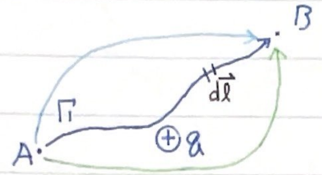
Electric Potential

Electric Potential Energy



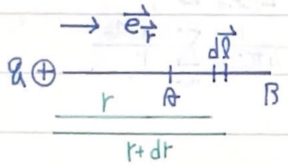
$$\vec{F} = q\vec{E}$$

$$\int_A^B -\vec{F}\cdot d\vec{l} = U(B) - U(A)$$

$$U(A) - U(B) = \int_A^B \vec{F}\cdot d\vec{l} = \int_A^B q\vec{E}\cdot d\vec{l}$$

$$\frac{1}{q}\left[U(A) - U(B)\right] = \int_A^B \vec{E}\cdot d\vec{l} = \frac{U(A)}{q} - \frac{U(B)}{q} = V(A) - V(B)$$

I one point charge OAB collinearity



$$\vec{E} = \frac{Q}{4\pi\epsilon_0}\frac{1}{|\vec{r}|^2}\vec{e_r} \qquad \vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{|\vec{r}|^2}$$

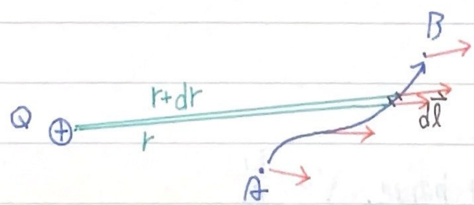
$$d\vec{l} = d\vec{r}\ \vec{e_r}$$

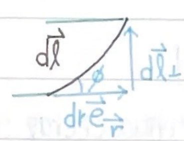
$$W_{a\to b} = \int_A^B \vec{F}\cdot d\vec{r} = \int_A^B \frac{1}{4\pi\epsilon_0}\frac{qq_0}{|\vec{r}|^2}d\vec{r} = \frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$V(A) - V(B) = \int_A^B \vec{E}\cdot d\vec{l} = \int_A^B \frac{Q}{4\pi\epsilon_0}\frac{1}{|\vec{r}|^2}d\vec{r} = \frac{Q}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)\Big|_{r_A}^{r_B} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

let $r_B \to \infty$, $V(B) \to 0$, $V(A) = \frac{Q}{4\pi\epsilon_0}\frac{1}{r_A}$

II





$$d\vec{l} = d\vec{l}_\perp + dr\vec{e_r}$$

$$\vec{E}\cdot d\vec{l} = \frac{Q}{4\pi\epsilon_0}\frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

let $r_B \to \infty$, $V(B) = 0$, $V(A) = \frac{Q}{4\pi\epsilon_0}\frac{1}{r_A}$

$$W_{a\to b} = \int_{r_A}^{r_B} \vec{F}\cos\phi\, d\vec{l} = \int_{r_A}^{r_B} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{|\vec{r}|^2}\cos\phi\, d\vec{l}$$

$$U = \frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

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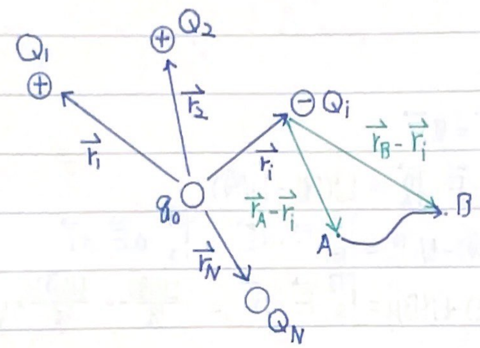
III



$$\vec{E} = \sum_{i=1}^{N} \vec{E}_i$$

$$V(A) - V(B) = \int_A^B \vec{E} \cdot d\vec{l} = \sum_{i=1}^{N} \int_A^B \vec{E}_i \cdot d\vec{l}$$

$$= \sum_{i=1}^{N} \frac{Q_i}{4\pi\epsilon_0} \left(\frac{1}{|\vec{r}_A - \vec{r}_i|} - \frac{1}{|\vec{r}_B - \vec{r}_i|} \right)$$

$$U = \frac{q_0}{4\pi\epsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \cdots \right)$$

$$= \frac{q_0}{4\pi\epsilon_0} \sum \frac{q_i}{r_i}$$

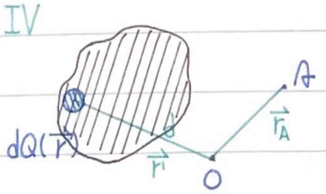
$$= \frac{1}{4\pi\epsilon_0} \sum_{i<j} \frac{q_i q_j}{r_{ij}}$$

let $r_B \to \infty$

$$V(B) = \sum_{i=1}^{N} \frac{Q_i}{4\pi\epsilon_0} \frac{1}{|\vec{r}_B - \vec{r}_i|} = 0 \quad \text{and} \quad \sum_{i=1}^{N} Q_i < \infty$$

$$V(A) = \sum_{i=1}^{N} \frac{Q_i}{4\pi\epsilon_0} \frac{1}{|\vec{r}_A - \vec{r}_i|}$$

