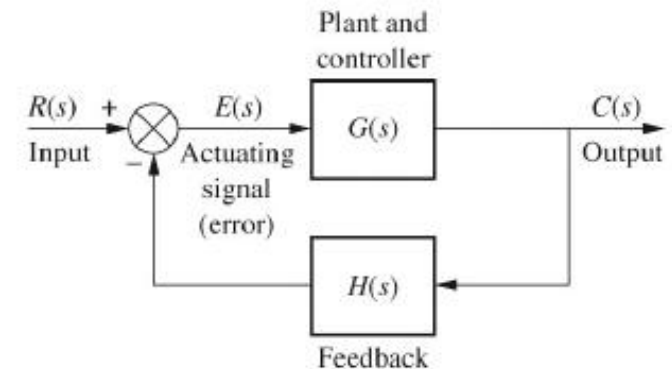
$$E(s) = \frac{1}{1 + G(s)H(s)} R(s)$$

And

$$C(s) = \frac{G}{1 + G(s)H(s)} R(s)$$

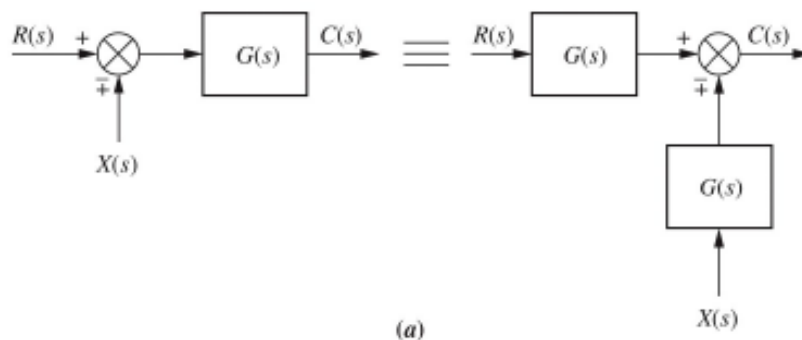
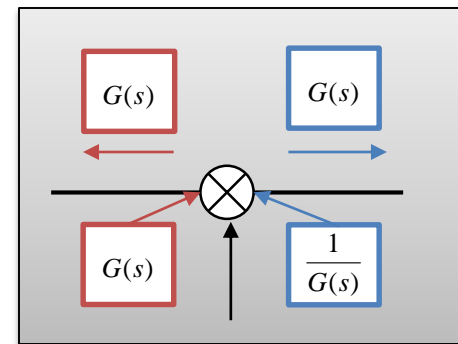
■ Transfer function

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$



REDUCTION TECHNIQUES

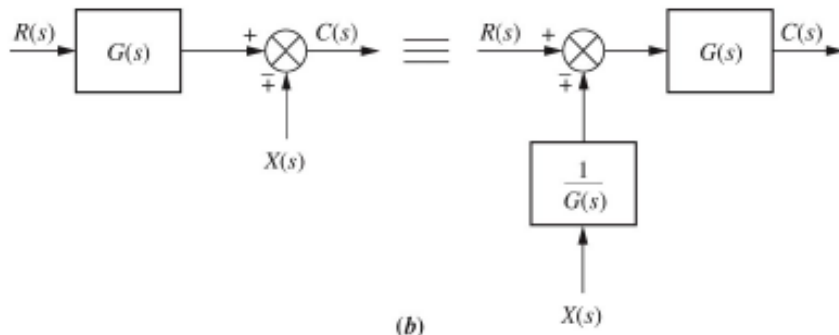
Moving a block before or after a summing junction



(a)

$$C(s) = R(s)G(s) \mp X(s)G(s)$$

$$C(s) = R(s)G(s) \mp X(s)G(s)$$



(b)

