守恆量

$\overrightarrow{LLR}$

$$= \frac{m\vec{v}(t)\times 2A\vec{e}_z}{-k} - \vec{e}_r(t)$$

力學能

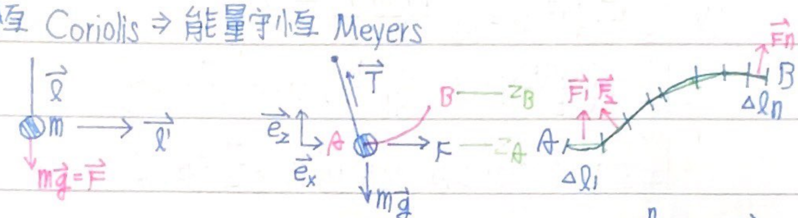
$$e^2 = |\overrightarrow{LLR}|^2 = 1 + \frac{4mA^2}{|k^2|}\left[2\left(\frac{1}{2}m|\vec{v}(t)|^2 + \frac{k}{r(t)}\right)\right]$$

面積率

(1)力學能守恆 (時間平移)　(2)面積率→角動量守恆 (空間中方定轉)　(3)動量守恆 (空間平移)

力學能守恆 Coriolis ⇒ 能量守恆 Meyers

作功



$\vec{T}+\vec{F}+m\vec{g}=0$

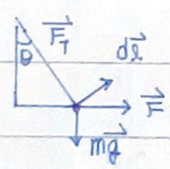
$W_{AB} = \sum_{i=1}^{n} \vec{F}_i\cdot\overrightarrow{\Delta l_i} \Rightarrow \lim_{\substack{N\to\infty \\ |\Delta l_i|\to 0}} \sum_{i=1}^{N} \vec{F}_i\cdot\overrightarrow{\Delta l_i}$

$= \int_A^B \vec{F}\cdot d\vec{l} = W_{A\to B}$

$d\vec{l} = dx\,\vec{e}_x + dz\,\vec{e}_z$　　$W_{T\,A\to B} \int_A^B \vec{F}_T\cdot d\vec{l} = 0$

$m\vec{g} = -mg\,\vec{e}_z$　　$W_{mg\,A\to B} \int_A^B m\vec{g}\cdot d\vec{l}$

$m\vec{g}\cdot d\vec{l} = -mg\,dz$　　$= -mg(z_B - z_A)$



$W_{\vec{F},A\to B} = \int \vec{F}\cdot d\vec{l}$

$= \int mg\tan\theta\,dx$

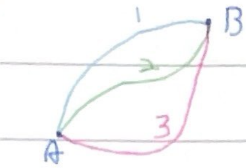
$\vec{F} = +m\vec{g}\tan\theta\,\vec{e}_x,\ \tan\theta = \frac{dz}{dx}$　　$= \int mg\frac{dz}{dx}dx$

$= mg(z_B - z_A)$

$\Rightarrow W_{\vec{F}_T\,A\to B} + W_{\vec{F}\,A\to B} + W_{m\vec{g}\,A\to B} = 0$, $\vec{F}_T + \vec{F} + m\vec{g} = 0$, $\int_A^B(\vec{F}_T + \vec{F} + m\vec{g})\cdot d\vec{l}$

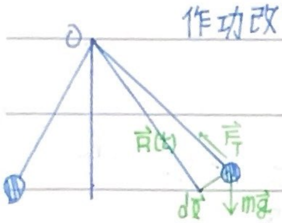
$= \int_A^B \vec{F}_T\,dl + \int_A^B \vec{F}\,dl + \int_A^B m\vec{g}\,dl = 0$

保守力



$W_{AB_1}=W_{AB_2}=W_{AB_3}=\int_A^B \vec{F}\cdot d\vec{\ell}$

作功改變物體動能



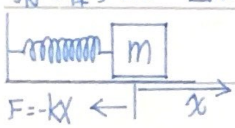
$\vec{F}$是使物作R(t)軌跡的運動 $d\vec{\ell}=d\vec{R}(t)$, $\frac{d\vec{R}(t)}{dt}=\vec{v}(t)$

