ENME665: Nonlinear Oscillations

- Material covered last class
  - Examples of nonlinear systems

- Today's class
  - ❖ Examples (to be continued) ; nonlinear damping
  - ❖ Qualitative Analyses (Chapter 2, Nayfeh and Mook)
    - ➢ Seek information about all solutions, would like to know whether a certain property of these solutions remain unchanged if the system is subjected to various types of changes

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•Qualitative Analyses (Chapter 2, Nayfeh and Mook)

- ❖ First integral of motion
- ❖ Phase portraits
- ❖ Examples: Undamped and damped pendulum systems

$$\ddot{\theta} + \frac{g}{l}\left(\theta - \frac{\theta^3}{6}\right) = 0; \quad \text{undamped case}$$

$$\ddot{\theta} + \frac{g}{l}\sin\theta = 0; \quad \text{undamped case}$$

$$\ddot{\theta} + 2\mu\dot{\theta} + \frac{g}{l}\sin\theta = 0; \quad \text{Damped case}$$

- ❖ Ship-roll motions
- ❖ Duffing oscillator

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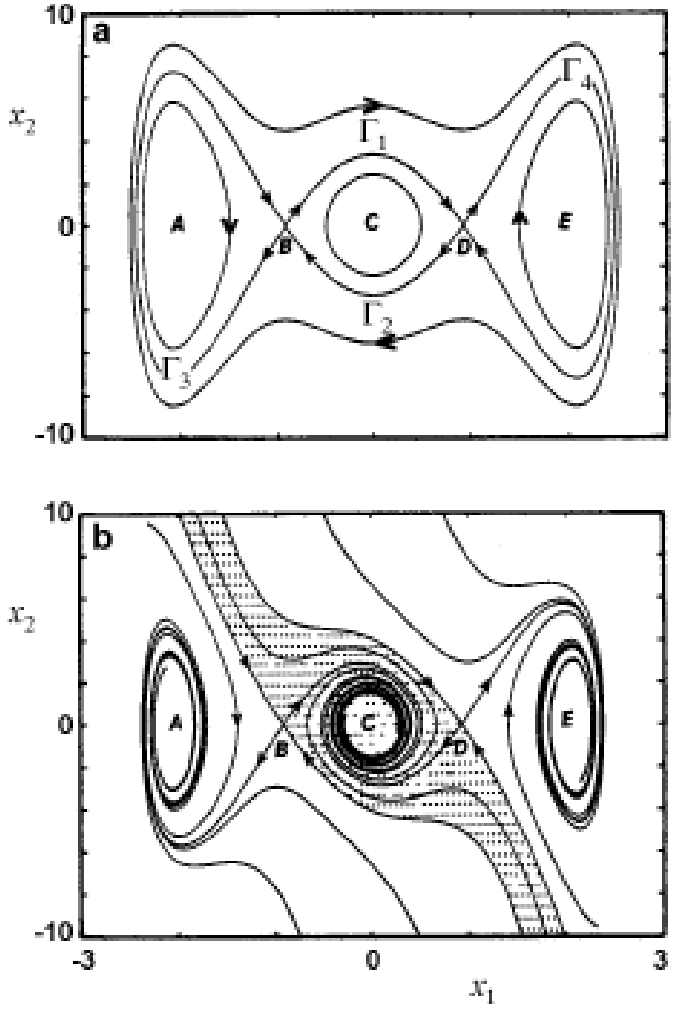
- Example 2.8 (Nayfeh and Balachandran, 1995): Ship-Roll Motions



$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\left(\omega_0^2 x_1 + \alpha_3 x_1^3 + \alpha_5 x_1^5\right) - \left(2\mu_1 x_2 + \mu_3 x_2^3\right)$$

$\omega_0 = 5.278;\ \alpha_3 = -1.402\omega_0^2;\ \alpha_5 = 0.271\omega_0^2$

undamped case: $\mu_1 = \mu_3 = 0$

damped case: $\mu_1 = 0.086$ and $\mu_3 = 0.108$

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•Duffing oscillator $\ddot{x} + ax + bx^3 = 0$

•Georg Duffing (1861-1944)

•Experimental prototype: Dynamics and Control Laboratory, University of Maryland

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- Check out the nonlinear dynamics and chaos laboratory demonstrations at the Duke University site

  http://nonlineardynamics.pratt.duke.edu/video

- Material to be covered next class

  - Quantitative Analyses (Chapter 2, Nayfeh and Mook)
    - ❖ Landau symbols and ordering
    - ❖ Straightforward expansions
    - ❖ Lindstedt-Poincaré technique
    - ❖ Method of multiple scales