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Experimental Comparison of the van der Pol and Rayleigh Nonlinear Oscillators for a Robotic Swinging Task

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Abstract

In this paper, the effects of different lower-level building blocks of a robotic swinging system are explored, from the perspective of motor skill acquisition. The van der Pol and Rayleigh oscillators are used to entrain to the system's natural dynamics, with two different network topologies being used: a symmetric and a hierarchical one. Rayleigh outperformed van der Pol regarding maximum oscillation amplitudes for every morphological configuration examined. However, van der Pol started large amplitude relaxation oscillations faster, attaining better performance during the first half of the transient period. Hence, even though there are great similarities between the oscillators, differences in their resultant behaviours are more pronounced than originally expected.

1 Introduction

Various neural oscillators have been used in the past to implement several rhythmic motor control tasks. Mutually-inhibiting neurons (Matsuoka 1985) have been used to entrain humanoid arms with a slinky toy and turn a crank (Williamson 1998), bipedal walking (Taga 1991; Taga 1995), swinging (Lungarella and Berthouze 2002; Matsuoka, Ohyama et al. 2005), and bouncing (Lungarella and Berthouze 2004), while the van der Pol and Rayleigh oscillators have been utilised for the purposes of planar bipedal walkers (Zielinska 1996; Dutra, de Pina Filho et al. 2003; de Pina Filho, Dutra et al. 2005). In motor control studies, systems are often treated at a more abstract level of behaviour and less attention is paid to the impact the low level components have on the overall functioning of the system. In a previous study (Veskos and Demiris 2005a) we investigated the use of the van der Pol oscillator for a robotic swinging task. In this paper, we implement an additional oscillator, known as the Rayleigh oscillator. The two oscillators have a similar mathematical structure, thus allowing us to make direct comparisons between them and the resultant behaviours. We are specifically interested in determining whether this similar basic building block alters the higher-level behaviours of the system. Furthermore, we also wish to investigate the influence of different oscillatory network topologies. We therefore experimented with a hierarchical network structure, in addition to the previously used symmetric one.

2 Experimental Setup

We utilise two similar nonlinear oscillators to build the neural control system for our experiments: van der Pol and Rayleigh. Additionally, we connect these in two different manners, using a symmetric and a hierarchical topology.

2.1 Nonlinear Oscillators

The equations of the van der Pol (vdP) oscillator, as used in our experiments, are of the form:

$$\ddot{x}_i + \mu\left(x_i^2 - 1\right)\cdot\dot{x}_i + \omega^2 x_i = G_{in}\cdot fb + G_{i-j}\cdot x_j \quad (1)$$

where $i, j = \{\text{hip,knee}\}, \mu \geq 0$ is a parameter controlling the damping term, ω is the natural frequency of the oscillator, fb is the feedback from the vision system, G_{in} is the feedback gain, while $G_{hip-knee}$ and $G_{knee-hip}$ are the cross-coupling term gains. The final output given to the position-controlled motors activating the joints, is:

$$\theta_i = G_{out}\cdot sign\left(\dot{x}_i\right),\ i = \{hip, knee\} \quad (2)$$

where G_{out} is the output gain.

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For the Rayleigh oscillator, $\dot{x}$ is inserted in the $\left(x_i^2 - 1\right)$ term to yield:

$$\ddot{x}_i + \mu\left(\dot{x}_i^2 - 1\right)\cdot \dot{x}_i + \omega^2 x_i = G_{in}\cdot fb + G_{i-j}\cdot x_j \tag{3}$$

This difference alters the response of the two oscillators to changes in their natural frequency. For the vdP, increasing ω increases the oscillator's output *frequency*, while for Rayleigh has the effect of increasing output *amplitude*. Given that we only make use of the timing information and discard the amplitude in equation (2), it should be easier for Rayleigh to achieve entrainment to mechanical systems as its own natural dynamics are less pronounced. Furthermore, simulations of a planar bipedal walker task have shown Rayleigh to recover from random perturbations faster than van der Pol (Roy and Demiris 2005).

