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A Review of Nonlinear Dynamics of Mechanical Systems in Year 2008*

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Abstract

In this article, we provide an overview of a selection of topics of current interest in nonlinear dynamics and vibrations of mechanical systems. Specifically, we cover the traditional topics of structural dynamics, rotating systems, vibration control, vehicle dynamics, and machining, as well as some topics that have emerged more recently, namely micro- and nano-electromechanical systems, piecewise smooth systems, and structural health monitoring and system identification. In the introductory and closing sections, we offer our perspective on the state of affairs in nonlinear dynamics and its utility in the study of mechanical systems.

Key words : Nonlinear, Dynamics, Vibrations

1. Introduction

Nonlinear dynamics has a long history of applications in mechanical systems. One of the first class of nonlinear dynamics problems, which dates back to the times of Kepler, Newton, and Lagrange, is the n-body problem in celestial mechanics. In the "modern era" of dynamics (that is, since the invention of the calculus), the two-body problem can be claimed as the first problem to be solved, paving the way to the determination of lunar landing trajectories and orbits for space missions. Attempts to solve the three-body problem furthered the development of dynamics for two centuries, leading up to Poincaré's recognition of motions, which we now know as chaos[(1)]. Experimental evidence of nonlinear oscillatory behavior in earth-bound systems was also being collected, starting with Mersenne and Huygen's observations of amplitude dependent pendulum periods in the mid 1600s. This continued into the industrial age, and included observations of the fluttering of throttle valves in centrifugal governors that regulated steam engines used in the 1800s, chatter of the machine tool-workpiece systems during manufacturing operations, lateral swaying (also known as "hunting motions") of railway cars, gearbox rattle, and whirling of rotors. These applications brought more attention to the study of dynamics in engineering applications. Much of the early mechanical design work in this area was related to vibrations, for which linear theory is widely applicable. However, it was also recognized that nonlinearity was important to consider in many applications, and many books addressed problems in nonlinear oscillations[(2)–(7)]. Initially, the approaches relied on perturbation techniques, which were restricted primarily to weakly nonlinear systems, and topological methods, which were restricted primarily to low-order systems. In the area of machine dynamics, problems are inherently more nonlinear, and special solutions were known and used (for example, in steady operations), but many important applications were left unaddressed, due to the inability to solve, even approximately, the equations of motion. It is worth noting that the book from this era by Hayashi[(8)], which dealt with the dynamics of nonlinear electrical circuits, was pioneering in its view of how to treat nonlinear oscillators,

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and included extensive results on strange attractors.

There was a resurgence of interest in these problems starting in the 1980s, due largely to the recognition that chaos can occur in simple models of mechanical systems. A flurry of research activities and books[(9)–(12)] ensued, where tools from dynamical systems were applied to problems of interest to mechanical engineers. One outcome of this was a common language for addressing these problems, for example, terminology from bifurcation analyses and invariant manifold theory. Experiments also helped understand the qualitative changes associated with bifurcations and uncover a plethora of nonlinear phenomena in mechanical systems. Virgin[(13)] describes experiments that demonstrate a wide variety of nonlinear behavior and many of the tools used to deal with nonlinear system response data. The availability of computers has also changed the field in a fundamental way. Problems that were once intractable became solvable, at least numerically. This opened the floodgates to applications across many fields, including mechanical systems. This trend has also led to a revival in analytical methods and experimental investigations in dynamics, the interplay between them, and their complementary use with simulations. Experimental studies are increasingly being used to develop reduced-order models of complex mechanical systems that exhibit characteristics of a well understood low-order nonlinear system, with a prototypical example being the Düffing oscillator. The result is where we stand today, where problems of tremendous complexity, in terms of the numbers of degrees of freedom, in terms of force models, in terms of features such as stick-slip interactions, loss of contact dynamics, and time delays, and in terms of the nature of solutions, are being investigated. In fact, one might claim that our ability to solve the equations of motion is well beyond our ability to accurately model systems, and this is certainly true in many cases.

The state of affairs in nonlinear dynamics in mechanical systems is exciting. The general and powerful tools from dynamical systems theory, combined with a knowledge of mechanics, have opened the doors to many new areas of research, with researchers increasingly trying to take advantage of nonlinear phenomena to design mechanical systems rather than merely understand them. This is particularly true when nonlinear dynamicists combine talents with researchers from other disciplines, resulting in synergistic efforts that bring a fresh approach to problems. It is our intention to bring some flavor of these developments to the reader.

The purpose of this review is not to provide a thorough survey of nonlinear dynamics in mechanical systems, since such an undertaking would require multiple volumes. Rather, our aim is to focus on current trends and topics, with particular emphasis on a few areas. On a related note, one should not read this paper hoping to find a complete set of references to the literature on the selected topics (even if such a thing were possible), but we will point the reader to early papers and discuss a few recent papers in each field, from which the interested reader can follow the trail of a previous effort.

The paper is organized as follows. In Section 2, we describe current trends in applications of nonlinear dynamics in some of the more traditional areas of mechanical systems, including structural systems, rotating systems, vehicle dynamics, machining, and vibration control. Next, in Section 3, we cover a few areas that are relatively new for applications of nonlinear dynamics, including micro-electromechanical systems (MEMS) and nano-electromechanical systems (NEMS), piecewise smooth mechanical system models, and system identification and its relation to structural health monitoring. Many readers will note that their favorite area of research has not been included in our coverage. This is simply a reflection of the interests and expertise of the authors, and in no way implies the objective importance of an area covered versus one not covered. After all, we do not cover, for example, spacecraft dynamics, flow-induced vibrations, stochastic systems, and many others, which are obviously as interesting and important as any of the topics considered herein. We close in Section 4, with a perspective about the past and future roles of nonlinear dynamics in the analysis and design of mechanical systems.

2. Trends in Traditional Areas of Mechanical Systems

2.1. Structural Systems

Structural systems, which are typically modeled by distributed-parameter models, arise in a wide range of applications and they have been studied at length scales ranging from the micro- and nano-scales to the macro-scale. For example, structural elements in the form of strings or cables can be found in musical instruments, power lines, mooring lines, towing cables, tether lines, and aerospace and offshore applications. Structural elements in the form of beams can be found in micro-scale resonators, slider-crank mechanisms, rotor blades, positioning mechanisms in disk drives, robotic manipulators, crane booms, and civil and aerospace engineering structures. Structural members such as membranes and plates can be found in micro-phone diaphragms, micro-pumps, wing structures, and so on. There is also a host of applications for ring-type and shell-type structural elements.

Nonlinear effects in structural systems can arise from a variety of sources including mid-plane stretching, geometry (for example, curvature, shape, large displacements, and large rotations), damping and inertial effects, elastic and anisotropic properties of materials, electromagnetic, magnetoelastic, and other properties of active materials, boundary conditions, loading (aerodynamic and hydrodynamic loading, forces associated with magnetic fields), and damage in structures such as cracks. Early studies were experimental in nature, and also included weakly nonlinear analyses of autonomous and non-autonomous single degree-of-freedom and multi-degree-of-freedom models, derived from Galerkin reductions of continuous systems. More recent avenues of investigation include the following: i) careful experimental investigations of chaotic attractors, the qualitative changes experienced with respect to the quasi-static variation of control parameters, and routes to chaos including period-doubling sequences, torus breakdown, crises, and so on; ii) uncovering of slow-scale and fast-scale phenomena including energy transfer between modes, a variety of sub-harmonic and super-harmonic responses, modulated motions, and resonance effects; iii) applications of geometrical techniques to seek special oscillatory solutions of these systems as well as to simplify them; and iv) analyses of local and global bifurcations experienced by these systems.

Experimental observations of nonlinear motions in strings were first reported in the late 1940s(14) and mid-1960s(15). It was reported that a string excited by a planar force transverse to the axis of the string produced a whirling response, a response that cannot be explained by linear analysis. Consideration of nonlinearities and coupling between in-plane and out-of-plane motions helped explain the instabilities that led to the energy transfer from in-plane to out-of-plane motions(7),(16). Since then, chaotic motions and phenomena such as torus doubling and crises have been documented through careful experiments with strings(17), illustrating the connections between the dynamics of low-dimensional systems in which such behavior had been earlier found and the dynamics of continuous systems. Nonlinear responses of cables, which can be considered as strings with an initial sag, has been the focus of many investigations since the early analytical treatment of Yamaguchi, Miyata, and Ito(18). Strings and cables can be viewed as representative examples of systems with more than one degree of freedom and cubic and quadratic nonlinearities, and a host of experimental, analytical, and numerical studies have followed since the 1970s and 1980s to understand the dynamics in the presence of different nonlinear resonances (internal and external resonances), interactions between different modes of oscillation, and local and global instabilities associated with the motions of these structural systems(19)–(22).

Since the 1970s, nonlinear oscillations of beams have been experimentally, analytically, and numerically studied(7),(23),(24) and these investigations have helped recognize the prevalence of possibilities for saddle-node bifurcations, pitchfork bifurcations, period-doubling bifurcations, and Hopf bifurcations in these systems. In terms of new experimental phenomena observed over the last two decades, of particular note are the experimental observations of energy transfer from a high-frequency excitation to a low-frequency response through qualitative changes that occur on a slow-time scale(25),(26). This work paved the way to the construction of a new energy-transfer paradigm in later studies[27),(28)]. Buckled beams of various configurations (e.g., clamped-clamped beams[29], metallic cantilever beams with a magnet on either side of the beam[30]) have served to illustrate the rich dynamics exhibited by mechanical systems with multiple equilibrium positions. Nonlinear instabilities (jumps from oscillations about one equilibrium position to oscillations about the other equilibrium position in the state space) experienced by such systems and the resulting aperiodic motions have been extensively studied. It could be argued that buckled beams were one of the first mechanical prototypes in which chaotic motions were observed and systematically investigated. Furthermore, the use of Poincaré maps by Moon[31] to understand the fractal characteristics of strange attractors associated with the chaotic motions of buckled beams has fashioned the experimental nonlinear dynamics investigations that have since followed. Apart from local bifurcation analysis, global analysis of reduced-order versions of beam equations have also been carried out to better understand the complex dynamics exhibited by such systems. For example, Yagasaki, Sakata, and Kimura[32] carried out Melnikov analysis with a single-mode reduction of a beam subjected to external and parametric excitations to show the existence of transverse homoclinic orbits and consequent chaotic motions, and in a subsequent study[33], bifurcations of tori solutions and bifurcation sets were studied. Unlike the treatment of dynamics of isotropic beams, the treatment of dynamics of composite beams is primarily numerical in nature. This treatment is necessitated by the anisotropy and heterogeneity of composite materials. Appropriate mechanics formulations that can be used for studying the nonlinear dynamics of composite beams and other structures is still a topic of current research, as discussed in the excellent book of Nayfeh and Pai[20]. The nonlinear dynamics of different configurations of beam structures such as L-shaped, T-shaped, and frame structures have also been analytically and experimentally studied by a number of researchers, as discussed in references[7),(19),(20)]. A glimpse of buckling-influenced dynamics at the micro-scale is provided by recent studies on micro-scale piezoelectric resonators[34),(35)].

Since the early experimental study of Tobias[36] and analytical study of Yamaki[37], many studies have been undertaken to examine the nonlinear dynamic behavior of plates. Qualitative aspects of the dynamics exhibited by buckled beams are also exhibited by fluttering plates in a supersonic flow[38),(39)]. (Fluttering panels have been found to occur in aircraft structures and rocket boosters.) In this context, the analytical treatment of Holmes[40] serves as an elegant example of local bifurcation analyses in nonlinear autonomous systems with finite degrees of freedom. Yang and Sethna[41] and Feng and Sethna[42] provided some of the first examples for the possibility for global bifurcations in plate-type systems. They used Melnikov and Shilnikov analyses to examine the possibility for chaos in thin square plates with one-to-one internal resonances. A comprehensive review of various studies on the dynamics of isotropic and anisotropic plates and membrane structures with different boundary conditions is provided in the work of Nayfeh[19] and Nayfeh and Pai[20]. These references also include a review of the studies that have examined the nonlinear dynamics of different types of ring structures and shell structures (infinitely long cylindrical shells and closed spherical shells) through experimental, analytical, and numerical means. These structural systems have served as illustrative examples for understanding how these systems behave in the presence of internal and external resonances and different types of forcing.

Apart from the application of perturbation methods to simplify reduced-order versions of structural systems, geometrical methods have also been extensively used since the 1990s. In this regard, much work has been carried out in the area of nonlinear normal modes, which extend the idea of normal mode vibrations to nonlinear systems. For an extended discussion of this and related topics, the reader is referred to the book by Vakakis, Manevitch, Mikhlin, Pilipchuk, and Zevin[43]. The simplification scheme used by Georgiou, Bajaj, and Corless [44),(45)], which is applicable to structural systems with multiple equilibria and soft and stiff characteristics, makes use of a combination of singular perturbation methods (for example, averaging) and invariant manifold techniques to separate slow-scale and fast-scale dynamics.

