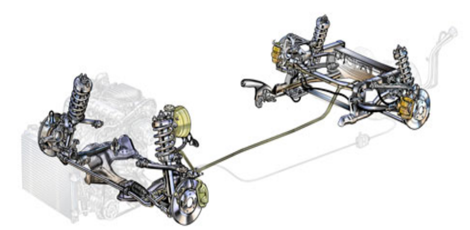
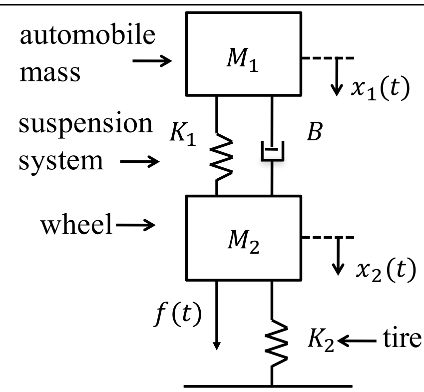
Studio #3: Modeling of Mechanical and Electrical Systems

The first objective of this studio is to reinforce dynamic modeling of mechanical and electrical systems covered in lectures. The second objective is to utilize Matlab to solve for the required transfer function from the obtained equations.

3.1. Automobile Suspension System



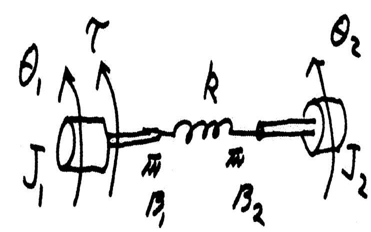


Show that the equations of motion in the Laplace domain are given as

$$(\mathrm{M_1 s^2 + Bs + K_1})\,\mathrm{X_1(s)} - (\mathrm{Bs + K_1})\mathrm{X_2(s)} = 0$$
$$(\mathrm{M_2 s^2 + Bs + K_1 + K_2})\,\mathrm{X_2(s)} - (\mathrm{Bs + K_1})\mathrm{X_1(s)} = \mathrm{F(s)}$$

3.2. Flexible Transmission in Robotic Systems

The figure below illustrates a motor ($\mathrm{J_1}$) connected to a load ($\mathrm{J_2}$) through a flexible coupling (gears, belt, cables, etc.).

