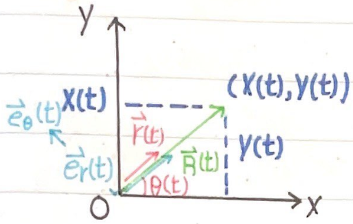
NO.

DATE

平面的極座標



$$\vec{R}(t)=x(t)\vec{e}_x+y(t)\vec{e}_y+z(t)\vec{e}_z$$

$$x(t)=r(t)\cos[\theta(t)]$$

$$y(t)=r(t)\sin[\theta(t)]$$

$$r^2(t)=x^2(t)+y^2(t) \qquad \theta(t)=\tan^{-1}\frac{y(t)}{x(t)}$$

$$\vec{R}(t)=r(t)\cos\theta(t)\vec{e}_x+r(t)\sin\theta(t)\vec{e}_y$$

$$\vec{v}(t)=\frac{d\vec{R}(t)}{dt}=\left[\frac{dr(t)}{dt}\cos\theta(t)-r(t)\frac{d\theta(t)}{dt}\sin\theta(t)\right]\vec{e}_x+$$

$$\frac{dr(t)}{dt}\sin\theta(t)+r(t)\frac{d\theta(t)}{dt}\cos\theta(t)]\vec{e}_y$$

$$=\frac{d\vec{r}(t)}{dt}\left[\cos\theta(t)\vec{e}_x+\sin\theta(t)\vec{e}_y\right]+r(t)\frac{d\theta(t)}{dt}\left[-\sin\theta(t)\vec{e}_x+\cos\theta(t)\vec{e}_y\right]$$

$$=\frac{d\vec{r}(t)}{dt}\vec{e}_r(t)+r(t)\frac{d\theta(t)}{dt}\vec{e}_\theta(t)$$

$$\vec{R}(t)=r(t)\left[\cos\theta(t)\vec{e}_x+\sin\theta(t)\vec{e}_y\right]=r(t)\vec{e}_r(t)$$

$$\frac{d\vec{R}(t)}{dt}=\frac{dr(t)}{dt}\vec{e}_r(t)+r(t)\frac{d\vec{e}_r(t)}{dt}$$

$$=\frac{dr(t)}{dt}\vec{e}_r(t)+r(t)\frac{d\theta(t)}{dt}\vec{e}_\theta(t)$$

$$\frac{d\vec{e}_r(t)}{dt}=\frac{d}{dt}\left[\cos\theta(t)\vec{e}_x+\sin\theta(t)\vec{e}_y\right]$$

$$=\frac{-d\theta(t)}{dt}\sin\theta(t)\vec{e}_x+\frac{d\theta(t)}{dt}\cos\theta(t)\vec{e}_y$$

$$=\frac{d\theta(t)}{dt}\vec{e}_\theta(t)$$

$$\vec{a}(t)=\frac{d\vec{v}(t)}{dt}=\frac{d}{dt}\left[\frac{dr(t)}{dt}\vec{e}_r(t)+r(t)\frac{d\theta(t)}{dt}\vec{e}_\theta(t)\right]$$

$$=\frac{d^2r(t)}{dt}\vec{e}_r(t)+\frac{dr(t)}{dt}\frac{d\vec{e}_r(t)}{dt}+\frac{dr(t)}{dt}\frac{d\theta(t)}{dt}\vec{e}_\theta(t)+r(t)\frac{d^2\theta(t)}{dt^2}\vec{e}_\theta(t)$$

$$+r(t)\frac{d\theta(t)}{dt}\frac{d\vec{e}_\theta(t)}{dt}$$

$$=\frac{d^2r(t)}{dt}\vec{e}_r(t)+\frac{dr(t)}{dt}\frac{d\theta(t)}{dt}\vec{e}_\theta(t)+\frac{dr(t)}{dt}\frac{d\theta(t)}{dt}\vec{e}_\theta(t)+r(t)\frac{d^2\theta(t)}{dt^2}\vec{e}_\theta(t)-$$

$$r(t)\left(\frac{d\theta(t)}{dt}\right)^2\vec{e}_r(\theta)$$

$$\frac{d\vec{e}_\theta(t)}{dt} = \frac{d}{dt}\left[-\underset{-\cos\theta}{\sin\theta(t)}\,\vec{e}_x + \underset{-\sin\theta}{\cos\theta(t)}\,\vec{e}_y\right]$$