2.2 Neural Topologies

Two different neural topologies were investigated by altering the values of the cross-coupling gains. By equating them, the topology is symmetric, where both degrees of freedom affect each other and strong neural entrainment takes place. This is shown in Figure 1.
To arrive at a hierarchical topology where only the hip oscillators directly receive the feedback signal, the vision feedback is not forwarded to the knee oscillator. Additionally, the intra-neural connection sending information back to the hip oscillator is severed. The knee oscillator can then entrain to the mechanical system solely by means of the hip-knee connection. This is better illustrated in Figure 2.

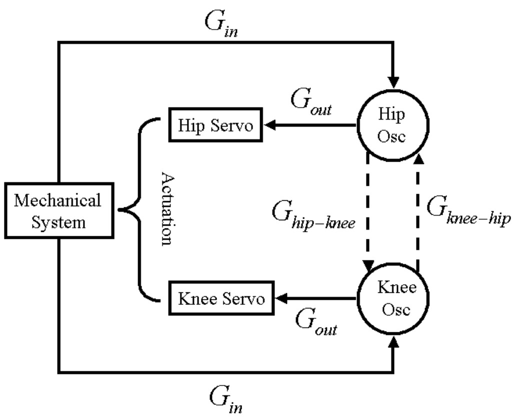

Figure 1: A functional overview of our experimental system for the symmetric neural topology. Both oscillators receive vision feedback and strong neural entrainment is facilitated by the intra-neural connections (shown with the dashed line).

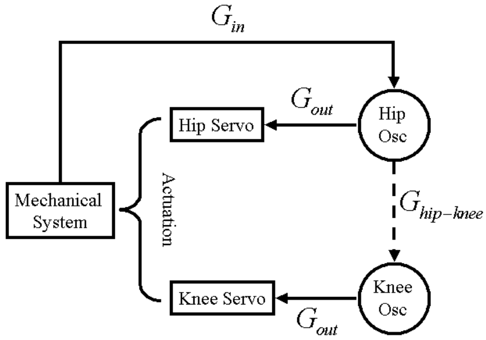

Figure 2: In the hierarchical topology, only the proximal (hip) oscillator directly receives vision feedback, which is then propagated to the distal (knee) oscillator, as shown by the dashed line.

2.3 Mechanical Setup

Experiments were performed on the robotic platform previously described in (Veskos and Demiris 2005b), shown in Figure 3.

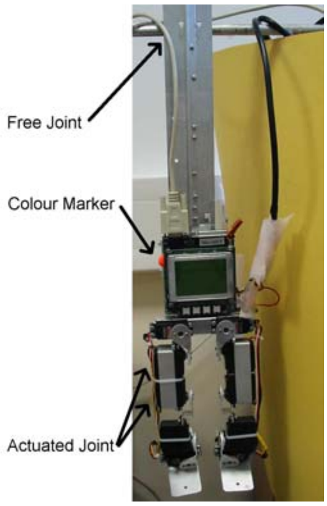

Figure 3: The robotic setup

The robot can be thought of as an underactuated triple pendulum with the top joint being free while the bottom two joints are totally forced to the output of the nonlinear oscillator. A coloured marker on the robot is tracked by a webcam viewing the setup from the side. The x coordinate of this marker is then used as feedback for the neural oscillator (*fb* term). In this study, only the hip and knee joints were actuated, while all others on the robot were held stiff.

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3 Experiments

Actuating the hip joint with the van der Pol and Rayleigh oscillators result in the phase plots of Figure 4. The maximum amplitude of oscillation of the robot is 39% larger using Rayleigh (179 instead of 129 units), for the same value of the natural frequency, $\omega^2 = 3.0$. Although the van der Pol oscillator has a more consistent neural limit cycle with less variation in amplitude, the mechanical system operates more smoothly with the Rayleigh oscillator. This is evidenced by the more even mechanical system plot; the phase portrait (a projection of the 3D plot on 2D, by removing the time axis) resembles a circle rather than an hourglass-like shape. The irregularities distorting the uniformity of the limit cycle occur at the point corresponding to the robot's flight phase past the midway position. Its speed there should be maximum, but the van der Pol shows a relative reduction in the value of the derivative, thus causing this "dent". While this improves as the system reaches the steady state, it does not disappear and is an indication of task suboptimality.

