Lagrangian Function in Special Relativity

$$p_i = \frac{\partial L}{\partial u_i} = \frac{m u_i}{\sqrt{1-\beta^2}} \qquad L = T^* - U$$

$$\frac{\partial T^*}{\partial u_i} = \frac{m u_i}{\sqrt{1-\beta^2}}, \quad T^* = -mc^2\sqrt{1-\beta^2}, \quad L = -mc^2\sqrt{1-\beta^2} - U$$

$$H = \sum_i u_i p_i - L = \sum_i \frac{p_i^2 c^2}{\gamma m c^2} + \frac{mc^2}{\gamma} + U$$

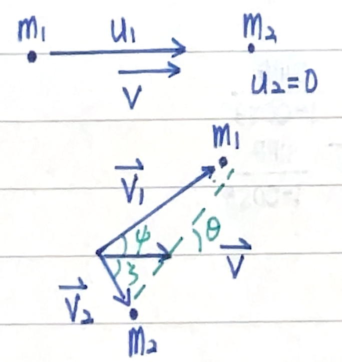
$$= \frac{1}{\gamma m c^2}\left(p^2c^2 + m^2c^4\right) + U$$

$$= \frac{E^2}{\gamma m c^2} + U = E + U = T + U + E_0$$

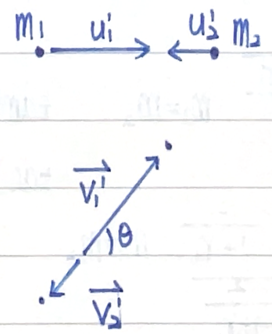
Relativistic Kinematics

$$p_1' = p_2', \quad m_1 u_1' \gamma_1' = m_2 u_2' \gamma_2'$$

system K



K'



$$p_1' = m_1 u_1' \gamma_1' = m_1 c \beta_1' \gamma_1'$$

$$= m_1 c\sqrt{(\gamma_1')^2 - 1} = m_2 c\sqrt{(\gamma_2')^2 - 1}$$

$$= p_2'$$

$$p_1' = \left(p_1 - \frac{u_2'}{c^2}E'\right)\gamma_2'$$

$$p_1 = m_1 u_1 \gamma_1$$

$$E_1 = m_1 c^2 \gamma_1$$

$$m_1 c\sqrt{(\gamma_1')^2-1} = (m_1 c\beta_1\gamma_1 - \beta_2' m_1 c\gamma_1)\gamma_2'$$
$$= m_1 c\left[\gamma_2'\sqrt{\gamma_1^2-1} - \gamma_1\sqrt{(\gamma_2')^2-1}\right]$$
$$= m_2 c\sqrt{(\gamma_2')^2-1}$$

$$\gamma_1' = \frac{\gamma_1 + \frac{m_1}{m_2}}{\sqrt{1+2\gamma_1\frac{m_1}{m_2}+\left(\frac{m_1}{m_2}\right)^2}} \qquad \gamma_2' = \frac{\gamma_1 + \frac{m_2}{m_1}}{\sqrt{1+2\gamma_1\frac{m_2}{m_1}+\left(\frac{m_2}{m_1}\right)^2}}$$

$$P_{1,x} = \left(p_{1,x}' + \frac{u_2'}{c^2}E_1'\right)\gamma_2' = (m_1 c\beta_1'\gamma_1'\cos\theta + m_1 c\beta_2'\gamma_1')\gamma_2'$$
$$= m_1 c\gamma_1'\gamma_2'(\beta_1'\cos\theta + \beta_2')$$

$$P_{1,y} = m_1 c\beta_1'\gamma_1'\sin\theta$$

$$\tan\psi = \frac{1}{\gamma_2'}\,\frac{\sin\theta}{\cos\theta + \frac{\beta_2'}{\beta_1'}} = \frac{1}{\gamma_2'}\,\frac{\sin\theta}{\cos\theta + \frac{m_1\gamma_1'}{m_2\gamma_2'}}$$

$$P_{2x} = \left(P_{2x}' + \frac{u_2'}{c^2}E_2'\right)\gamma_2' = (-m_2 c\beta_2'\gamma_2'\cos\theta + m_2 c\beta_2'\gamma_2')\gamma_2'$$
$$= m_2 c\beta_2'\gamma_2'(1-\cos\theta)$$

