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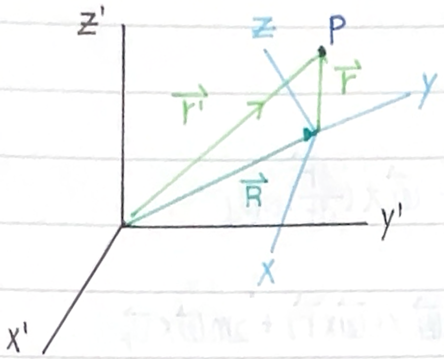
Motion in a Noninertical Reference Frame

Rotating Coordinate Systems



$$\vec{r'} = \vec{R} + \vec{r}$$

$$(d\vec{r})_{fixed} = d\vec{\theta} \times \vec{r} \qquad \left(\frac{d\vec{r}}{dt}\right)_{fixed} = \frac{d\vec{\theta}}{dt} \times \vec{r} = \vec{\omega} \times \vec{r}$$

$$\left(\frac{d\vec{r}}{dt}\right)_{fixed} = \left(\frac{d\vec{r}}{dt}\right)_{rotation} + \vec{\omega} \times \vec{r}$$ if P have a velocity with respect to the x system

$$\left(\frac{d\vec{r}}{dt}\right)_{fixed} = \frac{d}{dt}(x\vec{e}_x + y\vec{e}_y + z\vec{e}_z) = \underbrace{\frac{dx}{dt}\vec{e}_x + \frac{dy}{dt}\vec{e}_y + \frac{dz}{dt}\vec{e}_z}_{\left(\frac{d\vec{r}}{dt}\right)_{rotation}} + x\frac{d\vec{e}_x}{dt} + y\frac{d\vec{e}_y}{dt} + z\frac{d\vec{e}_z}{dt}$$

$$\frac{d\vec{e}_x}{dt} = \omega_3\vec{e}_y - \omega_2\vec{e}_z \qquad \frac{d\vec{e}_y}{dt} = -\omega_3\vec{e}_x + \omega_1\vec{e}_z \qquad \frac{d\vec{e}_z}{dt} = \omega_2\vec{e}_x - \omega_1\vec{e}_y$$

$$\Rightarrow \left(\frac{d\vec{r}}{dt}\right)_f = \left(\frac{d\vec{r}}{dt}\right)_r + \vec{\omega} \times \vec{r}$$

$$\vec{r'} = \vec{R} + \vec{r} \Rightarrow \underbrace{\left(\frac{d\vec{r'}}{dt}\right)_f}_{\substack{\vec{V}_f \\ \text{velocity relative} \\ \text{to the fixed axes}}} = \underbrace{\left(\frac{d\vec{R}}{dt}\right)_f}_{\substack{\vec{V} \\ \text{linear velocity} \\ \text{of the moving origin}}} + \left(\frac{d\vec{r}}{dt}\right)_f = \left(\frac{d\vec{R}}{dt}\right)_f + \underbrace{\left(\frac{d\vec{r}}{dt}\right)_r}_{\substack{\vec{v}_r \\ \text{velocity relative} \\ \text{to the rotating axes}}} + \vec{\omega} \times \vec{r}$$

$$\Rightarrow \vec{V}_f = \vec{V} + \vec{v}_r + \vec{\omega} \times \vec{r}$$

$$= \left[\left(\frac{d\vec{\omega}}{dt}\right)_r + \vec{\omega} \times \vec{\omega}\right] \times \vec{r} = \frac{d\vec{\omega}}{dt} \times \vec{r} = 0$$

$$\Rightarrow \vec{a}_f = \left(\frac{d\vec{V}_f}{dt}\right)_f = \underbrace{\left(\frac{d\vec{V}}{dt}\right)_f}_{0} + \left(\frac{d\vec{v}_r}{dt}\right)_f + \left(\frac{d\vec{\omega}}{dt}\right)_f \times \vec{r} + \vec{\omega} \times \left(\frac{d\vec{r}}{dt}\right)_f = \vec{\omega} \times \left[\left(\frac{d\vec{r}}{dt}\right)_r + \vec{\omega} \times \vec{r}\right]$$

$$\left(\frac{d\vec{v}_r}{dt}\right)_f = \left(\frac{d\vec{v}_r}{dt}\right)_r + \vec{\omega} \times \vec{v}_r = \vec{a}_r + \vec{\omega} \times \vec{v}_r \qquad = \vec{\omega} \times \vec{v}_r + \vec{\omega} \times \vec{\omega} \times \vec{r}$$

$$= \vec{a}_r + 2\vec{\omega} \times \vec{v}_r + \vec{\omega} \times \vec{\omega} \times \vec{r}$$

$$\Rightarrow \left(\frac{d\vec{\omega}}{dt}\right)_f = \left(\frac{d\vec{\omega}}{dt}\right)_r + \overbrace{\vec{\omega} \times \vec{\omega}}^{0} = \frac{d\vec{\omega}}{dt}$$