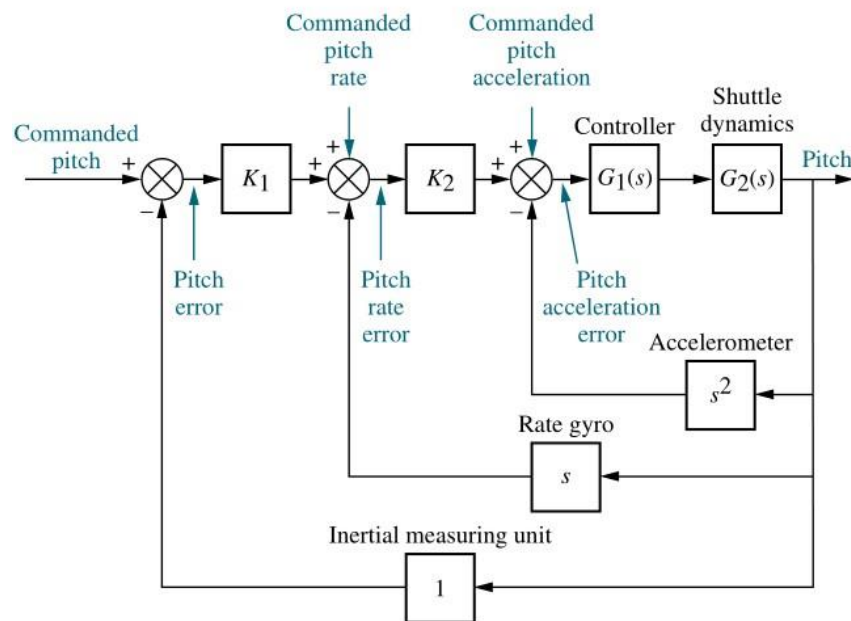
$$C(s) = R(s)G(s) \mp X(s)$$

$$C(s) = R(s)G(s) \mp X(s) \frac{1}{G(s)} G(s)$$

SPACE SHUTTLE PITCH CONTROL

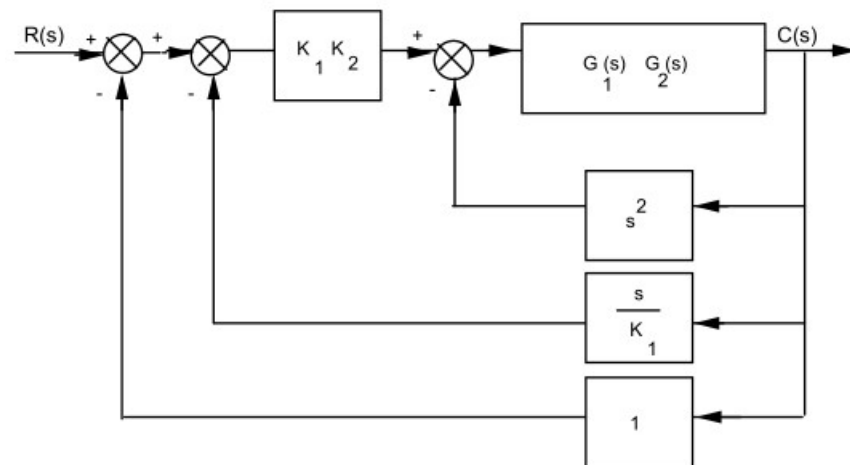
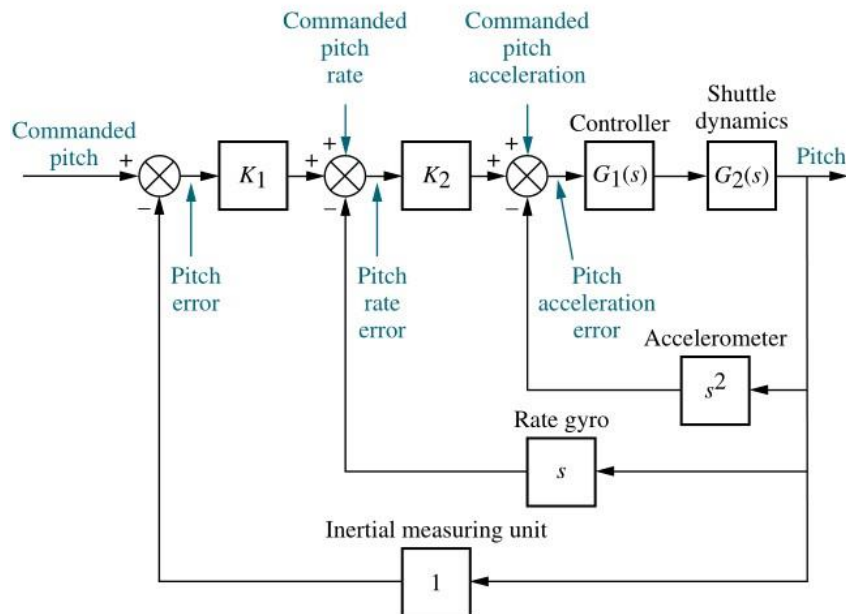
- Verify that the closed-loop transfer function from the commanded pitch input to actual pitch output is given by (assume all other inputs are zeros)

$$T(s) = \frac{K_1 K_2 G_1(s) G_2(s)}{1 + K_1 K_2 G_1(s) G_2(s) \left[1 + \frac{s}{K_1} + \frac{s^2}{K_1 K_2} \right]}$$



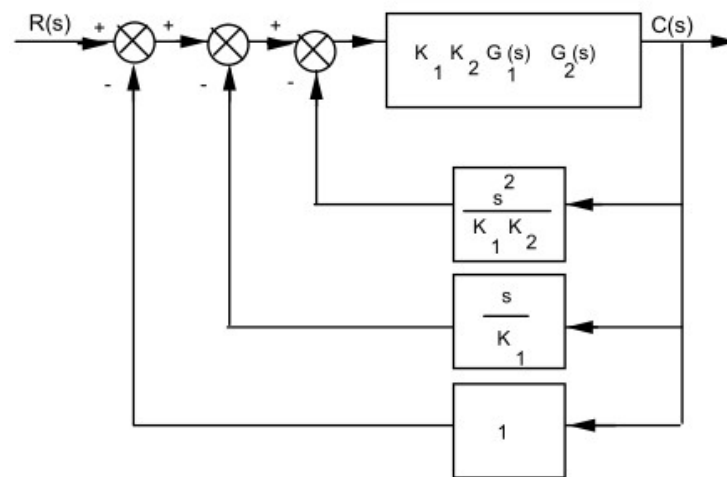
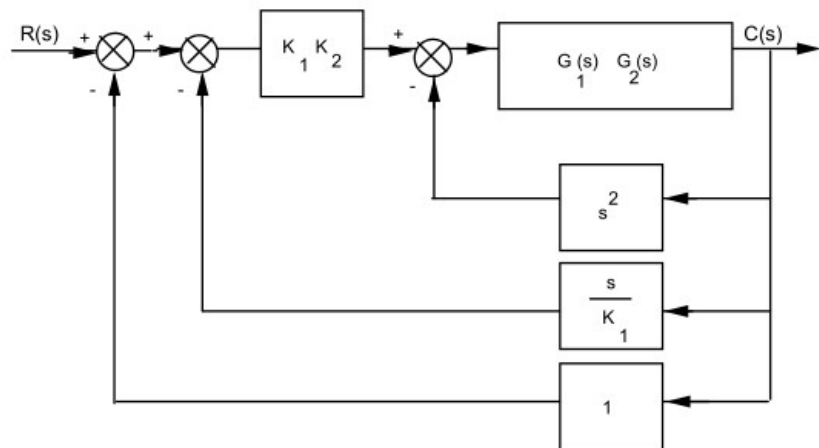
SPACE SHUTTLE PITCH CONTROL

- a) combine G_1 and G_2 . Then push K_1 to the right past the summing junction



SPACE SHUTTLE PITCH CONTROL

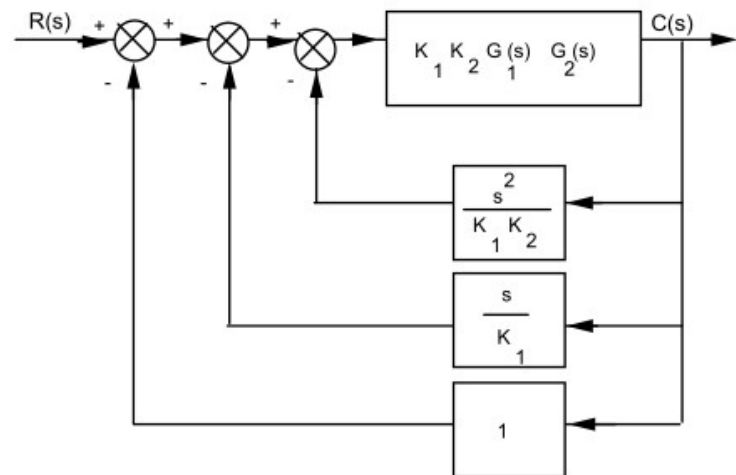
- b) Push $K_1 K_2$ to the right past the summing junction



SPACE SHUTTLE PITCH CONTROL

- c) Write the transfer function

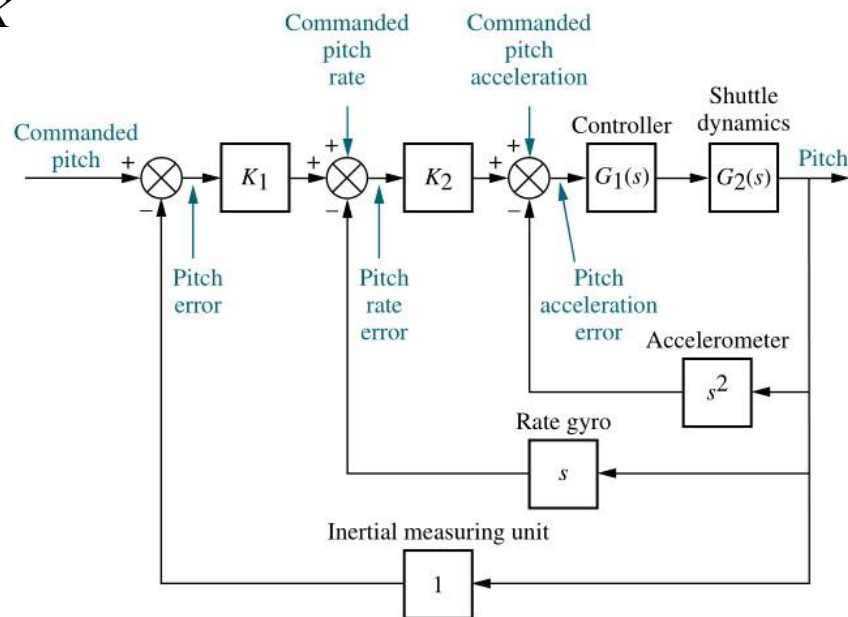
$$T(s) = \frac{K_1 K_2 G_1(s) G_2(s)}{1 + K_1 K_2 G_1(s) G_2(s) \left[1 + \frac{s}{K_1} + \frac{s^2}{K_1 K_2} \right]}$$



