



# ENME 462 STUDIO 5

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[https://github.com/XieGT/Control\\_Studios](https://github.com/XieGT/Control_Studios)





# OUTLINES

- ❖ Block Diagrams Algebra
- Space Shuttle pitch control
- Hybrid electronical vehicle
- UFSS pitch control

# CLOSED-LOOP TRANSFER FUNCTION

From the block diagram

$$C(s) = G(s)E(s)$$

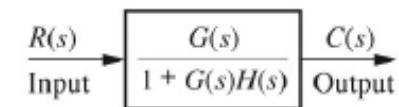
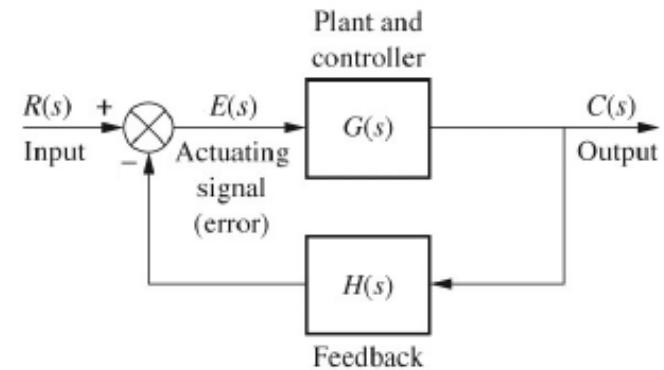
$$\begin{aligned} E(s) &= R(s) - C(s)H(s) \\ &= R(s) - G(s)H(s)E(s) \end{aligned}$$

Therefore

$$E(s) = \frac{1}{1 + G(s)H(s)} R(s)$$

And

$$C(s) = \frac{G}{1 + G(s)H(s)} R(s)$$

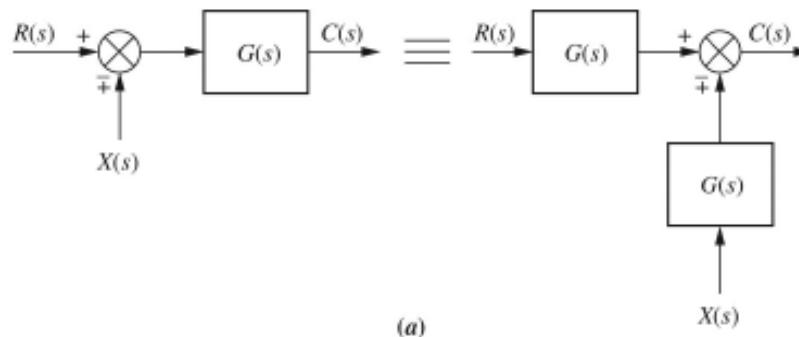
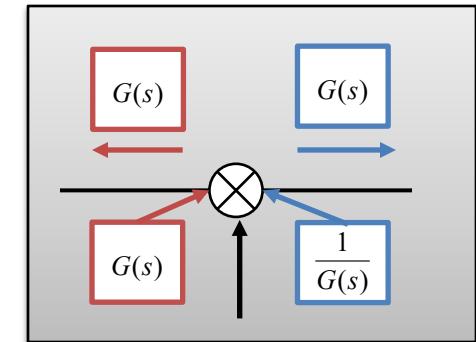


