ENME665: Advanced Topics in Vibrations (Nonlinear Oscillations)

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Textbooks:

- Nayfeh, A. H. and Mook, D. T. (1979). *Nonlinear Oscillations*, Wiley, New York (Available in EPSL Library)
- Nayfeh, A. H. and Balachandran, B. (1995, 2006). *Applied Nonlinear Dynamics*: *Analytical, Computational, and Experimental Methods*, Wiley, New York

Other Material: Other books in the field of nonlinear oscillations and nonlinear dynamics will also be used to illustrate concepts and ideas. Articles such as the following will also be used in this course: i) Balachandran, B. *et al.* (1994), "On the identification of nonlinear interactions in structures," AIAA Journal, Vol. 17(2), pp. 257—262; ii) Turner, K. L. *et al*. (1998). "Five parametric resonances in a microelectromechanical system," *Nature*, Vol. 396, pp. 149—152; iii) Voth, G. A. *et al*. (2002). "Experimental measurements of stretching fields in fluid mixing," *Physical Review Letters*, Vol. 88(25), pp. 254501-1-254501-4; and iv) Shaw, S. W. and Balachandran, B. (2008). ``A review of nonlinear dynamics of mechanical systems in Year 2008," *Japanese Society of Mechanical Engineers*, Vol. 2(No. 3), pp. 611-640.

Class Notes: Copies of certain lectures will be provided on Canvas

Time and Place: **Tu, Th** – 3:30 PM to 4:45 PM; EGR3114

Instructor Office Hours: **Tu, Th** – 5.00 PM to 6.00 PM or by appointment

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Course Outline

- The theme of the course will be nonlinear oscillations and dynamics of structural and mechanical systems.

- Starting with classical methods, a blend of computational, geometrical, and analytical methods will be used to provide a unified treatment of nonlinear oscillations and nonlinear dynamics.

- Bifurcations (qualitative changes) with respect to quasi-stationary variations of one or more control parameters will be considered and instabilities such as flutter and divergence will be discussed. The phenomenon called chaos will be explored.

- Specific topics to be considered include the following: 1) nonlinear oscillations of pendulum and structural systems; 2) perturbation methods such as the method of multiple scales; 3) phase plane analysis and Poincare' maps; 4) external, parametric, and internal resonances; 5) stability notions and instabilities such as saddle-node, pitchfork, and Hopf bifurcations of equilibria and periodic solutions; and 6) tools such as dimension calculations and Lyapunov exponents for analyzing nonlinear motions.

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Grading:

- Assignments: **20 %**
- One Mid-Term Exam (TBA): **40%**
- Project: **40%**

Things to Review

- Oscillator Physics; Single and Multiple Degree of Freedom Systems
- Single Variable Calculus: Taylor-series expansions, Graphs
- Multi-Variable Calculus: Partial Derivatives, Jacobian Matrix, Divergence Theorem
- Linear Algebra: Eigenvalues & Eigenvectors

Introductory Material

- Nonlinear: Negation of the word "Linear"

- Nonlinear Systems:

  - Principle of superposition does not hold
  - Multiple equilibria; Isolated periodic solutions
  - Sensitivity to initial conditions
  - Phenomena such as solitons
  - Bifurcations, chaos, nonlinear interactions, nonlinear phenomena

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- Qualitative Analyses
  - ❖ Seek information about all solutions, would like to know whether a certain property of these solutions remain unchanged if the system is subjected to various types of changes
  - ❖ Stability, one qualitative aspect of interest
  - ❖ Example: Phase portrait analyses
  - ❖ Numerical integrations of governing equations, a tool

- Quantitative Analyses

  - ❖ Perturbation methods

- Tools for analyses:
  - ❖ Numerical integration
  - ❖ Geometrical tools – e.g., Poincaré sections

Revisit Linear Systems

- Linear, unforced oscillators
  – no isolated periodic solutions

$$m\ddot{x} + kx = 0; \;\; m > 0; \;\; k > 0$$

- Linear, forced oscillators
  – limit cycles possible

$$m\ddot{x} + kx + c\dot{x} = F\cos(\Omega t); \;\; m > 0; \;\; k > 0; c > 0$$

Examples

i) <u>Poincaré Oscillator</u>

$$\frac{dr}{dt} = kr(1-r)$$

$$\frac{d\phi}{dt} = 2\pi$$

<u>References on Biological Applications</u>:

