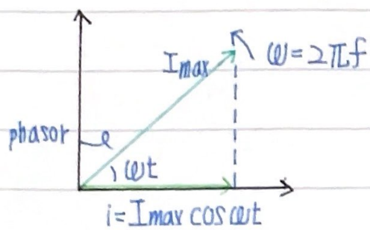
Alternating Current

Phasor and Alternating Current

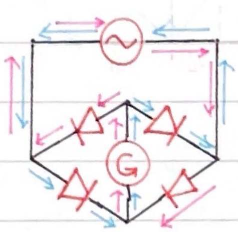
$v = V_{max}\cos\omega t$

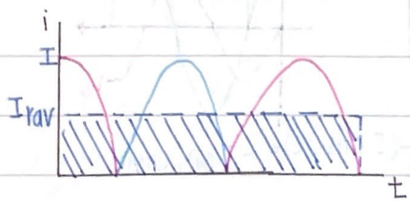
$i = I_{max}\cos\omega t$

phasor diagram



rectified alternating current





$I_{rav} = |\cos\omega t|_{av}\, I = \frac{2}{\pi} I$

root-mean-square current

$i = I_{max}\cos\omega t$, $i^2 = I_{max}^2\cos^2\omega t$

$= I_{max}^2\,\frac{1}{2}(1+\cos 2\omega t)$

$= \frac{1}{2}I_{max}^2 + \frac{1}{2}I_{max}^2\cos 2\omega t$

$\langle i^2\rangle = \langle \frac{1}{2}I_{max}^2\rangle + \langle \frac{1}{2}I_{max}^2\cos 2\omega t\rangle = \frac{1}{2}I_{max}^2$

$I_{rms} = \frac{I}{\sqrt{2}}$ $P_{avg} = I_{rms}^2 R$

$V_{rms} = \frac{V}{\sqrt{2}}$

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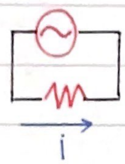
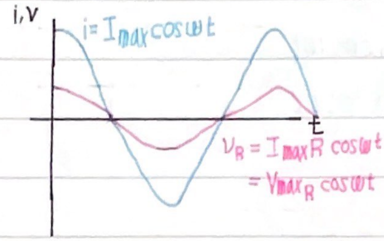
Resistance and Reactance

resistor in an AC circuit

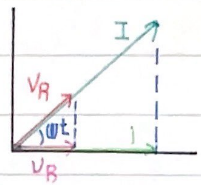




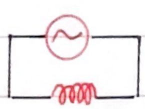
$v_R = iR = IR\cos\omega t$

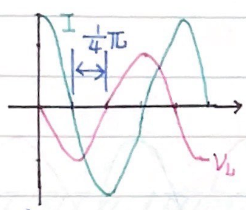
$V_{max_R} = I_{max} R$

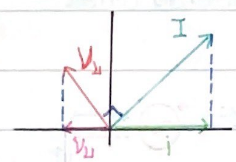
$v_R = V_{max_R}\cos\omega t$



inductor in an AC circuit







$v_L = L\frac{di}{dt} = L\frac{d}{dt}(I\cos\omega t) = -I\omega L\sin\omega t$

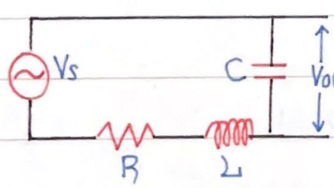
$= I\omega L\cos(\omega t + 90^\circ)$

$= V_{max}\cos(\omega t + \phi)$, $V_{max_L} = I\omega L$, for pure inductor, $\phi = 90^\circ$

inductive reactance $X_L = \omega L$

$\Rightarrow V_{max_L} = I_{max} X_L$

when $\omega\uparrow$ or $L\uparrow$, $X_L\uparrow$, $I_{max}\downarrow$: low-pass filter



$V_{out} = V_C = \frac{1}{\omega C}$, $\frac{V_{out}}{V_s} = \frac{1}{\omega C\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

if ω is large, $\frac{V_{out}}{V_s} = \frac{1}{\omega C\sqrt{(\omega L)^2}} = \frac{1}{LC\omega^2}$

if ω is small, $\frac{V_{out}}{V_s} = \frac{1}{\omega C\sqrt{(\frac{1}{\omega C})^2}} = \frac{\omega C}{\omega C} = 1$

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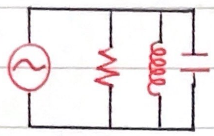
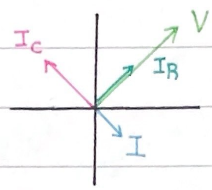
The L-R-C Parallel Circuit