- Transfer function

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

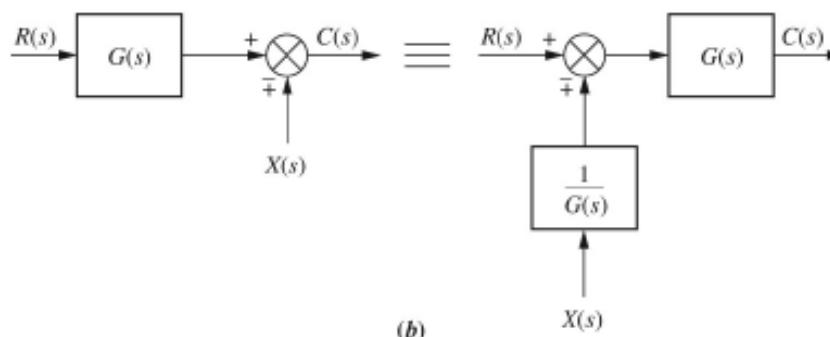
# REDUCTION TECHNIQUES

Moving a block before or after a summing junction



$$C(s) = R(s)G(s) \mp X(s)G(s)$$

$$C(s) = R(s)G(s) \mp X(s)G(s)$$



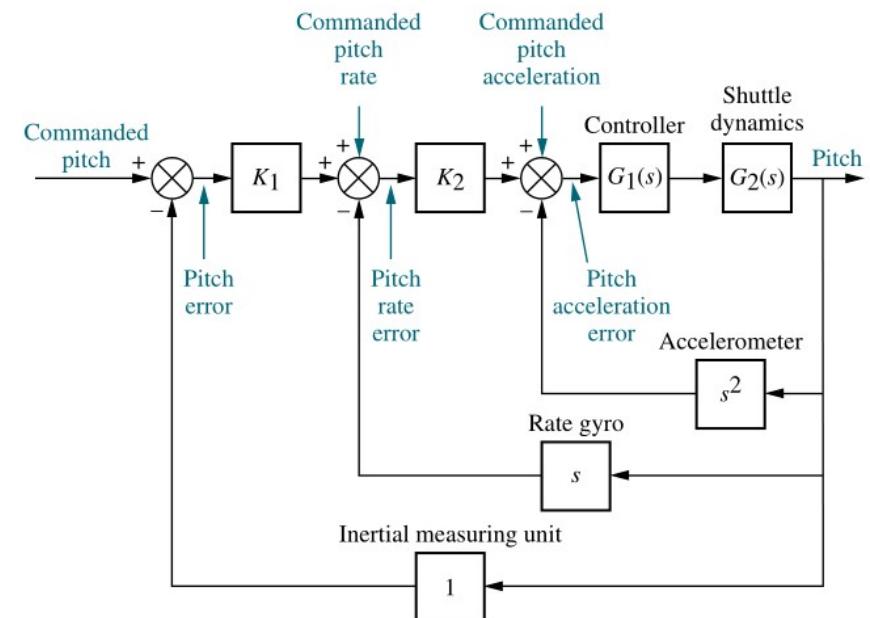
$$C(s) = R(s)G(s) \mp X(s)$$

$$C(s) = R(s)G(s) \mp X(s) \frac{1}{G(s)} G(s)$$

# SPACE SHUTTLE PITCH CONTROL

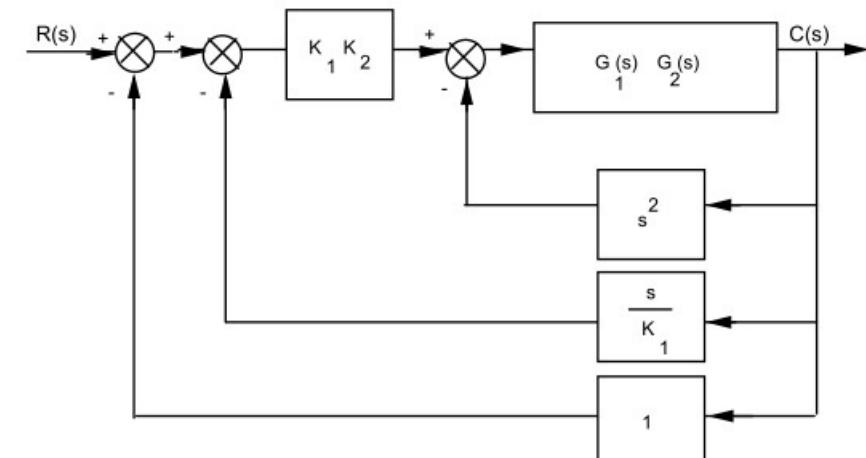
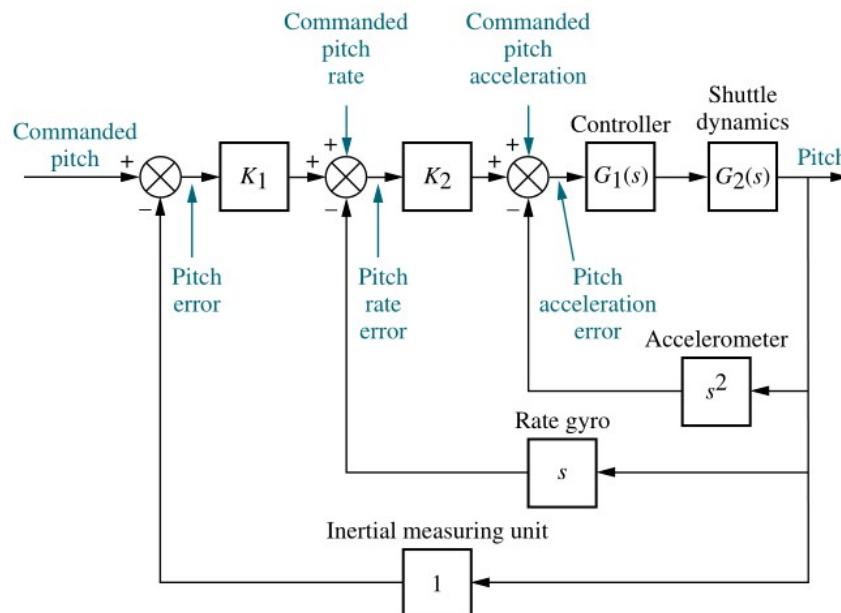
- Verify that the closed-loop transfer function from the commanded pitch input to actual pitch output is given by (assume all other inputs are zeros)

$$T(s) = \frac{K_1 K_2 G_1(s) G_2(s)}{1 + K_1 K_2 G_1(s) G_2(s) [1 + \frac{s}{K_1} + \frac{s^2}{K_1 K_2}]}$$



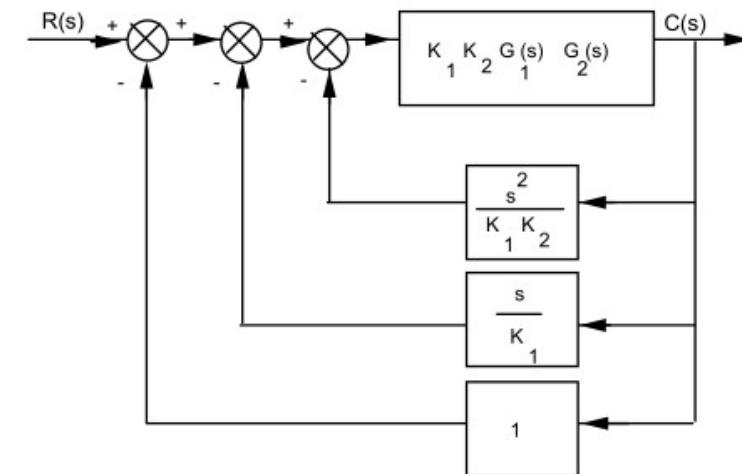
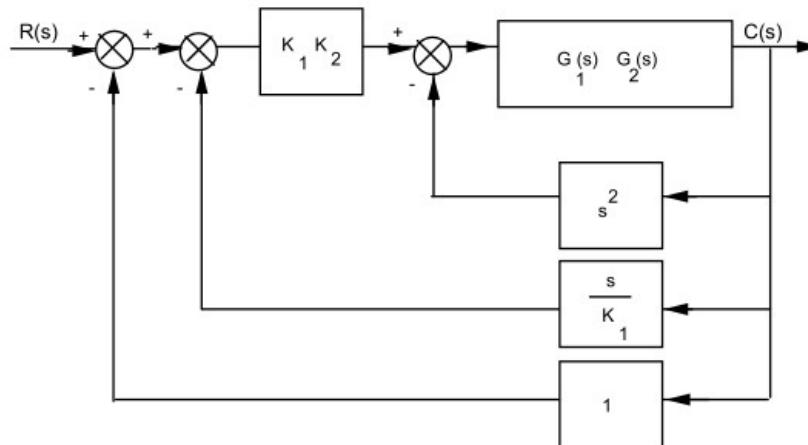
# SPACE SHUTTLE PITCH CONTROL

- a) combine  $G_1$  and  $G_2$ . Then push  $K_1$  to the right past the summing junction



# SPACE SHUTTLE PITCH CONTROL

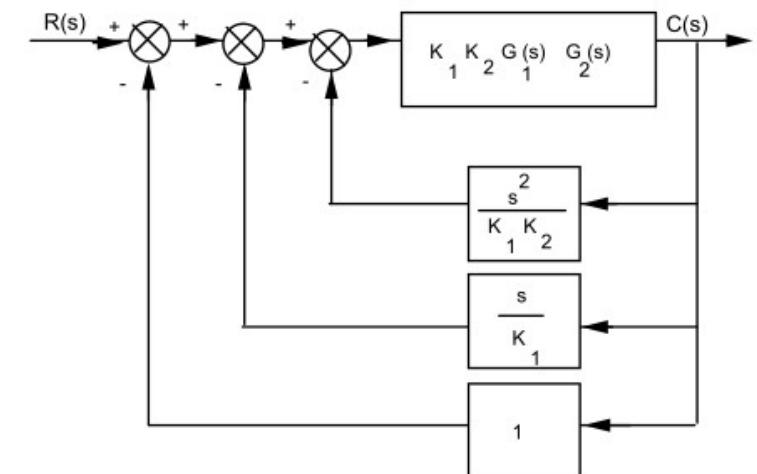
- b) Push  $K_1 K_2$  to the right past the summing junction



# SPACE SHUTTLE PITCH CONTROL

- c) Write the transfer function

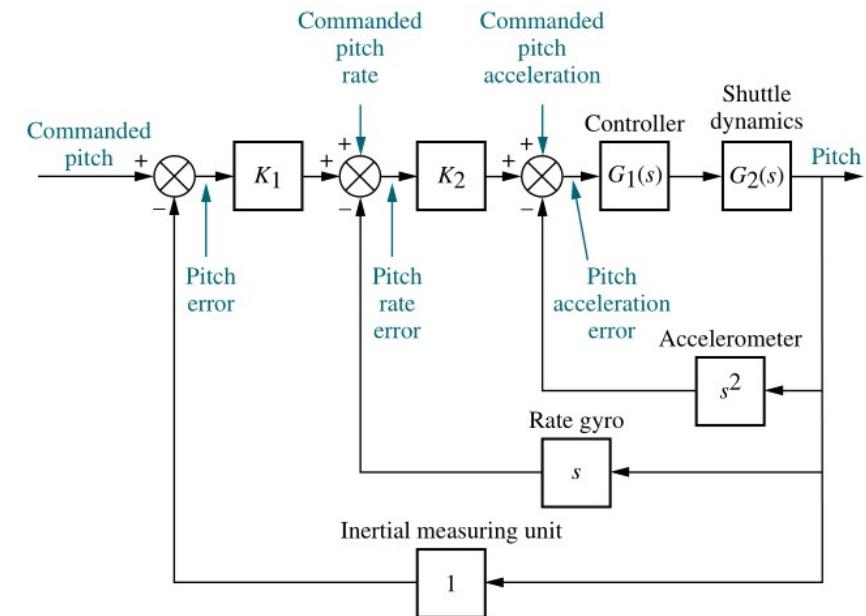
$$T(s) = \frac{K_1 K_2 G_1(s) G_2(s)}{1 + K_1 K_2 G_1(s) G_2(s) \left[ 1 + \frac{s}{K_1} + \frac{s^2}{K_1 K_2} \right]}$$



