AIP Chaos
An Interdisciplinary Journal of Nonlinear Science

Parameter estimation for chaotic systems using a hybrid adaptive cuckoo search with simulated annealing algorithm

Zheng Sheng, Jun Wang, Shudao Zhou, and Bihua Zhou

Citation: Chaos: An Interdisciplinary Journal of Nonlinear Science **24**, 013133 (2014); doi: 10.1063/1.4867989
View online: http://dx.doi.org/10.1063/1.4867989
View Table of Contents: http://scitation.aip.org/content/aip/journal/chaos/24/1?ver=pdfcov
Published by the AIP Publishing

CHAOS **24**, 013133 (2014)

Parameter estimation for chaotic systems using a hybrid adaptive cuckoo search with simulated annealing algorithm

Zheng Sheng,[1,a)] Jun Wang,[2] Shudao Zhou,[1,3] and Bihua Zhou[2]
[1]*College of Meteorology and Oceanography, PLA University of Science and Technology, Nanjing 211101, China*
[2]*National Defense Key Laboratory on Lightning Protection and Electromagnetic Camouflage, PLA University of Science and Technology, Nanjing 210007, China*
[3]*Collaborative Innovation Center on Forecast and Evaluation of Meteorological Disasters, Nanjing University of Information Science & Technology, Nanjing 210044, China*

(Received 2 October 2013; accepted 22 February 2014; published online 17 March 2014)

This paper introduces a novel hybrid optimization algorithm to establish the parameters of chaotic systems. In order to deal with the weaknesses of the traditional cuckoo search algorithm, the proposed adaptive cuckoo search with simulated annealing algorithm is presented, which incorporates the adaptive parameters adjusting operation and the simulated annealing operation in the cuckoo search algorithm. Normally, the parameters of the cuckoo search algorithm are kept constant that may result in decreasing the efficiency of the algorithm. For the purpose of balancing and enhancing the accuracy and convergence rate of the cuckoo search algorithm, the adaptive operation is presented to tune the parameters properly. Besides, the local search capability of cuckoo search algorithm is relatively weak that may decrease the quality of optimization. So the simulated annealing operation is merged into the cuckoo search algorithm to enhance the local search ability and improve the accuracy and reliability of the results. The functionality of the proposed hybrid algorithm is investigated through the Lorenz chaotic system under the noiseless and noise condition, respectively. The numerical results demonstrate that the method can estimate parameters efficiently and accurately in the noiseless and noise condition. Finally, the results are compared with the traditional cuckoo search algorithm, genetic algorithm, and particle swarm optimization algorithm. Simulation results demonstrate the effectiveness and superior performance of the proposed algorithm. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4867989]

Recently, parameter estimation for chaotic systems has been a subject of great interest and importance, in theory and various fields of application, such as geophysics prediction and communication system. At the same time, various optimization algorithms for parameter estimation of chaotic systems have been proposed, for example, particle swarm optimization (PSO), genetic algorithm (GA), artificial bee colony (ABC), differential evolution (DE), and etc. However, almost all the above optimization algorithms have some drawbacks and affect the quality of global optimization solutions. In this contribution, we study a novel and efficient hybrid optimization algorithm adaptive cuckoo search with simulated annealing (*ACS-SA*) to estimate the parameters of chaotic system, the proposed adaptive cuckoo search with simulated annealing algorithm is presented which incorporates the adaptive parameters adjusting operation and the simulated annealing operation in the cuckoo search algorithm. The results obtained from CS, GA, and PSO are also included. The corresponding results verify the accurateness of ACS-SA and the strong capability of optimization, the superiority of the proposed algorithm is obvious.

[a)]Author to whom correspondence should be addressed. Electronic mail: 19994035@sina.com. Phone: +86 13915955593

I. INTRODUCTION

Chaos, a universal complex dynamical phenomenon, lurks in many nonlinear systems, such as communication system, biological system, meteorological system, etc. Chaos phenomenon is generated by determinate equations but the appearance follows an apparently unpredictable non-periodic stochastic pattern,[1] it is sensitive to the initial state. So the control and synchronization of chaos have been widely studied and many researches are done.[2–10] Parameter estimation is a prerequisite to accomplish the control and synchronization of chaos. In the actual systems, many problems lack continuity of variables which is a chief necessity for parameter estimation. However, direct measurement of system parameters is very difficult, so estimating system parameters from an observed chaotic scalar time series based on parameter estimation methods is feasible and valuable. During the recent years, there has been an increasing interest in investigating parameter estimation methods and various optimization algorithms have been proposed.[1,11–19] Most of those algorithms are nature-inspired meta-heuristic algorithms that compromise the principles and the stochastic properties of the natural phenomena, such as PSO,[11,12] GA,[13,14] ABC,[15–17] DE,[18,19] and etc. Those algorithms are widely used and expose certain ability of optimization. However, there are shortcomings of those methods. For example, the GA and PSO methods are easily trapped into local-best solution that affects the quality of solutions. The stochastic behavior capacity of ABC method needs to be improved. DE method is inefficient in searching local-best solution and the processes are complex. Recently, a novel and robust meta-heuristic based method called cuckoo search algorithm has been proposed by Yang and Deb.[20–26] The algorithm is proved to be very promising and could outperform existing algorithms, such as GA and PSO.[21] Compared with GA and PSO, the CS algorithm has several advantages: first, the randomization is more efficient as the step length is heavy tailed, and any large step is possible. Besides, the number of parameters to be tuned is less than GA and PSO, and thus it is potentially more generic to adapt to a wider class of optimization problems. However, there are some shortcomings that limit the performance of the algorithm, such as the fixed value of parameters which limits the convergence rate and performance of the algorithm, and the relatively poor ability of local searching is also a drawback. So it is necessary to improve the performance of CS algorithm to obtain a higher-quality solution of optimization.