$V = V_R = V_L = V_C$

$i = i_R + i_L + i_C$

$$I^2 = I_R^2 + (I_C - I_L)^2 = \left(\frac{V}{R}\right)^2 + \left(V\omega C - \frac{V}{\omega L}\right)^2$$

$$I = V\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

large ω, $Z \to \frac{1}{\omega C}$, the current in the capacitor branch is much larger

small ω, $Z \to \omega L$ the current in the inductive branch is much larger

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Power in Alternating-Current Circuits

power in resistor

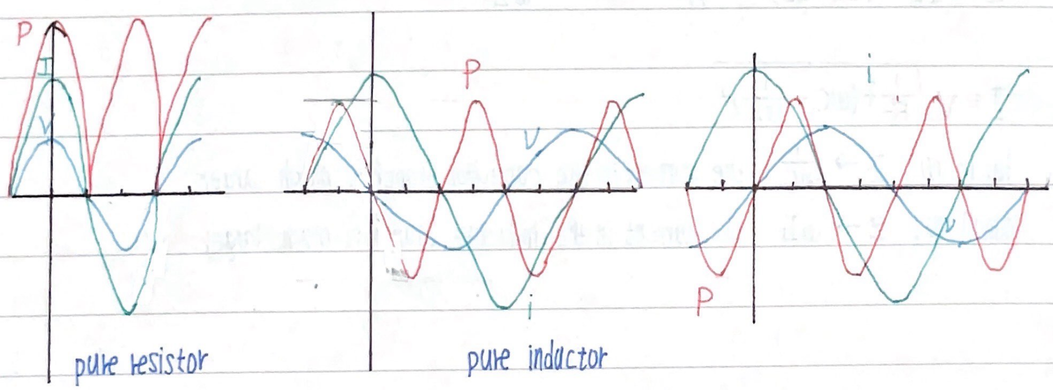
$$P_{av} = \frac{1}{2}VI = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}} = V_{rms}I_{rms} = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

power in inductor

$P_{av} = 0$

power in capacitor

$P_{av} = 0$



$$P_{av} = \langle P \rangle = \langle [V(\cos\omega t + \phi)][I\cos\omega t] \rangle$$

$$= \langle V_{max}(\cos\omega t\cos\phi - \sin\omega t\sin\phi)\, I_{max}\cos\omega t \rangle$$

$$= \langle V_{max} I_{max}\cos\phi \underbrace{\cos^2\omega t}_{\frac{1}{2}} - V_{max}I_{max}\sin\phi \underbrace{\cos\omega t\sin\omega t}_{0\,=\,\frac{1}{2}\sin 2\omega t} \rangle$$

$$= \frac{1}{2}V_{max}I_{max}\cos\phi$$

$$= V_{rms}I_{rms}\underline{\cos\phi}$$ power factor

for a pure resistance, $\phi = 0$

pure inductor or capacitor, $\phi = \pm 90°$

L-R-C series $\cos\phi = \frac{R}{Z}$

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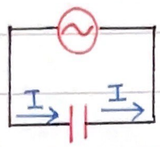
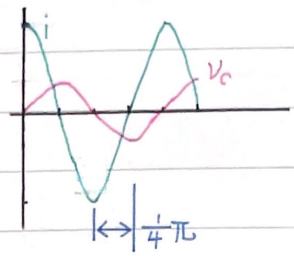
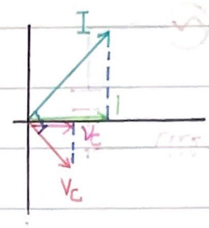
Capacitor in an AC circuits

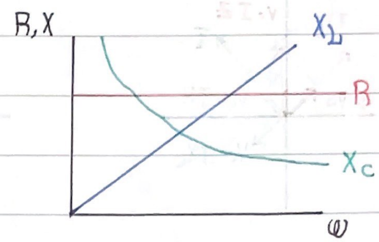






$i = \frac{dq}{dt} = I\cos\omega t$, $q = \frac{I}{\omega}\sin\omega t$

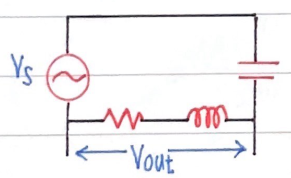
$v_c = \frac{q}{C} = \frac{I}{\omega C}\sin\omega t = \frac{I}{\omega C}\cos(\omega t - 90^\circ)$



$V_{max_C} = \frac{1}{\omega C}$ capacitive reactance $X_C = \frac{1}{\omega C}$

$V_{max_c} = IX_C$

when $\omega\uparrow$ or $C\uparrow$, $X_C\downarrow$, $I_{max}\uparrow$: high-pass filter



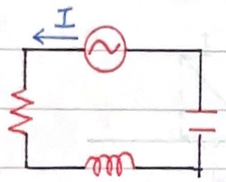
$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$, $I = \frac{V_s}{Z} = \frac{V_s}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

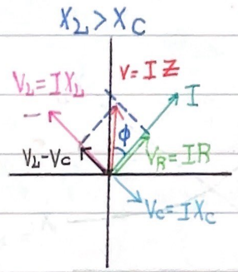
$V_{out} = I\sqrt{R^2 + X_L^2} = I\sqrt{R^2 + \omega^2 L^2} = V_s\sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + (\omega L - \frac{1}{\omega C})^2}}$ $\frac{V_{out}}{V_s} = \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