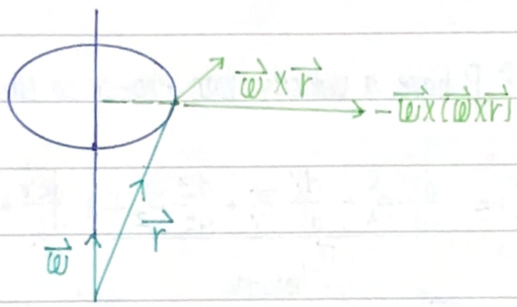
Centrifugal and Coriolis Forces

$$\vec{F} = m\vec{a}_f = m\left(\frac{d\vec{v}_f}{dt}\right)_{fixed}$$

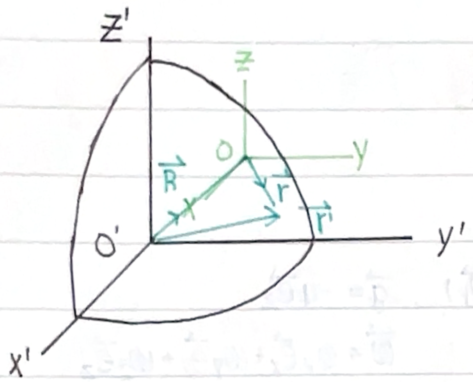
$$\left(\frac{d\vec{v}_f}{dt}\right)_{fixed} = \left(\frac{d\vec{V}}{dt}\right)_{fixed} + \left(\frac{d\vec{v}_r}{dt}\right)_{fixed} + \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\left(\frac{d\vec{r}}{dt}\right)_{fixed}$$

$$\Rightarrow \vec{F} = m\vec{a}_f = m\left(\frac{d^2\vec{R}}{dt^2}\right)_f + m\vec{a}_r + m\frac{d\vec{\omega}}{dt}\times\vec{r} + m\vec{\omega}\times(\vec{\omega}\times\vec{r}) + 2m\vec{\omega}\times\vec{v}_r$$

$$\vec{F}_{eff} \equiv m\vec{a}_r = F - \underbrace{m\left(\frac{d^2\vec{R}}{dt^2}\right)_f}_{\text{translational}} - \underbrace{m\frac{d\vec{\omega}}{dt}\times\vec{r}}_{\text{angular acceleration}} \overbrace{-m\vec{\omega}\times(\vec{\omega}\times\vec{r})}^{\text{centrifugal force}} \underbrace{-2m\vec{\omega}\times\vec{v}_r}_{\text{Coriolis force}}$$



Motion Relative to the Earth

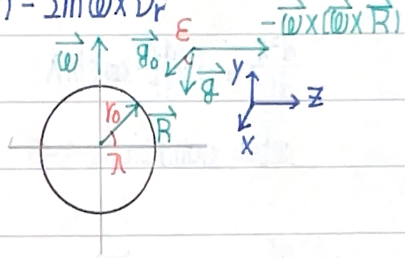


$$\vec{F} = \underbrace{\vec{S}}_{\text{the sum of the external forces}} + m\vec{g}_0 \qquad \vec{g}_0 = -G\frac{M_E}{R^2}\vec{e}_R$$

$$\vec{F}_{eff} = \vec{S} + m\vec{g}_0 - m\underbrace{\left(\frac{d^2\vec{R}}{dt^2}\right)_f}_{\vec{\omega}\times\left(\frac{d\vec{R}}{dt}\right)_f = \vec{\omega}\times(\vec{\omega}\times\vec{R})} - m\overbrace{\cancel{\frac{d\vec{\omega}}{dt}}}^{0 \text{ due to } \vec{\omega} \text{ constant in time}}\times\vec{r} - m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2m\vec{\omega}\times\vec{v}_r$$

$$= \vec{S} + \underbrace{m\vec{g}_0 - m\vec{\omega}\times[\vec{\omega}\times(\vec{r}+\vec{R})]}_{\vec{g} = \vec{g}_0 - \vec{\omega}[\vec{\omega}\times(\vec{r}+\vec{R})]} - 2m\vec{\omega}\times\vec{v}_r$$

$$= \vec{S} + m\vec{g} - 2m\vec{\omega}\times\vec{v}_r$$



$$\vec{F}_{eff} = m\vec{a}_f - m\vec{\omega}\times\underbrace{(\vec{\omega}\times\vec{r})}_{R\omega\cos\lambda\,\vec{e}_y} - 2m\vec{\omega}\times\vec{v}_r\,,$$

