Single Qubit Operations

$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $|\circlearrowright\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

$|1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$ $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle \underline{-} |1\rangle)$ $|\circlearrowleft\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$

angle π $e^{i\pi} = -1$

Pauli Matrices

$X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

$|0\rangle —\oplus— |1\rangle$

$—\boxed{X}—$

NOT operation
bit flip

$X\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} = |1\rangle$

$X := |0\rangle\langle 1| + |1\rangle\langle 0|$

$X|j\rangle = |j \oplus 1\rangle$

$X\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} = -\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$

$X\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$

$X = \frac{1}{2}(|0\rangle + |1\rangle)(\langle 0| + \langle 1|) - \frac{1}{2}(|0\rangle - |1\rangle)(\langle 0| - \langle 1|$

$Y = \begin{bmatrix}0 & -i\\1 & 0\end{bmatrix}$ $Y\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-i\\0\end{bmatrix} = -i|0\rangle$

$Y = iXZ$ $XY = -YX$

$ZY = -YZ$

$XYX = iZX = -Y$

$Y\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix} = -\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$

$Y\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$

$Y = \frac{1}{2}(|0\rangle + i|1\rangle)(\langle 0| + i\langle 1|) - \frac{1}{2}(|0\rangle - i|1\rangle)(\langle 0| - i\langle 1|)$

$Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$

$|+\rangle \longleftrightarrow |-\rangle$

$|\circlearrowright\rangle \longleftrightarrow |\circlearrowleft\rangle$

rotate π around z axis
phase flip

$X|j\rangle = (-1)^j|j\rangle$

$XZ = -ZX$

$ZXZ = -X$

$ZYZ = -Y$

$ZZZ = Z$

$Z\begin{bmatrix}0\\1\end{bmatrix} = -\begin{bmatrix}0\\1\end{bmatrix}$

$Z\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$

$Z = |0\rangle\langle 0| - |1\rangle\langle 1|$

Other

$S = \begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$ phase gate

$T = \begin{bmatrix}1 & 0\\0 & e^{\frac{i\pi}{4}}\end{bmatrix}$ $\frac{\pi}{8}$ gate

$= e^{\frac{i\pi}{8}}\begin{bmatrix}e^{-\frac{i\pi}{8}} & 0\\0 & e^{\frac{i\pi}{8}}\end{bmatrix}$

$S = T^2 = \sqrt{Z}$

$SXS^\dagger = Y$ $SYS^\dagger = -X$ $SZS^\dagger = Z$

$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$ Clifford or Hadamard

$H\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$H\begin{bmatrix}0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

$H\frac{1}{\sqrt{4+2\sqrt{2}}}\begin{bmatrix}1\\-1-\sqrt{2}\end{bmatrix} = -\frac{1}{\sqrt{4+2\sqrt{2}}}\begin{bmatrix}1\\-1-\sqrt{2}\end{bmatrix}$

$H\frac{1}{\sqrt{4-2\sqrt{2}}}\begin{bmatrix}1\\-1+\sqrt{2}\end{bmatrix} = \frac{1}{\sqrt{4-2\sqrt{2}}}\begin{bmatrix}1\\-1+\sqrt{2}\end{bmatrix}$

$H^2 = I$ $H = \frac{X+Z}{\sqrt{2}}$

$HXH = Z$

$HYH = -Y$

$HZH = X$

$$HTH = e^{\frac{i\pi}{8}}HR_z(\tfrac{\pi}{4})H = e^{\frac{i\pi}{8}}He^{-i\pi\frac{Z}{8}}H = e^{\frac{i\pi}{8}}H\left[1 - \tfrac{\pi Z}{8}i - \tfrac{1}{2!}(\tfrac{\pi Z}{8})^3 + \cdots\right]H$$

$$= e^{\frac{i\pi}{8}}\left[H^2 - i\tfrac{\pi}{8}HZH - \tfrac{1}{2!}(\tfrac{\pi}{8})^2 HZHHZH + \cdots\right]$$

$$= e^{\frac{i\pi}{8}}\left[1 - i\tfrac{\pi}{8}X - \tfrac{1}{2!}(\tfrac{\pi}{8})^2X^2 + \cdots\right] = e^{\frac{i\pi}{8}}e^{-\frac{i\pi}{8}X} = e^{\frac{i\pi}{8}}R_X(\tfrac{\pi}{4})$$

NO.