IV



$$V(A) = \int \frac{dQ(\vec{r})}{4\pi\epsilon_0} \frac{1}{|\vec{r}_A - \vec{r}'|}$$

Electric Potential

potential is potential energy per unit charge $V = \frac{U}{q_0}$

$$\frac{W_{a \to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0} \right) = -(V_b - V_a) = V_a - V_b$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

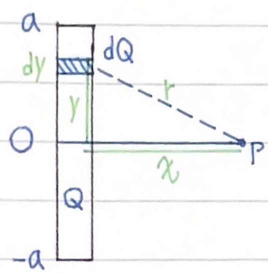
$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$$

$$= \int_a^b \vec{E} \cos\phi \, d\vec{l}$$

$$= -\int_b^a \vec{E} \cdot d\vec{l}$$

Calculating Electric Potential

case I electric potential due to a finite line of charge



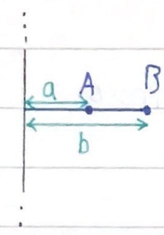
dQ

$$dV = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{\sqrt{x^2+y^2}} \qquad dQ = \frac{Q}{2a}dy$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{a}\frac{dy}{\sqrt{x^2+y^2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\ln\frac{\sqrt{a^2+x^2}+a}{\sqrt{a^2+x^2}-a}$$

case II

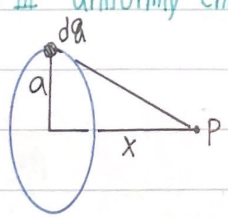


$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$V_{AB} = V_A - V_B = -\int_A^B \vec{E}\cdot d\vec{r} = -\int_a^b \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}dr$$

$$= -\frac{\lambda}{2\pi\epsilon_0}\ln r\Big|_a^b = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{a}{b}$$

case III uniformly charged ring



$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{\sqrt{a^2+x^2}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{a^2+x^2}}$$

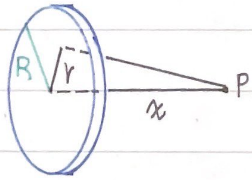
$$E_x = -\frac{dV}{dx} = \frac{1}{4\pi\epsilon_0}\frac{x}{(a^2+x^2)^{3/2}}Q \qquad (a^2+x^2)^{-\frac{1}{2}}$$

case IV uniformly disk

$$dq = \sigma dA = 2\pi\sigma r dr$$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{\sqrt{r^2+x^2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{2\pi\sigma r dr}{\sqrt{r^2+x^2}}$$



$$V = \frac{1}{4\pi\epsilon_0}\pi\sigma\int_0^R\frac{2r\,dr}{\sqrt{r^2+x^2}}$$

$$= \frac{\sigma}{4\epsilon_0}\int_0^R\frac{1}{\sqrt{r^2+x^2}}2r\,dr$$

$$= \frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+x^2}-x\right)$$

$$E_x = \frac{\sigma}{2\epsilon_0}\left(1-\frac{x}{\sqrt{R^2+x^2}}\right)$$

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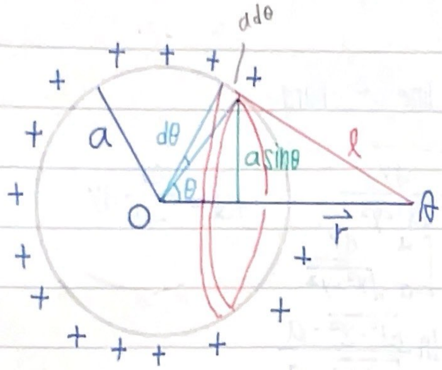
case V