Structural systems have been extensively studied over the last fifty years, and they have been shown to exhibit different types of nonlinear phenomena associated with low-order finite dimensional systems as well as help uncover new nonlinear phenomena such as energy transfer from a high-frequency excitation to a low-frequency response. These systems have also served as a vehicle for examining the application of different analytical as well as experimental tools of nonlinear dynamics, and it is expected that this trend will continue. With the growing use of anisotropic materials in structural systems, it is anticipated that numerical investigations will be extensively used to explore the nonlinear dynamics of such systems. Nonlinear spatio-temporal dynamics has not been discussed in this section, but also continues to be a topic of interest in a wide number of applications.

2.2. Rotating Systems and Machines

Recently, the U.S. National Academy of Engineering ranked "electrification" as the number one engineering achievement of the 20th century[(46)]. Crucial to this culture-changing development was the generation and transmission of power, which at that time was, and still today is, based almost entirely on rotating mechanical systems. In fact, it can be argued that the first need for modern vibration engineering was related to the need to understand the dynamic behavior of rotating systems, in particular, various types of turbines and generators. The earliest work in this area was restricted to linear problems, such as computing resonance frequencies and mode shapes, but nonlinear phenomena are quite ubiquitous in such systems, and eventually had to be dealt with. Nonlinear effects can arise from local sources such as rotor-stator contact and nonlinear bearing forces, squeeze-film effects, and these effects can also stem from finite deformations of flexible elements and/or gyroscopic effects. In rotating shaft problems, the effects of these nonlinearities can be exacerbated by internal resonances that occur as rotor speeds vary. These topics have been considered by many researchers, and there exist several excellent sources that cover a wide range of nonlinear rotor dynamic problems, including the books of Tondl[(47)] and Yamamoto and Ishida[(48)], and the review article by Ishida[(49)].

As cited by Ishida[(49)], more than fifty years ago, Yamamoto made the earliest systematic observations of nonlinear behavior in rotor systems, starting with the nonlinear resonance behavior caused by ball bearings. This work was followed by a string of results in which Yamamoto observed and analyzed a variety of nonlinear resonance phenomena, including subharmonic and combination resonances. Similar results were also uncovered in industry, primarily by the ground-breaking investigations of Ehrich and co-workers who have observed a variety of responses in gas turbines, including subharmonics, superharmonics, and chaos, which arise from rotor/stator contact nonlinearity[(50), (51)]. Tondl has also considered a number of problems on nonlinear behavior in rotor dynamics, and in his early book on the subject[(47)], a number of these problems are described, including an extensive set of test results that support the analytical work. While each of these areas is deserving of an expanded discussion, we focus on a specific set of problems where modern nonlinear methods have provided an elegant solution to a longstanding problem.

One of the most important topics in rotor dynamics is the behavior of a system during its passage through a resonance (typically, the first resonance mode of the rotor system is of interest). To describe this behavior there are two basic models. The first is to assume that the rotor speed is a specified function of time, which is a classical problem in the theory of "nonstationary" problem in dynamics, extensively covered in the book by Mitropolskii[(52)]. This is known as the case of "ideal excitation." In such problems, the main issue is the speed at which one passes through the resonance zone and the nature of the nonlinear resonance. If the passage speed is slow, the vibratory response will roughly follow the steady-state version of the nonlinear response curve. From this viewpoint, it is clear that the nonlinear nature of the resonance plays an important role in this dynamics, since the particular branch that is followed may lead to a large amplitude response, depending on whether the nonlinearity is hardening or softening, and whether the rotor is spinning up or down in speed. If the passage speed is sufficiently fast, the response can "punch through" the resonance and not become large in amplitude, in which case the nonlinearity plays a less critical role. In these cases, beating of the response amplitude envelope occurs, due to frequency differences. The details of these situations are worked out in great detail in the work of Mitropolskii(52), and reviewed for the case of rotors by Ishida(49). Nayfeh and Mook(7) also discuss these dynamics, as well as the non-ideal case, considered next.

The second type of model assumes that the source of the rotor spin up is an applied torque that has finite power, and this leads to a problem in the area traditionally known as a "non-ideal excitation." Nayfeh and Mook(7) and Balthazar et al.(53) have provided the background and references for these systems, from a classical nonlinear resonance point of view. The fundamental phenomenon is known as "the Sommerfeld effect," after the first recorded observations of Sommerfeld in the early 1900's, and it has been studied by a number of researchers (as cited in Nayfeh and Mook(7)). A basic model for these problems typically consists of a rotor mounted on a flexible support, wherein the rotor is acted on by a torque, and the translational and rotational motions of the rotor are dynamically coupled. The dynamics of these systems becomes interesting when the rotor speed interacts in a resonant manner with the translational system resonance. In such systems, one can encounter the phenomenon of "resonance capture," wherein the energy from the torque, which is intended to provide an angular acceleration of the rotor, is transferred through the nonlinear interactions into the translational motion, which can then become large. In these situations, it is observed that the rotor speed sometimes remains trapped near resonance, never to escape, leading to a large amplitude resonance response. However, in other cases, the amplitude begins to grow, but the system escapes from resonance, and the ultimate outcome is that the rotor spins up through resonance whereupon the response amplitudes become small. Traditional approaches to analyzing these problems include local perturbation techniques, through which one is able to describe the existence and stability of quasi-stationary solutions(54). However, these approaches are not successful in describing all details of the subtle dynamics observed, in particular, the nature of the basins of attraction for the various system responses. The fact that very small changes in system parameters and/or initial conditions separate systems that successfully pass through resonance from those that experience resonance capture necessarily requires consideration of global dynamics, which is characterized by responses far from a specific steady-state response. Such problems are well understood in the context of phase-plane analyses, wherein a separatrix, or, the stable manifold of a saddle point, splits the plane into basins of attraction for two distinct ultimate outcomes. The basic models for resonance capture are autonomous, but these models have at least three dimensions, and the subtleties of resonance capture are beautifully described by an understanding of the structure of this phase space, in particular, the nature of a two-dimensional stable manifold (separatrix) in this space. This description of the behavior requires a non-local analysis, specifically the computation of time-varying invariant manifolds in the system phase space, which can be well approximated in many systems by using a Melnikov-type analysis. The literature on this line of global analysis can be followed by considering the early work of Rand(55) and his co-workers, followed by the more recent work of Quinn et al.(56). Applications of this behavior can be found in simple rotor systems mounted on flexible supports, as well as in satellite dynamics(57).

Other problems where nonlinearity is important in rotating systems is the dynamics of geared systems, in which, apart from bearing stiffness nonlinearities, one encounters gear backlash nonlinearities due to lack of contact (another example of piecewise smooth systems; see Section 3.2). Periodic solutions of these systems and the qualitative changes experienced by them have been extensively studied, and the work of Pfeiffer and Glocker(58) is a representative example from a large literature on this class of problems. Also, experimental investigations into whirling motions of rotors (forward and backward), and how a backward whirling may be caused due to a combination of the effects of gravity loading and shaft/bearing flexibility continues to be a topic of interest[59].

At the current time, an understanding of the dynamics of rotors continues to be of importance for systems that generate and transmit power, and the rich source of nonlinearities in these systems continues to provide a fertile ground for basic and applied research, which is being pursued by using a wide variety of tools, including those from modern dynamical systems theory.

2.3. Vehicle Dynamics

Vehicles of different types, including railway vehicles, motorcycles, tractor-trailer vehicles, ships, aircraft, and underwater vehicles, have been shown to exhibit nonlinear behavior of different types. Over the last two decades and more, extensive experimental, analytical, and numerical research has been carried out to explain wheel shimmy (lateral oscillations of wheels), hunting motions (lateral swaying motions) of railway cars, motorcycle weave motions, vehicle spin outs, and fluttering of vehicle trailers. Here, as representative examples, the accent of this section is on the wheel shimmy and hunting phenomenon.

Wheel shimmy refers to a phenomenon where the wheels exhibit a periodic or aperiodic lateral dance at certain forward speeds of the vehicle. Lateral oscillations of aircraft nose landing gear sparked an early interest in this phenomenon around the 1930s and 1940s[60]. Since then, their occurrence has been studied through experimental, analytical, and numerical means, and they still continue to be of interest to the aircraft industry[61]–[63]. Apart from aircraft landing gears, wheel shimmy has been shown to occur in motorcycles[64]–[66], tractor-trailers, and other vehicles[67]. In certain speed ranges, these oscillations can be described as self-excited oscillations, similar to those that occur in manufacturing systems, as discussed in Section 2.4. Models with rigid tires and elastic casters as well as models with rigid casters and elastic tires (typically, pneumatic tires) have been used to study wheel shimmy. The nonlinearities that arise in these models can be attributed to one or more of the following: i) tire elasticity in the lateral direction, ii) suspension elasticity in the lateral direction, iii) contact between wheel and the ground, and iv) friction and free play. When contact nonlinearities and creep-force nonlinearities for the tire forces are considered, the models do not lend themselves to an analytical approach and one has to carry out numerical simulations to study this phenomenon. Although different models have been used, from a dynamics standpoint, wheel shimmy has been interpreted as limit-cycle oscillations that follow a supercritical Hopf bifurcation of a fixed point[62] or a subcritical Hopf bifurcation of a fixed point[68]. In recent work[69], the occurrence of subcritical Hopf and supercritical Hopf bifurcations of a fixed point have been investigated with respect to the system lateral damping, and it has been shown that an increase in this damping level leads to a supercritical Hopf bifurcation. Quasi-periodic shimmy oscillations have also been shown to occur by using models with time delays, where the delay allows one to account for the wheel-motion history in the contact patch[70]; these oscillations have been experimentally confirmed. Supercritical bifurcations of the wobble mode of motorcycles associated with wheel shimmying and subcritical bifurcations of the weave mode of motorcycles have also been shown to occur through the use of nonlinear multi-body models[71] and application of continuation tools such as AUTO97[72].

Like wheel shimmy motions, hunting motions are self-excited oscillations that occur in the lateral plane of a railway vehicle, the stability of which has been studied for nearly 100 years. It is believed that the nonlinearities that arise due to the contact forces between the wheel and the rail, the suspension system, and the contact kinematics are important for modeling this phenomena[73],[74]. Nonlinear motions of different types of vehicles including passenger vehicles that have highly developed suspensions and freight wagons that have less developed suspensions have been studied, and it is understood that Hopf bifurcations would be important to understand the onset of hunting motions. As dry friction is used for damping the motions of freight wagons, possibilities for stick-slip oscillations arise in these systems. Qualitative changes of motions with respect to the vehicle speed leading to chaotic motions have been documented[75]. True[74] has covered a broad range of topics related to rail vehicle dynamics. No doubt, with dry friction, tools that apply to non-smooth systems would need to be brought to bear upon freight vehicle dynamics. In recent times, gryoscopic dampers have been explored to suppress hunting motions by Yabuno et al.[76].

Nonlinear dynamics also plays a prominent role in other types of vehicles. For example, in the context of ships[77]–[79], one needs to consider nonlinear dynamics to identify regions where a vessel can be operated safely without capsizing. In recent years, nonlinear dynamics of advanced underwater vehicles such as supercavitating vehicles has also received considerable attention[80]. Supercavitating vehicles represent examples of piecewise smooth systems, which are further discussed in Section 3.2. With the growing complexity and sophistication of models being used to study the dynamics of different land based vehicles and others, it is quite likely that most of the studies will be numerical in nature.

2.4. Machining and Manufacturing Systems

The art of metal cutting extends to more than a century[81], and machining is still a principal manufacturing technology of importance to aerospace, automobile, electronics, and many other industries. In recent years, the focus has been on high-speed machining, high-precision machining, and machining of hard materials. High-speed milling has, among many benefits, the improved productivity associated with increased rates of metal removal[82],[83]. Due to their many attractive aspects, high-speed milling has been viewed as a viable alternative to other forms of manufacturing. For instance, in the aerospace and other industries, high-speed milling capabilities allow for design concepts such as unitized assemblies, thinner structural elements for weight reduction, and substantially reduced requirements for de-burring and hand-finishing machined components.

Manufacturing and machining systems serve as a prototypical example of mechanical systems with time delays. Nonlinear effects in machining systems can arise from a variety of sources including cutting forces, loss of contact between workpiece and tool, material properties, friction between tool and metal chip, and flexibility of tool and/or workpiece. The nonlinearity of the cutting force was experimentally documented by Taylor[81], who also recognized the undesirable dynamic state called chatter. This dynamic state, which is not necessarily unstable in the sense of dynamics, can be characterized by "large" displacements of the workpiece-tool system, and in certain operations, basic frequency-response components different from those of the source of excitation. In the 1940s and 1950s, some insightful experimental observations on chatter were made. Arnold[84] experimentally studied the occurrence of chatter with increase in cutting speed in lathe experiments, paving the way to later studies in which the occurrence of chatter was systematically studied with respect to other machining parameters. Doi and Kato[85] experimentally established the time-delay basis for the chatter problem. Since the 1960s, an extensive number of experimental, analytical, and numerical efforts have been devoted to predicting chatter, understanding chatter, and other nonlinear phenomena associated with drilling, turning, grinding, and milling operations. Through these efforts, it is widely recognized that chatter can be caused by regenerative (time-delay) mechanisms and/or other mechanisms such as friction, coupling between different vibration modes, and loss of contact effects.