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$$H = \frac{1}{\sqrt{2}}\left[(|0\rangle+|1\rangle)\langle 0| + (|0\rangle-|1\rangle)\langle 1|\right] = \frac{1}{\sqrt{2}}(|0\rangle\langle 0| + |1\rangle\langle 0| + |0\rangle\langle 1| - |1\rangle\langle 1|) = \frac{1}{\sqrt{2}}\sum_{x,y}(-1)^{x\cdot y}|x\rangle\langle y|$$

$$H^{\otimes n} = \frac{1}{\sqrt{2^n}}\sum_{x_1,y_1}(-1)^{x_1\cdot y_1}|x_1\rangle\langle y_1| \otimes \sum_{x_2,y_2}(-1)^{x_2\cdot y_2}|x_2\rangle\langle y_2| \otimes \cdots = \frac{1}{\sqrt{2^n}}\sum_{x,y}(-1)^{x\cdot y}|x\rangle\langle y|$$

$$H^{\otimes 2} = \frac{1}{2}\big[|00\rangle\langle 00| + |01\rangle\langle 00| + |00\rangle\langle 01| - |01\rangle\langle 01| + |10\rangle\langle 00| + |11\rangle\langle 00| + |10\rangle\langle 01| - |11\rangle\langle 01|$$
$$|00\rangle\langle 10| + |01\rangle\langle 10| + |00\rangle\langle 11| - |01\rangle\langle 11| - |10\rangle\langle 10| - |11\rangle\langle 10| - |10\rangle\langle 11| + |11\rangle\langle 11|\big]$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = |0\rangle\langle 1| + |1\rangle\langle 0|$$

$$R_X(\theta) = e^{-\frac{i\theta X}{2}} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}X = \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

$$R_Y(\theta) = e^{-\frac{i\theta Y}{2}} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Y = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \qquad e^{iAx} = \cos(x)I + i\sin(x)A$$

$$R_Z(\theta) = e^{-\frac{i\theta Z}{2}} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Z = \begin{bmatrix} e^{-\frac{i\theta}{2}} & 0 \\ 0 & e^{\frac{i\theta}{2}} \end{bmatrix}$$

$$H = R_Z(\tfrac{\pi}{2})R_X(\tfrac{\pi}{2})R_Z(\tfrac{\pi}{2}) = \begin{bmatrix} e^{-\frac{i\pi}{4}} & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix}\begin{bmatrix} \cos\frac{\pi}{4} & -i\sin\frac{\pi}{4} \\ i\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}\begin{bmatrix} e^{-\frac{i\pi}{4}} & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix} = \frac{e^{-\frac{i\pi}{2}}}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$(\hat{n}\cdot\hat{\sigma})^2 = (n_x^2+n_y^2+n_z^2)I + n_xn_y(XY+YX) + n_yn_z(YZ+ZY) + n_zn_x(ZX+XZ) = I$$

$|n| = I$

$\{\sigma_i, \sigma_j\} = 2I\delta_{ij}$

$$R_{\hat{n}}(\theta) = e^{-\frac{1}{2}i\theta\hat{n}\cdot\hat{\sigma}} = \left[1 - \frac{1}{2!}(\tfrac{\theta}{2})^2 + \frac{1}{4!}(\tfrac{\theta}{2})^4 - \cdots\right]I - i\left[\frac{\theta}{2} - (\tfrac{\theta}{2})^3 + \frac{1}{5!}(\tfrac{\theta}{2})^5 - \cdots\right]\hat{n}\cdot\hat{\sigma}$$
$$= \cos(\tfrac{\theta}{2})I - i\sin(\tfrac{\theta}{2})(n_xX + n_yY + n_zZ)$$

$$X R_Y(\theta) X = R_Y(-\theta)$$