The objective of this paper is to propose a novel and efficient hybrid optimization algorithm *"ACS-SA"* to estimate the parameters of chaotic system. The proposed hybrid algorithm is based on the CS algorithm and improved by integrating the adaptive operation and simulated annealing operation into the CS algorithm. When the hybrid algorithm *"ACS-SA"* is developed, it is utilized for the parameter estimation of Lorenz chaotic system under the noiseless and noise condition. In order to furnish a better insight into the capability of the ACS-SA algorithm, the results obtained from CS, GA, and POS are also included. The corresponding results verify the accurateness of ACS-SA and the strong capability of optimization, the superiority of the proposed algorithm is obvious.

The remaining sections of this paper are organized as follows. In Sec. II, a brief problem formulation is described. Section III elaborates the ACS-SA algorithm. The simulated numerical results established upon the proposed algorithm are given in Sec. IV, the compared results based on CS, GA, and PSO are also presented. The paper ends with conclusions in Sec. V.

II. PROBLEM FORMULATION

In fact, the problem of parameter estimation can be converted into the problem of multi-dimensional optimization by constructing the proper fitness function.

Let the following equation be a continuous nonlinear n-dimension chaotic system:

$$\dot{X} = F(X, X_0, \theta), \tag{1}$$

where $X = (x_1, x_2, \cdots, x_n)^T \in R^n$ denotes the state vector of the chaotic system, $\dot{X}$ is the derivative of X, $X_0 = (x_{10}, x_{2o}, \cdots, x_{n0})^T \in R^n$ denotes the initial state of system, and $\theta = (\theta_1, \theta_2, \cdots, \theta_d)^T$ is a set of original parameters. Suppose the structure of the system (1) is known, then the estimated system can be written as

$$\dot{\tilde{X}} = F(\tilde{X}, X_0, \tilde{\theta}), \tag{2}$$

where $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n)^T \in R^n$ denotes the state vector of the estimated system, $\tilde{\theta} = (\tilde{\theta}_1, \tilde{\theta}_2, \cdots, \tilde{\theta}_d)^T$ is a set of estimated parameters. In order to convert the parameter estimation problem into optimization problem, the following objective fitness function is defined:

$$F(\tilde{\theta}) = \sqrt{\frac{1}{M}\sum_{i=1}^{M}(X - \tilde{X})^2}, \tag{3}$$

where $i = 1, 2, \cdots, M$ is the sampling time point and Mdenotes the length of data used for parameter estimation. The parameter estimation of system (1) can be achieved by searching the most proper values of $\tilde{\theta}$ so that the objective function (3) is globally minimized.

It can be found that Eq. (3) is a multidimensional nonlinear function with multiple local search optima, it is easily trapped into local optimal solution and the computation amount is great, so it is not easy to search the globally optimal solution effectively and accurately using traditional general methods. In this paper, the hybrid ACS-SA algorithm is proposed to solve the complex optimization problem, the general principle of parameter estimation by ACS-SA is shown in Fig. 1.

III. ACS-SA ALGORITHM

The core thought of ACS-SA algorithm is integrating the parameter adaptive adjusting process and simulated annealing operation into the cuckoo search algorithm. Hoping the speed of convergence and the quality of solution can be improved.