- Glass, L. and Mackey, M. C. (1988). *From Clocks to Chaos: The Rythms of Life*, Princeton, Princeton University Press

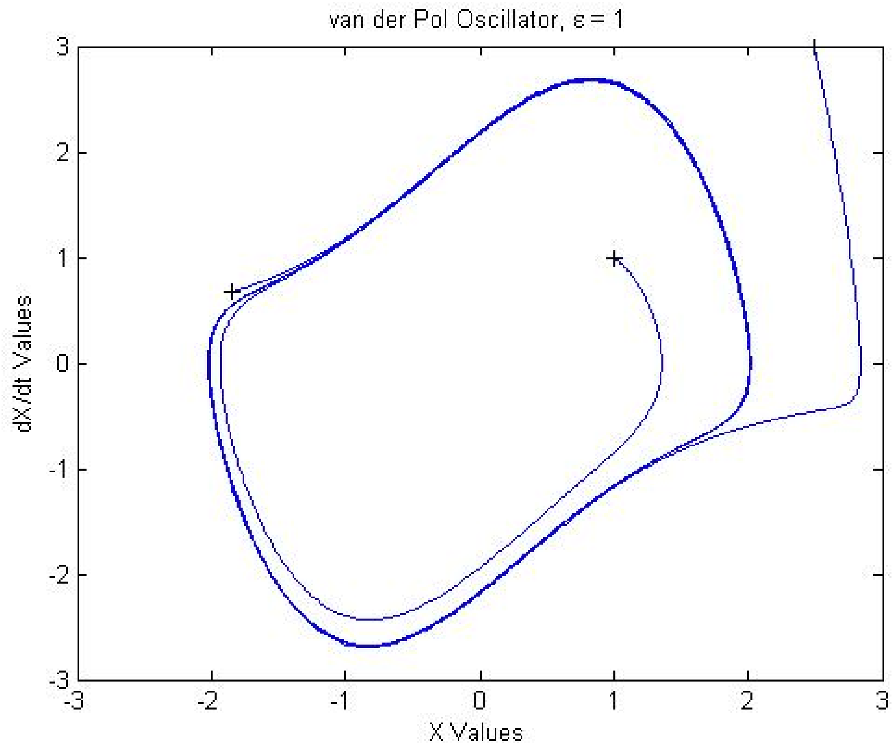
- Beuter, A., Glass, L., Mackey, M. C., Titcombe, M. S. (2003). N*onlinear Dynamics in Physiology and Medicine*, Springer-Verlag, Section 5.3

- Eguíluz, V. M., Ospeck, M. Choe, Y., Hudspeth, A. J., and Magnasco, M. O., "Essential Nonlinearities in Hearing," *Physical Review Letters*, Vol. 84(22), pp. 5232—5235, 2000.

Examples

ii) van der Pol Oscillator

$$\ddot{x} + \varepsilon\left(x^2 - 1\right)\dot{x} + x = 0$$



Numerically generated responses of the van der Pol oscillator are shown in this figure for ε=1. Trajectories generated from three different initial conditions are shown in this graph. It is seen that all of the trajectories are attracted to a periodic orbit. Since this orbit is an isolated periodic orbit, it is also called a *limit cycle*.

References on Biological Applications:

- Beuter, A., Glass, L., Mackey, M. C., Titcombe, M. S. (2003). *Nonlinear Dynamics in Physiology and Medicine*, Springer-Verlag, Section 10.5

--Parkinsonian Tremor

Examples: Nonlinear Phenomena and Instabilities

- **Period-doubling instability**
- **Hopf instability**
- **….**

Period-One and Period-Two Motions

In the following illustrations, two different types of periodic responses of a harmonically forced single degree-of-freedom nonlinear system generated through numerical integration are shown. One of them is a period-one response; that is, the period of the response is the same as that of the forcing period or the reference period. The other type of response is a period-two response; that is, the period of the response is twice that of the forcing or reference period.