$$P_{2y} = -m_2 c\beta_2'\gamma_2'\sin\theta \qquad \zeta = \frac{P_{2y}}{P_{2x}} = -\frac{1}{\gamma_2'}\,\frac{\sin\theta}{1-\cos\theta}$$

$$\gamma_1' = \gamma_2' = \sqrt{\frac{1+\gamma_1}{2}} \qquad m_1 = m_2 \qquad \tan\psi = \sqrt{\frac{2}{1+\gamma_1}}\,\frac{\sin\theta}{1+\cos\theta}$$
$$\tan\zeta = -\sqrt{\frac{2}{1+\gamma_1}}\,\frac{\sin\theta}{1-\cos\theta}$$

$$\tan\psi\tan\zeta = -\frac{2}{1+\gamma_1} \qquad m_1 = m_2$$
$$\tan\psi = \sqrt{\frac{2}{1+\gamma_1}}$$

$$\phi = \psi + \zeta + 2\psi = 2\tan^{-1}\left(\sqrt{\frac{2}{1+\gamma_1}}\right)$$

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$X_4 = ict \qquad X_4' = ict'$

$$\sum_{\mu=1}^{4} X_\mu^2 = 0 \qquad \sum_{\mu=1}^{4} (X_\mu')^2 = 0$$

$$\sum_j X_j^2 - c^2t^2 = \sum_\mu X_\mu^2 \equiv S^2 = 0 \text{ in } K$$

$$\sum_j (X_j')^2 - c^2(t')^2 = \sum_\mu (X_\mu')^2 \equiv (S')^2 = 0 \text{ in } K'$$

assume $(S')^2 = \kappa S^2$, $(S')^2 = \underbrace{\kappa(v) S^2 , S^2 = \kappa(-v)}_{\kappa^2 = 1 \text{ or } \kappa(v) = \pm 1}(S')^2$

in the limit of zero velocity, K and K' become identical $\kappa(v=0) = +1$

$\Rightarrow S^2 = (S')^2$

$\Rightarrow \sum_\mu X_\mu^2 = \sum_\mu (X_\mu')^2$

the Lorentz transformations are then orthogonal transformation in Minkowski space.

$X_\mu' = \sum_\nu \underline{\lambda_{\mu\nu}} X_\nu$ elements of the Lorentz transformation matrix

$$\vec{\lambda} \begin{bmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{bmatrix}$$

$A_\mu' = \sum_\nu \lambda_{\mu\nu} A_\nu \qquad$ four-vector $\mathbb{X} = (x, y, z, ict) = (\vec{X}, ict)$

$d\mathbb{X} = (d\vec{X}, icdt) \qquad ds = \sqrt{\sum_\mu dX_\mu^2} \qquad d\tau = \frac{i}{c}\sqrt{\sum_\mu dX_\mu^2} = \frac{i}{c} ds$

four-vector velocity $\mathbb{V} = \frac{d\mathbb{X}}{d\tau} = \left(\frac{d\vec{X}}{d\tau}, ic\frac{dt}{d\tau}\right)$

ordinary velocity $\vec{u}$, $u_j = \frac{dX_j}{dt}$

$$d\tau = dt\sqrt{1 - \frac{1}{c^2}\sum_j \frac{dX_j^2}{dt^2}} \quad \text{or} \quad d\tau = dt\sqrt{1-\beta^2}$$

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$$\mathbb{V} = \frac{1}{\sqrt{1-\beta^2}}(\vec{u}, ic)$$

$$\mathbb{P} = m\mathbb{V} = \left(\frac{m\vec{u}}{\sqrt{1-\beta^2}}, iP_4\right) \qquad P_4 \equiv \frac{mc}{\sqrt{1-\beta^2}}$$

$$P_j = \gamma m u_j \qquad P_4 = \gamma mc = \frac{E}{c} \qquad \mathbb{P} = \left(\vec{p}, i\frac{E}{c}\right)$$

$$P_1' = \frac{P_1 - \frac{v}{c^2}E}{\sqrt{1-\beta^2}} \qquad P_2' = P_2 \qquad P_3' = P_3$$

$$E' = \frac{E - vP_1}{\sqrt{1-\beta^2}}$$

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