First, we describe the CS algorithm briefly. The CS algorithm is a population based stochastic global search algorithm inspired by the obligate brood parasitism of some cuckoo species by laying their eggs in the nests of host bird. Yang and Deb[20] combined the cuckoo breeding behavior[27] with the Lévy flights[28–30] and proposed cucukoo search algorithm. In the algorithm, each egg in a nest represents a solution, and a cuckoo egg represents a new solution, the aim is to employ the new and potentially better solutions to replace not-so-good solutions in the nests until the optimal solution is found. During the search process, there are three idealized rules[30] that should be emphasized: (1) each cuckoo lays one egg at a time, and dumps it in a randomly chosen set; (2) the best nests with high quality of eggs will carry over to the next generations; (3) the number of available host nests is fixed, and the egg laid by a cuckoo is discovered by the host bird with a probability $p_a \in [0, 1]$, in this case, the host bird can either throw the egg away or abandon the nest to build a new nest (new solution) in a new location. Based on the above-mentioned three rules, the CS can be implemented simply. The detailed description of the procedures can refer to

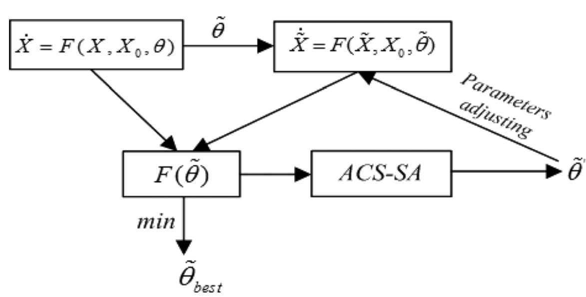

FIG. 1. The general principle of parameter estimation by the ACS-SA algorithm.

Refs. 20 and 31, and the repetitious details need not be given here.

Although the performance of searching the global optimal solution is good, the local searching ability of the CS algorithm is relatively poor, besides the performance is closely related to the parameter p_a and α, where p_a is the probability of abandoning the worsen nests and α is the step size. So the adaptive parameters adjusting operation and the simulated annealing operation are integrated into the CS algorithm to enhance the optimization performance. The detailed procedures are described as follows.

It is found that the traditional CS algorithm uses fixed values of parameters, and these values are set in the initial step and kept unchanged during the generations. However, it is difficult to establish a proper set of parameter values in the initialization step. If p_a is small and α is large, then the performance will be poor and the number of iterations will increase a lot. On the contrary, if p_a is large and α is small, then the speed of convergence is high but the quality of solution is decreased, it may be unable to search the optimal solution. In order to overcome the drawbacks of the fixed parameter values setting, the adaptive parameters adjusting operation is taken shown in the following equations:

$$\alpha(n) = (\alpha_{\max} - \alpha_{\min}) \cdot \sqrt{1 - \frac{n}{N}} + \alpha_{\min}, \tag{4}$$

$$p_a(n) = p_{a\_\max} - \frac{(p_{a\_\max} - p_{a\_\min})}{N} \cdot n, \tag{5}$$

where n and N are the number of the current iteration and the number of total iterations, respectively. It can be found from Eqs. (4) and (5) that the values of p_a and α are big enough in the early generations, so that the convergence speed is high and the diversity of solutions is increased. With the generations going on, the values of p_a and α are decreased gradually, although the convergence speed is reduced, the quality of solution is increased that result in a better fine-tuning of solutions. Overall, the convergence speed and the quality of solution are balanced and improved during the whole iterations.

In order to improve the local searching ability of the algorithm, the simulated annealing operation[32,33] is also integrated into the traditional CS algorithm. The annealing process is usually simulated using a Monte Carlo procedure. In this procedure, the thermal motion of atoms in contact with a heat bath at a given temperature is simulated. The procedure is simply stated here:[34] Given a configuration of the elements of the system, randomly displace the elements on a time, by a small amount and calculate the resulting change in the energy, ΔE. If $\Delta E < 0$, then accept the displacement and use the resulting configuration as the starting point for the next iteration. If $\Delta E \geq 0$, then the displacement is accepted with probability $P(\Delta E) = \exp(-\Delta E/k_b T)$ where T is the current temperature and k_b is Boltzmann's constant. Repetition of this step continues until equilibrium is achieved. For more detailed information on the simulated annealing, please refer to Refs. 32–34.

The whole graphical procedure of the *ACS-GA* algorithm is shown in Fig. 2.

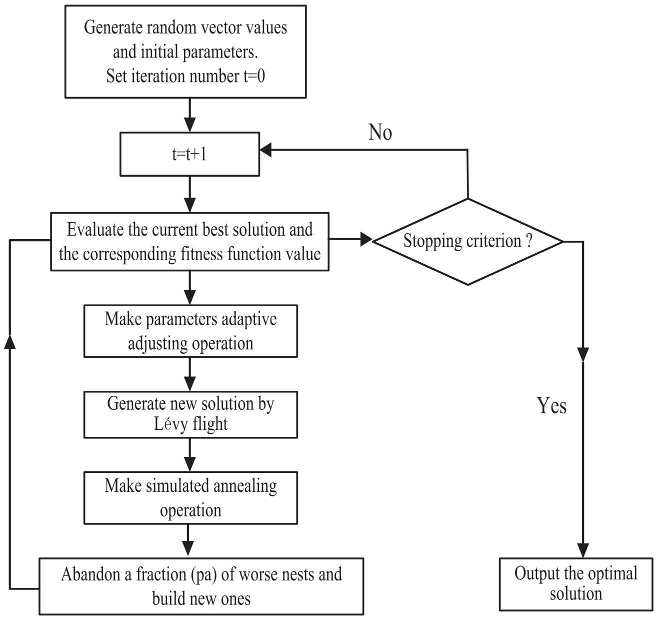

FIG. 2. The graphical procedure of the ACS-GA algorithm.