In the symmetric neural topology, strong neural entrainment takes place and due to the symmetry of the feedback system, the hip and knee oscillators essentially identical outputs, completely in phase. Again, Rayleigh was capable of producing larger amplitude oscillations than van der Pol, given the same system parameters: 214 versus 182 units, an 18% difference. These results are illustrated in Figure 5.

Results for the hierarchical topology in terms of maximum oscillation amplitudes were very similar, with the corresponding values being 214 and 181 units (Figure 6). In terms of the timing however, the symmetry in the neural topology makes the coupling between the two joint oscillators weaker and allows for delays to be introduced between the hip and knee. Rayleigh, however is much more resilient to this effect, as illustrated in Figure 7.

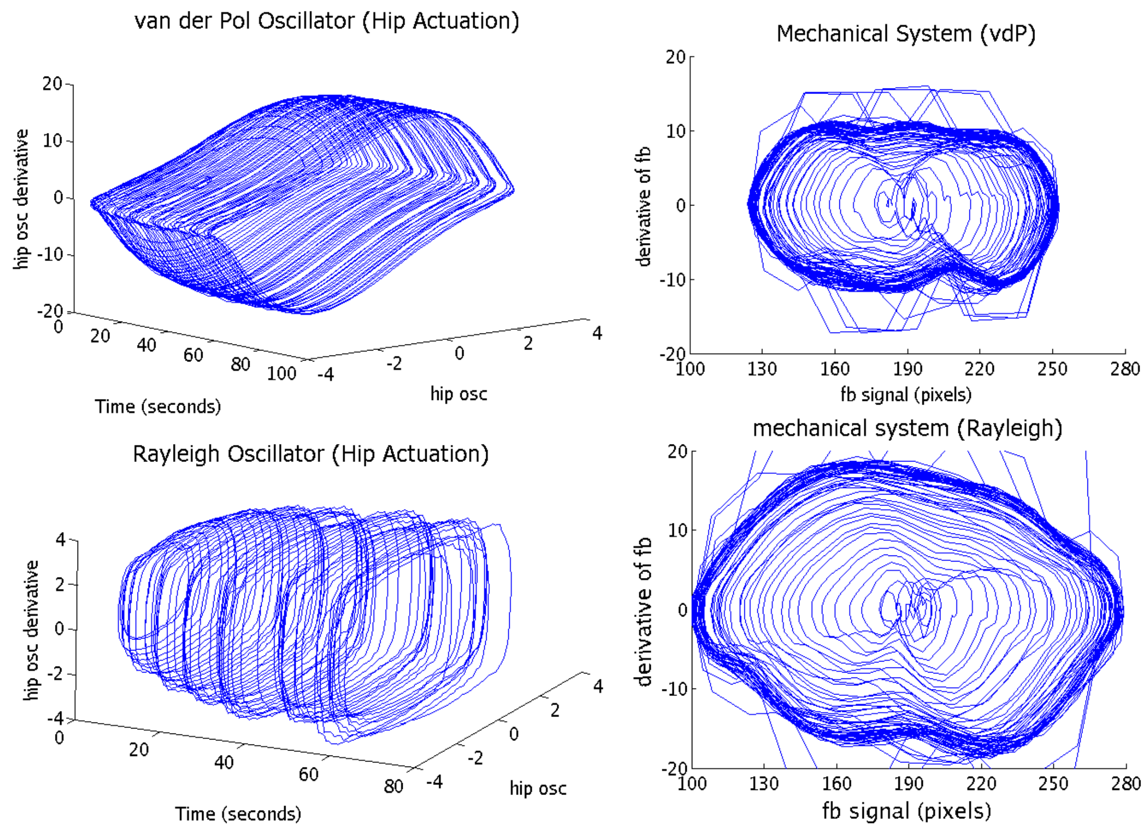

Figure 4: Phase plots for 1-DOF proximal (hip) actuation. The van der Pol oscillator is shown on the top row: the neural system on the left and a phase portrait of the mechanical system on the right. The plots for the Rayleigh oscillator are shown in the bottom row.

