

## Equilibrium and Elasticity

### Conditions for Equilibrium

First condition for equilibrium  $\Sigma \vec{F} = 0$

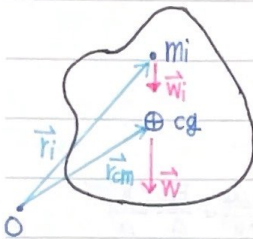
Second condition for equilibrium  $\Sigma \vec{\tau} = 0$

### Center of Gravity

$$x_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i}$$

$$y_{cm} = \frac{\Sigma m_i y_i}{\Sigma m_i}$$

$$z_{cm} = \frac{\Sigma m_i z_i}{\Sigma m_i}, \quad \vec{r}_{cm} = \frac{\Sigma m_i \vec{r}_i}{\Sigma m_i}$$

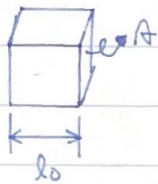


$$\vec{\tau}_i = \vec{r}_i \times \vec{w}_i = \vec{r}_i \times m_i \vec{g}, \quad \vec{\tau} = (\Sigma m_i \vec{r}_i) \times \vec{g} = \frac{\Sigma m_i \vec{r}_i}{\Sigma m_i} M \vec{g} \text{ which } \Sigma m_i = M$$

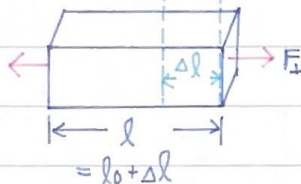
$$= \vec{r}_{cm} \times M \vec{g} = \vec{r}_{cm} \times \vec{W}$$

### Stress, Strain, and Elastic Moduli

initial



under tensile stress



$$\text{tensile stress} = \frac{F_\perp}{A}$$

$$\text{tensile strain} = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$$

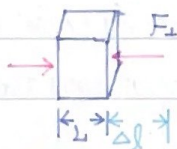
$$Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F_\perp / A}{\Delta l / l_0} = \frac{F_\perp}{A} \frac{l_0}{\Delta l}$$

Young's modulus

stress: strength of the forces causing the deformation

strain: resulting deformation

under compressive stress



$$\text{compressive stress} = \frac{F_\perp}{A}$$

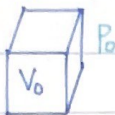
$$\text{compressive strain} = \frac{\Delta l}{l_0}$$

### bulk stress (volume stress) and bulk strain (volume strain)

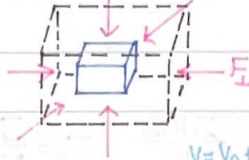
pressure in a fluid =  $\frac{F_{\perp}}{A}$

bulk strain =  $\frac{\Delta V}{V_0}$

initial



under bulk stress

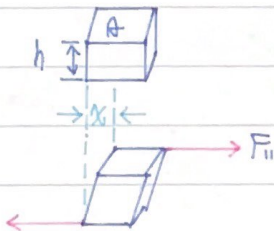


$\beta$ , bulk modulus =  $-\frac{\Delta P}{\frac{\Delta V}{V_0}}$

$V = V_0 + \Delta V (\Delta V < 0)$

$k = \frac{1}{\beta} = -\frac{1}{V_0} \frac{\Delta V}{\Delta P}$  compressibility

### shear stress and strain

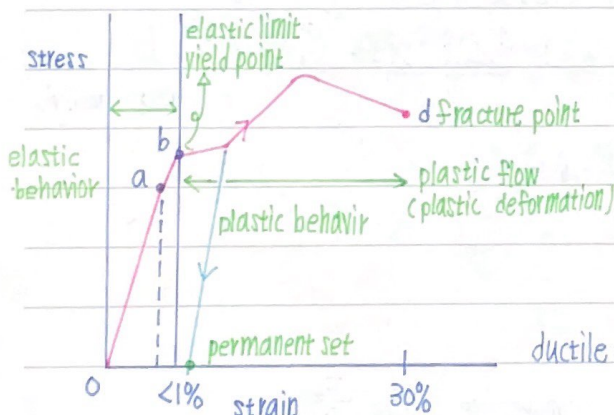


shear stress =  $\frac{F_{\parallel}}{A}$

shear strain =  $\frac{x}{h}$

shear modulus,  $S = \frac{F_{\parallel}/A}{x/h} = \frac{F_{\parallel}}{A} \cdot \frac{h}{x}$

### Elasticity and Plasticity



ductile: a large amount of plastic deformation take place between the elastic limit and the fracture point

brittle: fracture occurs soon after the elastic limit is passed