The detailed design steps for parameter estimation with the ACS-SA algorithm can be summarized as follows:

Step 1: Initial the parameter values of the algorithm, generate the random initial K vector values $\boldsymbol{X}_{ini} = [x_1, x_2 \cdots x_K]$, and set the iteration number $t = 0$.

Step 2: Begin the iteration process and update the iteration number $t = t + 1$.

Step 3: Evaluate fitness values of each individual and determine the current best individual x_{ibest} with the best objective value F_{ibest}.

Step 4: Adjust the parameter (p_a, α) values using Eqs. (4) and (5).

Step 5: Keep the best solution of the last iteration, and get a set of new solutions $\boldsymbol{X}_{new} = [x_1{}^{(t+1)}, \cdots x_i{}^{(t+1)}, \cdots, x_K{}^{(t+1)}]$ by Lévy flight, the Lévy flight is performed according to Eq. (6), where α is the step size, the product $\oplus$ means entry-wise multiplications location, $\mathrm{Le'vy}(\lambda)$ is the step-lengths that are distributed according to the following probability distribution shown in Eq. (7) which has an infinite variance. Here, the consecutive steps of a cuckoo essentially form a random walk process which obeys a power-law step-length distribution with a heavy tail

$$x_i{}^{(t+1)} = x_i{}^{(t)} + \alpha \oplus \mathrm{Le'vy}(\lambda), \tag{6}$$

$$\mathrm{Le'vy}(\lambda) \sim u = t^{-\lambda}, \;\; (1 < \lambda \leq 3). \tag{7}$$

Step 6: Evaluate the fitness value $F_i{}^{(t+1)}$ of the new solution $x_i{}^{(t+1)}$, and compare $F_i{}^{(t+1)}$ with $F_i{}^{(t)}$ which represents the solution of the tth iteration. If $F_i{}^{(t+1)}$ is better than $F_i{}^{(t)}$, then replace $x_i{}^{(t)}$ by $x_i{}^{(t+1)}$, otherwise, not abandoning the solution $x_i{}^{(t+1)}$ at once but accepting the solution with probability $p = e^{\left(-\frac{|\Delta f|}{K \cdot T}\right)}$, where Δf is the change in the fitness value $\Delta f = F_i{}^{(t+1)} - F_i{}^{(t)}$. K is Boltzmann's constant. T is the current temperature. Select a random variable $r_i \in [0, 1]$, $i \in [1, N]$, if $p \geq q$, then accept the new solution and use the

TABLE I. The mean value and best value of the 15 experiments based on the four different methods (ACS-SA, CS, PSO, and GA) in the noiseless condition are listed. The values contain the estimated parameters (σ_1,σ_2,σ_3) and fitness function values. The statistical results based on different methods in the noiseless condition.

	Mean value				Best value			
	ACS-SA	CS	PSO	GA	ACS-SA	CS	PSO	GA
σ_1	10.00000000	9.99995632	9.992814895	10.07905520	10.00000000	9.99996821	9.99642826	9.99712670
σ_2	28.00000000	28.00001956	28.006241569	27.88774721	28.00000000	28.00000685	28.00135772	28.00537790
σ_3	2.66666667	2.66666119	2.66291948	2.66955523	2.66666667	2.66666527	2.66671654	2.66538629
H	3.1744e-010	1.2406e-004	0.04945120	0.15141695	4.4661e-011	1.6885e-005	0.00204566	0.00771826

solution as the starting point for the next iteration; otherwise, abandon the solution. So a set of solution $\boldsymbol{X'}_{new} = [x_{11}{}^{(t+1)}, x_{22}{}^{(t+1)}, \cdots, x_{ii}{}^{(t+1)}]$ are obtained.

Step 7: Generate a set of random variables $r_i \in [0, 1]$, $i \in [1, N]$, if $r_i < p_a$, then keep the current solution $x_{ii}{}^{(t+1)}$; otherwise, it should abandon the solution and build the new solution $x'_{ii}{}^{(t+1)}$ by Lévy flight, calculate the fitness value $F'_i{}^{(t+1)}$ of $x'_{ii}{}^{(t+1)}$, compare $F'_i{}^{(t+1)}$ with $F_i{}^{(t+1)}$ and replace the worsen solution by the better one, then a set of better solutions are obtained $\boldsymbol{X}_{better} = [x_{11b}{}^{(t+1)}, x_{22b}{}^{(t+1)}, \cdots, x_{iib}{}^{(t+1)}]$.