$$= -\frac{d\theta(t)}{dt}\left[\cos\theta(t)\vec{e}_x + \sin\theta(t)\vec{e}_y\right]$$

$$= -\frac{d\theta(t)}{dt}\vec{e}_r(t)$$

$$\vec{a} = \left[\frac{d^2r(t)}{dt^2} - r(t)\left(\frac{d\theta(t)}{dt}\right)^2\right]\vec{e}_r(t) + \left[2\frac{dr(t)}{dt}\frac{d\theta(t)}{dt} + r(t)\frac{d^2\theta(t)}{dt^2}\right]\vec{e}_\theta(t)$$

$$= \left[\frac{d^2r(t)}{dt^2} - r(t)\left(\frac{d\theta(t)}{dt}\right)^2\right]\vec{e}_r(t) + \frac{1}{r(t)}\frac{d}{dt}\left[r^2(t)\frac{d\theta(t)}{dt}\right]\vec{e}_\theta(t)$$

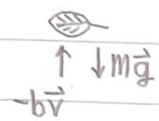
$$\frac{1}{r(t)}\left[2r(t)\frac{dr(t)}{dt}\frac{d\theta(t)}{dt} + r^2(t)\frac{d^2\theta(t)}{dt}\right]$$

$$T(x) = f(x)g(x)h(x)$$

$$\frac{dT(x)}{dt} = \frac{f(x+\Delta x)g(x+\Delta x)h(x+\Delta x) - f(x)g(x)h(x)}{\Delta x}$$

$$= \Big[f(x+\Delta x)g(x+\Delta x)h(x+\Delta x) - f(x)g(x+\Delta x)h(x+\Delta) + f(x)g(x+\Delta x)h(x+\Delta x) -$$
$$f(x)g(x)h(x+\Delta x) + f(x)g(x)h(x+\Delta x) - f(x)g(x)h(x)\Big]/\Delta x$$

$$= \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}\cdot\lim_{\Delta x\to 0} g(x+\Delta x)h(x+\Delta x) + \lim_{\Delta x\to 0}\frac{g(x+\Delta x)-g(x)}{\Delta x}\cdot$$
$$\lim_{\Delta\to 0} f(x)h(x+\Delta x) + \lim_{\Delta\to 0}\frac{h(x+\Delta x)-h(x)}{\Delta x}\cdot\lim_{\Delta\to 0} f(x)g(x)$$



$$\frac{dmv(t)}{dt} = mg - bv_z(t)$$

$$\frac{dv_z(t)}{dt} = g - \frac{b}{m}v_z(t), \quad \frac{1}{g-\frac{b}{m}v_z(t)}\frac{dv_z(t)}{dt} = 1$$

if $f(t) = g - \frac{b}{m}v_z(t)$ $\quad \frac{df(t)}{dt} = -\frac{b}{m}\frac{dv_z(t)}{dt}$, $\quad \frac{dv_z(t)}{dt} = -\frac{m}{b}\frac{df(t)}{dt}$ $= -\frac{m}{b}\frac{d(g-\frac{b}{m}v_z(t))}{dt}$

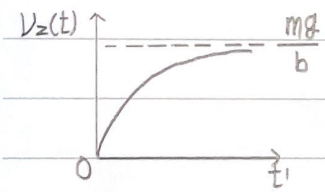
$$\int_0^{t'} \frac{1}{g-\frac{b}{m}v_z(t)}\frac{dv_z(t)}{dt}dt = \int_0^{t'} 1\,dt$$

$$\Rightarrow -\frac{m}{b}\int_0^{t'} \frac{1}{g-\frac{b}{m}v_z(t)}\frac{d(g-\frac{b}{m}v_z(t))}{dt}dt = t'$$

$$\Rightarrow -\frac{m}{b}\int_0^{t'} \frac{d(g-\frac{b}{m}v_z(t))}{g-\frac{b}{m}v_z(t)}\frac{dt}{dt} = t' \quad \Rightarrow \ln(g-\frac{b}{m}v_z(t) - \ln(g-\frac{b}{m}v_z(0)) = -\frac{b}{m}t'$$

$$\Rightarrow \frac{g-\frac{b}{m}v_z(t)}{g-\frac{b}{m}v_z(0)} = e^{-\frac{b}{m}t'}, \quad g-\frac{b}{m}v_z(t') = \left[g-\frac{b}{m}v_z(0)\right]e^{-\frac{b}{m}t'}$$

assume $v_z(0)=0$, then $g-\frac{b}{m}v_z(t) = ge^{-\frac{b}{m}t'}$, $v_z(t) = \frac{m}{b}g(1-e^{-\frac{b}{m}t'})$