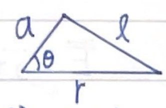


$$dQ = \frac{2\pi a\sin\theta\, a d\theta}{4\pi a^2} Q = \frac{\sin\theta\, d\theta}{2} Q$$

$$\int \frac{dQ(\vec{r})}{4\pi\epsilon_0} \frac{1}{|\vec{r}_A - \vec{r}|} = \frac{1}{4\pi\epsilon_0}\int \frac{Q\sin\theta}{2l} d\theta$$

$$= \frac{Q}{4\pi\epsilon_0}\int \frac{l}{2ar} \frac{\frac{dl}{d\theta}}{l} d\theta$$

$$= \frac{Q}{4\pi\epsilon_0} \frac{l}{2ar}$$



$$l^2 = a^2 + r^2 - 2ar\cos\theta$$

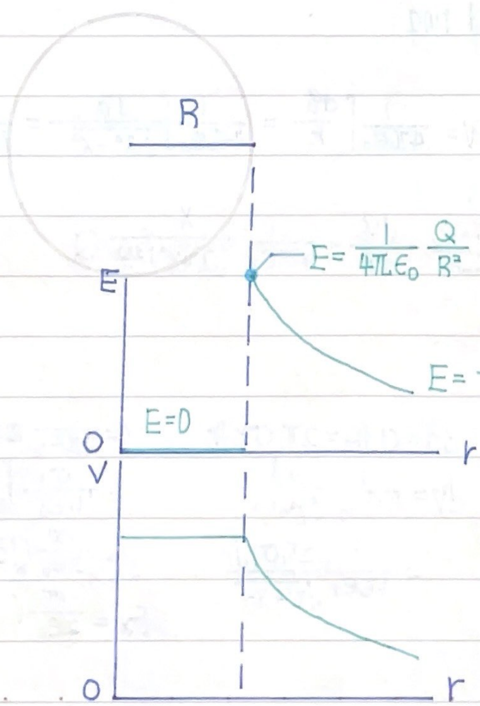
$$2l\frac{dl}{d\theta} = 2ar\sin\theta$$

$r > a$

$$\frac{Q}{4\pi\epsilon_0} \frac{1}{2ar} l \Big|_{r-a}^{r+a} = \frac{Q}{4\pi\epsilon_0} \frac{1}{r}$$

$r < a$

$$\frac{Q}{4\pi\epsilon_0} \frac{1}{2ar} l \Big|_{a-r}^{a+r} = \frac{Q}{4\pi\epsilon_0} \frac{1}{a}$$



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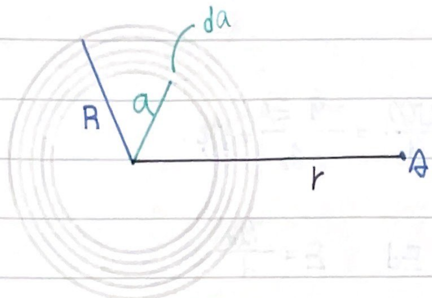
case VI



$r > R \geq a$

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{1}{r}\frac{4}{3}\pi R^3 \rho$$

$r < R$

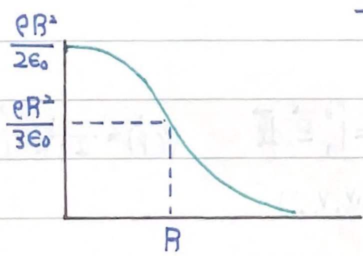
$a < r$

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{1}{r}\frac{4}{3}\pi r^3 \rho = \frac{\rho}{\epsilon_0}\frac{r^2}{3}$$

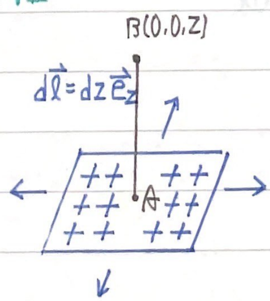
$r \leq a < R$

$$\int_r^R \frac{1}{4\pi\epsilon_0}\frac{1}{a}4\pi a^2 \, da \, \rho = \frac{\rho}{\epsilon_0}\int_r^R a\,da = \frac{\rho}{\epsilon_0}\left(\frac{R^2}{2} - \frac{r^2}{2}\right)$$

$$\frac{\rho R^2}{2\epsilon_0} - \frac{\rho r^2}{6\epsilon_0}$$



case VII



can't let $\vec{r}_B \to \infty$, $V(B) = 0$ since $Q = \infty$

let $Z = 0$ $V(Z) = 0$

$Z > 0$ $\vec{E} = \frac{\sigma}{2\epsilon_0}\vec{e}_z$

$Z < 0$ $\vec{E} = \frac{-\sigma}{2\epsilon_0}\vec{e}_z$

$d\vec{l} = dz\,\vec{e}_z + d\vec{l}_\perp$

$\vec{E} \cdot d\vec{l} = \frac{\sigma}{2\epsilon_0} dz$

$$V(A) - V(B) = \int_A^B \vec{E} \cdot d\vec{l} = \int_A^B \frac{\sigma}{2\epsilon_0} dz = \frac{\sigma}{2\epsilon_0}(Z_B - Z_A)$$