To understand the time-delay effect, let us consider a milling operation where a rotating cutter with multiple teeth is used to cut a workpiece. The rotation of the cutter is specified by what is called the spindle rotation speed and the amount of material to be removed during each pass is specified by what is called the chip thickness. The cutting tool produces a wavy workpiece surface, whose amplitude depends on the specified chip thickness and wavelength is determined by the spindle rotation speed. The current tooth that engages the workpiece cuts over the wavy surface left behind by the previous cutter tooth. So, what is uncut from a previous pass excites the workpiece-tool system, and the resulting vibrations modulate the chip thickness and thus also the cutting forces that in turn excite the workpiece-tool system.

The imprint of these vibrations is left on the workpiece surface. The cutting forces depend on the relative displacement between the tool and the workpiece at the current instant and a previous instant that is delayed by the period of the spindle rotation. Due to the physical mechanism involved, this effect is also referred to as a regenerative effect, and the associated oscillations are called self-excited oscillations.

For a stable milling operation, the periods of the waves produced on the workpiece surface correspond to the spindle rotation speed. However, when chatter occurs, in addition to these waves, there are other waves produced on the wavy surface, whose periods could be either a higher multiple (say two times) of the basic spindle rotation period or uncorrelated with the spindle rotation period. While post-chatter motions are accepted as nonlinear motions, many of the early efforts were focused on predicting chatter through a linear stability analysis. Most of the earliest stability treatments were aimed at determining the onset of chatter in turning operations. In this regard, frequency-domain analysis was used to examine the frequency-response function(s) between the system displacement(s) and cutting force(s) and determine the critical delay associated with the instability of a turning operation(86). From the standpoint of dynamics, this instability that was also investigated by Tobias(87) and Merritt(88), is the loss of stability of a fixed point of a delay differential system giving rise to a periodic motion. This is an example of a Hopf bifurcation(89), classically known as a flutter instability. While the dynamical systems used to describe turning operations are autonomous systems with time delays, the dynamical systems used to describe milling operations are non-autonomous systems with time delays. To predict the onset of chatter in a milling operation, one needs to examine the loss of stability of a periodic solution of a non-autonomous system(90). In many early efforts in the literature, milling operations were studied as equivalent turning operations and the stability analysis used for a turning operation was also used for the milling operation. In doing so, the possibility of period-doubling bifurcation of a periodic solution was missed in early studies on milling dynamics.

Hooke and Tobias(91) conducted metal cutting experiments in which a jump was observed from stable cutting to a chatter state, and their first experimental evidence of subcritical instability has been confirmed for turning(92)−(94) and boring operations(95). The presence of a subcritical instability is clearly not desirable from a machining operation standpoint, as stable operations could be perturbed leading to large changes in the response. Hanna and Tobias(92) used a model with a time delay and quadratic and cubic nonlinearities stemming from both the cutting force and the structure in their nonlinear analysis of chatter. This model and the loss of stability of solutions of this delayed system was examined in a comprehensive manner by Nayfeh, Chin, and Pratt(96), whose work provides an illustrative example of the use of perturbation methods to carry out local analysis to predict the onset of chatter and non-local analysis to examine post-chatter motions. They showed the possibility for a supercritical Hopf bifurcation of a fixed point. The motions that follow a supercritical Hopf bifurcation of a fixed point are not as severe as those that follow a subcritical Hopf bifurcation of a fixed point. However, through a non-local analysis, Nayfeh, Chin, and Pratt(96) show that the post-Hopf-bifurcation motions (or post-chatter motions) could loose stability through a cyclic-fold bifurcation. This is clearly undesirable from a machining operation standpoint. Considerable debate has occurred on whether the subcritical instability seen in the experiments is a local instability or one that follows immediately after a supercritical instability. Stépán and Kalmár-Nagy(97) show that a softening type cutting force model can result in a subcritical instability of a fixed point, which is further discussed in the articles of Moon and Kalmár-Nagy(98) and Stépán(99). Pratt and Nayfeh(100) experimentally and analytically show the possibility for subcritical Hopf instabilities in a boring problem, which is modeled by a system with two degrees of freedom plus time delays, extending the earlier work on turning operations in which planar nonlinear systems plus delays were investigated. Quasiperiodic oscillations and qualitative changes experienced by them were also examined in the context of turning operations. All of the abovementioned studies help establish the key role played by cutting force nonlinearities in the occurrence of chatter. Nonlinear feedback, which is based on bifurcation control ideas[101], has also been suggested for suppressing chatter in boring operations[100].

The role of friction in causing undesirable motions of a machining system have been examined in many studies, with good representative efforts being those due to Grabec[102], Wiercigroch[103], and Wiercigroch and Krivstov[104]. In these studies of turning operations, friction between the tool and metal-chip workpiece contact surfaces and the intermittent contact between the tool and the workpiece are taken into account in models of turning operations. Birth and death of periodic motions, and qualitative changes culminating in chaotic motions have been studied. The complexity of chip formation during metal cutting, the importance of thermal effects, and the possible qualitative change leading from a continuous to segmented chip formation were examined in the work of Davies and Burns[105].

Dynamics of milling operations is influenced by a number of factors including friction between the tool and chip, loss of contact between tool and workpiece, regenerative effects, and coupling between different modes of vibration[90]. As the rotation speed of the milling cutter is increased, the nature of cutting is highly interrupted and the amount of time spent cutting the workpiece to the time spent not cutting decreases. This has been studied in a number of elegant treatments, with a representative one being that due to Stépán and co-workers[106]. When the cutting time is short, discrete mathematical models can be developed to show the occurrence of period-doubling and Neimark-Sacker bifurcations. The subcritical nature of these bifurcations have also been examined, and the non-local behavior has been examined by using numerical simulations to help understand the complexity of motions seen in low-immersion milling experiments. Some of this behavior is also confirmed by the work of Long and Balachandran[107], who have also studied the influence of the feed rate on nonlinear dynamics of a milling operation. In related recent work[108], it has been pointed out that the feed rate leads to a variable delay term in the governing equations and period-doubling and Neimark-Sacker bifurcations are also possible in these systems. Systems with variable time delays and multiple time delays still remain a topic of active research.

2.5. Vibration Control

The general field of vibration control is quite broad and covers both active and passive approaches. Here, we focus on a subset of topics in which nonlinear dynamics has played a central role.

An important approach to passive vibration control has been the use of tuned vibration absorbers. In these systems, a secondary subsystem is attached to the system of interest, and this secondary subsystem is tuned in such a manner that the response of the modified system is improved when compared with that of the original system. In most applications, the absorber is tuned so that its natural frequency is close a problem resonance frequency of the original (primary) system, and the response of the combined system has a lower amplitude compared to that of the original system in the vicinity of the problem frequency. In order to reduce the response amplitudes over a frequency range encompassing the problem frequency, damping is employed in the absorber, thus, replacing one sharp resonance peak with two low amplitude peaks. The classic linear analysis for the simple two degree-of-freedom model can be found in the work of Den Hartog[2]. Many extensions have been explored in the linear realm, typically by using more degrees of freedom or by tuning parameters on the fly, leading to so-called "semi-active" absorber systems. A literature review of these topics and the state of the art as of 1995 can be found in the review article by Sun et al.[109]. Nonlinearities in tuned absorbers can be used to maximize the vibration suppression bandwidth or reduce the amplitude of the primary system, as described in many papers[110]–[114]. However, nonlinearities can also lead to combination resonances that are not desirable in the system response[115],[116]. Perhaps, the earliest passive device that makes use of a purely nonlinear response for vibration suppression is the "autoparametric" vibration absorber of Haxton and Barr[117], which has continued to receive attention[118]–[120]. The idea behind this absorber is to attach the absorber to the primary system in such a manner that it experiences a parametric base excitation, and therefore, one tunes the absorber frequency to be around one-half of the troublesome frequency value. The governing system of equations has quadratic nonlinearities, whose influence is activated by this tuning, and this enables a parametric resonance to be excited and the absorber to respond in the desired frequency range. This device does provide vibration suppression of the primary system response. However, a significant shortcoming of this absorber is that the force acting on the primary system from the dynamic response of the absorber scales like the square root of the excitation amplitude, so that large absorber motions are required to achieve acceptable vibration reduction. In addition, one also needs to take note of the complex dynamics exhibited by systems with autoparametric absorbers. As an example, we cite the study of Bajaj, Vyas, and Raman(120) in which a primary single degree-of-freedom system fitted with a multi-pendula autoparametric absorber system has been explored for wide bandwidth suppression.

Another novel method of vibration suppression, which is closely tied to the "autoparametric" vibration absorber, makes use of the so-called saturation effect in systems that possess a 2:1 internal resonance. This phenomenon, which was first uncovered in the context of ship motions in the 1970s by Nayfeh, Mook, and Marshall(7) and later studied in the context of structural systems by Haddow, Barr, and Mook(121), relies on the presence of quadratic nonlinearities in the system. In these systems, as the excitation amplitude increases, the response of the directly excited vibration mode saturates at an essentially fixed amplitude, while that of the other vibration mode (the indirectly excited mode whose natural frequency is one half of that the directly excited mode) grows and "soaks up" the vibration energy. An absorber based on this behavior was first described by Golnaraghi and co-workers(122) and subsequently investigated in detail by Nayfeh and colleagues(123). A novel aspect of this means of vibration suppression is that the primary structure or system does not need to have an inherent 2:1 internal resonance, since this resonance can be induced by including an actuator with the appropriate (quadratic) nonlinear characteristics to the structure and/or through the associated electronics, and implementing a control loop that is tuned so that the closed loop system has the desired properties. Thus, the system is tuned into the 2:1 internal resonance, with the saturation mode being the target vibration mode of the structure whose amplitude is to be suppressed. In this manner, a feedback control system is used to obtain a response that mimics a well-understood open-loop nonlinear response with desirable features. One has to be careful in using this type of suppression, since the saturation phenomenon can breakdown for certain disturbance parameter values resulting in aperiodic motions that may not be desirable(124). However, this approach has been shown to be appealing for different applications, including suppression of aeroelastic flutter(125). Implementations of the saturation phenomenon based controllers with active materials such as piezoceramic materials to actuate the structures have also been carried out(126).

Absorbers can also be used for reducing torsional vibrations in rotating systems. When such absorbers are tuned to a given frequency, they are similar in behavior to the translational absorbers described above. The more interesting case is when the absorbers are tuned to a given order of rotation, as in the case of the centrifugal pendulum vibration absorber. These absorbers consist of centrifugally-driven pendula that are tuned to a given order of rotation. The nonlinear behavior of these devices was first pointed out by Den Hartog(127), who noted that the frequency shift from softening nonlinear effects, arising from the circular path followed by the absorber mass, would be detrimental to absorber function. These properties were thoroughly fleshed out and translated into design recommendations by Newland(128), whose intentional over-tuning recomendations are still in use for designing some absorber systems. More recently, these absorbers have been designed such that they follow noncircular paths that alleviate some of the unwanted nonlinear effects. For example, cycloids, which have a slightly hardening nonlinearity, have been employed for helicopter rotors(129). More interesting are the special tautochronic (that is, equal period, or frequency) epicycloidal absorber paths that render the absorber motion linear out to large amplitudes. These have been proposed, analytically investigated[130], and experimentally studied[131]. Among the interesting features of these systems are the possibilities of instabilities that arise when multiple identical absorbers are employed[132)–(134], impact responses[135], and superharmonic resonances[136]. There have been very recent applications to automotive engines that employ cylinder deactivation[137]. It is mentioned that absorbers constrained to follow circular paths in non-rotating systems have also been explored to suppress ship crane-load oscillations and shown to help suppress bifurcations that could lead to large amplitude responses of the crane load[138],[139]. In many systems, nonlinear responses of a primary system are inevitable because of the disturbance parameter values, and the ideas of bifurcation control[101], wherein one shifts bifurcation points out of a range of interest or suppresses bifurcations to achieve the desired response could be useful.

Impacts that dissipate energy have also been used for vibration suppression. The impact damper is device that accomplishes this by the addition of a supplemental system, the absorber, to the primary system such that, when the primary system is excited, it undergoes impacts with the absorber. If these impacts dissipate energy, it is possible to reduce the vibration amplitudes of the primary system. These ideas have been around for many decades, at least since the work of Lieber and Jensen in 1945[140]. An early systematic and thorough treatment from a dynamic stability point of view is that of Masri and Caughey[141], who studied the steady-state responses and their stability (using point mapping techniques) for a two degree of freedom model. Subsequently researchers found that such vibro-impact systems can exhibit a wide range of motions, including chaos, and this led to many research endeavors in this area, including experimentation. A review of key works on this and other vibro-impact problems has been carried out by Peterka[142]. These problems, due to the abrupt nature of the forces involved in their dynamics, have been at the forefront of research into so-called piecewise smooth systems, which are briefly considered in Section 3.2.