Step 8: Go to step 3 and check whether the stopping criterion is met. If stopping criterion is met, then output the best solution x_{ibest}; otherwise go to step 2 and continue the iteration process until the stopping criterion is met.

IV. SIMULATION RESULTS

To demonstrate the effectiveness of ACS-SA algorithm, the Lorenz chaotic system is chosen to test the performance, the system which designates a mathematical model for thermal convection was proposed by Edward Lorenz. The algorithm is implemented using Matlab 7.0 and executed on Pentium 2.66 G, 1 G of memory personal computer. Lorenz chaotic system equation is expressed as follows:

$$\begin{cases} \dot{x} = \sigma_1(y - x) \\ \dot{y} = \sigma_2 x - xz - y, \\ \dot{z} = xy - \sigma_3 z \end{cases} \tag{8}$$

where (x, y, z) is the state variables; σ_1,σ_2,σ_3 are the unknown chaotic system parameters which need to be estimated. The true parameters of the system are $\sigma_1 = 10$, $\sigma_2 = 28$, $\sigma_3 = 8/3$ which ensure a chaotic behavior, in order to obtain the values of some state variables, the fourth-order Runge-Kutta algorithm is used to solve Eq. (8), the step is $h = 0.01$. Then a series of state variables values are obtained and 100 state variables of different times ($\{(x(n), y(n), z(n)),\ n = 1, 2, \cdots 100\}$) are chosen to be the sample data. The parameters of the algorithm are set as follows. $p_{a\_max} = 0.5$, $p_{a\_min} = 0.01$, $\alpha_{max} = 0.5$, $\alpha_{min} = 0.005$, the max iteration number is $N = 200$, the sample size is $M = 100$, the annealing mode is shown in Eq. (9) where n is the iteration number, the initial temperature is $T_0 = 100$

$$T(n) = \frac{T_0}{\ln(1 + n)}. \tag{9}$$

The objective (fitness) function H is shown in Eq. (10), where $(x(n), y(n), z(n))$ is the nth state variable that corresponds to the true system parameters, $(\tilde{x}(n), \tilde{y}(n), \tilde{z}(n))$ is the nth state variable that corresponds to the estimated system parameters

$$H = \sqrt{\frac{1}{M}\sum_{n=1}^{M}(\tilde{x}(n) - x(n))^2 + (\tilde{y}(n) - y(n))^2 + (\tilde{z}(n) - z(n))^2)}. \tag{10}$$

In order to eliminate the difference of each experiment, the algorithm is executed 15 times, then the mean value of the 15 experiments is taken as the final estimated value, the mean value and best value of the 15 experiments are listed in Table I. The results based on traditional cuckoo search (the best parameter setting is $p_a = 0.25$, $p_a = 0.01$), PSO (the best parameter setting is $w = 0.8$, $c = 1.5$, where w is the inertia weight and c is learning factor), and GA (the best parameter setting is $c_r = 0.8$, $m_u = 0.1$ where c_r is the crossover rate and m_u is the mutation rate) are also listed in Table I.

It can be seen from Table I that the best fitness values obtained by ACS-SA algorithm are quite better than the comparison algorithms. The mean values of the establish parameters are also with higher precision than other algorithms. The estimated values are close to the true values infinitely. The fitness values calculated by ACS-SA are much smaller than the others and close to zero infinitely. Fig. 3 shows the convergence process of the fitness values of all the algorithms during the iterations in a single run.

Fig. 4 shows the convergence processes of the three parameters (σ_1,σ_2,σ_3) during the iterations in a single run by different algorithms.

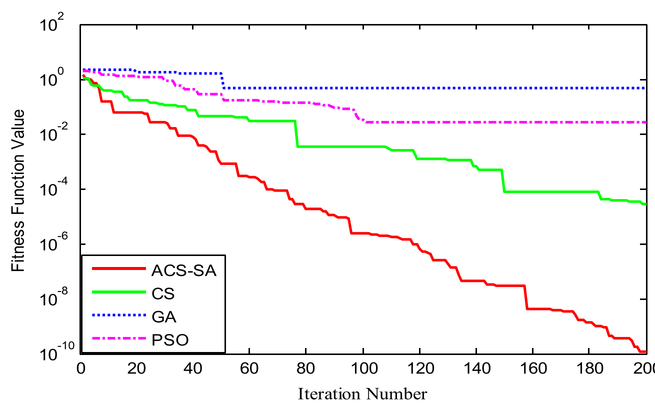

FIG. 3. The convergence process of fitness function value during the iterations in a single run by different algorithms.