$Z > 0$, $V(B) = -\frac{\sigma}{2\epsilon_0} Z_B$

$Z < 0$, $V(B) = \frac{\sigma}{2\epsilon_0} Z_B$

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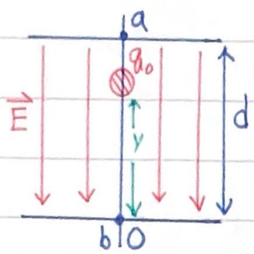
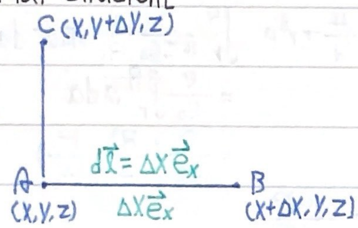
Case VIII oppositely charged parallel plates



$$V(y) = \frac{U(y)}{q_0} = \frac{q_0 E y}{q_0} = Ey$$

$$V_a - V_b = Ed \qquad E = \frac{V_{ab}}{d}$$

Potential Gradient



$$V(A) - V(B) = \int_b^a dV = -\int_a^b dV = \int_a^b \vec{E}\cdot d\vec{l} \qquad V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{dQ}{|\vec{r}-\vec{r}'|}$$

$$= V(x,y,z) - V(x+\Delta x, y, z)$$

$$= E_x \Delta x$$

$$E_x = \lim_{\Delta x \to 0} -\frac{V(x+\Delta x, y, z) - V(x,y,z)}{\Delta x} = -\frac{\partial V}{\partial x}$$

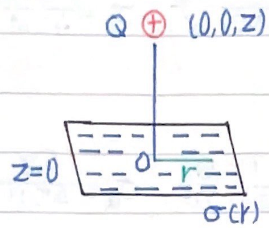
$$\vec{E} = -\frac{\partial V}{\partial x}\vec{e}_x - \frac{\partial V}{\partial y}\vec{e}_y - \frac{\partial V}{\partial z}\vec{e}_z$$

$$= -\left(\vec{e}_x \frac{\partial}{\partial x} + \vec{e}_y \frac{\partial}{\partial y} + \vec{e}_z \frac{\partial}{\partial z}\right) V(x,y,z)$$

$$= -\vec{\nabla}\cdot V(x,y,z)$$

Method of Image Charges

case I



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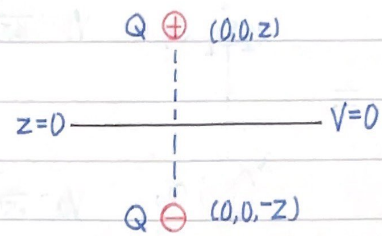


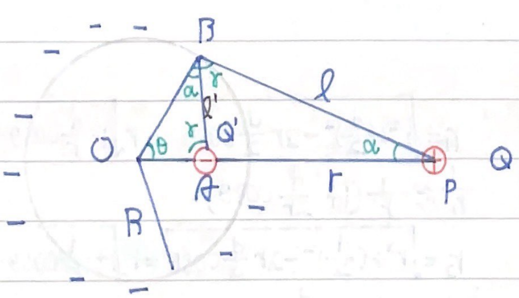
$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{\sqrt{r^2+(z-z)^2}} + \frac{-Q}{\sqrt{r^2+(z+z)^2}}\right)$$

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial z}\Big|_{z=0} = -\frac{Q}{2\pi}\frac{z}{(r^2+z^2)^{3/2}}$$

$$Q = \int_0^{2\pi}\int_0^{\infty} \sigma(r)\, r\,dr\,d\theta$$
$$= -\frac{zr}{2\pi}\int_0^{2\pi} d\theta \int_0^{\infty} \frac{r\,dr}{(r^2+z^2)^{\frac{3}{2}}}$$
$$= -Q$$