A recent development in the use of nonlinearity for vibration control is the idea of using targeted energy transfer (TET) (also known as nonlinear energy pumping) and nonlinear energy sinks (NES). Nonlinear energy pumping occurs when vibrational energy is transferred via TET to a particular subsystem, designed to be an NES, through nonlinear coupling effects. Since the NES dissipates the energy before it transfers back to the main system, this flow of energy is essentially one-way. In fact, this is an example of resonance capture, as discussed in Section 2.2 in the context of nonideal systems. The NES is designed to have a purely nonlinear stiffness (that is, zero linear stiffness), so that it acts as a broadband nonlinear absorber, which effectively traps and dissipates energy across a wide range of frequencies. The fundamental idea for this phenomenon for passive vibration suppression was first laid out by Gendleman and Vakakis[143],[144], and soon thereafter, developed for the purpose of transient vibration control by Vakakis and several co-workers. This area has blossomed in recent years, and now involves research groups from around the world. This line of work has led to fundamental contributions to the area of nonlinear resonances, for example as described by Quinn et al.[145], and also led to the development of devices that exploit these phenomena for applications, including suppression of aeroelastic flutter[146],[147] and isolation from seismic excitation[148]. A forthcoming review article[149] and book[150] survey these topics and provide a quite complete review of the literature on the subject.

There are other systems, such as particle impact dampers, in which nonlinearities play a key role in suppressing the motions of the primary system. The dynamics of these systems is quite challenging and it appears that aperiodic motions are preferred in order to realize an effective vibration damper. Also, guided by local analysis, ideas such as bifurcation control have been and are being explored. Nonlinear dynamics based vibration control is yet to be fully explored, and we expect this area to be an active and fascinating one of continued interest.

3. Emerging Areas of Mechanical Systems

3.1. Micro- and Nano-Electromechanical Systems

A rapidly developing field of technology that is benefiting from our understanding of nonlinear dynamics is that of micro- and nano-electromechanical systems, which we will collectively refer to here as MEMS. From a basic viewpoint, MEMS are nothing more than very small mechanical devices, fabricated by using methods developed originally and primarily for miniaturized integrated circuits in the early 1960s. In these mechanical devices, many forces come into play, such as electrostatic and van der Waals forces, which one does not need to consider in large scale devices. The possibility of such small mechanical devices was first popularized in a famous lecture by Feynman in 1959(151), and noted in a more specific way by Peterson two decades later(152). Since that time, the field has developed by leaps and bounds, and now it involves many thousands of researchers from physics, chemistry, mechanical and electrical engineering, materials science, and biology. A significant variety of MEMS utilize a dynamic mode of operation, typically, some type of high-frequency vibration. An example of such systems is micro-mechanical resonators, which have been realized in the form of electrostatically or piezoelectrically actuated beams, plates, membranes, tuning forks, folded beams, rings, and disk systems. These mechanical resonators have very high natural frequencies, some in the GHz range, and extremely small damping(153). These resonators have a number of applications, mostly, filters(154) and transmission lines in the area of communications and signal processing, and sensors, including rate gyros and accelerometers(155),(156). MEMS have also been used in optical systems, such as the digital micromirror device of Texas Instruments(157). A majority of these devices have been designed to operate in a linear response regime, exploiting the very sharp resonance and the attendant predictable response features of the devices. It is becoming clear that in several applications certain features of nonlinear responses may offer potential for an enhanced performance, and research on such topics is in full force. Furthermore, as pointed out in recent work, as one approaches the nano-scales, it is inevitable that the devices will operate in the nonlinear regime(158).

The sources of nonlinearities in MEMS include structural nonlinearities due to stretching, bending, and buckling, electrostatic or piezoelectric actuation, and dissipation experienced by these systems. While the possibility for nonlinear effects such as buckling in micro-scale structures was recognized in the early 1990s(159), investigations of nonlinear dynamics in MEMS started with the observations of a nonlinear parametric resonance in the torsional vibrations of micro-mirrors by Turner and co-workers in the late 1990s(160),(161). Since then, work has focused on basic characterization of nonlinear behavior in MEMS, and attempts have been made to exploit such behavior in order to enhance the desired functionality of devices. Arguably, the most promising application area is that of sensors, in particular, for mass detection and for measuring rotation rates using MEMS gyroscopes. Mass sensing in MEMS make use of shifts in dynamic properties that occur when certain target substances (analytes) are present in the environment that a MEMS sensor is exposed to(162). For example, a common approach is to coat the surface of a micro-resonator with a substance that attracts a specific compound, making it adhere to the device, thereby changing its mass (and/or stiffness). The resulting shift in the location a linear resonance peak is then used to detect the presence of the analyte(163). This approach has nonlinear versions, since changes in mass also result in shifts in bifurcation points that can be measured. This approach has been extensively investigated on the basis of the resonances of Düffing type systems, for example, by Buks and Yurke(164), and using parametric resonances by Turner and co-workers(165). These changes are very abrupt and they are useful for on-off type detection, but they are not so useful for determining how much of a substance is present. For example, consider the case in which the excitation frequency is swept past a bifurcation point, resulting in a sudden jump in the response amplitude, such as that occurs near a typical nonlinear resonance (external or parametric resonance). The bifurcation point depends in a known manner on the resonator natural frequency ω_n, and thus, changes in the bifurcation point can be correlated with changes in ω_n, and through that, the presence of an analyte. One very appealing feature of using a parametric resonance is that the location of the bifurcation point does not have a strong dependence on the resonator damping(165) (so long as the damping and forcing levels are in the range where the parametric instability can occur). One interesting and rather unique feature of parametric resonances in MEMS is that they quite often exhibit competing nonlinear effects, typically mechanical hardening and electrostatic softening, which can lead to a rich variety of behaviors, including systems that can experience mixed softening and hardening responses(166).

Related to the use of bifurcations for sensing is the notion of "bifurcation amplifiers." The idea behind these systems is to use the very sharp transitions across bifurcations to amplify signals(167). For example, consider the case where a parameter, say forcing amplitude or frequency, is nominally held very close to a pair of coexisting jump points, where the response can go back and forth between two stable response branches. (This often requires special tuning of the system parameters.) If that parameter is oscillated back and forth across such a point, the output signal, that is, the system response, will, under certain conditions, move back and forth across the jumps, providing very large gains of the input signal. MEMS examples of such amplifiers, which make use of nonlinear mechanical behavior, include nano-scale beams that behave as Düffing-like systems(168), and coupled nano-beams with internal resonances(169). The operations of these devices requires that one sweep parameters across bifurcations, which are non-stationary problems of the type considered by Mitropolskii(52) and many others. However, in this case, noise is also an essential ingredient for understanding the system dynamics, since the sweep rate and noise both contribute to the system's propensity to jump across response branches. These problems have been considered by many researchers(170). Note that such "amplifiers" do not receive a signal amplitude and simply increase it by some multiple, as done by a linear amplifier. Rather, these amplifiers can have very large gains, but the output amplitude is not generally proportional to the input amplitude. Thus, they are useful for detection of specific events, rather than for general amplification.

Another specific application where nonlinear effects in MEMS may be beneficial is in gyroscopes used to measure rates of rotation. These rate gyroscopes make use of two (nearly) uncoupled vibrational modes with (nearly) equal frequencies that are coupled by Coriolis effects arising from rotation. The basic operation of such gyroscopes involves resonance excitation of the drive mode, and measurement of the sense (undriven) mode. Complications arise in the practical implementation of these sensors due to the following: (i) static mechanical mode coupling, known as quadrature error, which leads to spurious signals not arising from the rotation, and (ii) mismatched modal frequencies, which limits response sensitivity and output(171). One means of overcoming the mismatch problem is to use multiple degrees of freedom for the drive mode, thereby widening its bandwidth(172). Another means of achieving the same goal, by using only a single degree of freedom, is to exploit nonlinearities in the drive and/or sense mode. This has been considered for cases in which the drive mode experiences a direct excitation(173),(174), as well as a parametric excitation(175). In these cases, the hardening effects of the mechanical suspension(174) and the softening or hardening effects of electrostatic actuation(166), can be used to tune the nonlinear response so that the stable branch of the resonance response curve covers a wide frequency range. Thus, the system can be driven at any frequency in this range with a resonance response, and so one simply needs to excite the system at the resonance of the sense mode in order to obtain the desired response. This has been experimentally achieved(173),(175), and it offers a practical solution to the mismatch problem in rate gyroscopes.

Bistable MEMS devices have also been conceived for a number of applications including micromirrors(157), micro-pumps(176), and relays(177). Interesting observations about jumps from oscillations about one equilibrium position to oscillations about the other equilibrium position have been reported through simulations and experiments(178). Dynamically bistable MEMS have also been of recent interest, and have provided a nice platform for investigating basins of attraction(179). As noted previously, as many of the micro-scale systems involve structural elements of some type, one has to be careful in developing models that are used for studies of nonlinear dynamics of these systems. In this context, modeling efforts undertaken and the reduced-order models developed for micro-scale beam and plate structures with electrostatic actuation are reviewed in the book by Nayfeh and Pai[20]. It has also been recognized in some recent efforts that buckling influenced dynamics is important to take into account in clamped-clamped micro-scale structures used for filtering and other applications[34],[35].

Nonlinear phenomena such as synchronization and localization have also been receiving a lot of attention in the context of micro-mechanical and nano-mechanical oscillator arrays, which are being viewed for computing applications. Frequency entrainment and synchronization during subharmonic excitations, and excitations with rational commensurate frequencies, were experimentally demonstrated in a coupled array of two interconnected clamped-clamped beams whose lengths were in the micro-scale range and widths were in the nano-scale range[180]. There are also exist connections between localization phenomena and nonlinear vibration modes, as discussed in the recent work of Dick, Balachandran, and Mote Jr.[181]. Experimental work on arrays of microbeams have demonstrated a wide range of interesting behavior, including so-called "intrinsic localized modes"[182],[183].

Nonlinear dynamics of beam-type structures is of considerable interest in the context of atomic force microscopy (AFM). Different types of bifurcations unique to smooth and non-smooth mechanical systems (see Section 3.2) have been shown to occur in beam models of AFM cantilevers[184],[185]. It is interesting to note that a commercial product[186] is now available, in which higher beam modes excited by impacts are used in tapping mode AFM. This approach has been used to obtain surface material properties along with the usual topographical information[187].

Limit cycles or self-oscillations are also of interest in different MEMS applications. These include an accelerometer that has linear mechanical behavior and uses nonlinear electronics to provide self-oscillations[188], and an optically driven disk resonator that makes use of both mechanical and thermal dynamics[189]. These are but two of many such systems that have potential applications for sensing and signal processing. No doubt, a considerable amount of interesting work is going on nonlinear dynamics of MEMS. This remains a fertile ground and much remains to be done, particularly in the arena of coupled oscillators at the micro-scale and nano-scale.

3.2. Piecewise Smooth Systems

A piecewise smooth system is one for which the mathematical model describing its dynamics consists of a collection of smooth models that cover the system phase space, and these regions are separated by boundaries in the phase space. This leads to a non-smooth model that is not amenable to the traditional tools used and/or developed for investigating smooth systems. These models are convenient representations for abrupt changes in force, or in the slopes of forces, and they naturally arise in commonly-used models for contact problems that involve friction and/or impacts. In the area of nonlinear vibrations, perhaps the earliest work on such problems was carried out by Den Hartog in the context of Coulomb friction[190], who employed the technique of patching together along boundaries the solutions of a piecewise linear Coulomb system model in order to determine the steady-state response of a simple oscillator with dry friction. Similar methods have since been successfully employed in a wide variety of piecewise smooth problems, for example, systems with piecewise linear stiffness characteristics[191]–[195], which includes as a limiting case vibro-impact systems[196]–[198]. If one searches the literature on this subject, they will find a wide range of applications, including some mentioned elsewhere in this article, specifically, the impact damper and models for intermittent contact in rotor and gear systems, models for manufacturing systems with dry friction, and models for studying atomic force microscope cantilever dynamics. Arguably, the most important feature of piecewise smooth systems is that they can experience bifurcations that have no analog in smooth systems[199]–[201]. These arise due to the immediate transitions that can occur when certain features of the system phase space encounter a boundary as system parameters or initial conditions are varied. The most important of these are grazing bifurcations and border-collision bifurcations. As an example of what is possible, as one tunes a parameter past a certain type of grazing bifurcation, a system can immediately transition from a periodic response to a chaotic response, without any sequence of intermediate bifurcations(200).

The systematic analysis of these bifurcations began with the work of Nordmark(202), which was based on some intriguing observations of certain global features observed in single degree of freedom models for vibro-impact(194), (203). Peterka studied several systems of this type in the 1970s, but his work was largely unknown until his more recent publications in the Western literature(142), (204). Since that time there has been wide interest in the field, from basic mathematical studies to a wide variety of applications. Noteworthy among the mechanical engineering applications work is the work of Pfeiffer and co-workers, who developed a sophisticated and systematic theory for the dynamics of multi-body systems with contacts, translated this into powerful computer codes, and used these to investigate a number of important application problems in machine dynamics, including transmissions(205), (206). The recent book by di Bernardo et al.(200) provides a thorough, modern, and accessible treatment of this general class of problems, covering the fundamental analytical techniques and a number of applications, with a focus on mechanical systems and an ample collection of references. A forthcoming review article provides a more focused discussion of vibro-impact problems from an applications viewpoint(207).

