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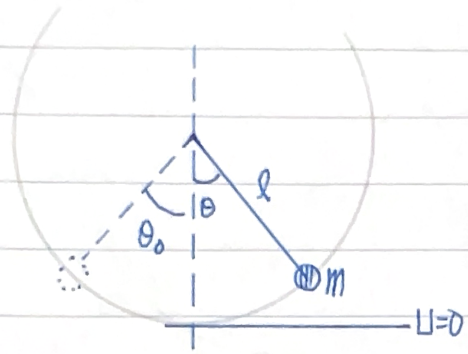
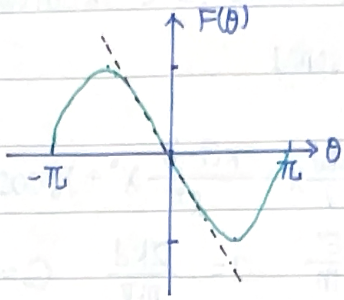
Plane Pendulum





$F(\theta) = -mg\sin\theta$

$I\frac{d^2\theta}{dt^2} = \ell F$ where $I = m\ell^2$

$\Rightarrow \frac{d^2\theta}{dt^2} + \omega_0^2 \sin\theta = 0$ where $\omega_0^2 = \frac{g}{\ell}$ if the amplitude of the motion is small

$\Rightarrow \frac{d^2\theta}{dt^2} + \omega_0^2\theta = 0$ $\quad T \cong 2\pi\sqrt{\frac{\ell}{g}}$

$\left[\begin{array}{l} E_k = \frac{1}{2}I\omega^2 = \frac{1}{2}m\ell^2\left(\frac{d\theta}{dt}\right)^2 \\ U = mg\ell(1-\cos\theta) \end{array}\right.$ if we let $\theta = \theta_0$ at the highest point $\left[\begin{array}{l} E(\theta=\theta_0) = 0 \\ U(\theta=\theta_0) = E_k = mg\ell(1-\cos\theta_0) \end{array}\right.$

$= 2mg\ell\sin^2\frac{\theta}{2}$

$\Rightarrow \frac{1}{2}m\ell^2\left(\frac{d\theta}{dt}\right)^2 = 2mg\ell\left(\sin^2\frac{\theta_0}{2} - \sin^2\frac{\theta}{2}\right)$, $\frac{d\theta}{dt} = 2\sqrt{\frac{g}{\ell}}\sqrt{\sin^2\frac{\theta_0}{2} - \sin^2\frac{\theta}{2}}$

$dt = \frac{1}{2}\sqrt{\frac{\ell}{g}}\frac{1}{\sqrt{\sin^2\frac{\theta_0}{2} - \sin^2\frac{\theta}{2}}}d\theta$ $\quad T = 2\sqrt{\frac{\ell}{g}}\int_0^{\theta_0}\frac{d\theta}{\sqrt{\sin^2\frac{\theta_0}{2} - \sin^2\frac{\theta}{2}}}$

$F(k,\phi) = \int_0^{\phi}\frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}$ $\quad k^2<1$ or if $z = \sin\theta$ $F(k,x) = \int_0^{x}\frac{dz}{\sqrt{1-z^2}\sqrt{1-k^2z^2}}$

$z = \frac{\sin\frac{\theta}{2}}{\sin\frac{\theta_0}{2}}$ $\quad k = \sin\frac{\theta_0}{2}$, $dz = \frac{\cos\frac{\theta}{2}}{2\sin\frac{\theta_0}{2}}$ $\quad d\theta = \frac{\sqrt{1-k^2z^2}}{2k}d\theta$