$\frac{mg}{b} - \frac{mg}{b}e^{-\frac{b}{m}t}$

$$\frac{dZ(t)}{dt} = v_z(t) = \frac{mg}{b}(1-e^{-\frac{b}{m}t}), \quad z(t)-z(0) = \int \frac{mg}{b}(1-e^{-\frac{b}{m}t})dt$$

$$= \frac{mg}{b}t' - \frac{mg}{m}\frac{1}{-\frac{b}{m}}(e^{-\frac{b}{m}t} - 1)$$

$$\int \frac{mg}{b}e^{-\frac{b}{m}t} = \frac{mg}{b}\int e^{-\frac{b}{m}t}$$

$$\int \underbrace{e^{-\frac{b}{m}t}}_{u}\underbrace{\times 1}_{dv} = e^{-\frac{b}{m}t} - \int 1\times e^{-\frac{b}{m}t}(-\frac{b}{m})$$

$\frac{mg}{b}t - \frac{mg}{b}($

$$\int e^{-\frac{b}{m}t}dt = \int e^u(-\frac{m}{b})du = -\frac{m}{b}\int e^u du = -\frac{m}{b}e^{-\frac{b}{m}t}$$

$u = -\frac{b}{m}t$

$du = -\frac{b}{m}dt$

$-\frac{m}{b}du = dt$

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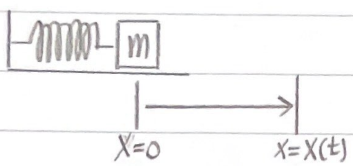


$F=-kX(t)$, $\dfrac{d\,m v_x(t)}{dt}=-kX(t)$, $m\dfrac{d^2X(t)}{dt^2}=-kX(t)$

$$\frac{d^2X(t)}{dt^2}=-\frac{k}{m}X(t)$$

$\omega^2=\dfrac{k}{m}$, ω 自然振盪相位頻率

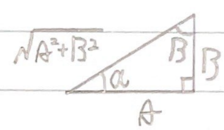
$\dfrac{d^2X(t)}{dt^2}=-\omega^2X(t)$, $\dfrac{d^2\cos\omega t}{dt^2}=-\omega^2\cos\omega t$

$\dfrac{d^2\sin\omega t}{dt^2}=-\omega^2\sin\omega t$

$A\cos\omega t+B\sin\omega t=x(t)$,

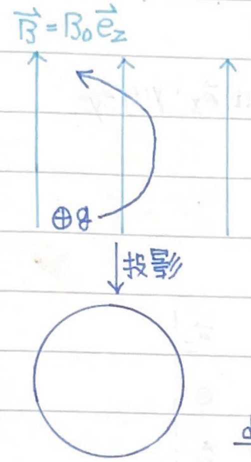
起始條件 $\left\{\begin{array}{l} t=0 \quad x(0)=A,\ \dfrac{dx(t)}{dt}=-A\sin\omega t+B\cos\omega t \\ \left.\dfrac{dx(t)}{dt}\right|_{t=0}=B\omega,\ B=\dfrac{1}{\omega}\left.\dfrac{dx(t)}{dt}\right|_{t=0} \end{array}\right.$

$$x(t)=\sqrt{A^2+B^2}\left[\frac{A}{\sqrt{A^2+B^2}}\cos\omega t+\frac{B}{\sqrt{A^2+B^2}}\sin\omega t\right]$$



$$=\sqrt{A^2+B^2}\left[\cos\alpha\cos\omega t+\sin\alpha\sin\omega t\right]$$

$$=\sqrt{A^2+B^2}\cos(\omega t-\alpha)=\sqrt{A^2+B^2}\sin(\omega t+\beta)$$



$$q\overrightarrow{v(t)}\times\vec{B}=\vec{F}=\frac{dm\vec{v}(t)}{dt} \qquad \begin{bmatrix}\vec{e}_x & \vec{e}_y & \vec{e}_z\\ v_x & v_y & v_z\\ 0 & 0 & B_0\end{bmatrix}$$