Piecewise smooth systems form a subset of nonsmooth systems, for whose analyses, several continuation tools have been recently developed and are being developed. As examples, we cite the continuation tool of Dercole and Kuznetsov, SlideCont, intended for planar autonomous systems(208), and the work of Dankowicz and co-workers(209) and their continuation tool TC-HAT used for continuation of periodic solutions. These types of models arise in a number of applications beyond mechanical systems, and we expect this area to remain very active.

3.3. System Identification and Structural Health Monitoring

A fundamental scientific problem in engineering is system identification. This problem is of paramount importance for the prediction of responses of mechanical and structural systems subjected to various loading conditions and design of control schemes for these systems. The identification of linear systems has been extensively addressed in the literature, and since the 1970s, identification of nonlinear systems has also received considerable attention. In this section, the emphasis is on reviewing some recent efforts in the area of system identification and the developments in the area of nonlinear dynamics based structural health monitoring.

Nonlinear system identification efforts can be broadly classified into one of the following three categories: i) parametric identification schemes, ii) nonparametric identification schemes, and iii) schemes based on artificial neural networks (ANN), which are discrete models based on neural networks in biology. In a broad sense, one may consider an ANN as a high-order nonlinear function fit between the system output and input. Different types of parametric and nonparametric identification schemes have been proposed for nonlinear mechanical and structural systems since the early 1970s. In general, with a parametric scheme, one seeks to determine the values of parameters in an assumed model of the system to be identified, while with a nonparametric scheme, one seeks to determine the functional representation of the system to be identified. Nonparametric schemes prove to be useful when the considered system is not well understood. Since the mid 1980s, there has been a great spurt of activity in developing ANN models for systems in many disciplines, because they are believed to have a high degree of fault tolerance and adaptive or learning capabilities.

Parametric identification schemes that have been used include direct approaches, statistical quasilinearization, extended Kalman filtering, and maximum likelihood estimation methods. In several of these schemes, to ascertain the best or optimal estimate of the parameters of an assumed model from noisy data, filtering and estimation techniques are used to estimate an augmented state vector and the nominal response is determined through a numerical integration of the governing system of equations[(210)–(212)]. Parametric identification of a weakly nonlinear system was reported in the work of Hanagud and co-workers[(213)], and in this work, the nominal response is determined by using the method of multiple scales and an objective function is optimized on the basis of the Levenberg-Marquardt scheme. As opposed to time-domain parametric identification schemes that have been predominantly used, frequency-domain parametric identification schemes have been also been used by different researchers. Yasuda, Kawamura, and Watanabe[(214)] and Yasuda and Kamiya[(215)] make use of the method of harmonic balance to determine the response of multi-degree-of-freedom and continuous systems to periodic excitations. They make the fundamental assumption that the response of the periodically forced nonlinear system is periodic, which may not always be true. In subsequent work, Yasuda and Kamiya[(216)] proposed a time-domain technique suitable for estimation of parameters in models of elastic structures such as beams. Apart from structural applications, other applications of nonlinear parametric identification schemes include ship motions[(217)], aircraft landing gears[(218)], and others.

Parametric identification schemes that seek to take advantage of nonlinear resonances and known behaviors of nonlinear systems have been proposed by Nayfeh[(219)]. In recent years, there has been a trend to exploit the chaotic behavior exhibited by a nonlinear system. For example, in the work of Yuan and Feeny[(220)], the identification scheme relies on the extraction of unstable periodic orbits in a chaotic response and the use of the harmonic balance method for determining the system parameters based on the extracted periodic orbit. Prompted by observations that a chaotic signal may be used to probe a nonlinear system[(221)], the feasibility of using a chaotic excitation to drive the system to be identified has been explored by Narayanan and co-workers[(222)]. Noting that a periodically forced nonlinear system does not always exhibit a chaotic response, a motivation to use a chaotic excitation has been to ensure that the driven system exhibits features of a chaotic response irrespective of the linear or nonlinear nature of the system. The chaotic driving signal is obtained as the response of a nonlinear oscillator and scaled appropriately before it is used to excite the system of interest. In the scheme developed by Narayanan and co-workers, the response of the system to be identified is assumed to be composed of several unstable periodic orbits, a feature of a chaotic response, and one of these unstable periodic orbits is extracted. Subsequently, on the basis of this extracted response, the method of harmonic balance is used to setup a system of algebraic equations to determine the system parameters.

The nonparametric schemes used in the 1980s made use of Volterra-series and Weiner-series representations, and these schemes are complex and computationally intensive. To alleviate some of these problems, in recent schemes, the functional representation is made up of Chebyshev polynomials or other sets of orthogonal functions. The parameters of the polynomials that best fit the data are determined by making use of the orthogonality of the polynomials. In a series of efforts, best represented by the foundational work of Masri and co-workers [(223)–(226)], nonparametric identification schemes have been presented for single-degree-of-freedom and multi-degree-of-freedom systems. Masri, Miller, Saud, and Caughey[(225),(226)] first used a recursive time-domain technique to identify the linear properties of the system, and subsequently, building on this step, they used a nonparametric identification scheme that needs accurate measurements of system accelerations in response to a random, an impulse, or a deterministic excitation. In another effort, Natke and Zamirowski[(227)], proposed a scheme to identify the functional forms (polynomial representations) of damping and stiffness terms in nonlinear multi-degree-of-freedom mechanical systems. This scheme is based on measurements of the acceleration, velocity, and displacement of the system to an arbitrary excitation. Given the complexity of many mechanical systems, such as shock absorbers, isolators, and mounts, research into nonparametric identification schemes is active with efforts aimed at determining the best functional form from limited experimental data(228) and capturing behavior such as hysteresis(229) and excitation parameter dependent system characteristics(230). Despite the many developments, nonparametric identification schemes can still be computationally expensive, depending on the type and scale of the system.

As an example of the use of a ANN based scheme, we cite the work of Masri, Chassiakos, and Caughey(231). They use a three-layered feed-forward neural network to model a damped Düffing oscillator. Deterministic excitations were used during the training phase, and the generalization abilities of the network were evaluated by testing the network with selected random excitations. This study provides some important clues regarding the fault tolerance capacity and generalization capabilities of a neural network in the context of identification of a nonlinear single degree-of-freedom mechanical system. ANN based schemes have also been proposed for detecting damage in structures that exhibit a nonlinear behavior(232).

A closely related area where nonlinear dynamics based approaches are being increasingly used is structural health monitoring, wherein one seeks to assess the state of the health of a structural system with regard to an intended functionality of the considered system. These approaches are based in the time domain. In one class of these approaches, the structures are probed by a chaotic input and the resulting steady-state responses of the structural system are examined at different locations (including before and after a critical location such as a joint in a structure) by using nonlinear time series analysis tools, and the response features are compared with those of the input to detect possible changes in the health (due to damage) in the structure(233) – (235). Appropriate care needs to be exercised in choosing the chaotic input for these studies(236). In other related work, the changes in the geometrical features of an attractor associated with a structural response have been studied to detect damage in the considered system(237), (238). These structural health monitoring efforts point to new directions in the field and they promise exciting avenues for continued future research.

4. Closing

These are exciting times for nonlinear dynamics and its applications in mechanical systems. Traditional areas of mechanical systems are benefiting from modern treatments, and new areas are emerging in which mechanical system designs are being envisaged on the basis of nonlinear phenomena. Here, we have attempted to provide the reader a glimpse of some of the many interesting areas being studied in nonlinear dynamics and vibrations. This is an extremely active field of research that cuts across many disciplines, and it is felt that the selected topics would be of interest to mechanical engineers. Even within this limited scope, we have neglected many important and interesting topics. Similarly, we have no doubt overlooked many key references in those areas described herein. We hope that readers will forgive our oversights and omissions, which are inevitable given the task at hand.

It's clear that our understanding of nonlinear dynamics plays an important role in many applications. However, a question that is commonly asked of people working in this area: "Have there been any successful engineering applications of chaos?" While some applications are underway at least in an academic sense and others are waiting to be fully realized yet, one can answer in the affirmative. For example, chaotic motions are proving to be useful for system identification, and it is suspected that they would play an important role in the development of new tools for structural health monitoring. Chaotic motions could also be helpful in the design of particle impact dampers. Also, as speculated recently by Moon(239), chaos (or some type of randomness) at very small amplitudes at interfaces may be essential for the operations of some machinery. More generally, chaos is now recognized as a potential system response that occurs in many systems, and therefore, one should understand its basic features and be able to recognize it and design around it - or to include it, as needed. However, chaos is only a small part of this field.

It is our opinion that an important aspect of modern treatments of nonlinear dynamics and vibrations is the systematic treatment of systems by using a variety of analytical, geometric, and computational tools in synergy with experiments. This approach continues to bear fruit in many application areas, and it is expected to continue to do so as more engineering systems are designed to operate in and exploit nonlinear responses, and as problems become increasingly multidisciplinary.

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References

(1) P. Holmes, Ninety plus thirty years of nonlinear dynamics: Less is more and more is different, International Journal of Bifurcation and Chaos 15 (2005) 2703–2716.
(2) J. P. Den Hartog, Mechanical Vibrations, 4th Edition, McGraw-Hill, New York, 1956.
(3) A. A. Andronov, A. Vitt, S. S.E. Khaikin, Theory of Oscillations, Pergamon Press, 1966.
(4) N. Minorsky, Nonlinear Oscillations, D. Van Nostrand Company, Princeton, 1962.
(5) J. J. Stoker, Nonlinear Vibrations in Mechanical and Electrical Systems, John Wiley & Sons, New York, 1950.
(6) G. Schmidt, A. Tondl, Non-Linear Vibrations, Cambridge University Press, 1986.
(7) A. H. Nayfeh, D. T. Mook, Nonlinear Oscillations, Wiley, 1979.
(8) C. Hayashi, Nonlinear Oscillations in Physical Systems, McGraw - Hill, 1964.
(9) J. Guckenheimer, P. Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, Springer-Verlag, 1986.
(10) F. C. Moon, Chaotic Vibrations, John Wiley & Sons, 2004.
(11) J. M. T. Thompson, H. B. Stewart, Nonlinear Dynamics and Chaos, John Wiley & Sons, 2002.
(12) A. H. Nayfeh, B. Balachandran, Applied Nonlinear Dynamics, John Wiley & Sons, 1995.
(13) L. N. Virgin, Introduction to Experimental Nonlinear Dynamics, Cambridge University Press, 2000.
(14) H. Harrison, Plane and circular motion of a string, Journal of the Acoustical Society of America 20 (1948) 874–875.
(15) G. S. S. Murthy, B. S. Ramakrishnan, Nonlinear character of resonance in stretched strings, Journal of the Acoustical Society of America 38 (1965) 461–471.
(16) R. Narasimha, Nonlinear vibrations of an elastic string, Journal of Sound and Vibration 8 (1968) 134–146.
(17) T. C. A. Molteno, N. B. Tufillaro, Torus doubling and chaotic string vibrations: Experimental results, Journal of Sound and Vibration 137 (1990) 327–330.
(18) H. Yamaguchi, T. Miyata, M. Ito, Time response analysis of a cable under harmonic excitations, Proceedings of Japan Society of Civil Engineers 308 (1981) 37–45.
(19) A. H. Nayfeh, Nonlinear Interactions: Analytical, Computational, and Experimental Methods, Wiley Series in Nonlinear Science, Wiley-Interscience, New York, 2000.
(20) A. H. Nayfeh, P. F. Pai, Linear and Nonlinear Structural Mechanics, Wiley, 2004.