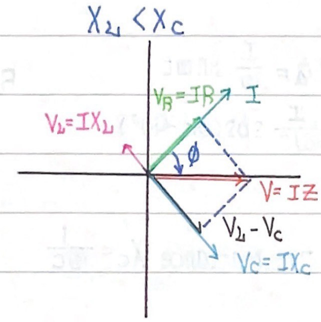
ω small : $\frac{V_{out}}{V_s} \rightarrow \sqrt{\frac{R^2}{\frac{1}{\omega^2 C^2}}} = \omega RC$

ω large : $\frac{V_{out}}{V_s} \rightarrow \sqrt{\frac{\omega^2 L^2}{\omega^2 L^2}} = 1$

The L-R-C Series Circuit







$V_R = IR \quad V_L = IX_L \quad V_C = IX_C$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$$

impedance $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

$\Rightarrow V = IZ$

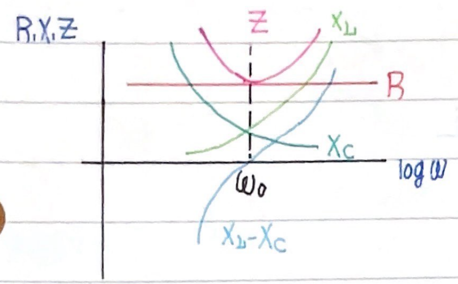
$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I(X_L - X_C)}{IR} = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

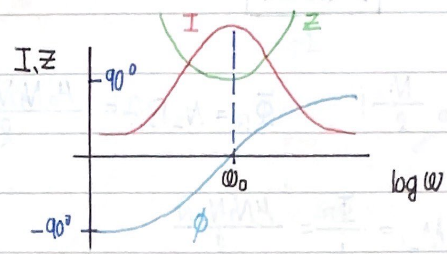
$\frac{V}{\sqrt{2}} = \frac{I}{\sqrt{2}} Z$, $V_{rms} = I_{rms} Z$

Resonance in AC Circuits

at resonance

$X_L = X_C$ $\quad \omega_0 L = \frac{1}{\omega_0 C}$, $\omega_0 = \frac{1}{\sqrt{LC}}$ $\quad f_0 = \frac{\omega_0}{2\pi}$

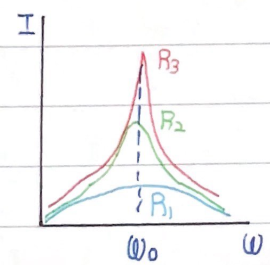




$$P_{avg} = I_{rms}^2 R = \frac{(\Delta V_{rms})^2}{Z^2} R = \frac{(\Delta V_{rms})^2 R}{R^2 + (X_L - X_C)^2} \quad , (X_L - X_C)^2 = \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{L^2}{\omega^2}(\omega^2 - \omega_0^2)^2$$

$$= \frac{(\Delta V_{rms})^2 R \omega^2}{R^2 \omega^2 + L^2(\omega^2 - \omega_0^2)^2}$$

when L and C fixed



$R_1 > R_2 > R_3$

curve sharpness described by quality factor, Q

$$Q = \frac{\omega_0}{\Delta \omega} = \frac{\omega_0 L}{R}$$

width of the curved measured between the two values of ω for which Pavg at half-power points

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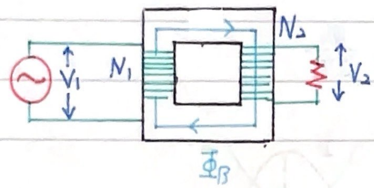
Transformers



$B = \mu_0 \frac{N_1}{l} i$ $\quad \Phi_B = N_2 B A = \frac{\mu_0 N_1 N_2 A i}{l}$

$M_{12} = M_{21} = \frac{\Phi_B}{i} = \frac{\mu N_1 N_2 A}{l}$

$L_1 = \mu_0 (\frac{N_1}{l})^2 A \cdot l = \frac{\mu N_1^2 A}{l}$ $\quad L_2 = \mu (\frac{N_2}{l})^2 A \cdot l = \frac{\mu N_2^2 A}{l}$, $M = \sqrt{L_1 L_2}$

$K = \frac{M}{\sqrt{L_1 L_2}}$ when $K \leq 1$, magnetic leakage

$\varepsilon_1 = -N_1 \frac{d\Phi_B}{dt}$ $\quad \varepsilon_2 = -N_2 \frac{d\Phi_B}{dt} \Rightarrow \frac{\varepsilon_1}{\varepsilon_2} = \frac{N_1}{N_2}$

$V_1 I_1 = V_2 I_2$, $I_2 = \frac{V_2}{R}$

$\frac{V_1}{I_1} = \frac{R}{(\frac{N_2}{N_1})^2}$