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{-QQ}{4z^2}\vec{e}_z$$

case II



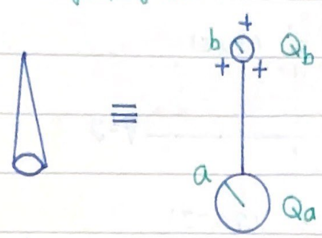
$\triangle OAB \simeq \triangle OBP$ $\quad \frac{\overline{OA}}{\overline{OB}} = \frac{\overline{OB}}{\overline{OP}} = \frac{\overline{AB}}{\overline{BP}} = \frac{l'}{l}$, $\overline{OA} = \frac{R^2}{r}$

put point charge Q' at point A $\quad V(B) = \frac{1}{4\pi\epsilon_0}\frac{Q'}{l'} + \frac{1}{4\pi\epsilon_0}\frac{Q}{l}$
$$= \frac{1}{4\pi\epsilon_0}\left(\frac{Q'}{l'} + \frac{Q}{l}\right)$$

hope $V(B)=0$, $Q' = -\frac{l'}{l}Q = -\frac{R}{r}Q$

Application

Case I lightning rod



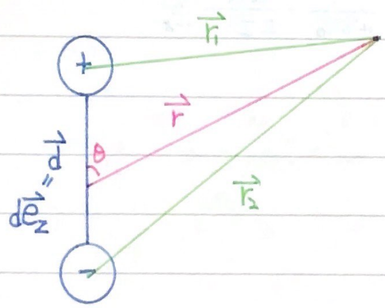
$V_b = \frac{1}{4\pi\epsilon_0}\frac{Q_b}{b}$ $\quad \vec{E}_b = \frac{1}{4\pi\epsilon_0}\frac{Q_b}{b^2}$ $\quad \sigma_b = \frac{Q_b}{4\pi b^2}$

$V_a = \frac{1}{4\pi\epsilon_0}\frac{Q_a}{a}$ $\quad \vec{E}_a = \frac{1}{4\pi\epsilon_0}\frac{Q_a}{a^2}$ $\quad \sigma_a = \frac{Q_a}{4\pi a^2}$

$\vec{E}_b b = V_b$
$\vec{E}_a a = V_a$ and $V_a = V_b$

$$\frac{\sigma_b}{\sigma_a} = \frac{a^2}{b^2}\frac{Q_a}{Q_b} = \frac{a}{b} \qquad \frac{\vec{E}_b}{\vec{E}_a} = \frac{Q_b}{Q_a} = \frac{a}{b}$$

Case II electric dipole



$$V = \frac{Q}{4\pi\epsilon_0}\frac{1}{r_1} + \frac{-Q}{4\pi\epsilon_0}\frac{1}{r_2}$$
$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{Q}{4\pi\epsilon_0}\frac{r_2 - r_1}{r_1 r_2}$$
$$\approx \frac{Q}{4\pi\epsilon_0}\frac{d\cos\theta}{|\vec{r}|^2}$$

Qd: electric dipole

$$r_1 = \sqrt{r^2 + \left(\frac{d}{2}\right)^2 - 2r\frac{d}{2}\cos\theta} = r\sqrt{1 - \frac{d}{r}\cos\theta + \left(\frac{d}{2r}\right)^2}$$
$$\frac{1}{r_1} \approx \frac{1}{r}\left(1 + \frac{d}{2r}\cos\theta\right)$$
$$r_2 = \sqrt{r^2 + \left(\frac{d}{2}\right)^2 - 2r\frac{d}{2}\cos\theta} = r\sqrt{1 + \frac{d}{r}\cos\theta + \left(\frac{d}{2r}\right)^2}$$
$$\frac{1}{r_2} \approx \frac{1}{r}\left(1 - \frac{d}{2r}\cos\theta\right)$$

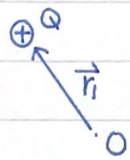
$$V(x,y,z) = \frac{Qd}{4\pi\epsilon_0}\frac{z}{(x^2+y^2+z^2)^{3/2}}$$

$$E_x = -\frac{\partial V}{\partial x} = \frac{Qd}{4\pi\epsilon_0}\frac{3xz}{(x^2+y^2+z^2)^{3/2}} \qquad E_z = \frac{Qd}{4\pi\epsilon_0}\frac{3z^2 - (x^2+y^2+z^2)}{(x^2+y^2+z^2)^{\frac{5}{2}}}$$

$$E_y = \frac{Qd}{4\pi\epsilon_0}\frac{3yz}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

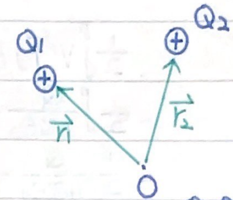
Electric- Field Energy

$i=1$



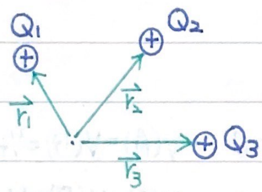
$\Delta W_1 = 0$

$i=2$



$$\Delta W_2 = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{|\vec{r}_1 - \vec{r}_2|}$$

$i=3$



$$\Delta W_3 = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_3}{|\vec{r}_3 - \vec{r}_1|} + \frac{1}{4\pi\epsilon_0}\frac{Q_2 Q_3}{|\vec{r}_3 - \vec{r}_2|}$$