$v_r = 0$, $\omega = (-\omega\cos\lambda, 0, \omega\sin\lambda)$, $\vec{r} = (0, 0, R)$, $m\vec{a}_f = (0, 0, -m g_0)$

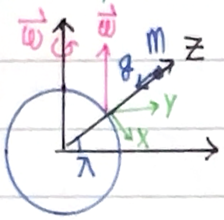
$$= -m g_0\vec{e}_z - m\begin{vmatrix} e_x & e_y & e_z \\ -\omega\cos\lambda & 0 & \omega\sin\lambda \\ 0 & R\omega\cos\lambda & 0 \end{vmatrix} = -m g_0\vec{e}_z + mR\omega^2\sin\lambda\cos\lambda\,\vec{e}_x + mR\omega^2\cos^2\lambda\,\vec{e}_z$$

$$\Rightarrow (F_f)_x = mR\omega^2\sin\lambda\cos\lambda \qquad (F_f)_z = -m g_0 + mR\omega^2\cos^2\lambda$$

$$\tan\varepsilon = \frac{|(F_f)_x|}{|(F_f)_z|} = \frac{R\omega^2\sin\lambda\cos\lambda}{g_0 - R\omega^2\cos^2\lambda}$$ if ε is very small

$$\varepsilon = \frac{R\omega^2\sin\lambda\cos\lambda}{g_0 - R\omega^2\cos^2\lambda}$$

falling freely, nelegect centrifugal force



$$\vec{F}_{eff} = m\vec{a}_r = m\frac{d^2\vec{r}}{dt^2} = m\vec{g} - 2m(\vec{\omega}\times\vec{v}_r)\ , \quad \vec{g} = -g\vec{e}_z$$

$$\vec{\omega} = \omega_x\vec{e}_x + \omega_y\vec{e}_y + \omega_z\vec{e}_z$$

$$= -\omega\cos\lambda\,\vec{e}_x + \omega\sin\lambda\,\vec{e}_z$$

$$\vec{v}_r = \frac{dx}{dt}\vec{e}_x + \frac{dy}{dt}\vec{e}_y + \frac{dz}{dt}\vec{e}_z$$

$$\vec{\omega}\times\vec{v}_r = -\frac{dy}{dt}\omega\sin\lambda\, e_x + \left(\frac{dx}{dt}\omega\sin\lambda + \frac{dz}{dt}\omega\cos\lambda\right)\vec{e}_y - \frac{dy}{dt}\omega\cos\lambda\,\vec{e}_z$$

$$\frac{d^2x}{dt^2} = 2\frac{dy}{dt}\omega\sin\lambda \qquad \frac{d^2y}{dt^2} = -2\left(\frac{dx}{dt}\omega\sin\lambda + \frac{dz}{dt}\omega\cos\lambda\right) \qquad \frac{d^2z}{dt^2} = -g + 2\frac{dy}{dt}\omega\cos\lambda$$

initial condition, $t=0$ $\quad \frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0 \quad x=y=0 \quad z=h$

$$\Rightarrow \frac{dx}{dt} = 2y\omega\sin\lambda \qquad \frac{dy}{dt} = -2\omega\left[x\sin\lambda + (z-h)\cos\lambda\right] \qquad \frac{dz}{dt} = -gt + 2y\omega\cos\lambda$$

when $\omega=0$, $\quad \frac{dx}{dt}=0 \quad \frac{dy}{dt}=0 \qquad \frac{dz}{dt} = -gt\ , \quad x=0 \quad y=0 \quad z = h-\frac{1}{2}gt^2$

$$\Rightarrow \frac{dx}{dt}=0 \qquad \frac{dy}{dt} = \omega g t^2\cos\lambda \qquad \frac{dz}{dt} = -gt\ , \quad x=0 \quad y=\frac{1}{3}\omega g t^3\cos\lambda \quad z = h-\frac{1}{2}gt^2$$

$$\Rightarrow \frac{dx}{dt} = \frac{2}{3}\omega^2 g t^3\sin\lambda\cos\lambda \qquad \frac{dy}{dt} = \omega g t^2\cos\lambda \qquad \frac{dz}{dt} = -gt + \frac{2}{3}\omega^2 g t^3\cos^2\lambda$$

$$x = \frac{1}{6}\omega^2 g t^4\sin\lambda\cos\lambda \qquad y = \frac{1}{3}\omega g t^3\cos\lambda \qquad z = h-\frac{1}{2}gt^2 + \frac{1}{6}\omega^2 g t^4\cos^2\lambda$$

$$t \cong \sqrt{\frac{2h}{g}}$$

$$y \cong \frac{1}{3}\omega\cos\lambda\sqrt{\frac{8h^3}{g}}$$