(21) O. M. O'Reilly, P. J. Holmes, Non-linear, non-planar and non-periodic vibrations of a string, Journal of Sound and Vibration 153 (1992) 413–435.
(22) O. M. O'Reilly, Global bifurcations in the forced vibration of a damped string, International Journal of Non-Linear Mechanics 28 (1993) 337–351.
(23) W. Y. Tseng, J. Dugundji, Nonlinear vibrations of a beam under harmonic excitation, ASME Journal of Applied Mechanics 37 (1970) 292–297.
(24) W. Y. Tseng, J. Dugundji, Nonlinear vibrations of a buckled beam under harmonic excitation, ASME Journal of Applied Mechanics 38 (1971) 467–476.
(25) S. M. Hasan, A. Haddow, Nonlinear oscillations of a flexible cantilever, in: Proceedings of the 2nd Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, 1988.
(26) T. J. Anderson, B. Balachandran, A. H. Nayfeh, Observations of nonlinear resonances in a flexible cantilever beam, in: Proceedings of the 33rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Dallas, Texas, 1992.
(27) S. A. Nayfeh, A. H. Nayfeh, Nonlinear interactions between two widely spaced modes - external excitation, International Journal of Bifurcation and Chaos 3 (1993) 417–427.
(28) T. J. Anderson, B. Balachandran, A. H. Nayfeh, Coupling between a high-frequency mode and a low-frequency mode: Theory and experiment, Nonlinear Dynamics 11 (1996) 17–36.
(29) J. Dugundji, Some aeroelastic and nonlinear vibration problems encountered in the journey to ithaca, AIAA Journal 46 (2008) 21–53.
(30) P. J. Holmes, F. C. Moon, Strange attractors and chaos in nonlinear mechanics, ASME Journal of Applied Mechanics 50 (1983) 1021–1032.
(31) F. C. Moon, Experiments on chaotic motions of a forced nonlinear oscillator: Strange attractors, ASME Journal of Applied Mechanics 47 (1980) 638–644.
(32) K. Yagasaki, M. Sakata, K. Kimura, Dynamics of a weakly nonlinear system subjected to combined parametric and external excitation, ASME Journal of Applied Mechanics 57 (1990) 209–217.
(33) K. Yagasaki, Bifurcations and chaos in a quasiperiodically forced beam: Theory, simulation, and experiment, Journal of Sound and Vibration 183 (1995) 1–31.
(34) H. Li, S. Preidikman, B. Balachandran, C. D. Mote Jr., Nonlinear free and forced oscillations of piezoelectric microresonators, Journal of Micromechanics and Microengineering 16 (2006) 356–367.
(35) H. Li, B. Piekarski, D. DeVoe, B. Balachandran, Nonlinear forced oscillations of piezoelectric microresonators with curved cross-sections, Sensors and Actuators A in press.
(36) S. A. Tobias, Non-linear forced vibrations of circular disks. an experimental investigation, Engineering 186 (1958) 51–56.
(37) N. Yamaki, Influence of large amplitudes on flexural vibrations of elastic plates, Zeitschrift fur Angewandte Mathematik and Mechanik (ZAMM) 41 (1961) 501–510.
(38) E. H. Dowell, Nonlinear oscillations of a fluttering plate, part 1, AIAA Journal 4 (1966) 1267–1275.
(39) E. H. Dowell, Flutter of a buckled plate as an example of a chaotic motion of a deterministic autonomous system, Journal of Sound and Vibration 85 (1982) 333–344.
(40) P. J. Holmes, Bifurcations to divergence and flutter in flow-induced oscillations: A finite dimensional analysis, Journal of Sound and Vibration 53 (1977) 471–503.
(41) X. L. Yang, P. R. Sethna, Local and global bifurcations in parametrically excited vibrations of nearly square plates, International Journal of Non-Linear Mechanics 26 (1991) 199–220.
(42) Z. C. Feng, P. R. Sethna, Global bifurcations in the motion of parametrically excited

thin plates, Nonlinear Dynamics 4 (1993) 389–408.
(43) A. F. Vakakis, L. I. Manevitch, Y. V. Mikhlin, V. N. Pilipchuk, A. A. Zevin, Normal Modes and Localization in Nonlinear Systems, Wiley, 1996.
(44) I. T. Georgiou, A. K. Bajaj, M. Corless, Slow and fast invariant manifolds, and normal modes in a two-degree-of-freedom structural dynamical system with multiple equilibrium states, International Journal of Non-Linear Mechanics 33 (1998) 275–300.
(45) I. T. Georgiou, I. B. Schwartz, Dynamics of large scale coupled structural/mechanical systems: A singular perturbation/proper orthogonal decomposition approach, SIAM Journal on Applied Mathematics 59 (4) (1999) 1178–1207.
(46) G. Constable, R. Sommerfeld, Greatest Engineering Achievements of the 20th Century, U.S. National Academy of Engineering, 2008.
(47) A. Tondl, Some Problems of Rotor Dynamics, Chapman and Hall, 1965.
(48) T. Yamamoto, Y. Ishida, Linear and Nonlinear Rotordynamics: A Modern Treatment with Applications, Wiley-Interscience, 2001.
(49) Y. Ishida, Nonlinear vibrations and chaos in rotordynamics, JSME International Series C- Dynamics, Control, Robotics, Design and Manufacturing 37 (1994) 237–245.
(50) F. F. Ehrich, Some observations of chaotic phenomena in high speed rotordynamics, Journal of Vibration and Acoustics 113 (1991) 50–57.
(51) F. F. Ehrich, Observations of subcritical superharmonic and chaotic response in rotordynamics, ASME Journal of Vibration and Acoustics 114 (1992) 93–100.
(52) Y. A. Mitropolsky, Problems of the Asymptotic Theory of Nonstationary Vibrations, Davey, 1965.
(53) J. M. Balthazar, D. T. Mook, H. I. Weber, R. M. Brasil, A. Fenili, D. Belato, J. Felix, An overview on non-ideal vibrations, Meccanica 38 (2003) 613–621.
(54) M. J. Dantas, J. M. Balthazar, On the existence and stability of periodic orbits in non ideal problems: General results, ZAMP 58 (2007) 940–958.
(55) R. H. Rand, R. J. Kinsey, D. L. Mingori, Dynamics of spinup through resonance, International Journal of Nonlinear Mechanics 27 (1992) 489–502.
(56) D. D. Quinn, R. H. Rand, J. Bridge, The dynamics of resonant capture, Nonlinear Dynamics 8 (1995) 1–20.
(57) R. Haberman, R. H. Rand, T. Yuster, Resonance capture and separatrix crossing in dual-spin spacecraft, Nonlinear Dynamics 18 (1999) 159–184.
(58) F. Pfeiffer, C. Glocker, Multibody Dynamics with Unilateral Constraints, Wiley, 1996.
(59) H. Yabuno, Y. Kunitho, T. Inoue, , Y. Ishida, Nonlinear analysis of rotor dynamics by using the method of multiple scales, in: H. Y. Yu, E. Kreuzer (Eds.), Dynamics and Control of Nonlinear Systems with Uncertainty, Springer, 2007, pp. 167–176.
(60) B. v. Schlippe, R. Dietrich, Shimmying of a pneumatic wheel, NACA TM 1365.
(61) J. Baumann, A nonlinear model for landing gear shimmy with applications to the mcdonnell douglas g/a-18a, 81st Meeting of the AGARD Structures and Materials Panel.
(62) G. Somieski, Shimmy analysis of a simple aircraft nose landing gear model using different mathematical models, Aerospace Science and Technology 1 (1997) 545–555.
(63) I. J. M. Besselink, Shimmy of aircraft main landing gears, Ph.d., University of Delft, Delft, The Netherlands (2000).
(64) R. S. Sharp, The stability and control of motorcycles, Journal of Mechanical Engineering Science 13 (1971) 316–329.
(65) R. S. Sharp, The lateral dynamics of motorcycles and bicycles, Vehicle System Dynamics 14 (1985) 265–283.
(66) D. J. N. Limebeer, R. S. Sharp, S. Evangelou, The stability of motorcycles under acceleration and braking, Proceedings of the Institute of Mechanical Engineers 215, Part C (2001) 1095–1109.

(67) H. B. Pacejka, Tire and Vehicle Dynamics, Butterworth-Heinemann, 2002.
(68) G. Stépán, Chaotic motion of wheels, Vehicle Dynamics 20 (1991) 341–351.
(69) D. Takacs, G. Stépán, S. J. Hogan, Isolated large amplitude periodic motions of towed rigid wheels, Nonlinear Dynamics 52 (2008) 27–34.
(70) D. Takacs, G. Stépán, Experiments on quasi-periodic wheel shimmy, in: Proceedings of the ASME IDETC/CIE 2007, Las Vegas, Nevada, 2007.
(71) J. P. Meijaard, A. A. Popov, Multi-body modelling and analysis into the non-linear beahvior of modern motorcycles, Proceedings of the Institute of Mechanical Engineers 221, Part K (2007) 63–76.
(72) E. Doedel, A. Champneys, T. Fairgrieve, Y. Kuznetsov, B. Sandstede, X. Wang, Auto 97: Continuation and bifurcation software for ordinary differential equations (1997).
(73) A. H. Wickens, The dynamics of railway vehicles - from stephenson to carter, Proceedings of the Institute of Mechanical Engineers 212, Part F (1998) 209–217.
(74) H. True, Recent advances in the fundamental understanding of railway vehicle dynamics, International Journal of Vehicle Dynamics 40 (2006) 251–264.
(75) K. Tanifuji, K.-I. Nagai, Chaotic oscillation of a wheelset rolling on a rail, in: L. Segel (Ed.), The Dynamics of Vehicles on Roads and Tracks, 1996.
(76) H. Yabuno, H. Takano, H. Okamoto, Stabilization control of hunting motion of railway vehicle wheelset using gyroscopic damper, Journal of Vibration and Control 14 (2008) 209–230.
(77) R. C. Rainey, J. M. Thompson, The transient capsize diagram: a new method of quantifying stability in waves, Journal of Ship Research 35 (1991) 58–62.
(78) J. M. Falzarano, S. W. Shaw, A. W. Troesch, Application of global methods for analyzing dynamical system to ship rolling motion and capsizing, International Journal of Bifurcation and Chaos 2 (1992) 101–115.
(79) K. J. Spyrou, J. M. T. Thompson, The nonlinear dynamics of ship motions: a field overview and some recent developments, Philosophical Transactions of the Royal Society, London A 358 (2000) 1735–1760.
(80) G. Lin, B. Balachandran, E. Abed, Nonlinear dynamics and bifurcations of a supercavitating vehicle, IEEE Journal of Oceanic Engineering 32 (2007) 753–761.
(81) F. W. Taylor, On the art of cutting metals, Transactions of the ASME 28 (1907) 31–350.
(82) J. F. Kahles, M. Field, S. M. Harvey, High speed machining possibilities and needs, Annals of the CIRP 27 (1978) 551–558.
(83) R. I. King, Handbook of High-Speed Machining Technology, Chapman and Hall, 1985.
(84) R. N. Arnold, Mechanism of tool vibration in cutting of steel, Proceedings of the Institute of Mechanical Engineers (London) 154 (1946) 261–276.
(85) S. Doi, S. Kato, Chatter vibration of lathe tools, Transactions of the ASME 78 (1956) 1127–1134.
(86) J. Tlusty, M. Polacek, The stability of a machine tool against self-excited vibration in machining, in: Proceedings of Conference on International Research in Production Engineering, Pittsburgh, PA, 1963.
(87) S. A. Tobias, Machine-Tool Vibration, Wiley, 1965.
(88) H. E. Merritt, Theory of self-excited machine-tool chatter, ASME Journal of Engineering for Industry 87 (1965) 447–454.
(89) G. Stépán, Retarded Dynamical Systems: Stability and Characteristic Functions, Longman, 1989.
(90) B. Balachandran, Nonlinear dynamics of milling processes, Philosophical Transactions of the Royal Society of London A 359 (2001) 793–819.
(91) C. J. Hooke, S. A. Tobias, Finite amplitude instability: A new type of chatter, in: Proceedings of the 4th International MTDR Conference, 1964.

(92) N. H. Hanna, S. A. Tobias, A theory of nonlinear regenerative chatter, ASME Journal of Engineering for Industry 96 (1974) 247–255.
(93) M. Shi, S. A. Tobias, Theory of finite amplitude machine tool instability, International Journal of Machine Tool Design and Research 24 (1984) 45–69.
(94) J. R. Pratt, M. A. Davies, C. J. Evans, M. D. Kennedy, Dynamic interrogation of a basic cutting process, Annals of CIRP 48 (1999) 39–42.
(95) A. H. Nayfeh, J. R. Pratt, Chatter identification and control for a boring process, in: F. C. Moon (Ed.), New Applications of Nonlinear and Chaotic Dynamics in Mechanics, Kluwer, 1997.
(96) A. H. Nayfeh, C. M. Chin, J. R. Pratt, Perturbation methods in nonlinear dynamics applications to machining dynamics, ASME Journal of Manufacturing Science and Engineering 119 (1997) 1–9.
(97) G. Stépán, T. Kalmár-Nagy, Nonlinear regenerative machine tool vibrations, in: Proceedings of the ASME International Design Engineering Technical Conferences, Sacramento, CA, 1997.
(98) F. C. Moon, T. Kalmár-Nagy, Nonlinear models for complex dynamics in cutting materials, Philosophical Transactions of the Royal Society of London A 359 (2001) 695–711.
(99) G. Stépán, Modelling nonlinear regenerative effects in metal cutting, Philosophical Transactions of the Royal Society of London A 359 (2001) 739–757.
(100) J. R. Pratt, A. H. Nayfeh, Chatter control and stability analysis of a cantilever boring bar under regenerative cutting conditions, Philosophical Transactions of the Royal Society of London A 359 (2001) 759–792.
(101) E. H. Abed, H. O. Wang, A. Tesi, Control of bifurcations and chaos, in: W. Levine (Ed.), The Control Handbook, CRC Press, 1996, pp. 951–966.
(102) I. Grabec, Chaotic dynamics of the cutting process, International Journal of Machine Tools and Manufacture 28 (1988) 19–32.
(103) M. Wiercigroch, Chaotic vibrations of a simple model of the machine-tool cutting process, ASME Journal of Vibration and Acoustics 119 (2001) 468–475.
(104) M. Wiercigroch, A. M. Kristov, Frictional chatter in orthogonal metal cutting, Philosophical Transactions of the Royal Society of London A 359 (2001) 713–738.
(105) M. A. Davies, T. J. Burns, Thermomechanical oscillations in material flow during high-speed machining, Philosophical Transactions of the Royal Society of London A 359 (2001) 821–846.
(106) G. Stépán, R. Szalai, B. P. Mann, P. V. Bayly, T. Insperger, J. Gradisek, E. Govekar, Nonlinear dynamics of high-speed milling - analyses, numerics, and experiments, ASME Journal of Vibration and Acoustics 127 (2005) 197–203.
(107) X. H. Long, B. Balachandran, Stability analysis for milling process, Nonlinear Dynamics 49 (2007) 349–359.
(108) X. H. Long, B. Balachandran, B. P. Mann, Dynamics of milling processes with variable time delays, Nonlinear Dynamics 47 (2007) 49–63.
(109) J. Q. Sun, M. R. Jolly, M. A. Norris, Passive, adaptive and active tuned vibration absorbers—A survey, Journal of Mechanical Design 117 (1995) 234–242.
(110) R. E. Roberson, Synthesis of a nonlinear dynamic vibration absorber, Journal of the Franklin Institute 254 (1952) 205–220.
(111) L. A. Pipes, Analysis of a nonlinear dynamic vibration absorber, Journal of Applied Mechanics 20 (1953) 515–518.
(112) F. R. Arnold, Steady-state behavior of systems provided with nonlinear dynamic vibration absorber, Journal of Applied Mechanics 55 (1955) 487–492.
(113) A. Soom, M. Lee, Optimal design of linear and nonlinear vibration absorbers for