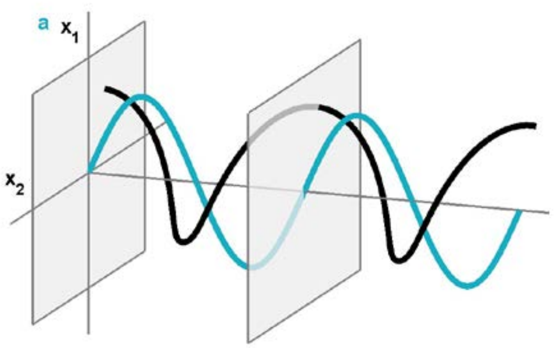


Illustration of a period-one motion. The trajectory of the forcing is illustrated by the blue solid line and the response of the nonlinear system is illustrated by the black solid line. A Poincaré section is constructed by using the forcing period for the clock period. The periodic response makes one intersection with this section.

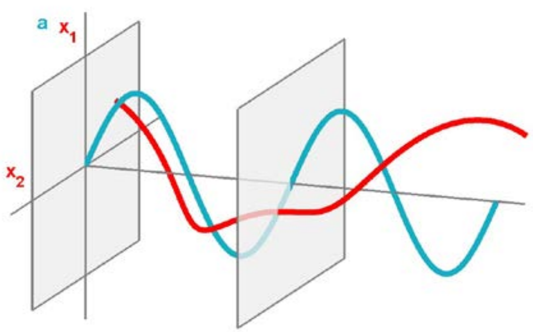
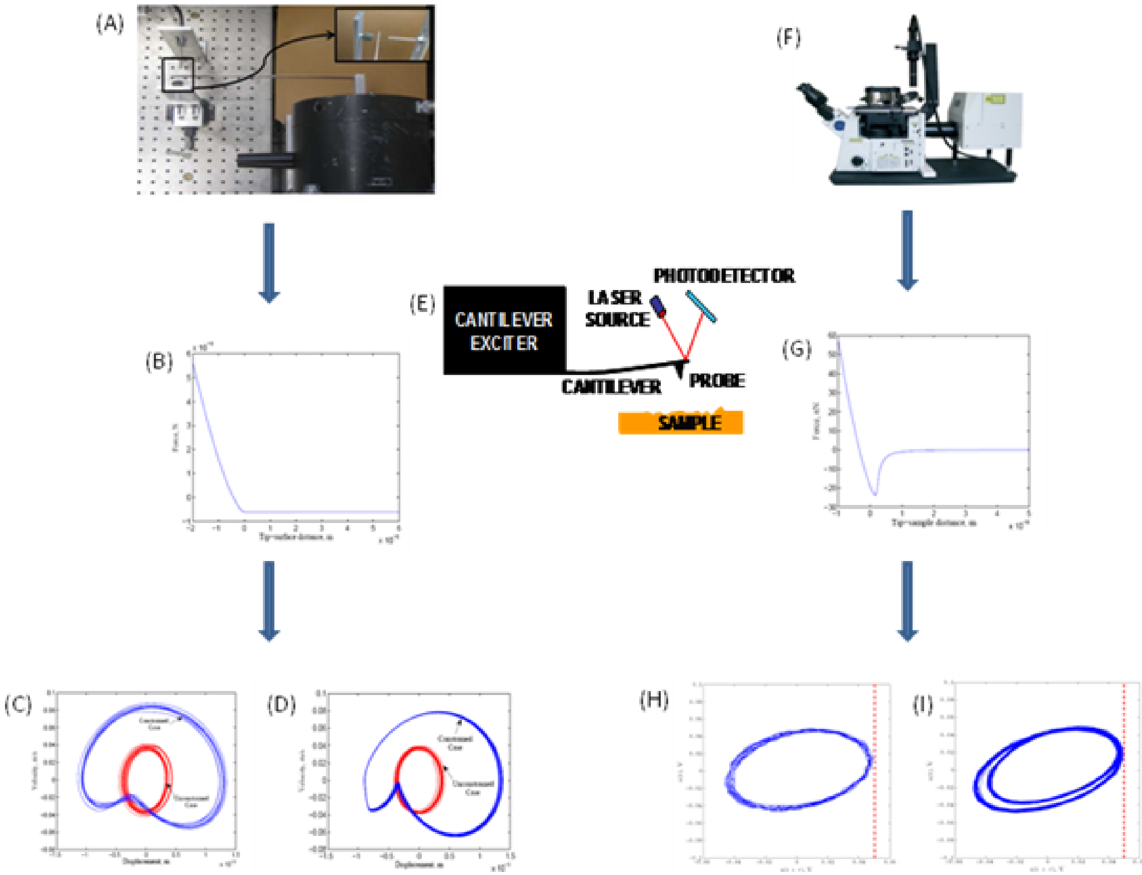


