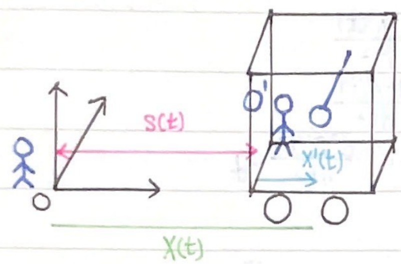
非慣性座標系中的運動



$x(t)=x'(t)+S(t)$

$y(t)=y'(t)$

$z(t)=z'(t)$

$S(t)=b$

$x(t)=x'(t)+b \quad \frac{dx(t)}{dt}=\frac{dx'(t)}{dt} \quad \frac{d^2x(t)}{dt^2}=\frac{d^2x'(t)}{dt^2}$ ⇒ 若O是慣性系 O'也是

$y'(t)=y(t) \quad \frac{dy(t)}{dt}=\frac{dy'(t)}{dt}$

$z(t)=z'(t) \quad \frac{dz(t)}{dt}=\frac{dz'(t)}{dt}$

$S(t)=v_0t$

$x(t)=x'(t)+v_0t \quad \frac{dx(t)}{dt}=\frac{dx'(t)}{dt}+v_0 \quad m\frac{d^2x(t)}{dt^2}=\frac{d^2x'(t)}{dt^2}m=F_x$

$y(t)=y'(t) \quad \frac{dy(t)}{dt}=\frac{dy'(t)}{dt} \quad m\frac{d^2y(t)}{dt^2}=\frac{d^2y'(t)}{dt^2}m=F_y$

$z(t)=z'(t) \quad \frac{dz(t)}{dt}=\frac{dz'(t)}{dt} \quad m\frac{d^2z(t)}{dt^2}=\frac{d^2z'(t)}{dt^2}m=F_z$

⇒伽利略相對原理

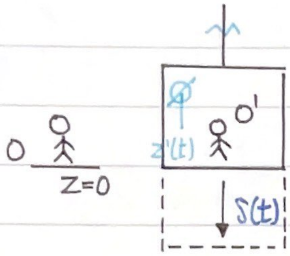
$S(t)=\frac{1}{2}at^2$

$x(t)=\frac{1}{2}at^2+x'(t) \quad \frac{dx(t)}{dt}=\frac{dx'(t)}{dt}+at \quad m\frac{d^2x(t)}{dt^2}=(\frac{d^2x'(t)}{dt}+a)m=F_x$

$y(t)=y'(t) \quad \frac{dy(t)}{dt}=\frac{dy'(t)}{dt} \quad m\frac{d^2y(t)}{dt^2}=\frac{d^2y'(t)}{dt}m=F_y$

$z(t)=z'(t) \quad \frac{dz(t)}{dt}=\frac{dz'(t)}{dt} \quad m\frac{d^2z(t)}{dt^2}=\frac{d^2z'(t)}{dt}m=F_z$

$m\frac{d^2x'(t)}{dt^2}=F_x-ma$ ⇒ 慣性力(假力) $\propto m$



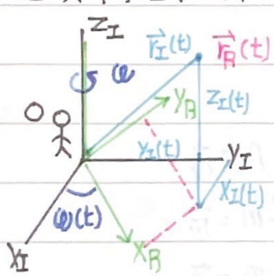
$x(t)=x'(t)$ $\qquad \dfrac{d^2x(t)}{dt^2}=\dfrac{d^2x'(t)}{dt^2}$

$y(t)=y'(t)$ $\qquad \dfrac{d^2y(t)}{dt^2}=\dfrac{d^2y'(t)}{dt^2}$

$z(t)=z'(t)-\frac{1}{2}gt^2$ $\qquad \dfrac{d^2z(t)}{dt^2}=\dfrac{d^2z'(t)}{dt^2}-g$

$$\frac{d^2z'(t)}{dt^2}m_I=m_I\frac{d^2z(t)}{dt^2}+m_Ig \qquad m_G=m_I \quad <10^{-10}$$
$$=F_z+m_Ig$$
$$=-m_Ig+m_Ig=0$$