SPACE SHUTTLE PITCH CONTROL

- Verify that the closed-loop transfer function from the **commanded pitch rate input** to **actual pitch rate** is given by (assume all other inputs are zeros)

$$T(s) = \frac{K_2 s G_1(s) G_2(s)}{1 + G_1(s) G_2(s) [s^2 + K_2 s + K_1 K]}$$

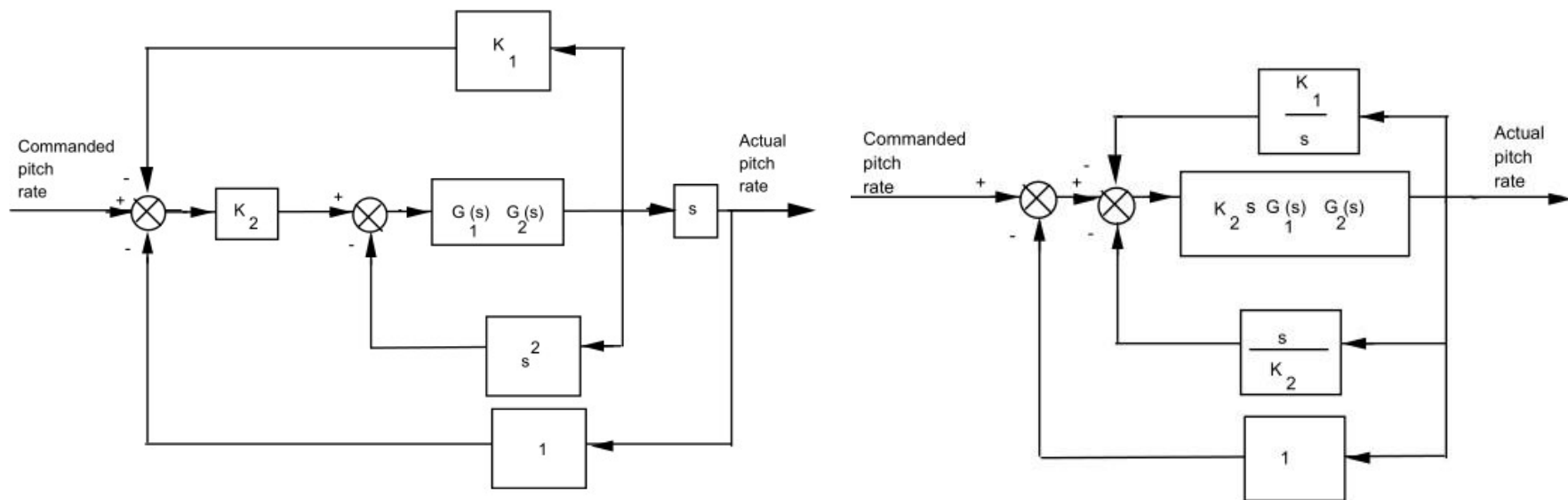




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- The figure contains two block diagrams. The left diagram is a detailed control system for shuttle pitch. It starts with a 'Commanded pitch' input, which is compared with the 'Actual pitch' (feedback from an 'Inertial measuring unit' with gain 1) at a summing junction. The resulting 'Pitch error' is fed into a gain block K_1 . The output of K_1 is compared with the 'Actual pitch rate' (feedback from a 'Rate gyro' with gain s) at a second summing junction. The output of this junction is fed into a gain block K_2 . The output of K_2 is compared with the 'Actual pitch acceleration' (feedback from an 'Accelerometer' with gain s^2) at a third summing junction. The output of this junction is the 'Pitch acceleration error', which is fed into a 'Controller' block $G_1(s)$. The output of $G_1(s)$ is fed into a 'Shuttle dynamics' block $G_2(s)$, which produces the 'Actual pitch' output. The right diagram is a simplified block diagram. It starts with a 'Commanded pitch rate' input, which is compared with the 'Actual pitch rate' (feedback from a block with gain 1) at a summing junction. The output of this junction is fed into a gain block K_2 . The output of K_2 is fed into a summing junction before a block containing $G_1(s)$ and $G_2(s)$ in series. The output of this block is fed into an integrator block s , which produces the 'Actual pitch rate' output. The 'Actual pitch rate' is also fed back to the first summing junction and through a block with gain s^2 to the second summing junction. Additionally, the 'Actual pitch rate' is fed back through a block with gain K_1 to the 'Commanded pitch rate' input.

SPACE SHUTTLE PITCH CONTROL

- Push K_2 to the right past the summing junction; and push s to the left past the pick-off point yields,

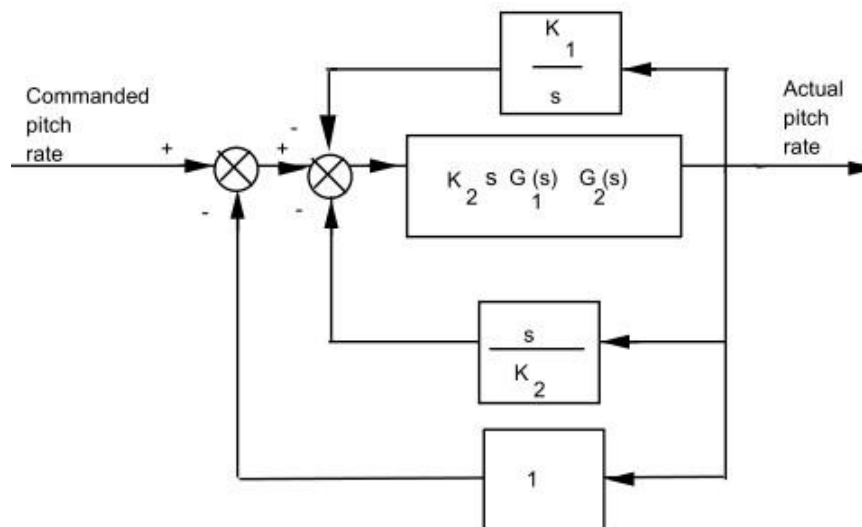




SPACE SHUTTLE PITCH CONTROL

- Then the closed-loop transfer function:

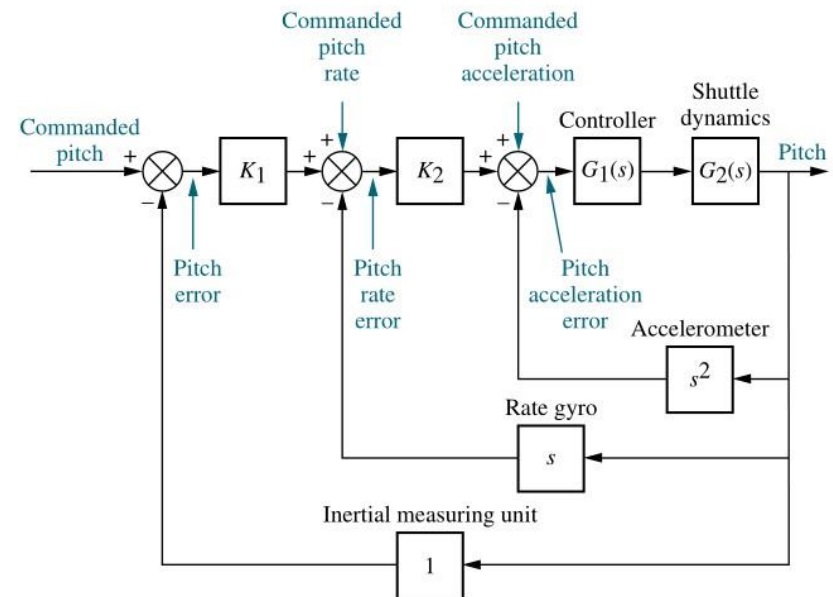
$$T(s) = \frac{K_2 s G_1(s) G_2(s)}{1 + K_2 s G_1(s) G_2(s) \left(1 + \frac{s}{K_2} + \frac{K_1}{s} \right)} = \frac{K_2 s G_1(s) G_2(s)}{1 + G_1(s) G_2(s) (s^2 + K_2 s + K_1 K_2)}$$



SPACE SHUTTLE PITCH CONTROL

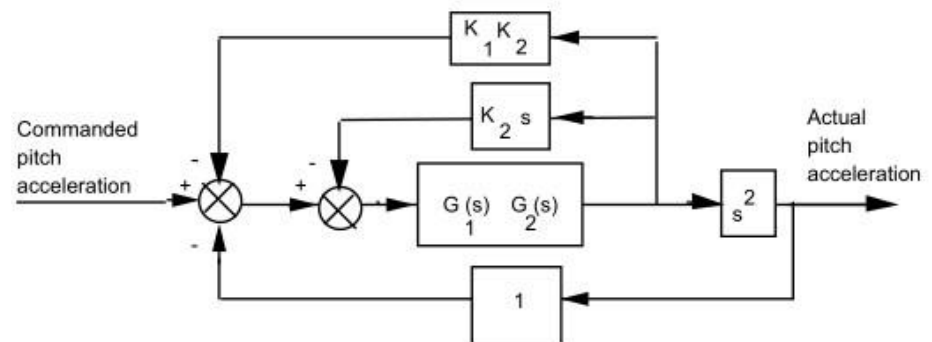
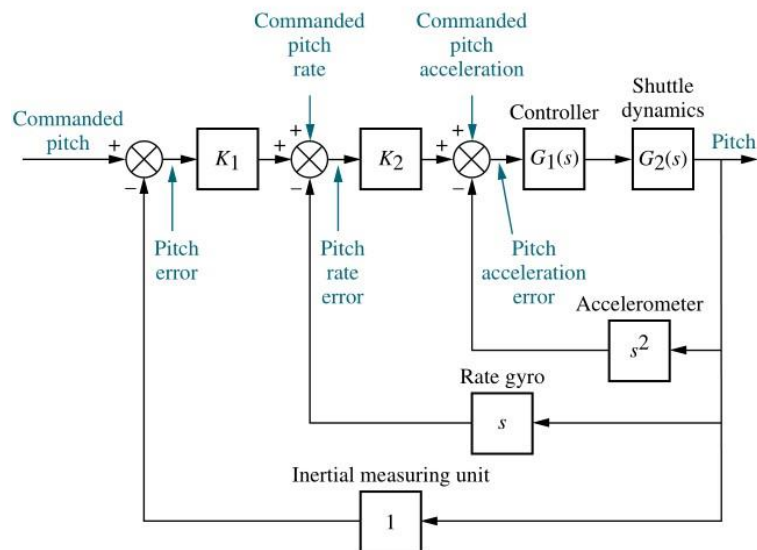
- Verify that the closed-loop transfer function from the **commanded pitch acceleration input** to **actual pitch acceleration** is given by (assume all other inputs are zeros)

$$T(s) = \frac{s^2 G_1(s) G_2(s)}{1 + G_1(s) G_2(s) [s^2 + K_2 s + K_1 K_2]}$$



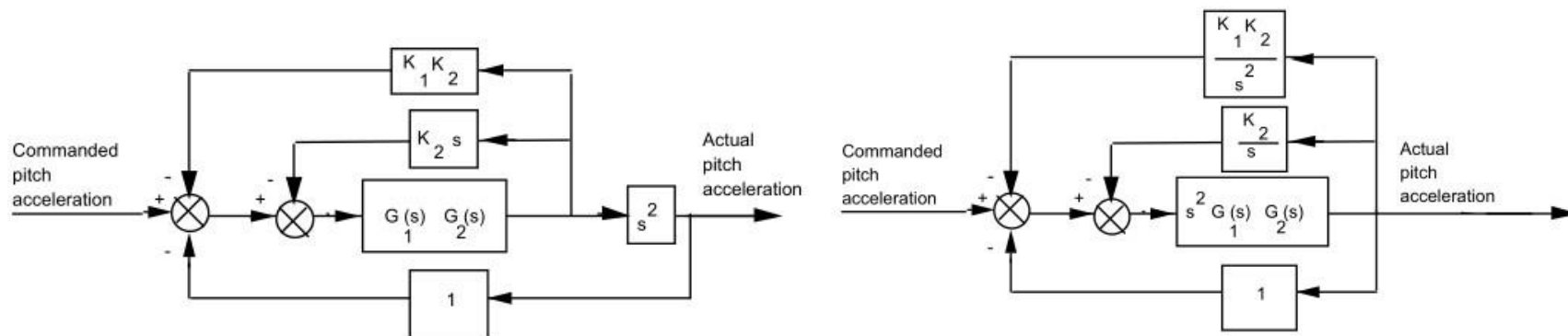
SPACE SHUTTLE PITCH CONTROL

- Rearranging the block diagram to show commanded pitch acceleration as the input and actual pitch acceleration as the output:



SPACE SHUTTLE PITCH CONTROL

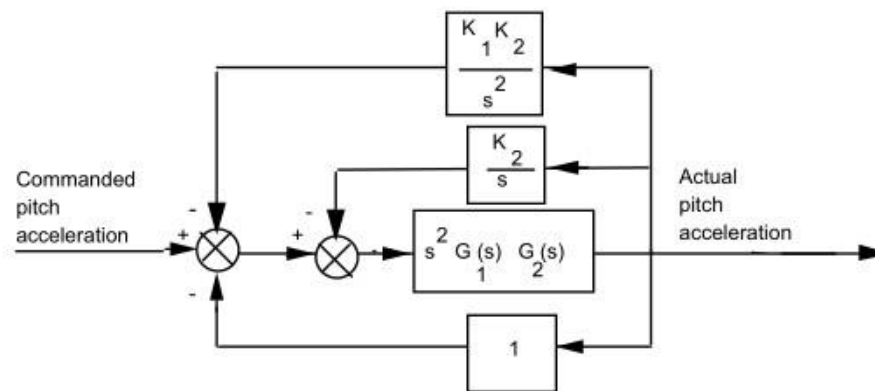
- Push s^2 to the left past the pick-off point yields,



SPACE SHUTTLE PITCH CONTROL

- Then the closed-loop transfer function:

$$T(s) = \frac{s^2 G_1(s) G_2(s)}{1 + s^2 G_1(s) G_2(s) \left(1 + \frac{K_1 K_2}{s^2} + \frac{K_2}{s} \right)} = \frac{s^2 G_1(s) G_2(s)}{1 + G_1(s) G_2(s) (s^2 + K_2 s + K_1 K_2)}$$



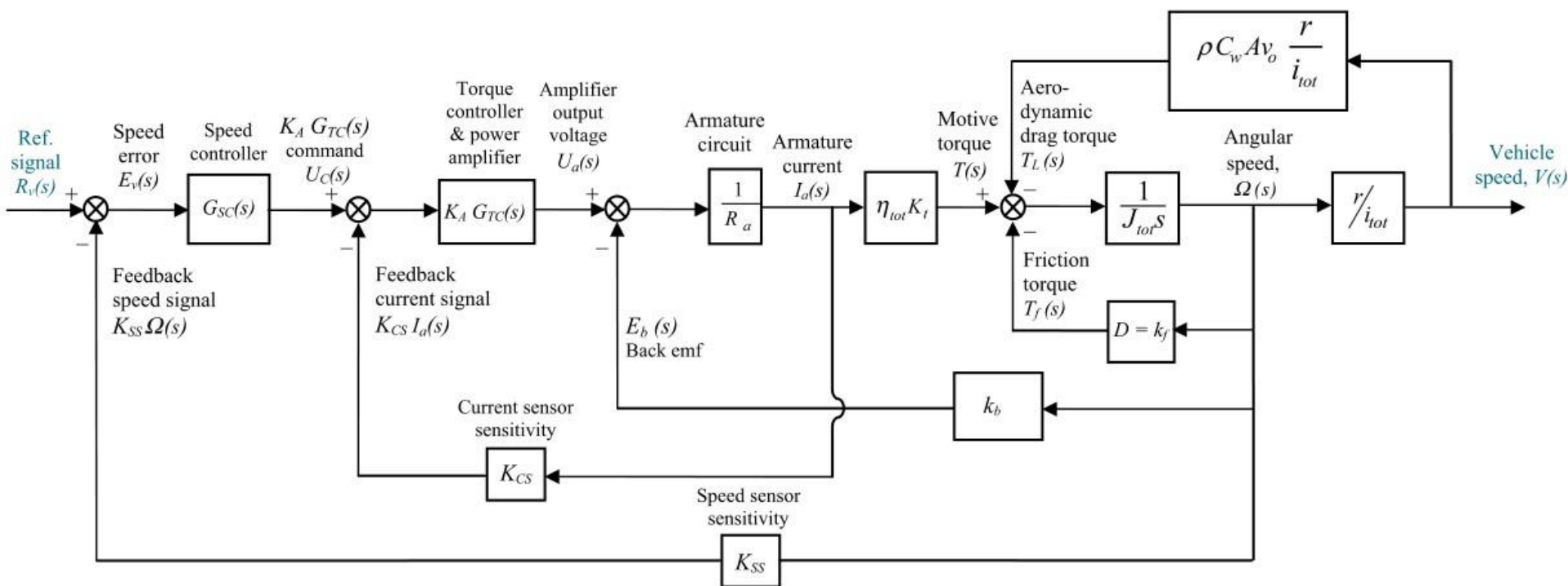


HYBRID ELECTRICAL VEHICLE

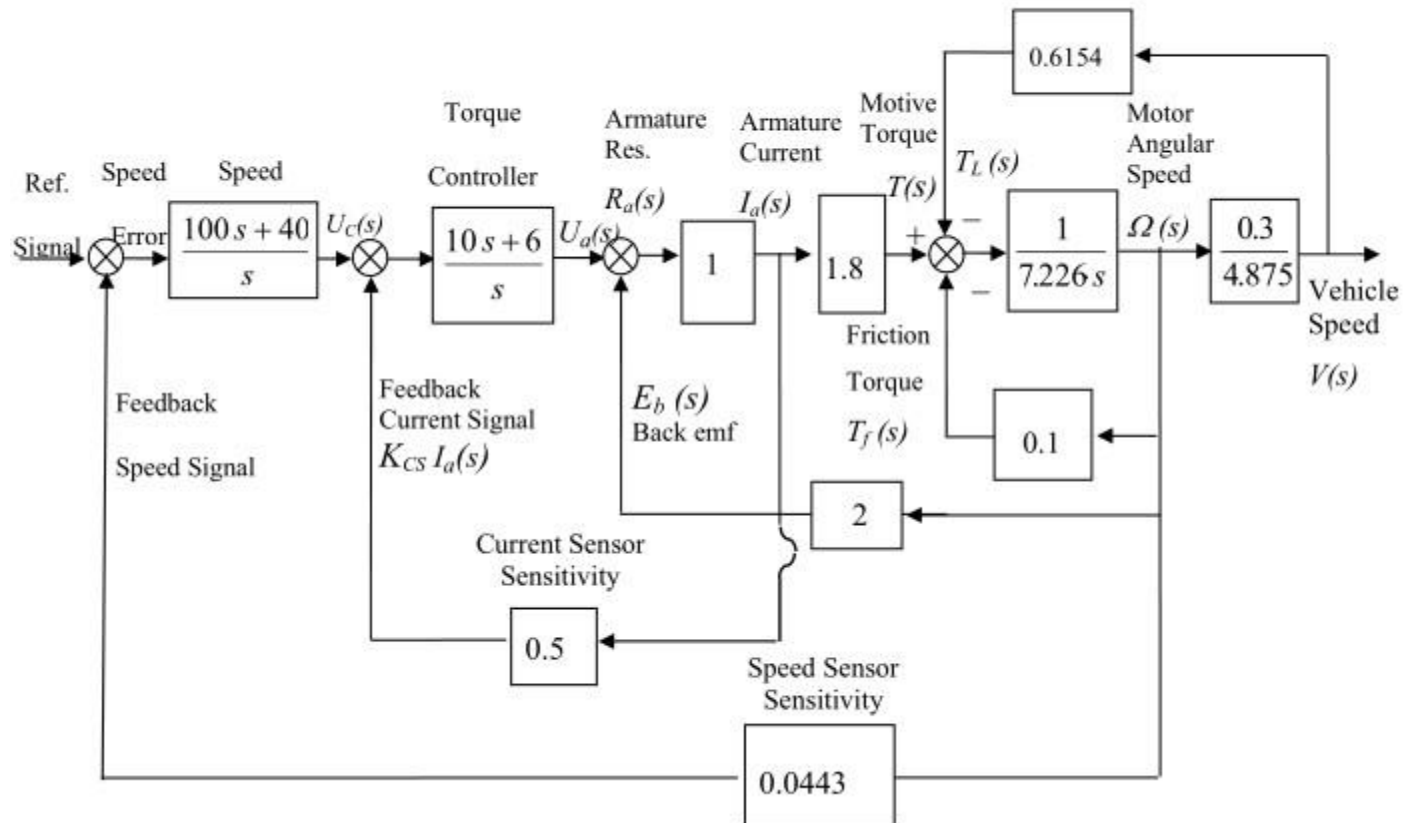
- The figure below shows the block diagram for a Hybrid Electrical Vehicle driven by a DC motor. Let the speed controller $G_{sc} = 100 + \frac{40}{s}$ the torque controller and the power amp $K_A G_{TC} = 10 + \frac{6}{s}$, the current sensor sensitivity $K_{SS} = 0.0433$.