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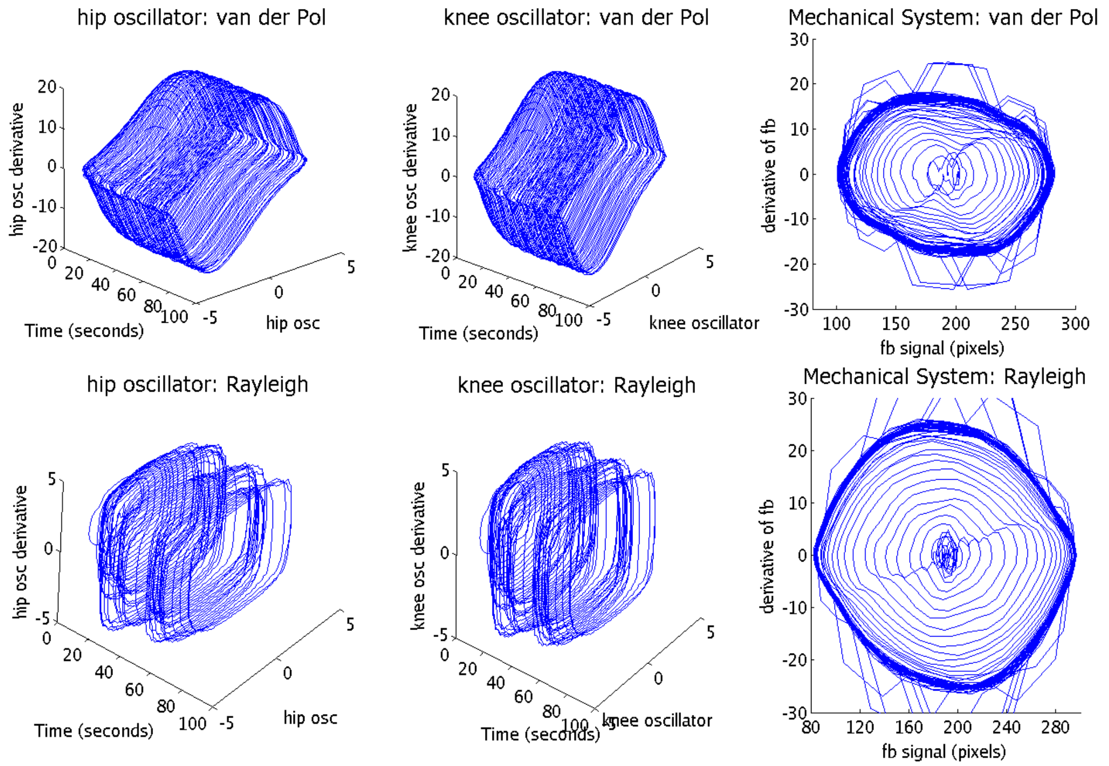

Figure 5: Phase plots for 2-DoF actuation with the symmetric neural topology. Results for van der Pol oscillator are shown on the top row and for Rayleigh on the bottom. From left to right, the columns are: hip oscillator phase plot, knee oscillator phase plot and mechanical system phase portrait.