Illustration of a period-two motion. The trajectory of the forcing is illustrated by the blue solid line and the response of the nonlinear system is illustrated by the red solid line. A Poincaré section is constructed by using the forcing period for the clock period. The periodic response makes two intersections with this section.

Period-Doubling Instability: An Example

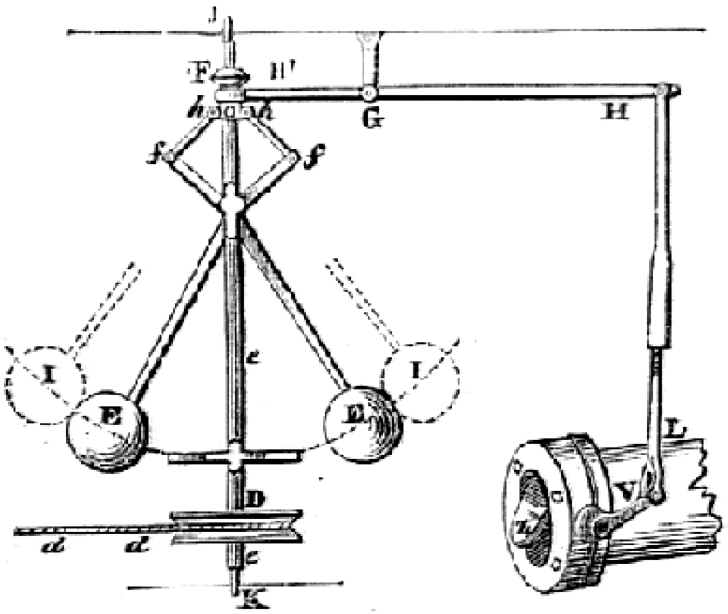


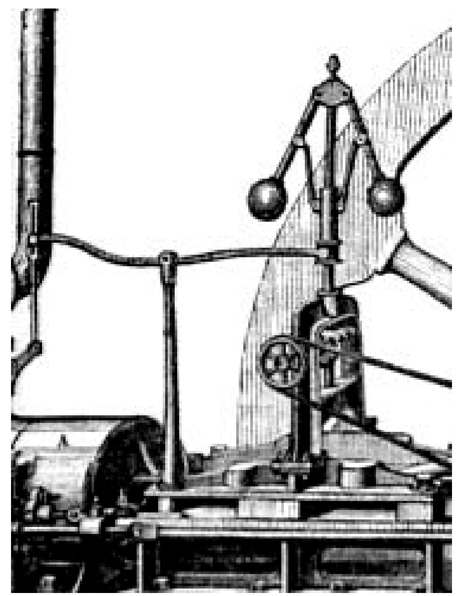
- **Dick, A. J., Balachandran, B., Yabuno, Yabuno, Numatsu, M., Hayashi, K., Kuroda, M., and Ashida, K.** (2009). "Utilizing Nonlinear Phenomenon to Locate Grazing in the Constrained Motion of a Cantilever Beam," *Nonlinear Dynamics*, 57, 335-349 (2009).
- **Chakraborty, I. and Balachandran, B.** (2009). "Cantilever Dynamics with Attractive and Repulsive Tip Interactions," *Proceedings of the ASME International Mechanical Engineering Congress and Exposition*, Lake Buena Vista, FL, Nov. 13-19, 2009. Paper No. IMECE 2009-10330.

