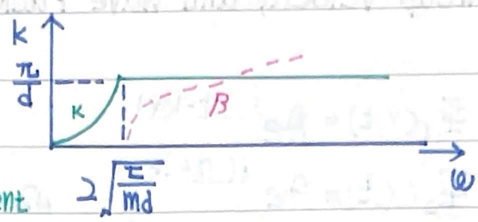
NO.

DATE / /

therefore have $\cos\frac{kd}{2}=0$ $\sin\frac{kd}{2}=1$

$$\omega = 2\sqrt{\frac{T}{md}}\cosh\frac{\beta d}{2}$$



attenuation coefficient

$$\Psi = Ae^{i(\omega t - kx)} = Ae^{-\beta x}e^{i(\omega t - kx)}$$

damping, or attenuation

⇒ wave is propagated without attenuation for $\omega \le 2\sqrt{\frac{T}{md}}$ passing band

and that attenuation sets in at $\omega_c = 2\sqrt{\frac{T}{md}}$ critical or cutoff

and increase with increasing frequency

$$V' = \frac{\omega}{k} = \frac{\omega}{\mathrm{Re}\,\kappa}$$

NO.

DATE / /

Group Velocity and Wave Packets

$$\Psi_1(x,t) = Ae^{i(\omega t-kx)}$$

$$\Psi_2(x,t) = Ae^{i(\Omega t-Kx)} \qquad \Omega=\omega+\Delta\omega \quad K=k+\Delta k$$

$$\Psi(x,t) = \Psi_1+\Psi_2 = A(e^{i\omega t}e^{-i\omega t} + e^{i(\omega+\Delta\omega)t}e^{-i(k+\Delta k)x})$$

$$= A\left[e^{i(\omega+\frac{\Delta\omega}{2})t}e^{-i(k+\frac{\Delta k}{2})x}\right]\left[e^{-i\frac{\Delta\omega t-\Delta kx}{2}} + e^{i\frac{\Delta\omega t-\Delta kx}{2}}\right]$$

$$= \underbrace{2A\cos\left(\frac{\Delta\omega t-\Delta kx}{2}\right)}_{\text{slowly varying amplitude}}\cos\left[\left(\omega+\frac{\Delta\omega}{2}\right)t-\left(k+\frac{\Delta k}{2}\right)x\right]$$

group velocity $U = \frac{dx}{dt} = \frac{\Delta\omega}{\Delta k}$

in a nondispersive medium $V=U$

$$\Psi(x,t) = \sum_{r=1}^{n} A_r e^{i(\omega_r t-k_r x)} = \int_{-\infty}^{+\infty} A(k)\, e^{i(\omega t-kx)}dk$$

$$= \int_{k_0-\Delta k}^{k_0+\Delta k} A(k)e^{i(\omega t-kx)}dk \quad \text{wave packet}$$

$$\omega(k) = \omega(k_0) + \left.\frac{d\omega}{dk}\right|_{k=k_0}\cdot(k-k_0)+\cdots$$

$$\omega t-kx = (\omega_0 t-k_0x)+\omega_0'(k-k_0)t-(k-k_0)x$$

$$= (\omega_0 t-k_0x)+(k-k_0)(\omega_0' t-x)$$

$$\Psi(x,t) = \int_{k_0-\Delta k}^{k_0+\Delta k} A(k)\, e^{i(k-k_0)(\omega_0' t-x)}\, e^{i(\omega_0 t-k_0x)}\, dk$$

$$U = \omega_0' = \left.\frac{d\omega}{dk}\right|_{k=k_0} \qquad \frac{1}{U} = \left.\frac{dk}{d\omega}\right|_0 = \left.\frac{d}{d\omega}\frac{\omega}{v}\right|_{k=k_0} = \frac{v_0-\left.\omega\frac{dv}{d\omega}\right|_{k=k_0}}{v_0^2}$$

$$\therefore U = \frac{v_0}{1-\left.\frac{\omega_0}{v_0}\frac{dv}{d\omega}\right|_{k=k_0}}$$

NO.

DATE / /

the normal coordinates of the system $\eta_r(t) \equiv B_r e^{i\omega_r t}$

$$q_j(t) = \sum_r \eta_r \sin(j \frac{r\pi}{n+1})$$

$$= \sum_r \sin(\frac{jr\pi}{n+1})(\mu_r \cos\omega_r t - \nu_r \sin\omega_r t) \qquad B_r = \mu_r + i\nu_r$$

$$q_j(0) = \sum_r \mu_r \sin(\frac{jr\pi}{n+1}) \qquad \frac{dq_j(0)}{dt} = -\sum_r \omega_r \nu_r \sin(\frac{jr\pi}{n+1})$$

$$\sum_j q_j(0) \sin(\frac{js\pi}{n+1}) = \sum_{j,r} \mu_r \sin(\frac{jr\pi}{n+1}) \sin(\frac{js\pi}{n+1})$$

$$\text{and } \sum_{j=1}^{n} \sin(\frac{jr\pi}{n+1}) \sin(\frac{js\pi}{n+1}) = \frac{n+1}{2}\delta_{rs} \qquad r,s = 1,2 \cdots n$$

$$\Rightarrow \sum_j q_j(0) \sin\frac{js\pi}{n+1} = \sum_r \mu_r \frac{n+1}{2}\delta_n = \frac{n+1}{2}\mu_s$$

$$\nu_s = -\frac{2}{\omega_s(n+1)} \sum_j \frac{dq_j(0)}{dt} \sin(\frac{js\pi}{n+1})$$