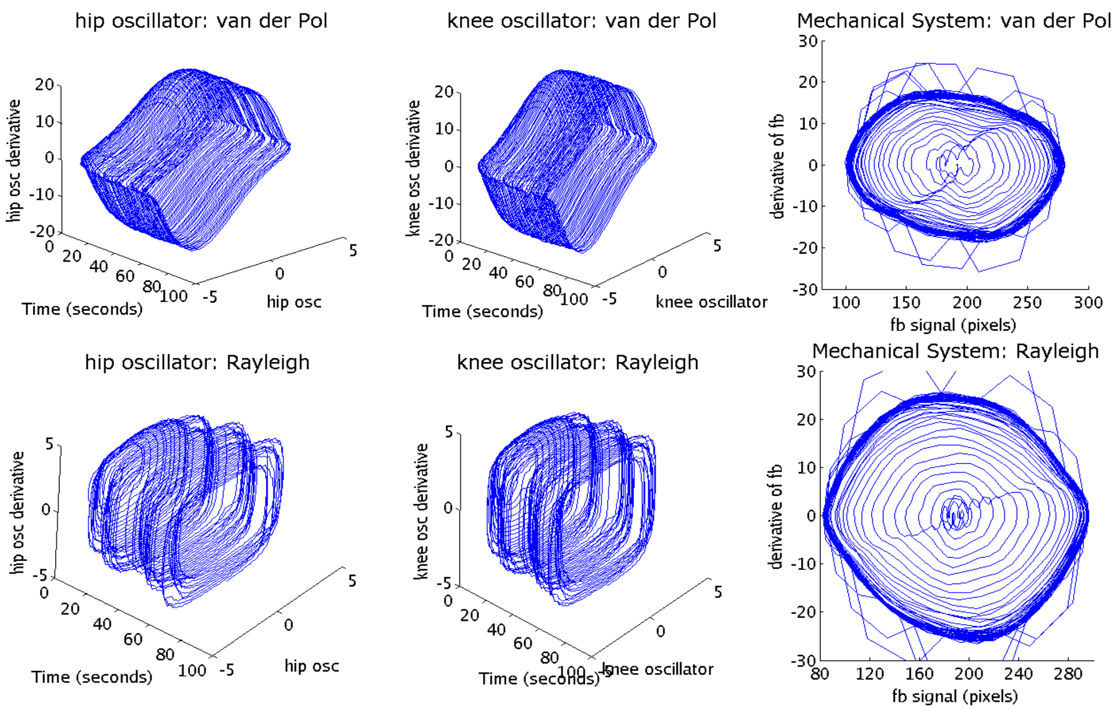

Figure 6: Phase plots for the hierarchical network topology. The asymmetry in the neural topology makes the coupling between the two joint oscillators weaker and introduces a small phase difference. This way the limit cycles are not identical for both joints as in the symmetric case.

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4 Discussion

Larger oscillation amplitudes were always achieved when both degrees of freedom were actuated. Injecting more energy in the system also made limit cycles smoother, eliminating the suboptimal speed drops observed for the hip-only actuation scheme.

To analyse the transient behaviour of the two different oscillators, a comparison of the envelopes of oscillation for the entire experiments was made. This is shown in Figure 9. Something that should be noted is that van der Pol started producing relaxation oscillations earlier, thus giving it an advantage over the first ten seconds of the trial. This phenomenon is more pronounced in the case of both degrees of freedom being activated. Another two trials were performed where the second degree of freedom (knee) was released at t=5s. This moment was chosen as an 'early' release point where the system was still in its transient state. The Rayleigh oscillator's behaviour is almost identical to the 1-dof case until t=9s and only manages to reach the performance of the vdP at t=12s.