等角速度轉動座標系



$$\vec{r}_I(t)=x_I(t)\vec{e}_x+y_I(t)\vec{e}_y+z_I(t)\vec{e}_z$$
$$\vec{r}_R(t)=x_R(t)\vec{e}_{xR}(t)+y_R(t)\vec{e}_{yR}(t)+z_R(t)\vec{e}_{zR}$$
$$\vec{e}_{xR}(t)=\cos\omega t\,\vec{e}_x+\sin\omega t\,\vec{e}_y$$
$$\vec{e}_{yR}(t)=-\sin\omega t\,\vec{e}_x+\cos\omega t\,\vec{e}_y$$
$$\vec{e}_{zR}(t)=\vec{e}_z$$

$$\vec{r}_I(t)=\vec{r}_R(t)$$
$$x_I(t)=x_R(t)\cos\omega t-y_R(t)\sin\omega t$$
$$y_I(t)=x_R(t)\sin\omega t+y_R(t)\cos\omega t$$

$$\vec{v}_I(t)=\frac{d\vec{r}_I(t)}{dt}=\frac{d\vec{r}_R(t)}{dt}=\frac{d}{dt}\left[x_R(t)\vec{e}_{xR}(t)+y_R(t)\vec{e}_{yR}+z_R(t)\vec{e}_{zR}\right]$$
$$=\frac{dx_R(t)}{dt}\vec{e}_{xR}(t)+x_R(t)\frac{d\vec{e}_{xR}(t)}{dt}+\frac{dy_R(t)}{dt}\vec{e}_{yR}(t)+y(t)\frac{d\vec{e}_{yR}(t)}{dt}+\frac{dz_R(t)}{dt}\vec{e}_{zR}+0$$

$$\frac{d\vec{e}_{xR}(t)}{dt}=-\omega\sin\omega t\,\vec{e}_x+\omega\cos\omega t\,\vec{e}_y=\omega\vec{e}_{yR}(t)=\vec{\omega}\times\vec{e}_{xR}(t)$$
$$\frac{d\vec{e}_{yR}(t)}{dt}=-\omega\cos\omega t\,\vec{e}_x-\omega\sin\omega t\,\vec{e}_y=-\omega\vec{e}_{xR}(t)=\vec{\omega}\times\vec{e}_{yR}(t)$$
$$\frac{d\vec{e}_{zR}(t)}{dt}=\vec{0}=\vec{\omega}\times\vec{e}_{zR}(t)$$

$$\frac{d_R \vec{r}_R(t)}{dt} \equiv \frac{dx_R(t)}{dt}\vec{e}_{xR}(t) + \frac{dy_R(t)}{dt}\vec{e}_{yR}(t) + \frac{dz_R(t)}{dt}\vec{e}_{zR}(t)$$

$$\frac{d\vec{r}_R(t)}{dt} = \frac{d_R\vec{r}_R(t)}{dt} + \vec{\omega}\times\vec{r}_R(t)$$

$$\begin{aligned}
&x_R(t)\vec{\omega}\times\vec{e}_{xR}(t) + y_R(t)\vec{\omega}\times\vec{e}_{yR}(t) + z_R\vec{\omega}\times\vec{e}_{zR}(t)\\
&= \vec{\omega}\times\left[x_R(t)\vec{e}_{xR}(t) + y_R(t)\vec{e}_{yR}(t) + z_R(t)\vec{e}_R(t)\right]\\
&= \vec{\omega}\times\vec{r}_R(t)
\end{aligned}$$

$$\begin{aligned}
\vec{a}_I(t) &= \frac{d\vec{v}_I(t)}{dt} = \frac{d}{dt}\frac{d\vec{r}_I(t)}{dt} = \frac{d}{dt}\frac{d\vec{r}_R(t)}{dt}\\
&= \frac{d}{dt}\left[\frac{d_R\vec{r}_R(t)}{dt} + \vec{\omega}\times\vec{r}_R(t)\right]\\
&= \frac{d_R}{dt}\frac{d_R\vec{r}_R(t)}{dt} + \vec{\omega}\times\frac{d_R\vec{r}_R(t)}{dt} + \underbrace{\frac{d_R\,\vec{\omega}\times\vec{r}_R(t)}{dt}}_{\Downarrow\ \vec{\omega}\times\frac{d_R\vec{r}_R(t)}{dt}} + \vec{\omega}\times\left[\vec{\omega}\times\vec{r}_R(t)\right]\\
&= \frac{d_R}{dt}\frac{d_R\vec{r}_R(t)}{dt} + 2\vec{\omega}\times\frac{d_R\vec{r}_R(t)}{dt} + \vec{\omega}\times\left[\vec{\omega}\times\vec{r}_R(t)\right]
\end{aligned}$$