# SPACE SHUTTLE PITCH CONTROL

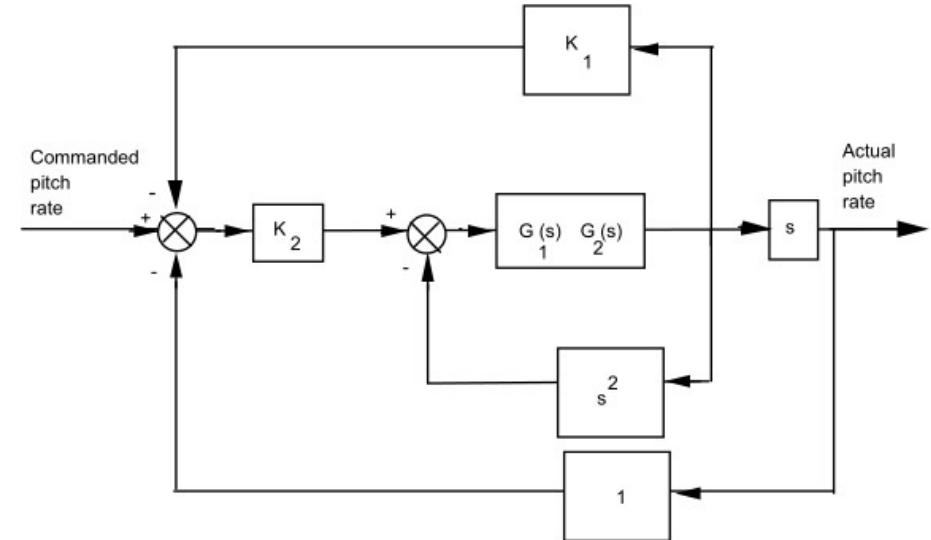
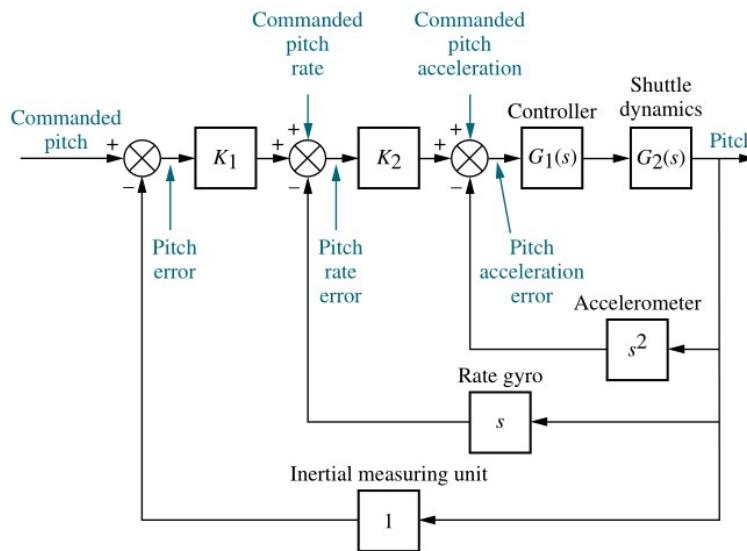
- Verify that the closed-loop transfer function from the ***commanded pitch rate input*** to ***actual pitch rate*** is given by (assume all other inputs are zeros)

$$T(s) = \frac{K_2 s G_1(s) G_2(s)}{1 + G_1(s) G_2(s)[s^2 + K_2 s + K_1 K_2]}$$



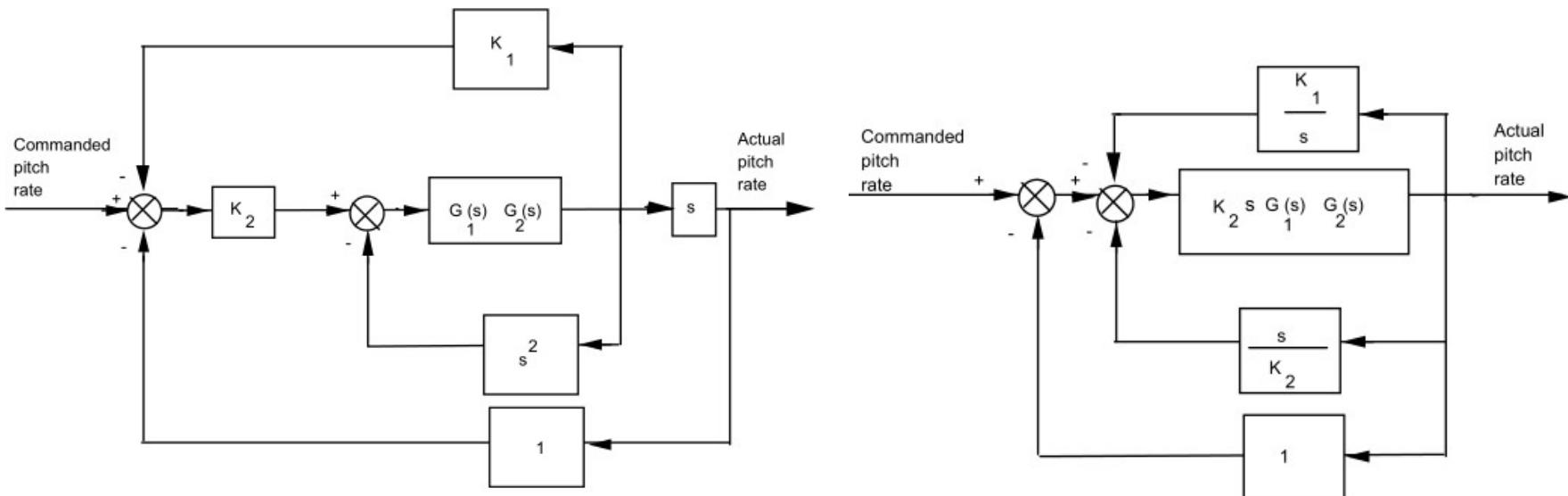
# SPACE SHUTTLE PITCH CONTROL

- Rearranging the block diagram to show commanded pitch rate as the input and actual pitch rate as the output:



# SPACE SHUTTLE PITCH CONTROL

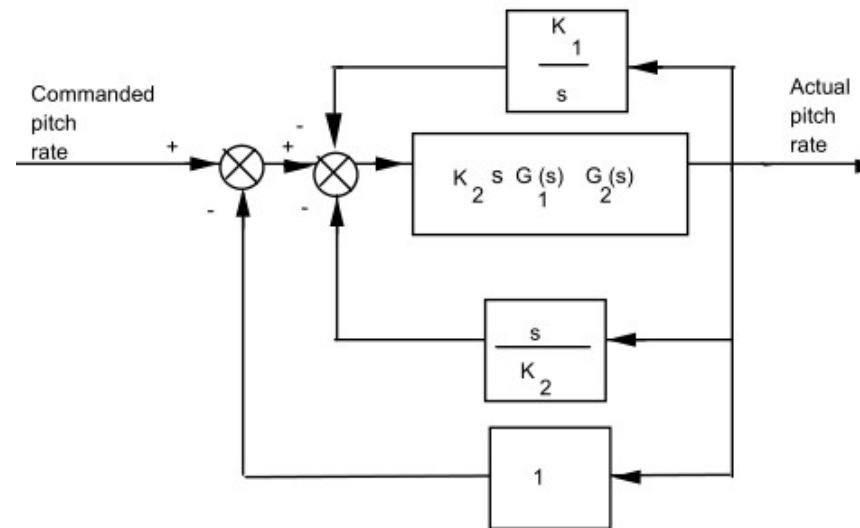
- Push  $K_2$  to the right past the summing junction; and push s to the left past the pick-off point yields,



# SPACE SHUTTLE PITCH CONTROL

- Then the closed-loop transfer function:

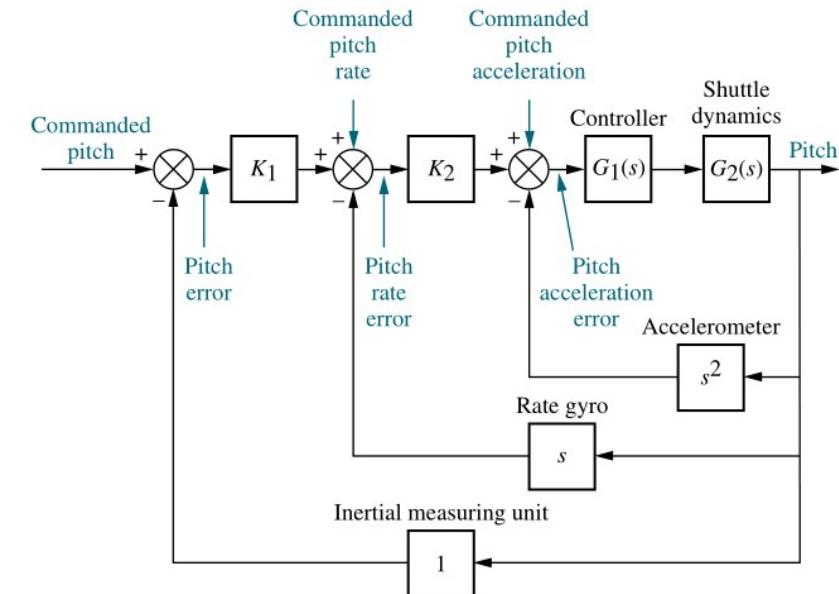
$$T(s) = \frac{K_2 s G_1(s) G_2(s)}{1 + K_2 s G_1(s) G_2(s) \left( 1 + \frac{s}{K_2} + \frac{K_1}{s} \right)} = \frac{K_2 s G_1(s) G_2(s)}{1 + G_1(s) G_2(s)(s^2 + K_2 s + K_1 K_2)}$$



# SPACE SHUTTLE PITCH CONTROL

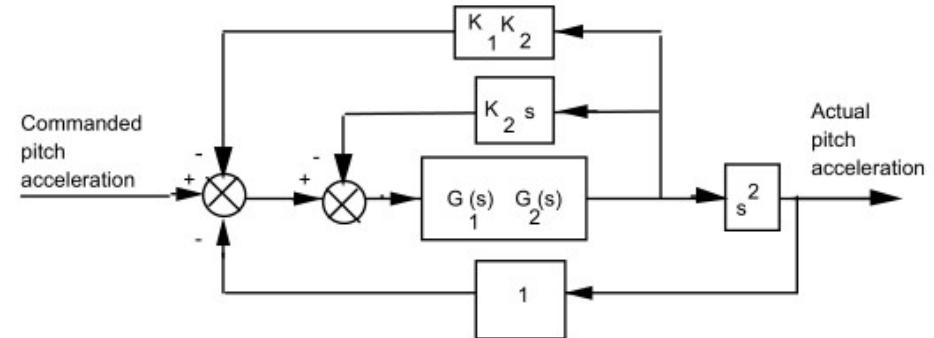
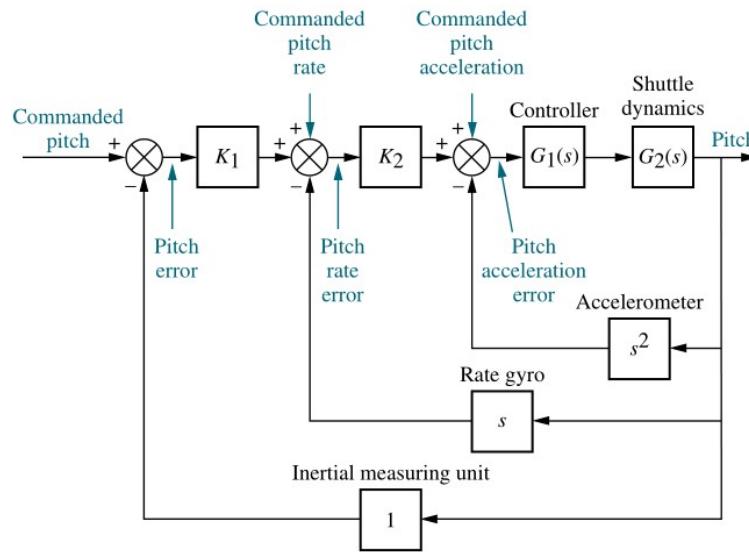
- Verify that the closed-loop transfer function from the *commanded pitch acceleration input* to *actual pitch acceleration* is given by (assume all other inputs are zeros)

$$T(s) = \frac{s^2 G_1(s) G_2(s)}{1 + G_1(s) G_2(s)[s^2 + K_2 s + K_1 K_2]}$$



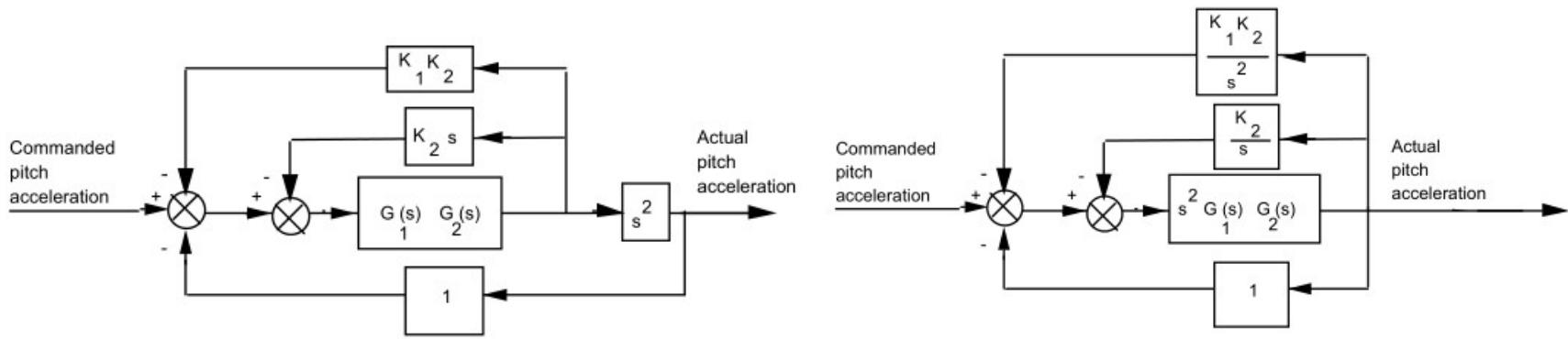
# SPACE SHUTTLE PITCH CONTROL

- Rearranging the block diagram to show commanded pitch acceleration as the input and actual pitch acceleration as the output:



# SPACE SHUTTLE PITCH CONTROL

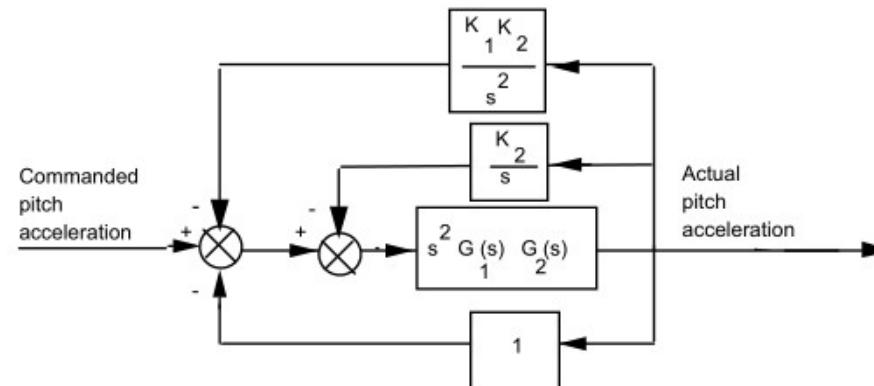
- Push  $s^2$  to the left past the pick-off point yields,



# SPACE SHUTTLE PITCH CONTROL

- Then the closed-loop transfer function:

$$T(s) = \frac{s^2 G_1(s) G_2(s)}{1 + s^2 G_1(s) G_2(s) \left( 1 + \frac{K_1 K_2}{s^2} + \frac{K_2}{s} \right)} = \frac{s^2 G_1(s) G_2(s)}{1 + G_1(s) G_2(s)(s^2 + K_2 s + K_1 K_2)}$$