damped systems, Journal of Vibration, Acoustics, Stress, and Reliability in Design 105 (1983) 112–119.
(114) B. S. J. C. Nissen, K. Popp, Optimization of a nonlinear dynamic vibration absorber, Journal of Sound and Vibration 99 (1985) 149–154.
(115) H. J. Rice, Combinational instability of the nonlinear vibration absorber, Journal of Sound and Vibration 108 (1986) 526–532.
(116) J. Shaw, S. W. Shaw, A. G. Haddow, On the response of the nonlinear vibration absorber, International Journal of Non-Linear Mechanics 24 (1989) 281–293.
(117) R. S. Haxton, A. D. S. Barr, The autoparametric vibration absorber, Journal of Engineering for Industry 94 (1972) 119–225.
(118) M. P. Cartmell, J. Lawson, Performance enhancement of an autoparametric vibration absorber, Journal of Sound and Vibration 177 (1994) 173–195.
(119) A. Vyas, A. K. Bajaj, Dynamics of autoparametric vibration absorbers using multiple pendulums, Journal of Sound and Vibration 246 (2001) 115–135.
(120) A. K. Bajaj, A. Vyas, A. Raman, Explorations into the nonlinear dynamics of a single dof system coupled to a wideband autoparametric vibration absorber, in: G. Rega, F. Vestroni (Eds.), Chaotic Dynamics and Control of Systems and Processes in Mechanics, Springer, 2005, pp. 17–26.
(121) A. G. Haddow, A. B. D. Mook, Theoretical and experimental study of modal interaction in a two-degree-of-freedom structure, Journal of Sound and Vibration 97 (1984) 451–473.
(122) M. F. Golnaraghi, Vibration suppression of flexible structures using internal resonance, Mechanics Research Communications 18 (1991) 135–143.
(123) S. S. Oueini, A. H. Nayfeh, M. Golnaraghi, A theoretical and experimental implementation of a control method based on saturation, Nonlinear Dynamics 13 (1997) 189–202.
(124) B. Balachandran, A. H. Nayfeh, Nonlinear motions of beam-mass structure, Nonlinear Dynamics 1 (1990) 39–61.
(125) B. Hall, D. T. Mook, A. H. Nayfeh, S. Preidikman, A novel strategy for suppressing the flutter oscillations of aircraft wings, AIAA Journal 39 (2001) 1843–1850.
(126) P. F. Pai, B. Wen, A. S. Naser, M. J. Schultz, Structural vibration control using pzt patches and non-linear phenomena, Journal of Sound and Vibration 215 (1998) 273–296.
(127) J. P. D. Hartog, Tuned pendulums as torsional vibration eliminators, in: Stephen Timoshenko 60th Anniversary Volume, The Macmillan Company, New York, 1938.
(128) D. Newland, Nonlinear aspects of the performance of centrifugal pendulum vibration absorbers, Journal of Engineering for Industry 86 (1964) 257–263.
(129) J. Madden, Constant frequency bifilar vibration absorber, United States Patent No. 4218187 (1980).
(130) H. H. Denman, Tautochronic bifilar pendulum torsion absorbers for reciprocating engines, Journal of Sound and Vibration 159 (2) (1992) 251–277.
(131) S. W. Shaw, P. M. Schmitz, A. G. Haddow, Dynamics of tautochronic pendulum vibration absorbers: Theory and experiment, Journal of Computational and Nonlinear Dynamics 1 (2006) 283–293.
(132) C. P. Chao, C. T. Lee, S. W. Shaw, Non-unison dynamics of multiple centrifugal pendulum vibration absorbers, Journal of Sound and Vibration 204 (5) (1997) 769–794.
(133) A. S. Alsuwaiyan, S. Shaw, Performance and dynamic stability of general-path centrifugal pendulum vibration absorbers, Journal of Sound and Vibration 252 (5) (2002) 791–815.
(134) T. Nester, A. G. Haddow, S. W. Shaw, Experimental investigation of a system with

nearly identical centrifugal pendulum vibration absorbers, in: Proceedings of the ASME 19th Biennial Conference on Mechanical Vibration and Noise, Chicago, Illinois, 2003.

(135) S. W. Shaw, C. Pierre, The dynamic response of tuned impact absorbers for rotating flexible structures, Journal of Computational and Nonlinear Dynamics 1 (2006) 13–24.

(136) Y. Ishida, T. Inoue, T. Kagaw, M. Ueda, Nonlinear analysis of a torsional vibration of a rotor with centrifugal pendulum vibration absorbers and its suppression, Transactions of the Japan Society of Mechanical Engineers C 71 (2005) 2431–2438.

(137) T. M. Nester, A. G. Haddow, S. W. Shaw, J. E. Brevick, V. J. Borowski., Vibration reduction in variable displacement engines using pendulum absorbers, in: Proceedings of the SAE Noise and Vibration Conference and Exhibition, no. 2003-01-1484, Traverse City, Michigan, 2003.

(138) B. Balachandran, Y. Y. Li, C. C. Fang, A mechanical filter concept for passive and active control of nonlinear oscillations, Journal of Sound and Vibration 228 (1999) 651–682.

(139) K. V. Kaipa, B. Balachandran, Suppression of crane load oscillations using shape controlled mechanical filters, Journal of Vibration and Control 8 (2002) 121–134.

(140) P. Lieber, D. Jensen, An acceleration damper: Development, design and some applications, ASME Transactions 67 (1945) 523–530.

(141) S. F. Masri, T. K. Caughey, On the stability of the impact damper, Journal of Applied Mechanics 88 (1966) 586–592.

(142) F. Peterka, Bifurcations and transition phenomena in an impact oscillator, Chaos, Solitons & Fractals 7 (1996) 1635–1647.

(143) O. V. Gendelman, Transition of energy to nonlinear localized mode in highly asymmetric system of nonlinear oscillators., Nonlinear Dynamics, v. 25, pp. 237-253, 2001 25 (2001) 237–253.

(144) A. F. Vakakis, O. V. Gendelman, Energy pumping in nonlinear mechanical oscillators ii: Resonance capture, Journal of Applied Mechanics 68 (2001) 42–48.

(145) D. S. Quinn, O. Gendleman, G. Kerschen, T. Sapsis, L. A. Bergman, A. Vakakis, Efficiency of targeted energy transfers in coupled nonlinear oscillators associated with 1:1 internal resonance captures: Part i, Journal of Sound and Vibration 311 (2008) 1228–1248.

(146) Y. Lee, A. F. Vakakis, L. A. Bergman, D. M. MacFarland, G. Kerschen, Suppression of aeroelastic instability by means of broadband passive targeted energy transfers, part i: Theory, AIAA Journal 45 (2007) 693–711.

(147) Y. S. Lee, G. Kerschen, D. M. McFarland, W. J. Hill, C. Nichkawde, T. W. Stganac, L. Bergman, A. Vakakis, Suppresion of aeroelastic instability by means of broadband passive targeted energy transfers: part ii: experiments, AIAA Journal 45 (2007) 29931–2400.

(148) F. Nuccra, D. M. McFarland, L. Bergman, A. Vakakis, Application of broadband nonlinear targeted energy transfers for seismic mitigation of a shear frame, preprint.

(149) Y. Lee, A. F. Vakakis, L. A. Bergman, D. M. MacFarland, G. Kerschen, P. N. Panagopoulos, Passive nonlinear targeted energy transfer (tet) and its application to vibration absorption: A review, Proceedings of the Institution of Mechanical Engineers, Part K, Journal of Multi-body Dynamics to appear.

(150) A. F. Vakakis, O. Gendleman, L. Bergman, D. McFarland, G. Kerschen, Y. Lee, Passive Nonlinear Targeted Energy Transfers in Mechanical and Structural Systems, Springer Verlag, 2008.

(151) R. P. Feynman, There's plenty of room at the bottom (reprint), Journal of Microelectromechanical Systems 1 (1) (1992) 60–66.

(152) K. Petersen, Silicon as a mechanical material, Proceedings of the IEEE 70 (5) (1982) 420–457.
(153) J. Wang, J. E. Butler, T. Feygelson, C. T.-C. Nguyen, 1.51-ghz polydiamond micromechanical disk resonator with impedance-mismatched isolating support, in: Proceedings, 17th International IEEE Micro Electro Mechanical Systems Conference, pp. 641-644, 2004.
(154) J. F. Rhoads, S. W. Shaw, K. L. Turner, R. Baskaran, Tunable microelectromechanical filters that exploit parametric resonance, Journal of Vibration and Acoustics 127 (5) (2005) 423–430.
(155) N. Yazdi, F. Ayazi, K. Najafi, Micromachined inertial sensors, Proceedings of the IEEE 86 (8) (1998) 1640–1659.
(156) N. Barbour, G. Schmidt, Inertial sensor technology trends, IEEE Sensors Journal 1 (2001) 332–339.
(157) L. Hornbeck, T. R. Howell, R. L. Knipe, M. A. Mignardi, Digital micromirror device - commercialization of massively parallel mems technology, in: Proceedings of the ASME International Mechanical Engineering Congress and Exposition, Dallas, TX, 1997.
(158) H. W. C. Postma, I. Kozinsky, A. Husain, M. L. Roukes, Dynamic range of nanotube and nanowire-based electromechanical systems, Applied Physics Letters 86 (2005) 223105.
(159) U. Lindeberg, J. Soderkvist, T. Lammerink, M. Elwenspoek, Quasi-buckling of micromachined beams, Journal of Micromechanics and Microengineering 3 (1993) 183–186.
(160) K. L. Turner, S. A. Miller, P. G. Hartwell, N. C. MacDonald, S. H. Strogatz, S. G. Adams, Five parametric resonances in a microelectromechanical system, Nature 396 (6707) (1998) 149–152.
(161) K. L. Turner, P. G. Hartwell, F. M. Bertsch, N. C. MacDonald, Parametric resonance in a microelectromechanical torsional oscillator, in: 1998 ASME International Mechanical Engineering Congress and Exposition, Micro-Electro-Mechanical Systems, Vol. DSC-Vol 66, 1998, pp. 353–340.
(162) T. Thundat, P. I. Oden, R. J. Warmack, Microcantilever sensors, Microscale Thermophysical Engineering 1 (3) (1997) 185–199.
(163) T. Thundat, E. A. Wachter, S. L. Sharp, R. J. Warmack, Detection of mercury vapor using resonating microcantilevers, Applied Physics Letters 66 (13) (1995) 1695–1697.
(164) E. Buks, B. Yurke, Mass detection with nonlinear nanomechanical resonator, Physical Review E 74.
(165) W. Zhang, K. L. Turner, Frequency-tuning for control of parametrically resonant mass sensors, Journal of Vacuum Science and Technology A 23 (4) (2005) 841–845.
(166) J. F. Rhoads, S. W. Shaw, K. L. Turner, The nonlinear response of resonant microbeam systems with purely-parametric electrostatic actuation, Journal of Micromechanics and Microengineering Submitted.
(167) P. Jung, P. Hänggi, Amplification of small signals via stochastic resonance, Physical Review A 44 (1991) 8032–8042.
(168) R. Almog, S. Zaitsev, O. Shtempluck, E. Buks, High intermodulation gain in a micromechanical duffing resonator, Applied Physics Letters 88.
(169) R. Lifshitz, Physics department, tel aviv university, personal communication.
(170) V. N. Smelyanskiy, M. I. Dykman, B. Golding, Time oscillations of escape rates in periodically driven systems, Physical Review Letters 82 (16) (1999) 3193–3197.
(171) A. M. Shkel, R. Horowitz, A. Seshia, R. T. Howe, Dynamics and control of micromachined gyroscopes, in: American Control Conference, San Diego, California, 1999.
(172) C. Acar, A. M. Shkel, An approach for increasing drive-mode bandwidth of mems