$$\frac{1}{R_a} = 1; \eta_{tot} K_t = 1.8; k_b = 2; D = k_f = 0.1; \frac{1}{J_{tot}} = \frac{1}{7.226}; \frac{r}{i_{tot}} = 0.0615; \rho C_w A v_0 \frac{r}{i_{tot}} = 0.6154$$

HYBRID ELECTRICAL VEHICLE



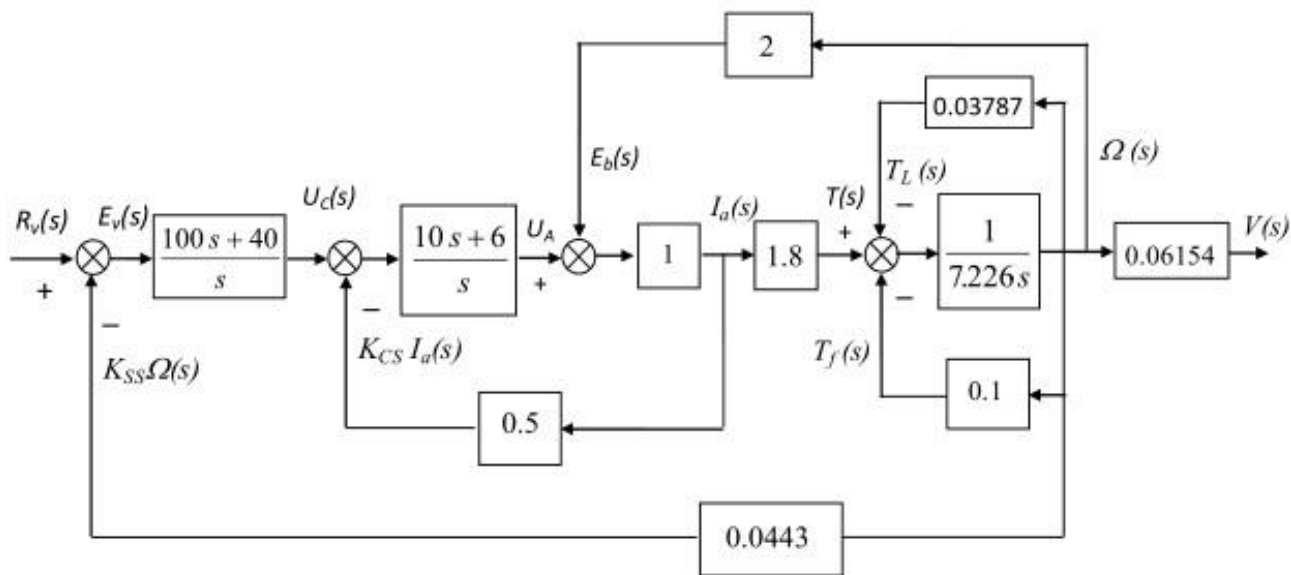
HYBRID ELECTRICAL VEHICLE



HYBRID ELECTRICAL VEHICLE

- Moving the last pick-off point to the left past the $\frac{r}{i_{tot}} = \frac{0.3}{4.875} = 0.06154$

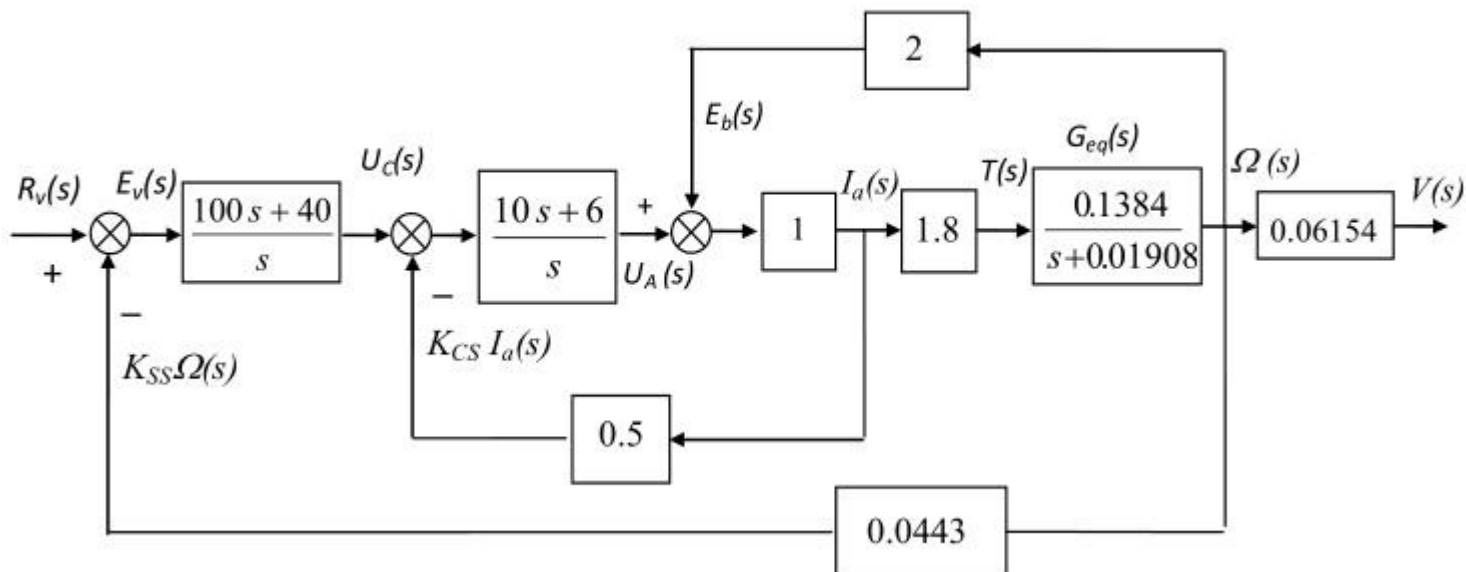
$$0.6154 \times \frac{0.3}{4.875} = 0.03787$$