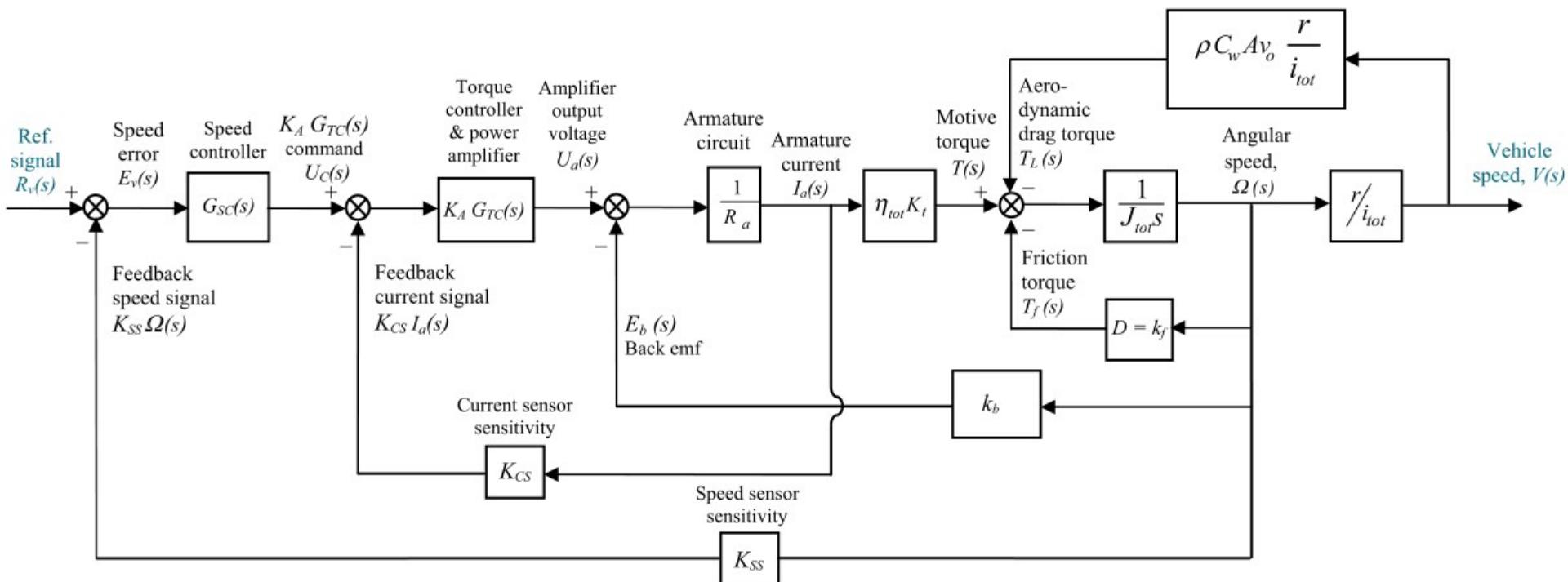


# HYBRID ELECTRICAL VEHICLE

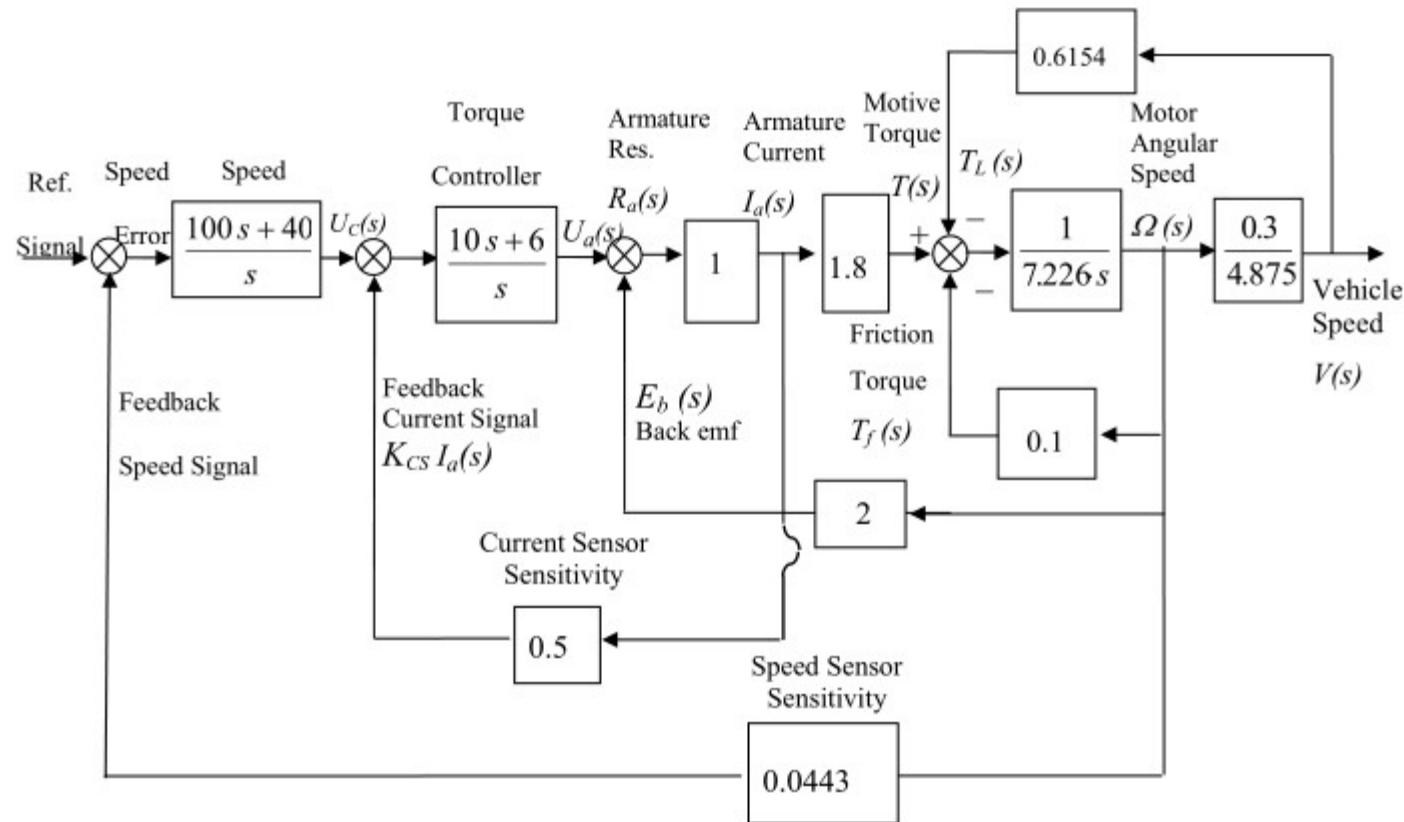
- The figure below shows the block diagram for a Hybrid Electrical Vehicle driven by a DC motor. Let the speed controller  $G_{sc} = 100 + \frac{40}{s}$ , the torque controller and the power amp  $K_A G_{TC} = 10 + \frac{6}{s}$ , the current sensor sensitivity  $K_{SS} = 0.0433$ .

$$\frac{1}{R_a} = 1; \eta_{tot} K_t = 1.8; k_b = 2; D = k_f = 0.1; \frac{1}{J_{tot}} = \frac{1}{7.226}; \frac{r}{i_{tot}} = 0.0615; \rho C_w A v_0 \frac{r}{i_{tot}} = 0.6154$$

# HYBRID ELECTRICAL VEHICLE



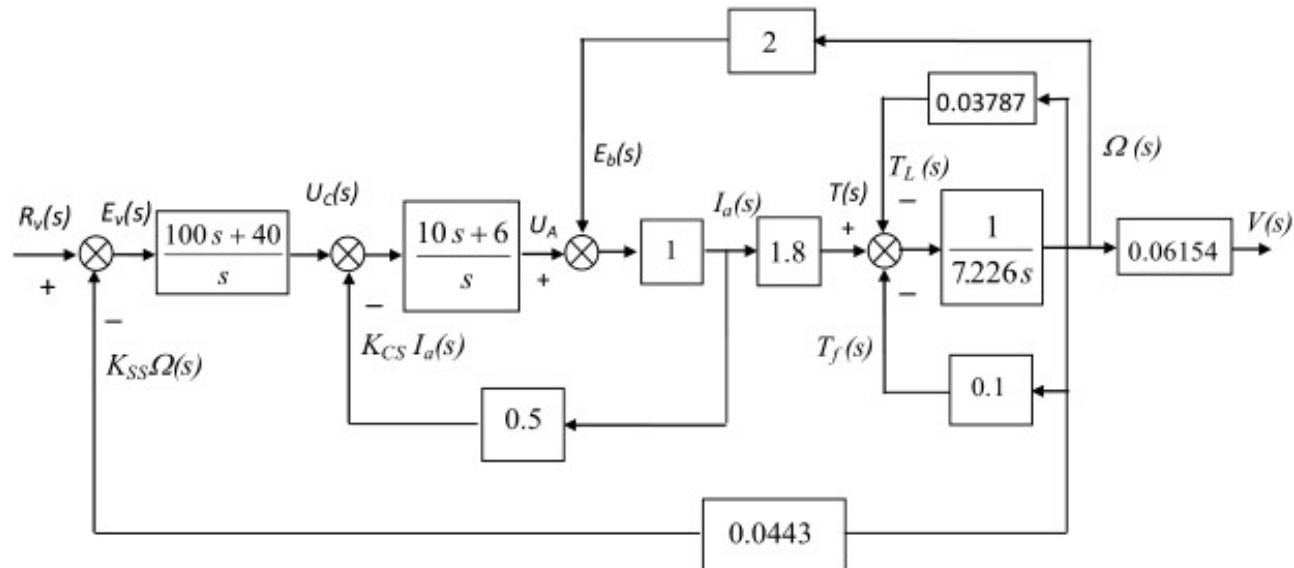
# HYBRID ELECTRICAL VEHICLE



# HYBRID ELECTRICAL VEHICLE

- Moving the last pick-off point to the left past the  $\frac{r}{i_{tot}} = \frac{0.3}{4.875} = 0.06154$

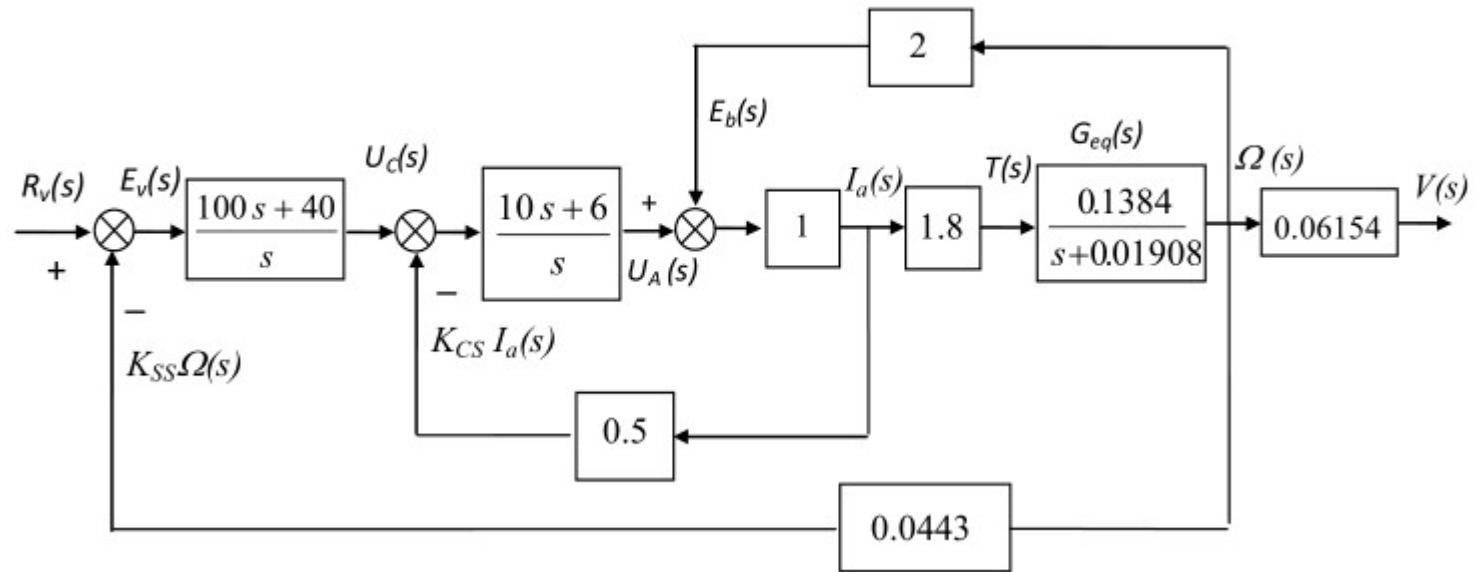
$$0.6154 \times \frac{0.3}{4.875} = 0.03787$$



