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Identical Particles

Two Particle System

$\Psi(\vec{r}_1, \vec{r}_2, t)$

$H = \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} + V(\vec{r}_1, \vec{r}_2)$

$\left[-\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 + V(\vec{r}_1, \vec{r}_2)\right]\psi = E\psi$

$\iint |\Psi(r_1, r_2)|^2 d^3r_1\, d^3r_2 = 1$

$|f\rangle = \psi(\vec{r})\chi(\vec{s})$

define exchange operator

$\hat{P} f(\vec{r}_1, \vec{r}_2) = f(\vec{r}_2, \vec{r}_1)$

$\hat{P}^2 f(\vec{r}_1, \vec{r}_2) = f(\vec{r}_1, \vec{r}_2)$

$P\psi = \lambda\psi$

$P^2\psi = P(\lambda\psi) = \lambda^2\psi \Rightarrow \lambda^2 = 1,\ \lambda = \pm 1$

$\begin{cases} \lambda = +1 & \text{bosons} & \text{spin} = N \\ \lambda = -1 & \text{fermions} & \text{spin} = \text{half } N \end{cases}$

↳ Pauli's exclusive principle

$\chi_S(x_1, x_2) = \frac{1}{\sqrt{2}}\left[\psi_a(x_1)\psi_b(x_2) + \psi_b(x_1)\psi_a(x_2)\right]$

$\chi_A(x_1, x_2) = \frac{1}{\sqrt{2}}\left[\psi_a(x_1)\psi_b(x_2) - \psi_b(x_1)\psi_a(x_2)\right]$ anti-Symmetry

a,b : states

1,2 : particle's "number"

Exchange Force

$f(x_1, \vec{S}_1; x_2, \vec{S}_2) = \underbrace{\psi(x_1, x_2)}_{\substack{\text{sym} \\ \text{anti-sym}}}\underbrace{\chi(\vec{S}_1, \vec{S}_2)}_{\substack{\text{sym} \\ \text{anti-sym}}}$ } bosons

$\psi_D(x_1, x_2) = \psi_a(x_1)\psi_b(x_1)$

distingnishable

$\langle (x_1 - x_2)^2 \rangle = \langle x_1^2 - 2x_1x_2 + x_2^2 \rangle = \langle x_1^2 \rangle + \langle x_2^2 \rangle - 2\langle x_1 x_2 \rangle$

Case D

$\langle x_1^2 \rangle = \iint \psi_a^*(x_1)\psi_b^*(x_2)\, x_1^2\, \psi_a(x_1)\psi_b(x_2)\, dx_1 dx_2 = \underbrace{\int \psi_a^*(x_1) x_1^2 \psi_a(x_1) dx}_{\langle x^2 \rangle_a} \cdot \overbrace{\int \psi_b^*(x_2)\psi_b(x_2) dx_2}^{1}$

$\langle x_2^2 \rangle = \langle x^2 \rangle_b$

$\langle x_1 x_2 \rangle = \int \psi_a^*(x_1) x_1 \psi_a(x_1) dx_1 \int \psi_b^*(x_2) x_2 \psi_b(x_2) dx_2 = \langle x \rangle_a \langle x \rangle_b$

$\Rightarrow \langle (x_1 - x_2)^2 \rangle_D = \langle x^2 \rangle_a + \langle x^2 \rangle_b - 2\langle x \rangle_a \langle x \rangle_b$

Case S

$\langle x_1^2 \rangle = \frac{1}{2}\iint \left[\psi_a^*(x_1)\psi_b^*(x_2) + \psi_b^*(x_1)\psi_a^*(x_2)\right] x_1^2 \left[\psi_a(x_1)\psi_b(x_2) + \psi_b(x_1)\psi_a(x_2)\right] dx_1 dx_2$

$= \frac{1}{2}(\langle x^2 \rangle_a + \langle x^2 \rangle_b + 0 + 0)$

$\langle x_2^2 \rangle = \frac{1}{2}(\langle x^2 \rangle_a + \langle x^2 \rangle_b)$

$\langle x_1 x_2 \rangle = \frac{1}{2}\iint \left[\psi_a^*(x_1)\psi_b^*(x_2) + \psi_b^*(x_1)\psi_a^*(x_2)\right] x_1 x_2 \left[\psi_a(x_1)\psi_b(x_2) + \psi_b(x_1)\psi_a(x_2)\right] dx_1 dx_2$

$= \frac{1}{2}\Big[\overbrace{\int \psi_a^*(x_1) x_1 \psi_a(x_1) dx_1}^{\langle x \rangle_a} \overbrace{\int \psi_b^*(x_2) x_2 \psi_b(x_2) dx_2}^{\langle x \rangle_b} + \overbrace{\int \psi_b^*(x_1) x_1 \psi_b(x_1) dx_1}^{\langle x \rangle_b} \overbrace{\int \psi_a^*(x_2) x_2 \psi_a(x_2) dx_2}^{\langle x \rangle_a}$

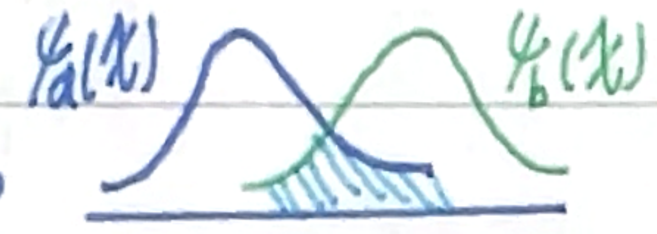
$+ \underbrace{\int \psi_a^*(x_1) x_1 \psi_b(x_1) dx_1}_{\langle x \rangle_{ab}} \underbrace{\int \psi_b^*(x_2) x_2 \psi_a(x_2) dx_2}_{\langle x \rangle_{ba}} + \underbrace{\int \psi_b^*(x_1) x_1 \psi_a(x_1) dx_1}_{\langle x \rangle_{ba}} \underbrace{\int \psi_a^*(x_2) x_2 \psi_b(x_2) dx_2}_{\langle x \rangle_{ab}}$

$$\langle (x_1-x_2)^2\rangle = \underbrace{\langle x^2\rangle_a + \langle x^2\rangle_b - 2\langle x\rangle_a\langle x_b\rangle}_{\langle (x_1-x_2)^2\rangle_D} - 2|\langle x\rangle_{ab}|^2$$ ← attractive exchange force

Case A

$$\langle (x_1-x_2)^2\rangle = \underbrace{\langle x^2\rangle_a + \langle x^2\rangle_b - 2\langle x\rangle_a\langle x\rangle_b}_{\langle (x_1-x_2)^2\rangle_D} + 2|\langle x\rangle_{ab}|^2$$ ← repulsive exchange force

$\langle x\rangle_{ab} = \int \psi_a^*(x)\, x\, \psi_b(x)\, dx$



overlap integral $\propto$ exchange interaction strength