HYBRID ELECTRICAL VEHICLE

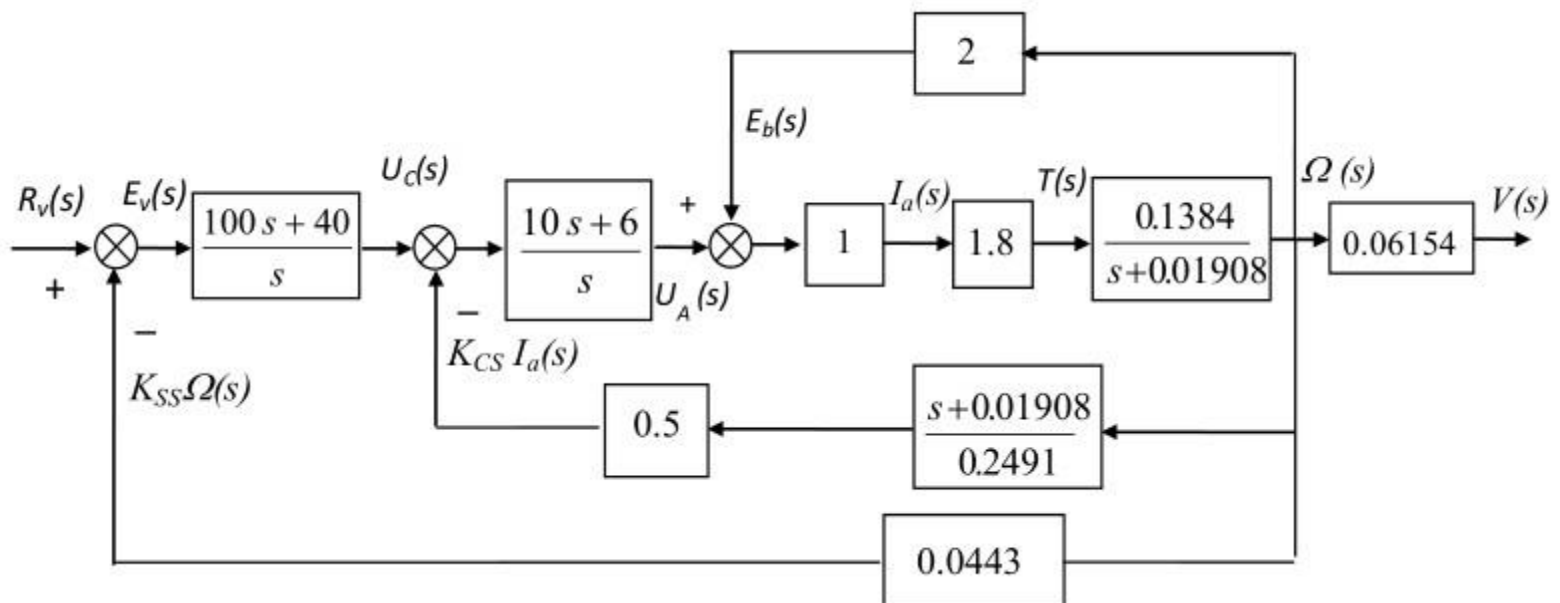
Transfer function

$$G_{eq}(s) = \frac{\Omega(s)}{T(s)} = \frac{1}{1 + \frac{7.226s}{0.13787}} = \frac{0.1384}{s + 0.01908}$$



HYBRID ELECTRICAL VEHICLE

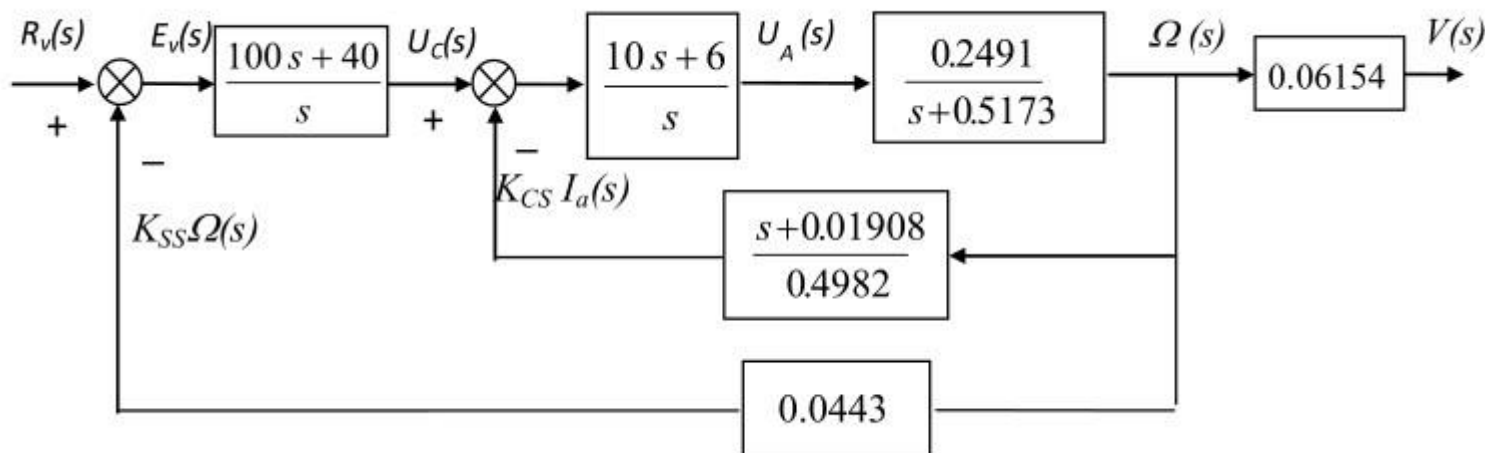
$$\frac{s + 0.01908}{1.8 \times 0.2491} = \frac{s + 0.01908}{0.2491}$$



