NO.

DATE / /

Hamilton's Principle

Hamilton's Principle

2nd century B.C.	Hero of Alexandria	reflection of light always takes the shortest path
1657	Fermat	light ray always travels from one point to another in a medium by a path that requires the least time.
1747	Maupertuis	dynamical motion takes place with minimum motion
1760	Lagrange	
1828	Gauss	least constraint
	Hertz	least curvature
1834, 1835	Hamilton	

of all the possible paths along which a dynamical system may move from one point to another point within a specified time interval (consistent with any constraints), the actual path followed is that which minimizes the time integral of the difference between the kinetic and potential energies

$$\delta\int_{t_1}^{t_2}(T-U)dt=0 \qquad T=T\left(\frac{dx_i}{dt}\right) \qquad U=U(x_i)$$

$$L=T-U=L\left(x_i,\frac{dx_i}{dt}\right)$$ Lagrange function

$$\Rightarrow \delta\int_{t_1}^{t_2} L(x_i,x_i')dt=0$$

$$\Rightarrow \frac{\partial L}{\partial x_i}-\frac{d}{dt}\frac{\partial L}{\partial x_i'}=0 \quad i=1,2,3\cdots$$

example : HSM

$$L=T-U=\frac{1}{2}m\left(\frac{dx}{dt}\right)^2-\frac{1}{2}kx^2$$

$$\frac{\partial L}{\partial x}=-kx \qquad \frac{\partial L}{\partial x'}=m\frac{dx}{dt} \qquad \frac{d}{dt}\frac{\partial L}{\partial x'}=m\frac{d^2x}{dt^2}$$

$$\Rightarrow m\frac{d^2x}{dt^2}+kx=0$$

Generalized Coordinates

degrees of freedom $3n$
generalized coordinates q_j
generalized velocities q'_j

N particles

$$X_{\alpha,i} = X_{\alpha i}(q_j, t) \qquad \alpha = 1,2,3 \dots n \qquad j = 1,2,3 \dots s$$
$$i = 1,2,3$$

Lagrange's Equations of Motion in Generalized Coordinates

$$L = T(X'_{\alpha i}) - U(X_{\alpha i}) = T(q_j, q'_j, t) - U(q_j, t)$$
$$= L(q_j, q'_j, t)$$

Hamilton's principle $\delta \int_{t_1}^{t_2} L(q_j, q'_j, t)\,dt = 0$

$$\frac{\partial L}{\partial q_j} - \frac{d}{dt}\frac{\partial L}{\partial q'_j} = 0 \qquad j = 1,2 \dots s$$

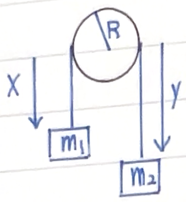
~~* the forces acting on the system must be derivable from potentials~~
~~* the equations of constraint must be relations that connect the coordinates of the particles and may be functions of the time → holonomic constraints~~
~~if the equations do not explictly contain the time : fixed or scleronomic~~
~~moving constraints are rheonomic~~

the equation of constraint can be expressed as an equation that involves only the coordinates of the system and the time t : holonomic constraints

the equation of constraint independent of time : scleronomic or fixed
contains time explicitly : rheonomic

SEASON

case I Atwood's machine



inextensible string l

$x+y+\pi R = l$, $y=-x+\text{constant}$ $\qquad \frac{dy}{dt}=-\frac{dx}{dt}$

$E_k = \frac{1}{2} m_1\left(\frac{dx}{dt}\right)^2+\frac{1}{2} m_2\left(\frac{dy}{dt}\right)^2 = \frac{1}{2}(m_1+m_2)\left(\frac{dx}{dt}\right)^2$

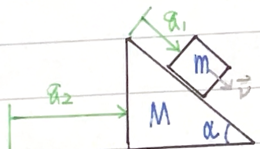
$U=-m_1gx-m_2gy \qquad = -(m_1-m_2)gx+\text{constant}$

$L=E_k-U \qquad = \frac{1}{2}(m_1+m_2)\left(\frac{dx}{dt}\right)^2+(m_1-m_2)gx$

$\frac{\partial L}{\partial x}=\frac{d}{dt}\frac{\partial L}{\partial \dot{x}}$, $(m_1-m_2)g=(m_1+m_2)\frac{dx^2}{dt^2}$, $\qquad \frac{dx^2}{dt^2}=\frac{m_1-m_2}{m_1+m_2}g$ elaldly measure for g

case II a block sliding on a wedge

$\left(\frac{dq_1}{dt}\right)^2\cos^2\alpha+\left(\frac{dq_2}{dt}\right)^2+2\frac{dq_1}{dt}\cos\alpha\frac{dq_2}{dt}+\left(\frac{dq_1}{dt}\right)^2\sin^2\alpha$



$T_M=\frac{1}{2}M\frac{dq_2}{dt} \qquad \vec{v}=(v_x,v_y)=\left(\frac{dq_1}{dt}\cos\alpha+\frac{dq_2}{dt}, \frac{dq_1}{dt}\sin\alpha\right)$

$T_m=\frac{1}{2}m(v_x^2+v_y^2)=\frac{1}{2}m\left[\left(\frac{dq_1}{dt}\right)^2+\left(\frac{dq_2}{dt}\right)^2+2\frac{dq_1}{dt}\frac{dq_2}{dt}\cos\alpha\right]$

$T=T_m+T_M \qquad =\frac{1}{2}(M+m)\left(\frac{dq_2}{dt}\right)^2+\frac{1}{2}m\left[\left(\frac{dq_1}{dt}\right)^2+2\frac{dq_1}{dt}\frac{dq_2}{dt}\cos\alpha\right]$

$U=-mgq_1\sin\alpha$

$L=E_k-U=\frac{1}{2}(M+m)\left(\frac{dq}{dt}\right)^2+\frac{1}{2}m\left[\left(\frac{dq_1}{dt}\right)^2+2\frac{dq_1}{dt}\frac{dq_2}{dt}\cos\alpha\right]+mgq_1\sin\alpha$

$\frac{\partial L}{\partial q_2}=\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_2}$

$M\frac{dq_2}{dt}+m\left(\frac{dq_2}{dt}+\frac{dq_1}{dt}\cos\alpha\right)=\text{constant}$

$\frac{d^2q_2}{dt^2}=-\frac{m}{M+m}\frac{d^2q_1}{dt^2}\cos\alpha$

$\frac{d^2q_1}{dt^2}=\frac{g\sin\alpha}{1-\frac{m\cos^2\alpha}{M+m}}$

$\frac{\partial L}{\partial q_1}=\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_1}$

$mg\sin\alpha=\frac{d}{dt}\left[m\left(\frac{dq_1}{dt}+\frac{dq_2}{dt}\cos\alpha\right)\right]$

$=m\left(\frac{d^2q_1}{dt^2}+\frac{d^2q_2}{dt^2}\cos\alpha\right)$