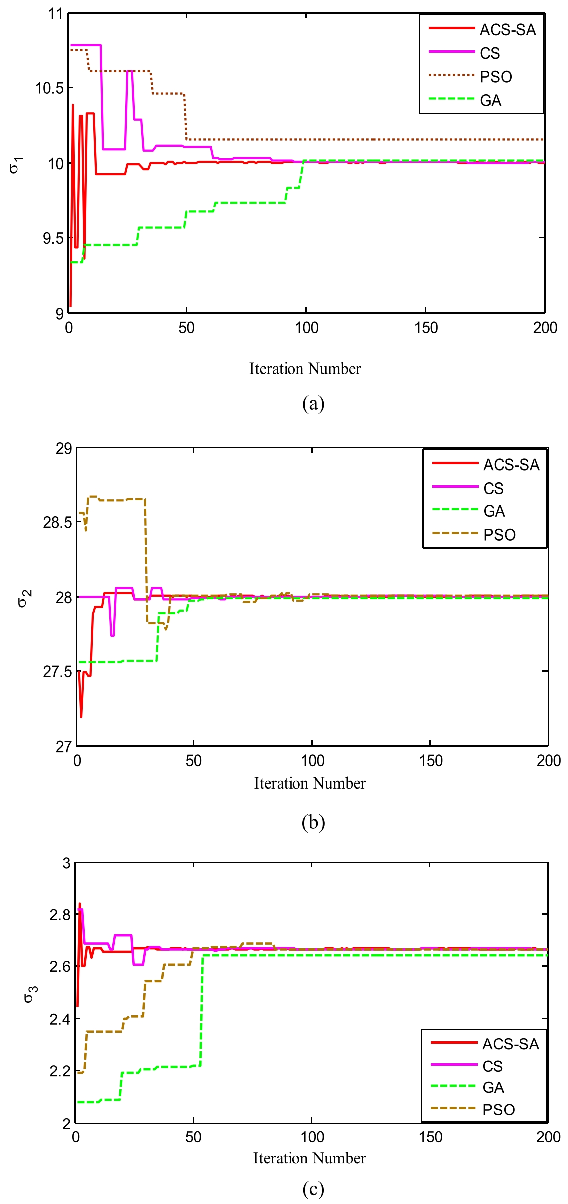

FIG. 4. The convergence processes of the three parameters (σ_1,σ_2,σ_3) during the iterations in a single run by different algorithms (a). The convergence process of σ_1 of the four algorithms (b). The convergence process of σ_2 of the four algorithms (c). The convergence process of σ_3 of the four algorithms.

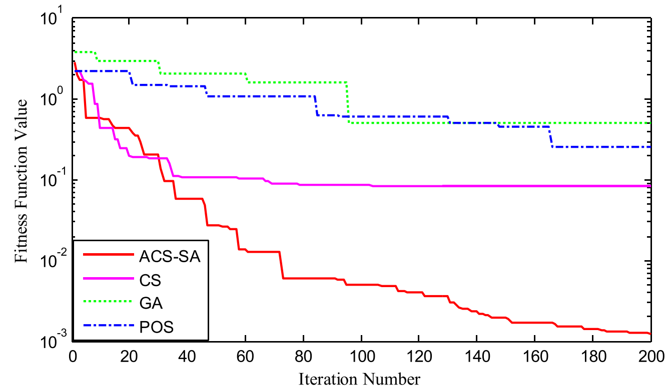

FIG. 5. The convergence process of the fitness value in a single run by the four algorithms in the noise condition.

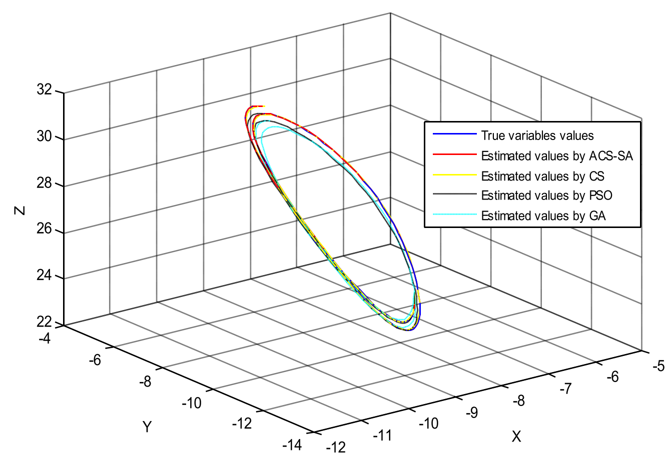

FIG. 6. The spatial distribution of the estimated variables by different algorithms in the noise condition.

It can be seen from Figs. 3 and 4 that the convergence processes of the fitness values and parameters of ACS-SA are much better than the other algorithms. The ACS-SA converges to the more optimal solution than the others; the estimated parameters converge to true value very fast. It can be concluded in general that the ACS-SA contributes to superior performance, CS performs nest-best, PSO is not as good as ACS-SA and CS but better than GA, and GA performs worst.