$W=\int_A^B \vec{F}\cdot d\vec{R}(t)=\int_A^B \frac{d\,m\vec{v}(t)}{dt}\cdot\vec{v}(t)dt$

$=\int_A^B m\left[\frac{dv(t)}{dt}v_x(t)+\frac{dv(t)}{dt}v_y(t)+\frac{dv(t)}{dt}v_z(t)\right]dt$

$=\int_A^B \frac{m}{2}\frac{d}{dt}\left[v_x^2(t)+v_y^2(t)+v_z^2(t)\right]dt=\frac{m}{2}\vec{v}_B\cdot\vec{v}_B-\frac{m}{2}\vec{v}_A\cdot\vec{v}_A$

$\int_A^B \vec{F}_{保守}\,d\vec{\ell}=U(A)-U(B)=\frac{m}{2}\vec{v}_B\cdot\vec{v}_B-\frac{m}{2}\vec{v}_A\cdot\vec{v}_A$, $U(A)+\frac{1}{2}m\vec{v}_A\cdot\vec{v}_A=U(B)+\frac{1}{2}m\vec{v}_B\cdot\vec{v}_B$



$d\vec{\ell}=dx\,\vec{e}_x$ $\quad\vec{F}=-kx\,\vec{e}_x$

$\int_A^B \vec{F}\cdot d\vec{\ell}=\int_A^B -kx\,dx=-\frac{k}{2}x_B^2+\frac{k}{2}x_A^2=U(A)-U(B)$

$E=\frac{1}{2}kx^2(t)+\frac{1}{2}m\left(\frac{dx(t)}{dt}\right)^2\Rightarrow\sqrt{\left[E-\frac{1}{2}kx^2(t)\right]\frac{2}{m}}=\frac{dx(t)}{dt}$, $1=\frac{1}{\sqrt{\frac{2}{m}(E-\frac{1}{2}kx^2(t))}}\frac{dx}{dt}$

$\int_0^t 1\,dt=\int_0^t\frac{1}{\sqrt{\frac{2}{m}(E-\frac{1}{2}kx^2(t))}}\frac{dx}{dt}dt=\int_{x(0)}^{x(t)}\frac{1}{\sqrt{\frac{2}{m}(E-\frac{1}{2}kx^2(t))}}dx=\int_{x(0)}^{x(t)}\frac{1}{\sqrt{\frac{2E}{m}-\frac{k}{m}x^2}}dx=\int_{x(0)}^{x(t)}\frac{1}{\sqrt{A^2-\omega^2x^2}}dx$

$=\int_{x(0)}^{x(t)}\frac{1}{A\sqrt{1-(\frac{\omega}{A})^2x^2}}dx=\int_{x(0)\frac{\omega}{A}}^{x(t)\frac{\omega}{A}}\frac{1}{A\sqrt{1-u^2}}\frac{A}{\omega}du$ $\qquad\frac{k}{m}=\omega^2\quad\frac{2E}{m}=A$

$u=\frac{\omega}{A}x\qquad=\frac{1}{\omega}\sin^{-1}u\Big|_{\frac{\omega}{A}x(0)}^{\frac{\omega}{A}x(t)}$

$=\sin^{-1}\left[\frac{\omega}{A}x(t)\right]-\overbrace{\sin^{-1}\left[\frac{\omega}{A}x(0)\right]}^{\alpha}=\omega t$

$\frac{\omega}{A}x(t)=\sin(\omega t+\alpha)$, $x(t)=\frac{A}{\omega}\sin(\omega t+\alpha)$

$\frac{d^2x(t)}{dt^2}=-\omega^2x(t)\qquad x(t)=B\sin(\omega t+\alpha)$