- Excitation away from first natural frequency is found to produce a qualitative change in the AFM cantilever's response
  - Period-doubling bifurcation close to grazing

Watt's Governor: Example of a Mechanical System that Shows the Nonlinear Instability called a Hopf Bifurcation

In 1787, James Watt adopted the centrifugal governor that was previously used for wind mills for regulating the speed of a steam engine.   In the arrangement of the flyball governor (see Figures shown below that have been obtained from the web).





Two heavy balls, also known as "flyballs" are attached through two arms to a rotating spindle, which is in turn connected to the engine. The movement of the flyball arms controls the opening and closing of a throttle valve for steam input to the engine. As, the spindle speed increases, the flyballs move up reducing the size of the valve opening and thus reducing the amount of steam input into the engine. This leads to a reduction of the speed. For a fixed spindle speed, the throttle opening is constant and this corresponds to an equilibrium position of the system. For a critical value of the spindle speed, the system undergoes a Hopf bifurcation leading to oscillations in the throttle valve opening.

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- Nonlinear Dynamics with LEGO
  http://www.inm.uni-stuttgart.de/mitarbeiter/leine/toys/index.en.html

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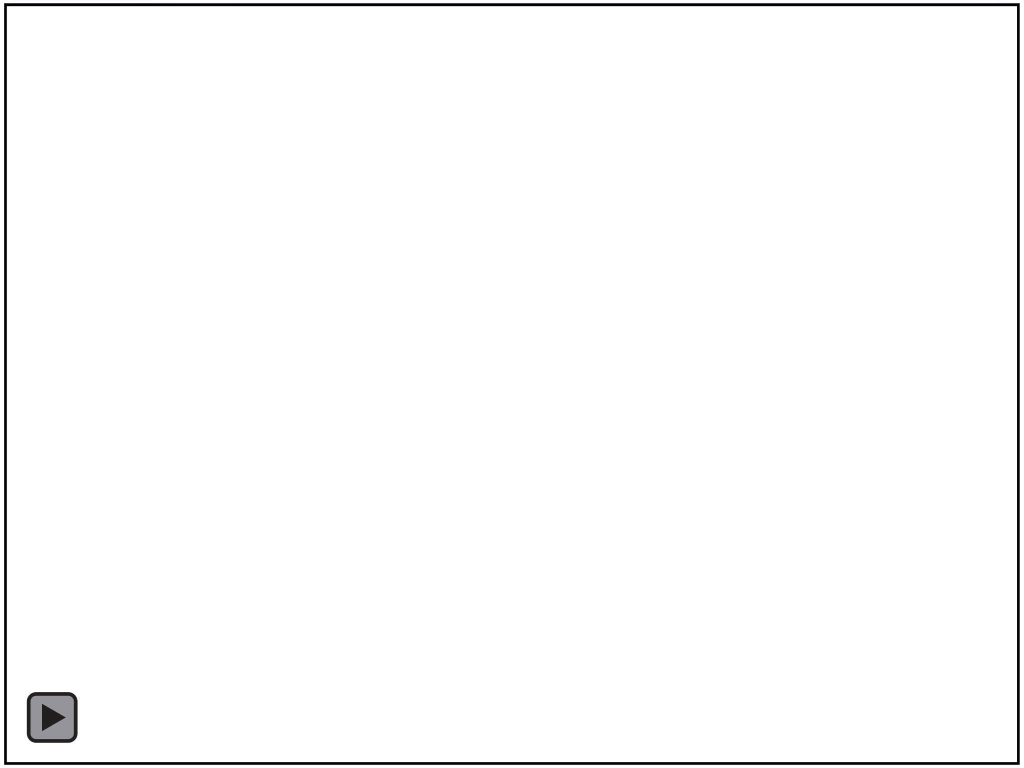
- Animation of nonlinear motions
  - ❖ Kapitza Pendulum
  - ❖ Hunting motions
  - ❖ Parametric resonance
  - ❖ Aircraft - Limit cycle motions
  - ❖ Fluid mixing

Limit Cycle Oscillations



(**Video, Courtesy of AFRL**)

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- Material to be covered next class
  - ❖ Examples (to be continued)