As the actual chaotic systems always associate with noise, in order to test the performance of parameter estimation in the noise condition, the noise sequences are added to the original sample data. We add the white noise to the state variables $\{(x(n),\ y(n),\ z(n)),\ \ n = 1,\ 2,\ \cdots 100\}$, the range of the noise sequences is from -0.1 to 0.1. Use the ACS-SA, CS, PSO, GA algorithms to estimate the chaotic system parameters, respectively, based on the same parameters setting of the algorithms as the previous experiments, the algorithms are executed 15 times, and the statistical results of the best fitness values, the mean fitness values over the 15 runs are listed in Table II.

TABLE II. The mean value and best value of the 15 experiments based on the four different methods (ACS-SA, CS, PSO, and GA) in the noise condition are listed. The values contain the estimated parameters (σ_1,σ_2,σ_3) and fitness function values. The statistical results by different algorithms in the noise condition.

	Mean value				Best value			
	ACS-SA	CS	PSO	GA	ACS-SA	CS	PSO	GA
σ_1	9.99736006	10.07854291	9.83103417	10.20841737	9.99934536	10.00102832	9.88098509	10.03926377
σ_2	28.00326286	27.97998389	27.85815637	27.65187747	28.00005123	28.00176048	28.02239873	27.89289181
σ_3	2.66655831	2.65932753	2.69847587	2.64385154	2.66675566	2.66643633	2.67475714	2.66004856
H	0.00862130	0.03820070	0.22082975	0.49669471	2.9152e-004	0.00113949	0.04956005	0.15516637

It can be seen from Table II that the four algorithms all have a certain capability of identification of parameters, and the performance of ACS-SA is much better than the other algorithms, it supplies more robust and precise results in the noise condition than the comparison algorithms, although the precision of the estimated parameters is declined compared with the estimated results in the noiseless condition, the precision is still satisfactory. So it can be concluded that the ACS-SA algorithm posses a powerful capability for parameters identification in the noise condition. Fig. 5 shows the convergence process of the fitness value in a single run by the four algorithms in the noise condition. The spatial distributions of the estimated variables by different algorithms in the noise condition are shown in Fig. 6. It can be seen from Figs. 5 and 6 that the estimated results in the noise condition by ACS-SA are better than CS, PSO, and GA, the noise immunity of ACS-SA is superior to others.

V. CONCLUSION

In this paper, a novel hybrid algorithm is proposed to estimate chaotic system parameters from the point of optimization. The hybrid algorithm is called "*ACS-SA*" that integrates the parameter adaptive adjusting process and simulated annealing operation into the cuckoo search algorithm. In order to verify the optimization capabilities of the "*ACS-SA*" algorithm, the Lorenz system is chosen to test the performance, the estimated results demonstrate the strong capabilities, and effectiveness of the proposed algorithm, the convergence speed and the quality of solution of the cuckoo search algorithm are balanced and improved by the parameter adaptive adjusting operation, and the local search ability is also improved by the simulated annealing operation. Compared with the CS, PSO, and GA algorithms, the "*ACS-SA*" algorithm supplies more robust and precise results than the comparison algorithms. In the end, the white noise is added into the system to test the performance of parameters identification in the noise condition, the simulated results indicate that the "*ACS-SA*" algorithm posses a powerful capability of noise immunity and can identify the parameters more accurately and stably than the CS, PSO, and GA algorithms in the noise condition. In general, the proposed "*ACS-SA*" algorithm is a feasible, effective, and promising method for parameters estimation of chaotic systems.

ACKNOWLEDGMENTS

This work is supported by National Natural Science Foundation of China under Grant Nos. 41105013, 41375028, 41305017, and 61271106.