HYBRID ELECTRICAL VEHICLE

- Transfer function for torque controller

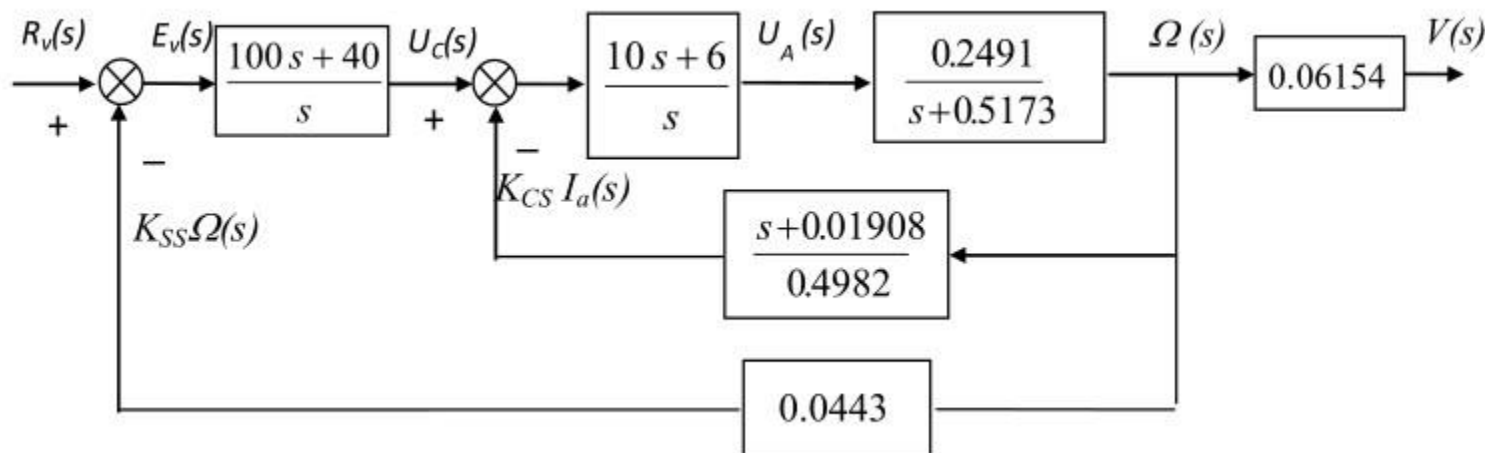
$$\frac{\Omega(s)}{U_A(s)} = \frac{\frac{0.2491}{s+0.01908}}{1 + \frac{0.2491}{s+0.01908} \times 2} = \frac{0.2491}{s+0.5173}$$



HYBRID ELECTRICAL VEHICLE

■ Thus

$$\frac{\Omega(s)}{U_c(s)} = \frac{\left(\frac{10s+6}{s}\right)\left(\frac{0.2491}{s+0.5173}\right)}{1 + \left(\frac{10s+6}{s}\right)\left(\frac{0.2491}{s+0.5173}\right)\left(\frac{s+0.01908}{0.4982}\right)} = \frac{0.2491(10s+6)}{s(s+0.5173) + 0.5(10s+6)(s+0.01908)}$$





HYBRID ELECTRICAL VEHICLE

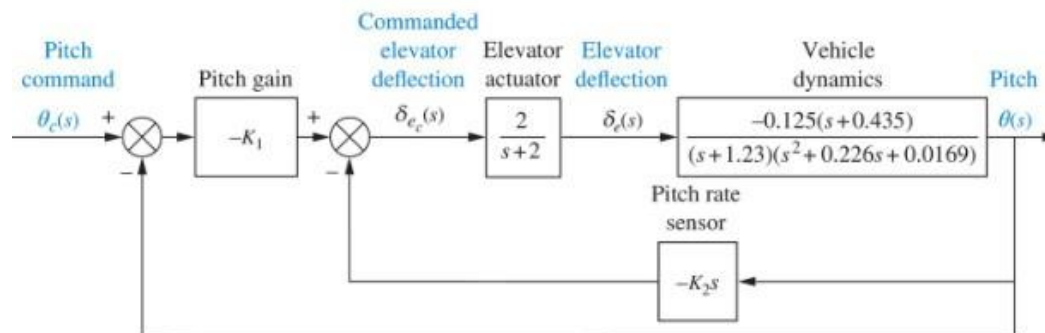
■ Thus

$$\frac{\Omega(s)}{R_v(s)} = \frac{\left(\frac{100s+40}{s}\right) \left(\frac{0.2491(10s+6)}{s(s+0.5173)+0.5(10s+6)(s+0.01908)}\right)}{1+0.0443\left(\frac{100s+40}{s}\right) \left(\frac{0.2491(10s+6)}{s(s+0.5173)+0.5(10s+6)(s+0.01908)}\right)}$$

$$\frac{\Omega(s)}{R_v(s)} = \frac{2491(s+0.4)(s+0.6)}{6s^3 + 14.644s^2 + 11.09s + 2.65}$$

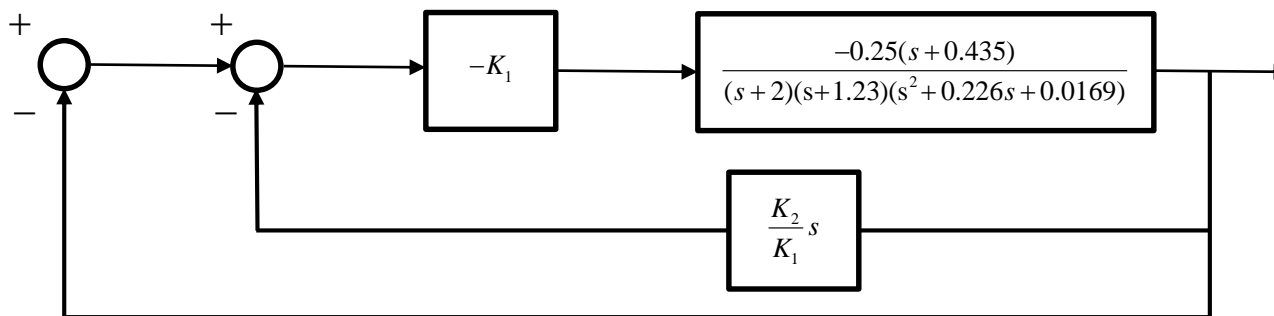
$$\frac{V(s)}{R_v(s)} = 0.06154 \frac{\Omega(s)}{R_v(s)} = \frac{15.33(s+0.4)(s+0.6)}{6s^3 + 14.644s^2 + 11.09s + 2.65}$$

- Transfer function $\theta(s)/\theta_c$ using block diagram reduction rules.



UFSS

- Transfer function $\theta(s)/\theta_c$ using block diagram reduction rules.

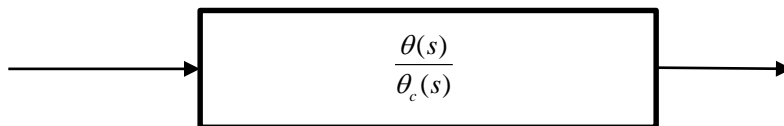


$$\frac{\theta(s)}{\theta_c(s)} = \frac{\frac{0.25K_1(s+0.435)}{(s+2)(s+1.23)(s^2+0.226s+0.169)}}{1 + \frac{0.25K_1(s+0.435)}{(s+2)(s+1.23)(s^2+0.226s+0.169)} \left(\frac{K_2}{K_1}s + 1 \right)}$$



UFSS

- Transfer function $\theta(s)/\theta_c(s)$ using block diagram reduction rules.



$$\frac{\theta(s)}{\theta_c(s)} = \frac{0.25K_1(s + 0.435)}{(s + 2)(s + 1.23)(s^2 + 0.226s + 0.169) + 0.25K_1(s + 0.435)(\frac{K_2}{K_1}s + 1)}$$

$$\frac{\theta(s)}{\theta_c(s)} = \frac{0.25K_1(s + 0.435)}{s^4 + 3.456s^3 + (3.359 + 0.25K_2)s^2 + (1.102 + 0.25K_1 + 0.109K_2)s + (0.416 + 0.109K_1)}$$