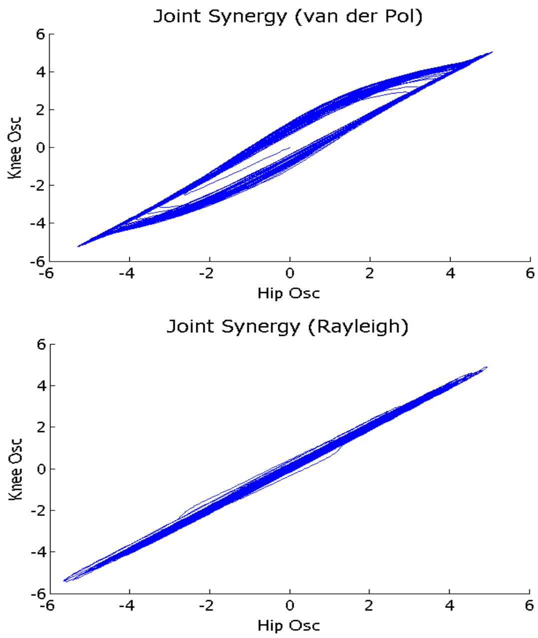

Figure 7: Hip-knee joint correlation plots for the van der Pol (top) and Rayleigh (bottom) oscillators in the hierarchical topology experiments. Rayleigh manages to maintain a 1:1 timing relationship between the two joints, while vdP introduces a phase difference.

Additionally, to compare the oscillators' frequency adaptation speed, the instantaneous period during the above experiments was plotted in Figure 8. Rayleigh consistently forces the mechanical system to oscillate at a lower frequency than van der Pol. This phenomenon is especially pronounced for the 2-DoF configurations. The difference in topology seems to have little effect on this matter; period of oscillation remains unaffected for a given oscillator.

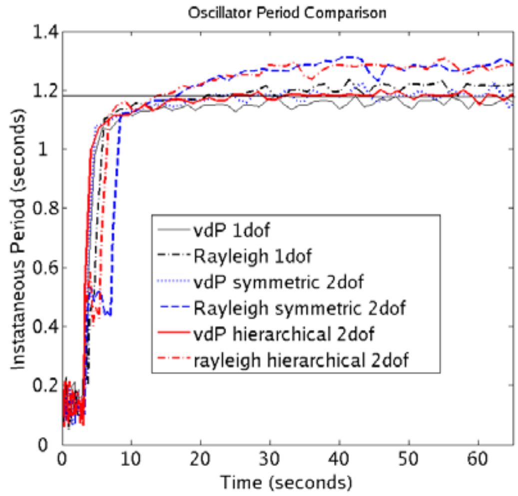

Figure 8: The instantaneous period of the mechanical system as driven by the different neural configurations. The Rayleigh oscillator consistently drives the system at a lower frequency than van der Pol, especially for the 2-DoF regimes. The mechanical system's natural period is denoted by the horizontal line at $T = 1.181\text{Hz}$.

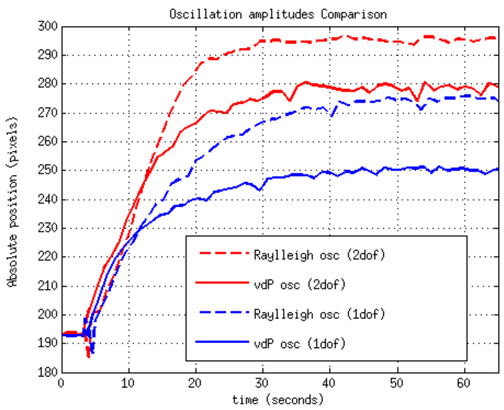

Figure 9: The envelopes of oscillation for four trials. The Rayleigh oscillator has a longer rise time than the vdP, but consistently reaches a larger oscillation amplitude given the same parameters, for both the 1-dof and staged-release 2-dof cases.

5 Conclusions

Our experiments have shown that even though the differences between the two oscillators studied are small, the nature of their dynamics altered the high-level behaviour of the system. Given the same experimental parameters, Rayleigh attained larger oscillation amplitudes for the mechanical system, at each morphological configuration. It also consistently forced the system to oscillate at frequencies lower than van der Pol. However, van der Pol starts large amplitude relaxation oscillations faster, attaining better performance during the first half of the transient period. This trade-off however is of limited scope, as it is of a fixed-offset nature; once Rayleigh has matched vdP's amplitude, it maintains its superior performance.

These experiments have shown that the effect of different oscillators, despite their great similarities are more pronounced than originally expected. Conversely, differing topologies that were expected to lead to stronger suboptimality had less effect.

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