vibratory gyroscopes, Journal of Microelectromechanical Systems 14 (2005) 520–528.
(173) F. Braghina, F. Restaa, E. Leoa, G. Spinola, Nonlinear dynamics of vibrating mems, Sensors and Actuators A: Physical 134 (2007) 98–108.
(174) W. O. Davis, O. M. O'Reilly, A. P. Pisano, On the nonlinear dynamics of tether suspensions for mems, Journal of Vibration and Acoustics 126 (2004) 326–331.
(175) L. A. Oropeza-Ramos, K. L. Turner, Parametric resonance amplification in a memgyroscope, in: IEEE Sensors 2005: The Fourth IEEE Conference on Sensors, Irvine, California, 2005.
(176) M. Capanu, J. G. Boyd IV, P. J. Hesketh, Design, fabrication, and testing of a bistable electromagnetically actuated microvalve, IEEE Journal of Microelectromechanical Systems 9 (2000) 181–189.
(177) E. J. J. Kruglick, K. S. J. Pister, Bistable mems relays and contact characterization, in: IEEE Solid State Sensor and Actuator Workshop, Hilton Head Island, SC, 1998.
(178) M. Sulfridge, T. Saif, N. Miller, M. Meinhart, Nonlinear dynamic study of a bistable mems: Model and experiment, IEEE Journal of Microelectromechanical Systems 13 (2004) 725–731.
(179) I. Kozinsky, H. W. Postma, O. Kogan, A. Hussain, M. Roukes, Basins of attraction of a nonlinear nanomechanical resonator, arXiv:0709.2169v1.
(180) S.-B. Shim, M. Imboden, P. Mohanty, Synchronized oscillation in coupled nanomechanical oscillators, Science 316 (2007) 95–99.
(181) A. J. Dick, B. Balachandran, C. D. Mote Jr, Intrinsic localized modes in microresonator arrays and their relationship to nonlinear vibration modes, Nonlinear Dynamics in press.
(182) M. Sato, B. E. Hubbard, L. Q. English, A. J. Sievers, B. Ilic, D. A. Czaplewski, H. G. Craighead, Study of intrinsic localized vibrational modes in micromechanical oscillator arrays, Chaos 13 (2) (2003) 702–715.
(183) M. Sato, B. E. Hubbard, A. J. Sievers, Nonlinear energy localization and its manipulation in micromechanical oscillator arrays, Reviews of Modern Physics 78 (1) (2006) 137–157.
(184) S. I. Lee, S. W. Howell, A. Raman, R. Reifenberger, Nonlinear dynamics of microcantilevers in tapping mode atomic force microscopy: A comparison between theory and experiment, Physics Review B 66.
(185) H. Dankowicz, Nonlinear dynamics as an essential tool for non-destructive characterization of soft nanostructures using tapping-mode atomic force microscopy, Philosophical Transactions of the Royal Society A 364 (2006) 3505–3520.
(186) B. Pittenger, A new view of materials, Small Times January.
(187) O. Sahin, G. Yaralioglu, R. Grow, S. F. Zappe, A. Atalar, C. Quate, O. Solgaard, High-resolution imaging of elastic properties using harmonic cantilevers, Sensors and Actuators A: Physical 114 (2-3) (2004) 183–190.
(188) S. Sung, J. G. Lee, T. Kang, Development and test of mems accelerometer with self-sustained oscillation loop, Sensors and Actuators A: Physical 109 (2003) 1–8.
(189) M. Pandey, K. Aubin, M. Zalalutdinov, R. B. Reichenbach, A. T. Zehnder, R. H. Rand, H. G. Craighead, Analysis of frequency locking in optically driven mems resonators, Journal of Microelectromechanical Systems 15 (6) (2006) 1546–1554.
(190) J. P. DenHartog, Forced vibration with combined coulomb and viscous friction, Transactions of the ASME 53 (1931) 107–115.
(191) J. P. Den Hartog, Forced vibration in nonlinear systems with various combinations of nonlinear springs, Journal of Applied Mechanics 3 (1936) 127–130.
(192) T. Watanabe, Forced vibrations of continuous system with nonlinear boundary conditions, Journal of Mechanical Design 100 (1978) 487–491.

(193) S. Maezawa, T. Watanabe, Steady impact vibrations in mechanical systems with broken-line collision characteristics, Nonlinear Vibration Problems 14 (1973) 473–500.

(194) S. W. Shaw, P. J. Holmes, A periodically forced piecewise linear oscillator, Journal of Sound and Vibration 90 (1983) 129–155.

(195) S. Natsiavas, Periodic response and stability of oscillators symmetric trilinear restoring force, Journal of Sound and Vibration 134 (1989) 315–331.

(196) J. M. T. Thompson, R. Ghaffari, Chaos after period doubling bifurcations in the resonance of an impact oscillator, Physics Letters A 91 (1982) 5–8.

(197) S. Natsiavas, Stability and bifurcation analysis for oscillators with motion limiting constraints, Journal of Sound and Vibration 141 (1990) 907–1002.

(198) P. V. Bayly, L. N. Virgin, An experimental study of an impacting pendulum, Journal of Sound and Vibration 164 (1993) 364–374.

(199) R. I. Leine, H. Nijmeijer, Dynamics and Bifurcations of Non-Smooth Mechanical Systems, Springer, 2004.

(200) M. di Bernardo, C. J. Budd, A. R. Champneys, P. Kowalczyk, Piecewise-smooth Dynamical Systems: Theory and Applications, Springer Verlag, 2007.

(201) A. Nordmark, Existence of periodic orbits in grazing bifurcations of impacting mechanical oscillators, Nonlinearity 14 (2001) 1517–1542.

(202) A. Nordmark, Non-periodic motion caused by grazing incidence in an impact oscillator, Journal of Sound and Vibration 145 (1991) 279–297.

(203) G. S. Whiston, Global dynamics of a vibro-impacting linear oscillator, Journal of Sound and Vibration 118 (1987) 396–429.

(204) F. Peterka, J. Vacik, Transition to chaotic motion in mechanical systems with impacts, Journal of Sound and Vibration 154 (1992) 95–115.

(205) F. Pfeiffer, C. Glocker, Multibody Dynamics with Unilateral Contacts, John Wiley & Sons, 1996.

(206) F. Pfeiffer, Applications of unilateral multibody dynamics, Philisophical Transactions of the Royal Society A 359 (2001) 2609–2628.

(207) R. A. Ibrahim, I. F. Grace, Selected problems in vibro-impact dynamics and applications, preprint, Wayne State University.

(208) F. Dercole, Y. Kuznetsov, Slidecont: An auto97 driver for bifurcation analysis of filippov systems, ACM Transactions on Mathematical Software 31 (2005) 95–119.

(209) W. Kang, P. Thota, B. Wilcox, H. Dankowicz, Bifurcation analysis of a microactuator using a new toolbox for continuation of hybrid system trajectories, in: Proceedings of the ASME IDETC/CIE 2007, Las Vegas, Nevada, 2007.

(210) N. Distefano, A. Rath, System identification in nonlinear structural seismic dynamics, Computer Methods in Applied Mechanics and Engineering 5 (1975) 353–372.

(211) P. M. T. Broersen, Estimation of parameters of nonlinear dynamical systems, International Journal of Non-Linear Mechanics 9 (1974) 355–361.

(212) C. B. Yun, M. Shinozuka, Identification of nonlinear structural dynamic systems, Journal of Structural Mechanics 8 (1980) 187–203.

(213) S. V. Hanagud, M. Meyappa, J. I. Craig, Method of multiple scales and identification of nonlinear structural dynamic systems, AIAA Journal 23 (1985) 802–807.

(214) K. Yasuda, S. Kawamura, K. Watanabe, Identification of nonlinear multi-degree-of-freedom systems (presentation of an identification technique), JSME International Journal, Series III 31 (1988) 8–14.

(215) K. Yasuda, K. Kamiya, Identification of a nonlinear beam (proposition of an identification technique), JSME International Journal, Series III 33 (1990) 535–540.

(216) K. Yasuda, K. Kamiya, Experimental identification technique of nonlinear beams in

time domain, Nonlinear Dynamics 18 (1999) 185–202.
(217) P. J. Gawthrop, A. Kountzeris, J. B. Roberts, Parametric identification of nonlinear ship roll motions from forced roll data, Journal of Ship Research 32 (1988) 101–111.
(218) S. M. Batill, J. M. Bacarro, Modeling and identification of nonlinear dynamic systems with application to aircraft landing gear, in: Proceedings of the 29th AIAA/ASME/ASCE/AHS Structures,Structural Dynamics, and Materials Conference, Williamsburg, VA, 1988.
(219) A. H. Nayfeh, Parametric identification of nonlinear dynamic systems, Computers & Strutures 20 (1985) 487–493.
(220) C. M. Yuan, B. F. Feeny, Parametric identification of chaotic systems, Journal of Vibration and Control 4 (1998) 405–426.
(221) L. Pecora, T. L. Carroll, Driving systems with chaotic signals, Physical Review A 44 (1991) 2374–2383.
(222) M. D. Narayanan, S. Narayanan, C. Padmanabhan, Parametric identification of nonlinear systems using chaotic excitation, ASME Journal of Computational and Nonlinear Dynamics 2 (2007) 225–231.
(223) S. F. Masri, T. K. Caughey, A nonparametric identification technique for nonlinear dynamic systems, ASME Journal of Applied Mechanics 46 (1979) 433–447.
(224) S. F. Masri, H. Sassi, T. K. Caughey, Nonparametric identification of nearly arbitrary nonlinear systems, ASME Journal of Applied Mechanics 49 (1982) 619–628.
(225) S. F. Masri, R. K. Miller, A. F. Saud, T. K. Caughey, Identification of nonlinear vibrating structures: Part i - formulation, ASME Journal of Applied Mechanics 54 (1987) 918–922.
(226) S. F. Masri, R. K. Miller, A. F. Saud, T. K. Caughey, Identification of nonlinear vibrating structures: Part ii - applications, ASME Journal of Applied Mechanics 54 (1987) 923–929.
(227) H. G. Natke, M. Zamirowski, On methods of structure identification for the class of polynomials within mechanical systems, ZAMM 70 (1990) 415–420.
(228) G. Kerschen, J.-C. Golvinal, F. M. Hemez, Bayesian model screening for the identification of nonlinear mechanical structures, ASME Journal of Vibration and Acoustics 125 (2003) 389–397.
(229) Y. Q. Ni, M. Ko, C. W. Wong, Nonparametric identification of nonlinear hysteretic systems, ASCE Journal of Engineering Mechanics 125 (1999) 206–215.
(230) M. Peifer, J. Timmer, T. U. Voss, Non-parametric identification of non-linear oscillating systems, Journal of Sound and Vibration 267 (2003) 1157–1167.
(231) S. F. Masri, A. G. Chassiakos, T. K. Caughey, Identification of nonlinear dynamic systems using neural networks, ASME Journal of Applied Mechanics 60 (1993) 123–133.
(232) S. Masri, A. W. Smyth, A. G. Chas, T. K. Caughey, N. F. Hunter, Application of neural networks for detection of changes in nonlinear systems, ASCE Journal of Engineering Mechanics 126 (2000) 666–676.
(233) L. Moniz, T. Carroll, L. Pecora, Assessment of damage in an eight-oscillator circuit using dynamical forcing, Physical Review E 68 (2003) 036215.
(234) M. D. Todd, K. Erickson, L. Chang, K. Lee, J. Nichols, Using chaotic interrogation and attractor nonlinear cross-prediction error to detect fastener preload loss in an aluminum frame, Chaos 14 (2004) 387–399.
(235) L. Moniz, J. Nichols, S. Trickey, M. Seaver, D. Pecora, L. Pecora, Using chaotic forcing to detect damage in a structure, Chaos 15 (2005) 023106.
(236) J. Nichols, S. T. Trickey, M. D. Todd, L. N. Virgin, Structural health monitoring through chaotic interrogation, Meccanica 38 (2003) 239–250.

(237) B. Epureanu, S.-H. Yin, M. M. Derriso, High-sensitivity damage detection based on enhanced nonlinear dynamics, Smart Materials and Structures 14 (2005) 321–327.

(238) S.-H. Yin, B. I. Epureanu, High-sensitivity mass sensing based on enhanced nonlinear dynamics and attractor morphing modes, in: IMECE 2006: The 2006 ASME International Mechanical Engineering Congress and Exposition, Chicago, Illinois, 2006.

(239) F. C. Moon, P. D. Stiefel, Coexisting chaotic and periodic dynamics in clock escapements, Philosophical Transactions of the Royal Society A 364 (2006) 2539–2563.