$i=i$

$$\Delta W_i = \sum_{j<i} \frac{1}{4\pi\epsilon_0}\frac{Q_j Q_i}{|\vec{r}_i - \vec{r}_j|}$$

$$W = \Delta W_1 + \Delta W_2 + \Delta W_3 + \cdots W_N$$

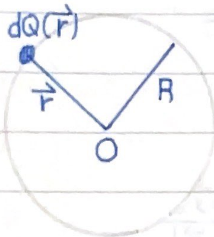
$$= \sum_{i=1}^{N} \Delta W_i = \sum_{i=1}^{N}\sum_{j<i} \frac{1}{4\pi\epsilon_0}\frac{Q_j Q_i}{|\vec{r}_i - \vec{r}_j|}$$

$$= \sum_{i=1}^{N}\sum_{\substack{j=1 \\ i\neq j}}^{N} \frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{Q_j Q_i}{|\vec{r}_i - \vec{r}_j|}$$

$$= \frac{1}{2}\sum_{i=1}^{N} Q_i \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{Q_j}{4\pi\epsilon_0 |\vec{r}_i - \vec{r}_j|} = \frac{1}{2}\sum_{i=1}^{N} Q_i V(\vec{r}_i)$$ energy in the electric field

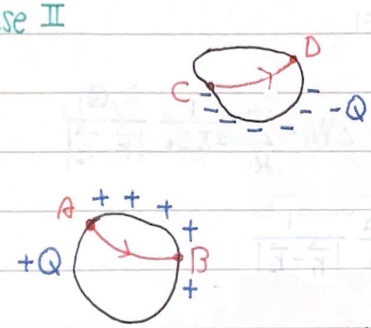
$$= U_E = \frac{1}{2}\int V(\vec{r})\, dQ(\vec{r})$$

case I



$$\frac{1}{2}\int V(\vec{r})\,dQ(\vec{r})$$
$$=\frac{1}{2}\int \frac{Q}{4\pi\epsilon_0}\frac{1}{R}\,dQ(\vec{r})$$
$$=\frac{1}{2}\frac{Q}{4\pi\epsilon_0}\frac{1}{R}\int dQ(\vec{r})$$
$$=\frac{1}{2}\frac{Q^2}{4\pi\epsilon_0}\frac{1}{R}$$

case II



$V(A)-V(B)=0 \qquad V(A)=V(B)=V_+$

$V(C)-V(D)=0 \qquad V(C)=V(D)=V_-$

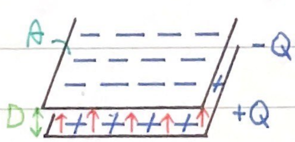
$$U_E=\frac{1}{2}\left[QV_+ + (-Q)V_-\right]=\frac{1}{2}Q(V_+-V_-)=\frac{1}{2}Q\,\Delta V$$

when $+Q$ and $-Q$ increase to $+nQ$ and $-nQ$, then

$\Delta V \longrightarrow n\,\Delta V \qquad |\vec{E}| \longrightarrow n\,|\vec{E}|$

let $\Delta V=\frac{1}{C}Q$ C: permittivity, then $U_E=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}C(\Delta V)^2$

case III



$$|\vec{E}|=\frac{\sigma}{\epsilon_0}=\frac{1}{\epsilon_0}\frac{Q}{A}$$
$$V_+-V_-=\int \vec{E}\cdot d\vec{l}=\frac{1}{\epsilon_0}\frac{Q}{A}D=\Delta V$$
$$C=\frac{\epsilon_0 A}{D}$$

$$U_E=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}\frac{D}{\epsilon_0 A}Q^2$$
$$=\frac{1}{2}D\epsilon_0 A\frac{Q^2}{(\epsilon_0 A)^2}$$
$$=\frac{\epsilon_0}{2}DA|\vec{E}|^2$$
$$=\underline{\frac{\epsilon_0}{2}|\vec{E}|^2}\cdot \text{volume}$$

energy density (not average density), u_E

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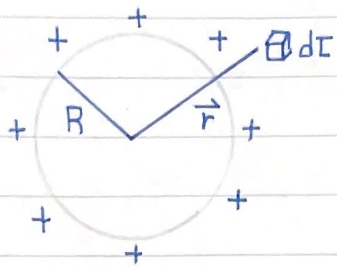
$$\Rightarrow U_E = \int u_E \, d\tau = \int \frac{\epsilon_0}{2} |\vec{E}|^2 d\tau = \frac{1}{2} \int V(\vec{r}) \, dQ(\vec{r})$$