$$\vec{F} = m\frac{d^2\vec{r}_I(t)}{dt^2} = m\left[\frac{d_R}{dt}\frac{d_R\vec{r}_R(t)}{dt} + 2\vec{\omega}\times\frac{d_R\vec{r}_R(t)}{dt} + \vec{\omega}\times\left[\vec{\omega}\times\vec{r}_R(t)\right]\right]$$

$$m\frac{d_R}{dt}\frac{d_R\vec{r}_R(t)}{dt} = \vec{F}\underbrace{-2m\vec{\omega}\times\frac{d_R\vec{r}_R(t)}{dt}}_{\text{科氏力}}\underbrace{-m\vec{\omega}\times\left[\vec{\omega}\times\vec{r}_R(t)\right]}_{\text{离心力}}$$

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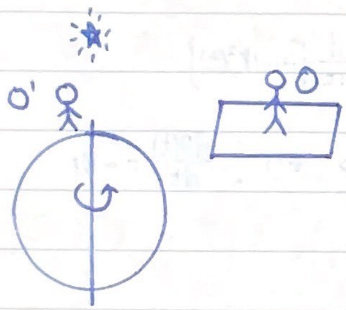
轉動座標系中牛頓第二定律的修正

$$m\frac{d_R}{dt}\frac{d_R\vec{r}_R(t)}{dt}=\vec{F}-2m\vec{\omega}\times\frac{d\vec{r}_R(t)}{dt}-m\vec{\omega}\times(\vec{\omega}\times\vec{r}_R(t))$$

科氏力　　離心力

$$\vec{r}_R(t)=x_R(t)\vec{e}_{xR}(t)+y_R(t)\vec{e}_{yR}(t)+z_R\vec{e}_{zR}(t)$$

$$\frac{d_R}{dt}\quad\frac{d_R\vec{r}_R(t)}{dt}=\frac{dx_R(t)}{dt}\vec{e}_{xR}(t)+\frac{dy_R(t)}{dt}\vec{e}_{yR}(t)+\frac{dz_R(t)}{dt}\vec{e}_{zR}(t)$$



$$0=\vec{V}_I(t)=\frac{d\vec{r}_I(t)}{dt}=\frac{d\vec{r}_R(t)}{dt}=\frac{d_R\vec{r}_R(t)}{dt}+\vec{\omega}\times\vec{r}_R(t)$$

$$\frac{d_R\vec{r}_R(t)}{dt}=-\vec{\omega}\times\vec{r}_R(t)$$

$$m\frac{d_R}{dt}\frac{d_R\vec{r}_R(t)}{dt}=\vec{F}-2m\vec{\omega}\times\frac{d_R\vec{r}_R(t)}{dt}-m\vec{\omega}\times(\vec{\omega}\times\vec{r}_R(t))$$

$$=m\vec{\omega}\times(\vec{\omega}\times\vec{r}_R(t))$$ 向心

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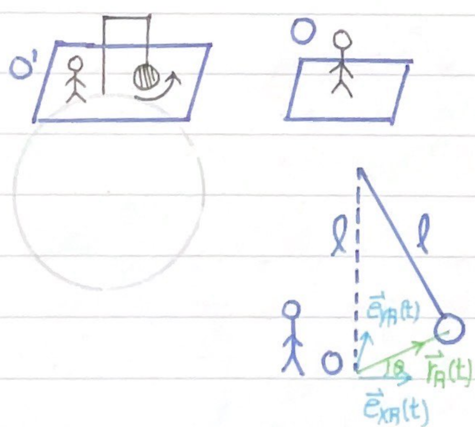
傅柯擺