# HYBRID ELECTRICAL VEHICLE

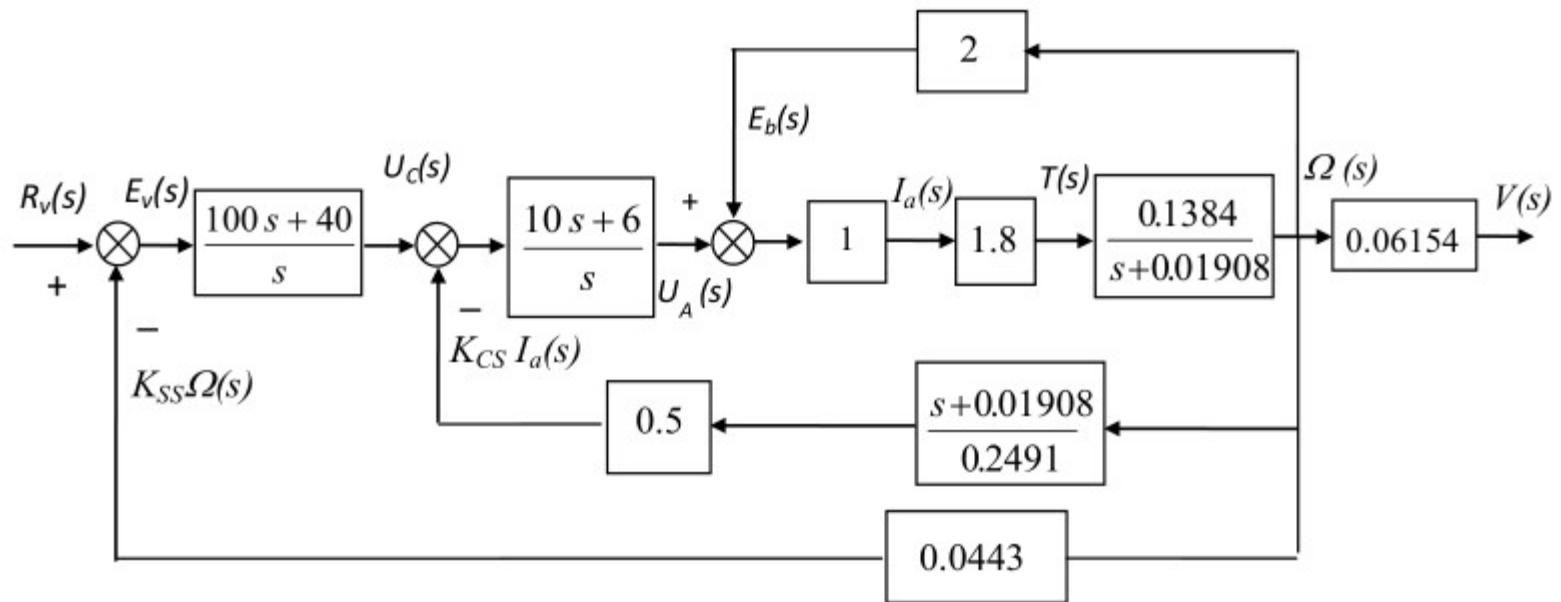
- Transfer function

$$G_{eq}(s) = \frac{\Omega(s)}{T(s)} = \frac{\frac{1}{7.226s}}{1 + \frac{0.13787}{7.226s}} = \frac{0.1384}{s + 0.01908}$$



# HYBRID ELECTRICAL VEHICLE

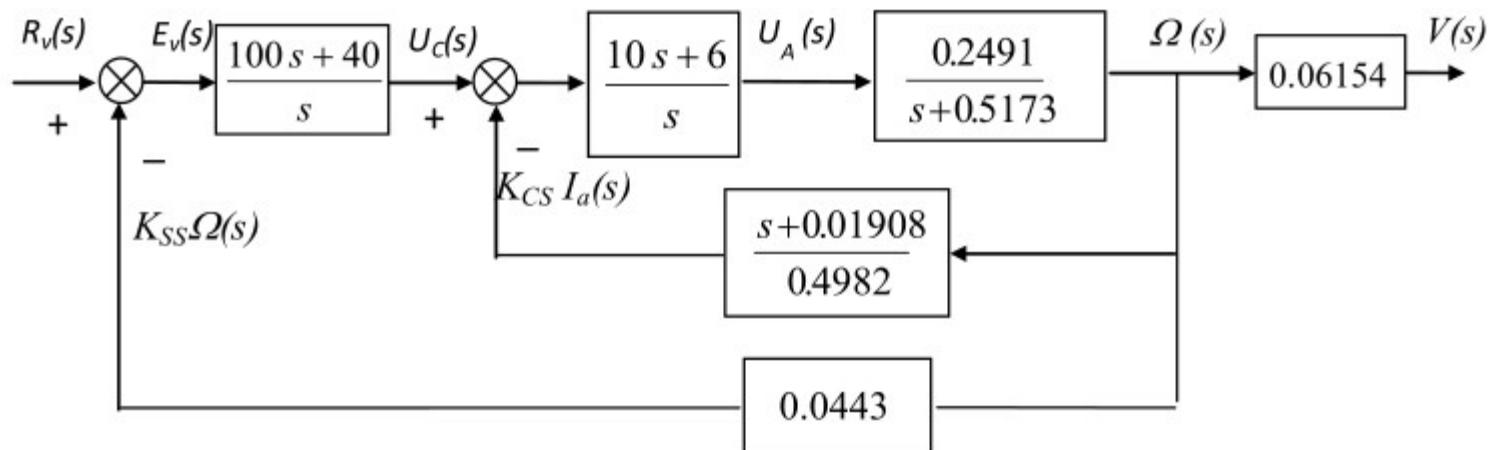
$$\frac{s + 0.01908}{1.8 \times 0.2491} = \frac{s + 0.01908}{0.2491}$$