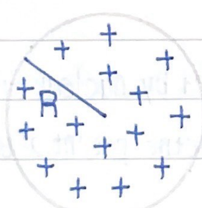
case IV use $U_E = \int \frac{\epsilon_0}{2} |\vec{E}|^2 d\tau$ calculate case I



$r > R$	$r < R$
$\vec{E} = \frac{Q}{4\pi\epsilon_0} \frac{1}{\lvert\vec{r}\rvert^2} \vec{e}_r$	$\vec{E} = 0$
$u_E = \frac{\epsilon_0}{2} \frac{Q^2}{(4\pi\epsilon_0)^2} \frac{1}{\lvert\vec{r}\rvert^4}$	$u_E = 0$

$$U_E = \int_R^\infty \frac{\epsilon_0}{2} \frac{Q^2}{(4\pi\epsilon_0)^2} \frac{1}{r^4} 4\pi r^2 dr$$
$$= \frac{1}{2} \frac{Q^2}{4\pi\epsilon_0} \int_R^\infty \frac{1}{r^2} dr$$
$$= \frac{1}{2} \frac{Q^2}{4\pi\epsilon_0} \frac{1}{R}$$

case V



$$Q = \frac{4\pi}{3} R^3 \rho$$

$r > R$	$r < R$
$\vec{E} = \frac{Q}{4\pi\epsilon_0} \frac{1}{\lvert\vec{r}\rvert^2} \vec{e}_r$	$\vec{E} = \frac{\rho}{3\epsilon_0} \vec{r}$
$u_E = \frac{\epsilon_0}{2} \frac{Q^2}{(4\pi\epsilon_0)^2} \frac{1}{r^4}$	$u_E = \frac{\epsilon_0}{2} \frac{\rho^2}{(3\epsilon_0)^2} r^2$

$$U_E = \int_{r \ge R}^\infty \frac{\epsilon_0}{2} \frac{Q^2}{(4\pi\epsilon_0)^2} \frac{1}{r^4} 4\pi r^2 dr + \int_0^R \frac{\epsilon_0}{2} \frac{\rho^2}{(3\epsilon_0)^2} r^2 4\pi r^2 dr$$
$$= \frac{1}{2} \frac{Q^2}{4\pi\epsilon_0} \frac{1}{R} + \frac{\epsilon_0}{2} \frac{\rho^2}{(3\epsilon_0)^2} 4\pi \int_0^R r^4 dr$$
$$= \frac{1}{2} \frac{Q^2}{4\pi\epsilon_0} \frac{1}{R} + \frac{1}{2} \frac{1}{\epsilon_0} \frac{4\pi}{3} \rho \frac{1}{5} R^5$$
$$= \frac{1}{2} \frac{Q^2}{4\pi\epsilon_0} \frac{1}{R} + \frac{1}{2} \frac{Q^2}{4\pi\epsilon_0} \frac{1}{5} \frac{1}{R}$$
$$= \frac{3}{5} \frac{Q^2}{4\pi\epsilon_0} \frac{1}{R}$$

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case VI use $U_E = \frac{1}{2}\int V(\vec{r})\,dQ(\vec{r})$ calculate case V

$r > R$

$$V(r) = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}$$

$$V(R) = \frac{Q}{4\pi\epsilon_0}\frac{1}{R}$$

$r < R$

$$V(r) = \frac{\rho R^2}{2\epsilon_0} - \frac{\rho R^2}{6\epsilon_0}$$

$$V(R) = \frac{\rho R^3}{3\epsilon_0}$$

$$U_E = \frac{1}{2}\int\left(\frac{\rho r^2}{2\epsilon_0} - \frac{\rho r^2}{6\epsilon_0}\right)4\pi r^2 \rho\, dr$$

$$= \frac{1}{2}\frac{\rho R^2}{2\epsilon_0}\rho\frac{4}{3}\pi R^3 - \frac{1}{2}\frac{\rho 4\pi}{6\epsilon_0}\rho\int_0^R r^4\, dr$$

$$= \frac{1}{2}\frac{\rho^2}{2\epsilon_0}\frac{4\pi}{3}\frac{R^6}{R} - \frac{1}{10}\frac{\rho^2}{2\epsilon_0}\frac{4\pi}{3}\frac{1}{5}\frac{R^6}{R}$$

$$= \frac{4}{10}\frac{1}{2\epsilon_0}\frac{3}{4\pi}\frac{Q^2}{R}$$

$$= \frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{Q^2}{R}$$

nucleus

element X_Z^A . $R = A^{\frac{1}{3}} r_0$, $r_0 \approx 1.2\times 10^{-15}$ m

$$U_E = \frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{Q^2}{R} \quad \sim \text{MeV}$$