$$\vec{r}_R(t)=x_R(t)\vec{e}_{xR}(t)+y_R(t)\vec{e}_{yR}(t)$$

$$r(t)=\sqrt{x_R^2(t)+y_R^2(t)}\qquad\vec{r}_R(t)=r(t)\vec{e}_r(t)$$

$$\theta(t)=\tan^{-1}\frac{y_R(t)}{x_R(t)}$$

$$\frac{d_R\vec{r}_R(t)}{dt}=\frac{dx_R(t)}{dt}\vec{e}_{xR}(t)+\frac{dy_R(t)}{dt}\vec{e}_{yR}(t)$$

$$=\frac{dr(t)}{dt}\vec{e}_r(t)+r(t)\frac{d\theta(t)}{dt}\vec{e}_\theta(t)$$

$$\frac{d_R}{dt}\frac{d_R\vec{r}_R(t)}{dt}=\left[\frac{d^2r(t)}{dt^2}-r(t)\left(\frac{d\theta(t)}{dt}\right)^2\right]\vec{e}_r(t)+\frac{1}{r(t)}\frac{d}{dt}\left[r^2(t)\frac{d\theta(t)}{dt}\right]\vec{e}_\theta(t)$$

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$$m\frac{d_R}{dt}\frac{d_R\vec{r}_R(t)}{dt} = -\frac{mg\,\vec{r}_R(t)}{\ell} - 2m\vec{\omega}\times\frac{d\vec{r}_R(t)}{dt} - m\vec{\omega}\times(\vec{\omega}\times\vec{r}_R(t))$$

$$= -\frac{mg}{\ell}r(t)\vec{e}_r(t) - 2m\omega\frac{dr(t)}{dt}\vec{e}_\theta(t) + 2m\omega r(t)\frac{d\theta(t)}{dt}\vec{e}_r(t) + m\omega^2 r(t)\vec{e}_r(t)$$

$$\|$$

$$m\left[\frac{d^2r(t)}{dt^2} - r(t)\left(\frac{d\theta(t)}{dt}\right)^2\right]\vec{e}_r(t) + m\frac{1}{r(t)}\frac{d}{dt}\left[r^2(t)\frac{d\theta(t)}{dt}\right]\vec{e}_\theta(t)$$

$\vec{e}_\theta(t)$: $$m\frac{1}{r(t)}\frac{d}{dt}\left[r^2(t)\frac{d\theta(t)}{dt}\right] = -2m\omega\frac{d\theta(t)}{dt}$$

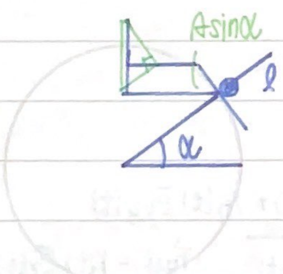
$$\frac{d}{dt}\left[r^2(t)\frac{d\theta(t)}{dt}\right] = -2\omega r(t)\frac{d\theta(t)}{dt} = \frac{d}{dt}\left[-\omega r^2(t)\right]$$

$$r^2(t)\frac{d\theta(t)}{dt} = -\omega r^2(t) + C \Rightarrow C=0,\quad \frac{d\theta(t)}{dt} = -\omega$$

$\vec{e}_r(t)$: $$\frac{d^2r(t)}{dt^2} - r(t)\omega^2 = -\frac{g}{\ell}r(t) - 2\omega^2 r + \omega^2 r(t)$$

$$= -\frac{g}{\ell}r(t) - \omega^2 r(t)$$

$$\frac{d^2r(t)}{dt^2} = -\frac{g}{\ell}r(t) \Rightarrow r(t) = A\sin(\Omega t),\quad \Omega^2 = \frac{g}{\ell}$$



$(\Delta V) = \omega A\sin\alpha$

$$\frac{2\pi A}{\omega A\sin\alpha} = \frac{2\pi}{\omega\sin\alpha} = \frac{T}{\sin\alpha}$$ 平面轉動週期

$$\frac{d\theta(t)}{dt} = -\omega\sin\alpha$$