# HYBRID ELECTRICAL VEHICLE

- Transfer function for torque controller

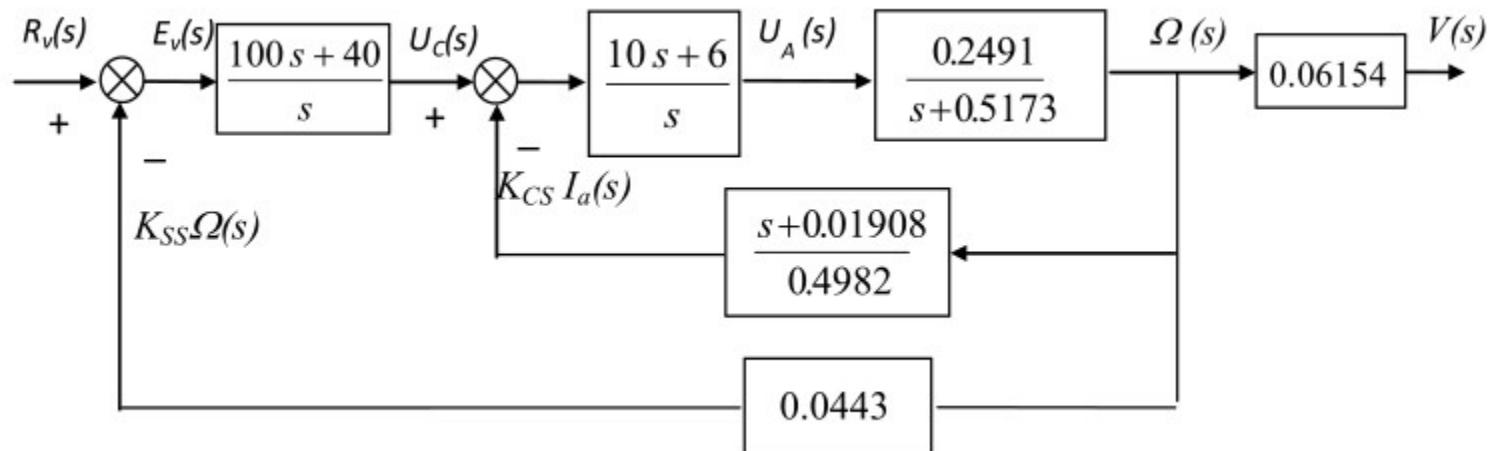
$$\frac{\Omega(s)}{U_A(s)} = \frac{\frac{0.2491}{s + 0.01908}}{1 + \frac{0.2491}{s + 0.01908} \times 2} = \frac{0.2491}{s + 0.5173}$$



# HYBRID ELECTRICAL VEHICLE

- Thus

$$\frac{\Omega(s)}{U_c(s)} = \frac{\left(\frac{10s+6}{s}\right)\left(\frac{0.2491}{s+0.5173}\right)}{1 + \left(\frac{10s+6}{s}\right)\left(\frac{0.2491}{s+0.5173}\right)\left(\frac{s+0.01908}{0.4982}\right)} = \frac{0.2491(10s+6)}{s(s+0.5173) + 0.5(10s+6)(s+0.01908)}$$



# HYBRID ELECTRICAL VEHICLE

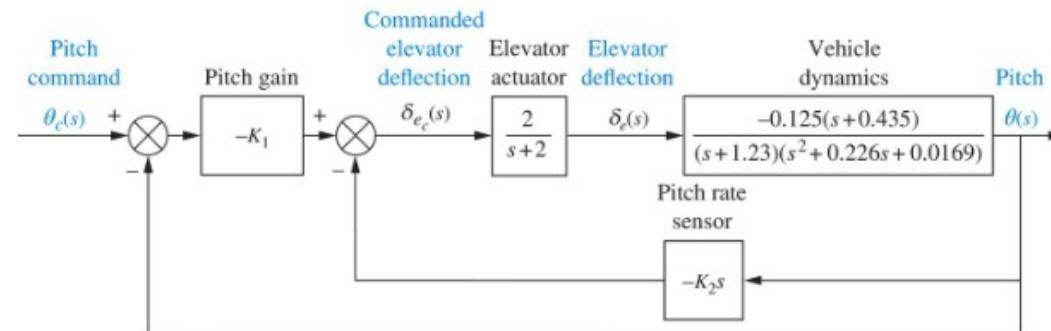
- Thus

$$\frac{\Omega(s)}{R_v(s)} = \frac{\left(\frac{100s+40}{s}\right) \left( \frac{0.2491(10s+6)}{s(s+0.5173)+0.5(10s+6)(s+0.01908)} \right)}{1 + 0.0443 \left(\frac{100s+40}{s}\right) \left( \frac{0.2491(10s+6)}{s(s+0.5173)+0.5(10s+6)(s+0.01908)} \right)}$$

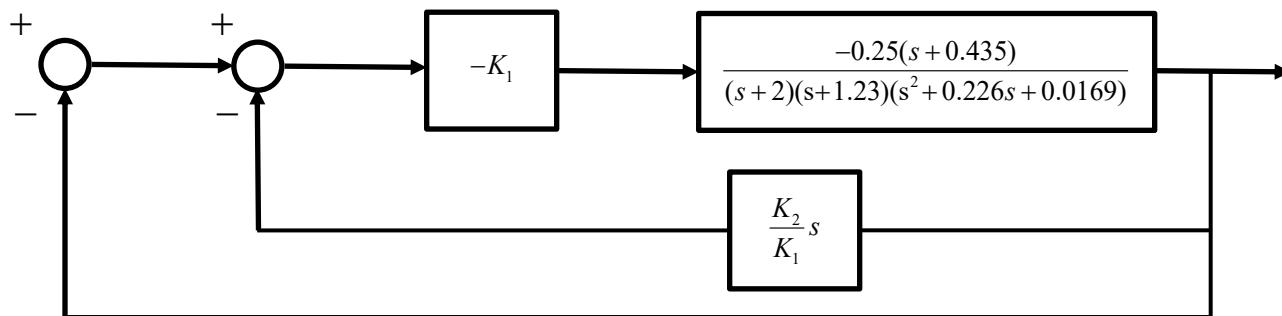
$$\frac{\Omega(s)}{R_v(s)} = \frac{2491(s+0.4)(s+0.6)}{6s^3 + 14.644s^2 + 11.09s + 2.65}$$

$$\frac{V(s)}{R_v(s)} = 0.06154 \frac{\Omega(s)}{R_v(s)} = \frac{15.33(s+0.4)(s+0.6)}{6s^3 + 14.644s^2 + 11.09s + 2.65}$$

- Transfer function  $\theta(s)/\theta_c$  using block diagram reduction rules.

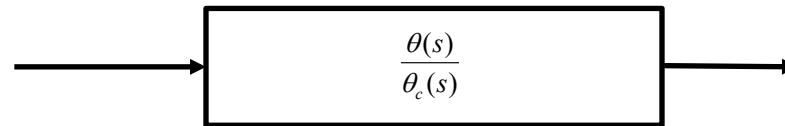


- Transfer function  $\theta(s)/\theta_c$  using block diagram reduction rules.



$$\frac{\theta(s)}{\theta_c(s)} = \frac{\frac{0.25K_1(s + 0.435)}{(s + 2)(s + 1.23)(s^2 + 0.226s + 0.0169)}}{1 + \frac{\frac{0.25K_1(s + 0.435)}{(s + 2)(s + 1.23)(s^2 + 0.226s + 0.0169)}(\frac{K_2}{K_1}s + 1)}{(s + 2)(s + 1.23)(s^2 + 0.226s + 0.0169)}}$$

- Transfer function  $\theta(s)/\theta_c$  using block diagram reduction rules.



$$\frac{\theta(s)}{\theta_c(s)} = \frac{0.25K_1(s + 0.435)}{(s + 2)(s + 1.23)(s^2 + 0.226s + 0.169) + 0.25K_1(s + 0.435)(\frac{K_2}{K_1}s + 1)}$$

$$\frac{\theta(s)}{\theta_c(s)} = \frac{0.25K_1(s + 0.435)}{s^4 + 3.456s^3 + (3.359 + 0.25K_2)s^2 + (1.102 + 0.25K_1 + 0.109K_2)s + (0.416 + 0.109K_1)}$$