$$\frac{dmv_x(t)}{dt}=qB_0v_y(t)$$

$$\frac{dmv_y(t)}{dt}=-qB_0v_x(t)$$

$$\frac{dmv_z(t)}{dt}=0$$

$\omega=\frac{qB_0}{m}$ 迴旋相位頻率

$$\frac{dv_x(t)}{dt}=\frac{qB_0}{m}v_y(t)=\omega v_y(t) \qquad \frac{d^2v_x(t)}{dt^2}=-\omega^2v_x(t),\ v_x(t)=A\sin(\omega t+B)$$

$$\frac{dv_y(t)}{dt}=-\frac{qB_0}{m}v_x(t)=\omega v_x(t) \qquad \frac{d^2v_y(t)}{dt^2}=-\omega^2v_y(t),\ v_y(t)=\frac{1}{\omega}\frac{dv_x(t)}{dt}$$

$$=A\cos(\omega t+B)$$

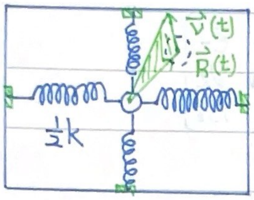
$$v_x^2(t)+v_y^2(t)=A^2,\ v_x^2(0)+v_y^2(0)=A^2,\ \frac{v_x(0)}{v_y(0)}=\tan B,\ B=\tan^{-1}\frac{v_x(0)}{v_y(0)}$$

$$x(t)-x(0)=\int_0^t\frac{dx(t)}{dt}dt=\int_0^t A\sin(\omega t+B)\,dt=-\frac{1}{\omega}A\left[\cos(\omega t+B)-\cos B\right]$$

$$y(t)-y(0)=\int_0^t\frac{dy(t)}{dt}dt=\int_0^t \qquad =+\frac{1}{\omega}A\left[\sin(\omega t+B)-\sin B\right]$$

$$x(t)-x(0)+\frac{A}{\omega}\cos B=-\frac{1}{\omega}A\cos(\omega t+B) \qquad \text{半徑 } R=\frac{A}{\omega}$$

$$y(t)-y(0)-\frac{A}{\omega}\sin B=\frac{1}{\omega}A\sin(\omega t+B)$$



$m\frac{d^2x(t)}{dt^2}=kx(t)$ $\quad m\frac{d^2y(t)}{dt^2}=ky(t)$ $\quad \vec{R}(t)=x(t)\vec{e}_x+y(t)\vec{e}_y$

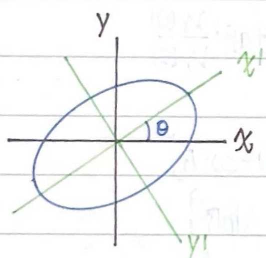
$\omega^2=\frac{k}{m}$

$x(t)=A\cos(\omega t+\delta)$ $\quad y(t)=B\sin(\omega t+\gamma)$

$$\frac{1}{2}\vec{R}(t)\times\vec{v}(t)=\frac{1}{2}\begin{vmatrix}\vec{e}_x & \vec{e}_y & \vec{e}_z\\ A\cos(\omega t+\delta) & B\sin(\omega t+\gamma) & 0\\ -A\omega\sin(\omega t+\delta) & B\omega\cos(\omega t+\gamma) & 0\end{vmatrix}$$

$$=AB\omega\cos(\delta-\gamma)\vec{e}_z=\text{constant}$$

if $\delta-\gamma=\frac{\pi}{2}(2k+1)$ 軌道是橢圓



$x'(t)=A'\cos\omega t$ $\quad \frac{\Delta\text{area}}{dt}=\frac{1}{2}\omega A'B'$

$y'(t)=B'\sin\omega t$ $\quad T=\frac{\pi A'B'}{\frac{1}{2}\omega A'B'}=\frac{2\pi}{\omega}$ 與橢圓形狀無關

$\frac{[x'(t)]^2}{A^2}+\frac{[y'(t)]^2}{B^2}=1$ $\quad$ 力指向橢圓中心

$r(t)=\sqrt{x'^2(t)+y'^2(t)}$ $\quad \frac{1}{2}r^2(t)\frac{d\theta(t)}{dt}$

$\theta=\tan^{-1}\frac{y'(t)}{x'(t)}$