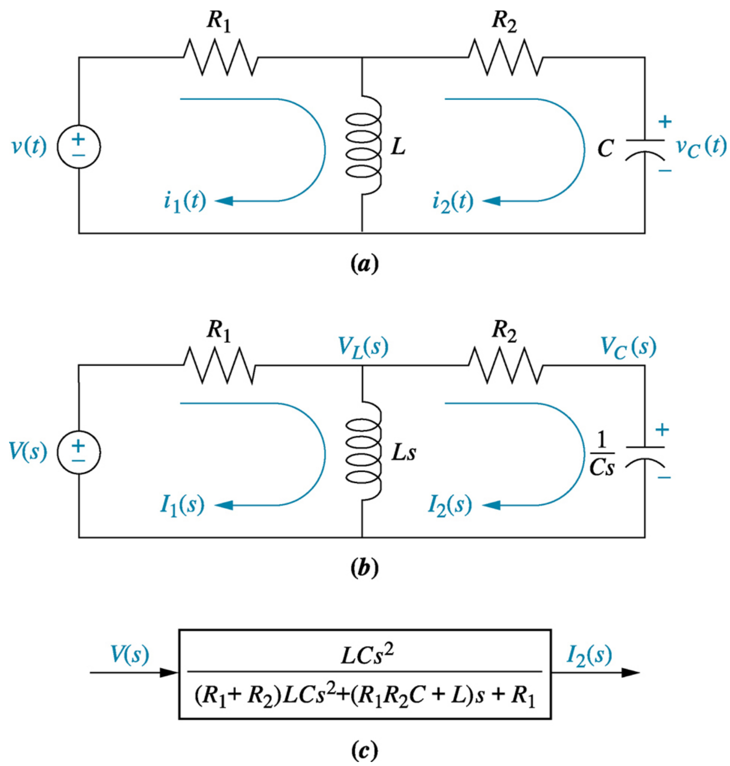


Show that the equations of motion in the Laplace domain can be written as

$$(\mathrm{J_1 s^2 + B_1 s + K})\theta_1(\mathrm{s}) - \mathrm{K}\theta_2(\mathrm{s}) = \tau(\mathrm{s})$$
$$(\mathrm{J_2 s^2 + B_2 s + K})\theta_2(\mathrm{s}) - \mathrm{K}\theta_1(\mathrm{s}) = 0$$

3.3. Analyzing Electrical Circuits

Given the electric circuit below, show that the transfer function $I_2(s)/V(s)$ is of the form given in the block diagram of part (c).



3.4. The answer verified in part 3, can also be obtained using Matlab's symbolic Math Toolbox. To that end, run the provided file ch2sp4.m. Note that this file is already uploaded on Canvas. The TA will now explain the code in the file.

MATLAB Exercises (To Be Completed In Class and Checked by the TA)

Remote Students and Students who do not finish within studio session:
Please save the output of Exercise 1, Exercise 2, and Exercise 3 in addition to your Matlab code in a word file and name it LastNameFirstNameStudio3.docx then upload to Canvas.

Exercise 1. Make a copy of the file ch2sp4.m and change the file name to stuido3exc1.m. Modify this file by utilizing results obtained in Part 3.1 (the Automobile Suspension System) to obtain the transfer function $T(s) = X_1(s)/F(s)$. That is, the transfer function between the vertical height of the vehicle and the disturbance on the road.

Show the TA your m-file and the Matlab output after you run the m-file.

Exercise 2. Make a copy of the file ch2sp4.m and change the file name to stuido3exc2.m. Using results obtained in Part 3.2 (Flexible Transmission in Robotic Systems), obtain the following transfer functions

(a) $T_1(s) = \frac{\theta_1(s)}{\tau(s)}$ (b) $T_2(s) = \frac{\theta_2(s)}{\tau(s)}$

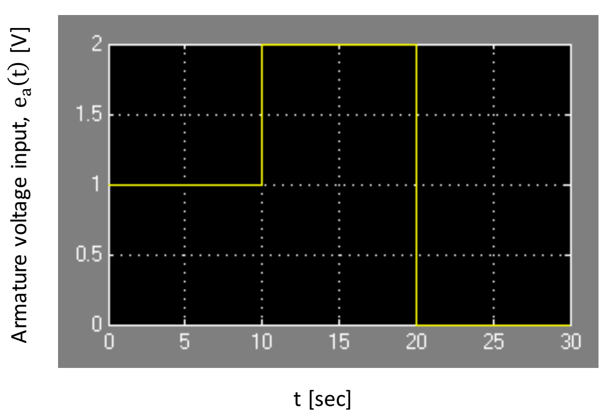
Show the TA your m-file and the Matlab output after you run the m-file for both parts (a) and (b).

Exercise 3. Write a Matlab script or use Simulink to simulate the angular displacement response of an armature-controlled DC servo-motor due to a voltage input shown below. The model of an armature-controlled DC motor is given by:

$$\frac{\theta(s)}{E_a(s)} = \frac{\alpha K}{s(s+\alpha)}$$

a. Assume that $\alpha = K = 1$. Present angular displacement and velocity responses and explain to the TA what the responses physically mean.
b. Present angular displacement and velocity responses when α is varied to 0.1 and 10. Explain to the TA what the responses physically mean.
c. Present angular displacement and velocity responses when K is varied to 0.1 and 10. Explain to the TA what the responses physically mean.

Hint: If you choose to write a script, one way is to use *lsim(system,input,time)* in Matlab.



Armature voltage input, $e_a(t)$ as a function of time.