$\Rightarrow T = 4\sqrt{\frac{\ell}{g}}\int_0^1\frac{dz}{\sqrt{(1-z^2)(1-k^2z^2)}}$

for $|\theta_0| < \pi$ $\quad \sin\frac{\theta_0}{2} = k$ $\quad -1<k<1$

$(1-k^2z^2)^{-\frac{1}{2}} = 1 + \frac{k^2z^2}{2} + \frac{3k^4z^4}{8} + \cdots$

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$$T = 4\sqrt{\frac{l}{g}}\int_0^1 \frac{dz}{\sqrt{1-z^2}}\left(1+\frac{k^2z^2}{2}+\frac{3k^4z^4}{8}+\cdots\right)$$

$$= 4\sqrt{\frac{l}{g}}\left(\frac{\pi}{2}+\frac{k^2}{2}\frac{1}{2}\frac{\pi}{2}+\frac{3k^4}{8}\frac{3}{8}\frac{\pi}{2}+\cdots\right)$$

$$= 2\pi\sqrt{\frac{l}{g}}\left(1+\frac{k^2}{4}+\frac{9k^4}{64}+\cdots\right)$$

if $|k| \leqq 1$ $\quad k = \sin\frac{\theta_0}{2}$

then $k \fallingdotseq \frac{\theta_0}{2} - \frac{\theta_3}{48}$

$$\fallingdotseq 2\pi\sqrt{\frac{l}{g}}\left(1+\frac{1}{16}\theta_0^2+\frac{11}{3072}\theta_0^4\right)$$

$$\frac{d\theta}{dt} = 2\sqrt{\frac{g}{l}}\sqrt{\sin^2\frac{\theta_0}{2}-\sin^2\frac{\theta}{2}}$$

if θ and θ_0 are small, $\left(\sqrt{\frac{l}{g}}\frac{d\theta}{dt}\right)^2+\theta^2 \cong \theta_0^2$

if the total energy $= E_0$, $\quad E = 2mgl\sin^2\frac{\theta_0}{2} \Rightarrow \theta_0 = \pm\pi$

$$\frac{d\theta}{dt} = \pm 2\sqrt{\frac{g}{l}}\cos\frac{\theta}{2}$$

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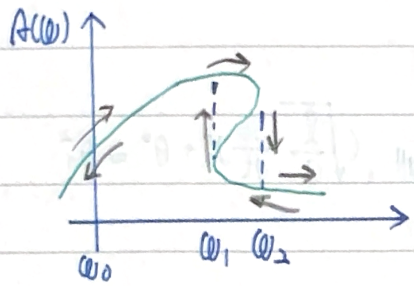
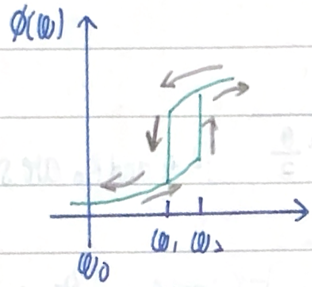
Jumps, Hysteresis, and Phase Lags

$$m\frac{d^2x}{dt^2} = -r\frac{dx}{dt} - kx + F_0\cos\omega t$$

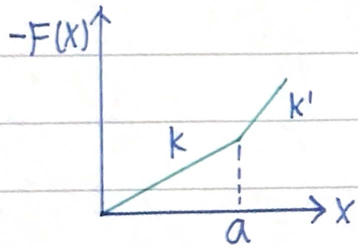
$$x(t) = A(\omega)\cos[\omega t - \phi(\omega)] \qquad A(\omega) = \frac{F_0}{\sqrt{(k-m\omega^2)^2 + (r\omega)^2}} \qquad \tan[\phi(\omega)] = \frac{r\omega}{k-m\omega^2}$$

$$k(x) = (1+\beta x^2)k_0$$

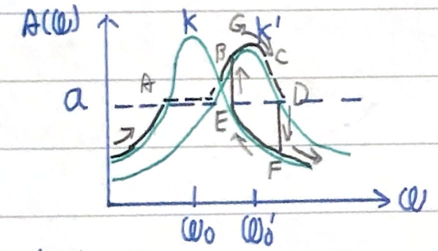




we consider a simpler dependence of k



$$F(x) = -kx \qquad x \le a \qquad k < k'$$
$$\simeq -k'x - c \qquad x \ge a$$



$\omega\uparrow$ ABGCDF

$\omega\downarrow$ FEGBA