$$X_Z^A \longrightarrow \oplus\, Y_{\frac{Z}{2}}^{\frac{A}{2}} \quad \oplus\, Y_{\frac{Z}{2}}^{\frac{A}{2}}$$

the energy released by nuclear fission was essentially electric potential energy

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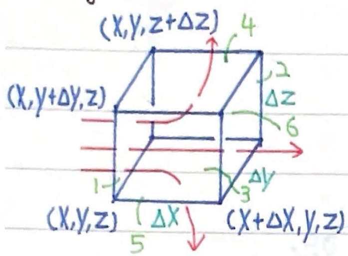
Divergence Theorem



flux of face 1 and 2 $E_x(2)\Delta y\Delta z - E_x(1)\Delta y\Delta z = [E_x(2)-E_x(1)]\Delta y\Delta z$

$= \frac{\partial E_x}{\partial x}\Delta x\Delta y\Delta z$

flux of face 3 and 4 $E_y(4)\Delta x\Delta z - E_y(3)\Delta x\Delta z = \frac{\partial E_y}{\partial y}\Delta x\Delta y\Delta z$

flux of face 5 and 6 $E_z(6)\Delta x\Delta y - E_z(5)\Delta x\Delta y = \frac{\partial E_z}{\partial z}\Delta x\Delta y\Delta z$

$$\int_{\square} \vec{E}\cdot d\vec{A} = \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right)\Delta x\Delta y\Delta z = \frac{1}{\epsilon_0}\left[\text{the charges in } \square\right]$$

$$\text{div } \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0 \\ \Delta z \to 0}} \frac{\int \vec{E}\cdot d\vec{A}}{\Delta x\Delta y\Delta z} = \text{div } \vec{E} = \lim \frac{\frac{1}{\epsilon_0} Q_{enel}}{\square \text{ volume}} = \frac{\rho(x,y,z)}{\epsilon_0}$$

$$\int_S \vec{E}\cdot d\vec{A} = \int \nabla\cdot\vec{E}\, d\tau \qquad \nabla\cdot\vec{E} = \text{div } \vec{E} = \frac{\rho}{\epsilon_0}$$

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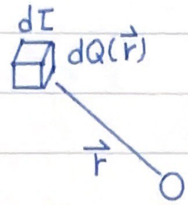

$$U_E = \frac{1}{2}\int V(\vec{r})\, dQ(\vec{r})$$

$$= \frac{1}{2}\int V(\vec{r})\, \rho(\vec{r})\, d\tau \quad = \frac{1}{2}\int V(\vec{r})\, \epsilon_0 \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) d\tau$$

$$= \frac{\epsilon_0}{2}\int V(\vec{r}) \left[\frac{\partial}{\partial x}\frac{\partial V}{\partial x} + \frac{\partial}{\partial y}\frac{\partial V}{\partial y} + \frac{\partial}{\partial z}\frac{\partial V}{\partial z}\right] d\tau \qquad \text{since } E_x = -\frac{\partial V}{\partial x}$$

let $\vec{G} = V\frac{\partial V}{\partial x}\vec{e}_x + V\frac{\partial V}{\partial y}\vec{e}_y + V\frac{\partial V}{\partial z}\vec{e}_z$

$$\nabla\cdot\vec{G} = \frac{\partial}{\partial x}V\frac{\partial V}{\partial x} + \frac{\partial}{\partial y}V\frac{\partial V}{\partial y} + \frac{\partial}{\partial z}V\frac{\partial V}{\partial z} = V\left(\frac{\partial}{\partial x}\frac{\partial V}{\partial x} + \frac{\partial}{\partial y}\frac{\partial V}{\partial y} + \frac{\partial}{\partial z}\frac{\partial V}{\partial z}\right)$$

$$= \frac{\epsilon_0}{2}\int \nabla\cdot\vec{G} - \left(\frac{\partial V}{\partial x}\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y}\frac{\partial V}{\partial y} + \frac{\partial V}{\partial z}\frac{\partial V}{\partial z}\right) d\tau$$

$$= \frac{\epsilon_0}{2}\int |\vec{E}|^2\, d\tau$$

$$U_E = \frac{1}{2}\int V(\vec{r})\, dQ(\vec{r}) = \frac{1}{2}\int V(\vec{r})\, \rho(\vec{r})\, d\tau = \int \frac{\epsilon_0}{2}(\vec{E}\cdot\vec{E})\, d\tau$$