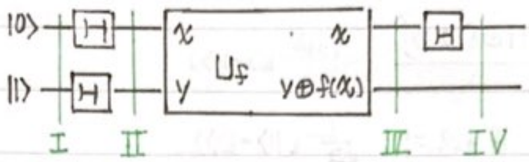
Quantum Search Algorithms

Introduction

Deutsch's algorithm



I $|01\rangle$

II $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

III $U_f|x\rangle(|0\rangle - |1\rangle)\frac{1}{\sqrt{2}} = (-1)^{f(x)}|x\rangle\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

if $f(0)=f(1)$ $\quad \pm\frac{|0\rangle+|1\rangle}{\sqrt{2}}\frac{|0\rangle-|1\rangle}{\sqrt{2}}$

if $f(0)\neq f(1)$ $\quad \pm\frac{|0\rangle-|1\rangle}{\sqrt{2}}\frac{|0\rangle-|1\rangle}{\sqrt{2}}$

IV if $f(0)=f(1)$ $\quad \pm|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}$

if $f(0)\neq f(1)$ $\quad \pm|1\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}$

$\pm|f(0)\oplus f(1)\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}$

Deutsch-Jozsa algorithm

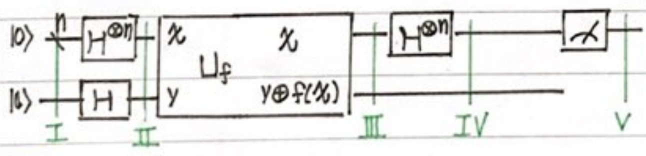
x	f_0	f_1	f_x	$f_{\bar{x}}$
0	0	1	0	1
1	0	1	1	0
	constant		balanced	

$f_0(x)=0 \qquad f_x(x)=x$

$f_1(x)=1 \qquad f_{\bar{x}}(x)=1-x$

determine by querying the function as few time as possible, whether the function is balanced or constant

$U_f(|x\rangle|y\rangle) = |x\rangle|y\overset{XOR}{\oplus}f(x)\rangle$, example $U_{f_x}(|00\rangle)=U_{f_x}(|0\rangle|0\rangle)=|0\rangle|0\oplus f_x(0)\rangle=|0\rangle|0\oplus 1\rangle$
$=|0\rangle|1\rangle$



I $|0\rangle^{\otimes n}|1\rangle$

II $\sum_{x\in\{0,1\}^n}\frac{|x\rangle}{\sqrt{2^n}}\frac{|0\rangle-|1\rangle}{\sqrt{2}}$

III $\sum_x\frac{(-1)^{f(x)}|x\rangle}{\sqrt{2^n}}\frac{|0\rangle-|1\rangle}{\sqrt{2}}$

IV $\sum_z\sum_x\frac{(-1)^{x\cdot z+f(x)}|z\rangle}{2^n}\frac{|0\rangle-|1\rangle}{\sqrt{2}}$

V z

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III : $U_f\left[\left(\frac{1}{\sqrt{2^n}}\sum_{x\in\{0,1\}^n}|x\rangle\right)\otimes\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)\right]=\frac{1}{\sqrt{2^n}}\sum_{x\in\{0,1\}^n}|x\rangle\otimes\frac{1}{\sqrt{2}}\left[|0\oplus f(x)\rangle-|1\oplus f(x)\rangle\right]$

$=\frac{1}{\sqrt{2^n}}\sum|x\rangle\otimes\underline{\frac{1}{\sqrt{2}}\left[|f(x)\rangle-|1\oplus f(x)\rangle\right]}$ $(-1)^{f(x)}(|0\rangle-|1\rangle)$

if $f(x)=0$, $\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$ if $f(x)=1$, $\frac{1}{\sqrt{2}}(|1\rangle-|0\rangle)$

$=\frac{1}{\sqrt{2^n}}\sum|x\rangle\otimes\frac{1}{\sqrt{2}}(-1)^{f(x)}(|0\rangle-|1\rangle)=\frac{1}{\sqrt{2^n}}\sum(-1)^{f(x)}|x\rangle\otimes\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$

IV $H^{\otimes n}|u\rangle=\frac{1}{\sqrt{2^n}}\sum_{v\in 0,1}(-1)^{u\cdot v}|v\rangle$

$\Rightarrow \frac{1}{2^n}\sum_{x\in 0,1}(-1)^{f(x)}\sum_{u\in 0,1}(-1)^{x\cdot v}|v\rangle$ $=\frac{1}{2^n}\sum_v\sum_x(-1)^{f(x)}(-1)^{x\cdot v}|v\rangle$