[1]A. Mohamadreza and M. Hamed, "Chaotic invasive weed optimization algorithm with application to parameter estimation of chaotic systems," Chaos, Solitons Fract. **45**, 1108–1120 (2012).
[2]S. Francesco and D. Pietro, "Estimation of communication-delays through adaptive synchronization of chaos," Chaos Solitons Fract. **45**, 35–46 (2012).
[3]L. X. Guo, M. F. Hu, and Z. Y. Xu, "Synchronization and chaos control by quorum sensing mechanism," Nonlinear Dyn. **73**, 1253–1269 (2013).
[4]S. Boccaletti, J. Kurths, and G. Osipov, "The synchronization of chaotic systems," Phys. Rep. **366**, 1–2 (2002).
[5]H. D. I. Abarbanel, N. F. Rulkov, and M. M. Sushchik, "Generalized synchronization of chaos: The auxiliary system approach," Phys. Rev. E **53**, 4528–4535 (1996).
[6]Y. Tang, Z. Wang, and J.-A. Fang, "Pinning control of fractional-order weighted complex networks," Chaos **19**, 013112 (2009).
[7]Y. Tang, Z. Wang, W. K. Wong, J. Kurths, and J.-A. Fang, "Multiobjective synchronization of coupled systems," Chaos **21**, 025114 (2011).
[8]A. W. Jayawardena, P. Xu, and W. K. Li, "A method of estimating the noise level in a chaotic time series," Chaos **18**, 023115 (2008).
[9]H. N. Agiza and M. T. Yassen, "Synchronization of Rossler and Chen chaotic dynamical systems using active control," Phys. Lett. A **278**, 191–197 (2001).
[10]S. J. Liao, "On the numerical simulation of propagation of micro-level inherent uncertainty for chaotic dynamic systems," Chaos Solitons Fract. **47**, 1–12 (2013).
[11]F. Gao and H. Q. Tong, "Parameter estimation for chaotic system based on particle swarm optimization," Acta Phys. Sin. **55**, 577–582 (2006).
[12]Q. He, L. Wang, and B. Liu, "Parameter estimation for chaotic systems by particle swarm optimization," Chaos Solitons Fract. **34**, 654–661 (2007).
[13]D. Dai, X. K. Ma, F. C. Li, and Y. You, "An approach of parameter estimation for a chaotic system based on genetic algorithm," Acta Phys. Sin. **51**, 2459–2462 (2002).
[14]T. Chao, Z. Yu, and J. J. Jack, "Estimating system parameters from chaotic time series with synchronization optimized by a genetic algorithm," Phys. Rev. E **76**, 016209 (2007).
[15]D. Karaboga and B. Akay, "A comparative study of artificial bee colony algorithm," Appl. Math. Comput. **214**(12), 108–132 (2009).
[16]D. Karaboga and B. Akay, "A survey: Algorithms simulating bee swarm intelligence," Artif. Intell. Rev. **31**, 61–85 (2009).
[17]D. Karaboga and B. Basturk, "Artificial been colony (abc) optimization algorithm for solving cinstreined optimization problems," Lect. Notes Comput. Sci. **4529**, 789–798 (2007).
[18]G. T. Ying and P. G. Xin, "Parameter estimation of chaotic system with time-delay: A differential evolution approach," Chaos Solitons Fract. **42**, 3132–3139 (2009).
[19]B. Peng, B. Liu, F. Y. Zhang, and L. Wang, "Differential evolution algorithm-based parameter estimation for chaotic systems," Chaos Solitons Fract. **39**, 2110–2118 (2009).
[20]X. S. Yang and S. Deb, "Cuckoo search via Lévy flights," in *Proceedings of World Congress on Nature & Biologically Inspired Computing* (IEEE Publications, USA, 2009), pp. 210–214.
[21]X. S. Yang and S. Deb, "Engineering optimisation by cuckoo search," Int. J. Math. Model. Numer. Optim. **1**(4), 330–343 (2010).
[22]H. G. Amir, X. S. Yang, and H. A. Amir, "Cuckoo search algorithm: A metaheuristic approach to solve structural optimization problems," Eng. Comput. **29**, 17–35 (2013).
[23]X. S. Yang, Z. H. Cui, R. B. Xiao, A. H. Gandomi, and M. Karamanoglu, *Swarm Intelligence and Bio-Inspired Computation: Theory and Applications* (Elsevier, 2013).
[24]A. Ouaarab, B. Ahiod, and X. S. Yang, "Discrete cuckoo search algorithm for the traveling salesman problem," Neural Comput. Appl. **2013**, 1–11 (2013).
[25]X. S. Yang and S. Deb, "Two-stage eagle strategy with differential evolution," Int. J. Bio-Inspired Comput. **4**, 1–5 (2012).
[26]X. S. Yang and S. Deb, "Multiobjective cuckoo search for design optimization," Comput. Oper. Res. **40**, 1616–1624 (2013).
[27]R. B. Payne, M. D. Sorenson, and K. Klitz, *The Cuckoos* (Oxford University Press, 2005).
[28]P. Barthelemy, J. Bertolotti, and D. S. Wiersma, "A Lévy flight for flight," Nature **453**, 495–498 (2008).
[29]I. Pavlyukevich, "Lévy flight, non-local search and simulated annealing," J. Comput. Phys. **226**, 1830–1844 (2007).
[30]X. T. Li and M. H. Yin, "Parameter estimation for chaotic systems using the cuckoo search algorithm with an orthogonal learning method," Chin. Phys. B **21**(5), 050507 (2012).
[31]C. Pinar and B. Erkan, "A conceptual comparison of the Cuckoo-search, particle swarm optimization, differential evolution and artificial bee colony algorithms," Artif. Intell. Rev. **39**, 315–346 (2013).
[32]S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, "Optimization by simulated annealing," Science **220**, 671–680 (1983).
[33]M. S. Kim, M. R. Feldman, and C. C. Guest, "Optimum encoring of binary phase-only filters with a simulated annealing algorithm," Opt. Lett. **14**(11), 545–547 (1989).
[34]L. G. Bruce and C. S. Christopher, "Using simulated annealing to solve routing and location problem," Naval Res. Logistics Q. **33**, 261–279 (1986).