V if f is constant

$\Rightarrow \frac{1}{2^n}(-1)^{f(0)}\underbrace{\sum_v\sum_x(-1)^{x\cdot v}}_{\substack{v=0,\ 2^n|0\rangle \\ v=1,\ 0}}|v\rangle=\frac{1}{2^n}(-1)^{f(0)}2^n|0\rangle^{\otimes n}=(-1)^{f(0)}|0\rangle^{\otimes n}$

if amplitude of $|0\rangle^{\otimes n}$ $=1$, f is constant
$=0$, is balanced

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for example : $f(x_0, x_1) = x_0 \oplus x_1$ $\begin{cases} f(0,0)=0 \\ f(0,1)=1 \\ f(1,0)=1 \\ f(1,1)=0 \end{cases}$

Ⅰ $|001\rangle$

Ⅱ $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Ⅲ $CX_{02}\, CX_{12}$

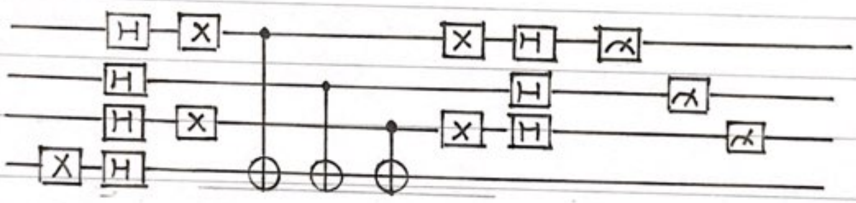
$$\frac{1}{2\sqrt{2}}\left[|00\rangle \otimes (|0\rangle - |1\rangle) + |01\rangle \otimes (|1\rangle - |0\rangle) + |10\rangle \otimes (|1\rangle - |0\rangle) + |11\rangle \otimes (|0\rangle - |1\rangle)\right]$$

$$= \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Ⅳ $|1\rangle \otimes |1\rangle \otimes (|0\rangle - |1\rangle)$

the amplitude of $|00\rangle$ is $0 \rightarrow f$ is balanced constant



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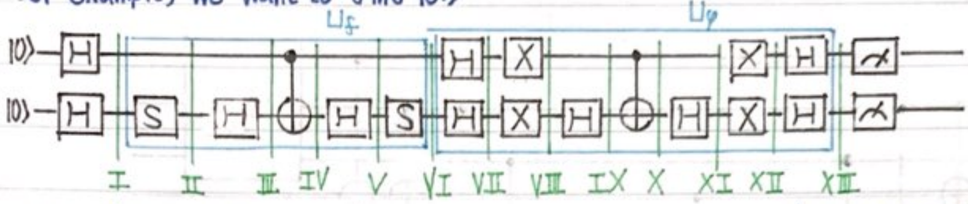
Grover's Algorithm

For example, we want to find $|01\rangle$



I $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

II $\frac{1}{2}(|00\rangle + i|01\rangle + |10\rangle + i|11\rangle)$

III $\frac{1}{2\sqrt{2}}\left[(1+i)|00\rangle + (1-i)|01\rangle + (1+i)|10\rangle + (1-i)|11\rangle\right]$

IV $\frac{1}{2\sqrt{2}}\left[(1+i)|00\rangle + (1-i)|01\rangle + (1-i)|10\rangle + (1+i)|11\rangle\right]$

V $\frac{1}{2}(|00\rangle + i|01\rangle + |10\rangle - i|11\rangle)$

VI $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$

VII $\frac{1}{2}(|00\rangle + |01\rangle - |10\rangle + |11\rangle)$

VIII $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$

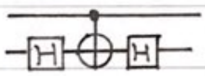
IX $\frac{\sqrt{2}}{2}(|01\rangle + |10\rangle)$

X $\frac{\sqrt{2}}{2}(|01\rangle + |11\rangle)$

XI $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$

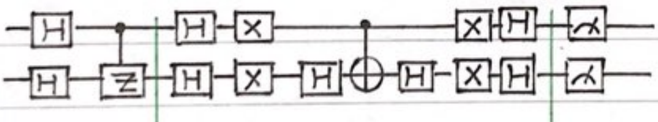
XII $\frac{1}{2}(-|00\rangle + |01\rangle - |10\rangle + |11\rangle)$

XIII $-|01\rangle$



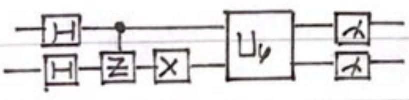
$|00\rangle \rightarrow |00\rangle$ $|10\rangle \rightarrow |10\rangle$
$|01\rangle \rightarrow |01\rangle$ $|11\rangle \rightarrow -|11\rangle$

Find $|11\rangle$



$\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$ $-|11\rangle$

Find $|01\rangle$



Find $|11\rangle$

find 010 or 101