NO.

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H atom

$\psi_1 = \sqrt{\frac{1}{\pi a_0^3}}\, e^{-\frac{r}{a_0}} \qquad \psi_2 = \sqrt{\frac{1}{32\pi a_0^3}}\,(2-\frac{r}{a_0})e^{-\frac{r}{2a_0}}$

$\psi_{trial} = C_1\psi_1 + C_2\psi_2 \;,\; S_{11}=S_{22}=0 \quad S_{12}=S_{21}=0$

$H = -\frac{\hbar^2}{2\mu}\nabla^2 - \frac{e^2}{4\pi\epsilon_0 r}$

$H_{11} = (\gamma-1)bcR \quad H_{22} = \frac{1}{4}(\gamma-1$

$\begin{vmatrix} H_{11}-ES_{11} & H_{12}-ES_{12} \\ H_{21}-ES_{21} & H_{22}-ES_{22} \end{vmatrix} =$

$E = \frac{1}{8}(\gamma-1)(5+3\sqrt{1+2\Gamma^3})b$

$= -0.999\ 46bcR$

$\Rightarrow \quad C_1(H_{11}-E)+C_2H_{21}=0$

$C_1H_{12} + C_2(H_{22}-E)=0$

$\frac{m_c}{m_n}$

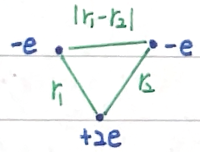
$\langle \frac{1}{|r_1-r_2|} \rangle = (\frac{8}{\pi a^3})^2 \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{\pi} \sin\theta_1 d\theta_1 \int_0^{\pi} \sin\theta_2 d\theta_2$

$\int_0^{\infty}\int_0^{\infty} e^{-\frac{4(r_1+r_2)}{a}} \frac{1}{r_{12}} r_1^2 r_2^2\, dr_1 dr_2$

$= (\frac{8}{\pi a^3})^2 (4\pi)^2 \int_0^{\infty} (\int_0^{r_2} \frac{r_1^2 e^{-\frac{4r_1}{a}}}{r_2} dr_1 + \int_{r_2}^{\infty} \frac{r_1^2 e^{-\frac{4r_1}{a}}}{r_1}) r_2 e^{-\frac{2Zr_2}{a}} dr_2$

$= (\frac{8}{\pi a^3})^2 (4\pi)^2 \frac{5}{2^7} (\frac{a_0}{2})^5 = \frac{5}{8}\frac{2}{a_0}$

The Ground State of Helium



$H = -\frac{\hbar^2}{2m}(\nabla_1^2+\nabla_2^2) - \frac{e^2}{4\pi\epsilon_0}(\frac{2}{r_1}+\frac{2}{r_2}-\frac{1}{|\vec{r_1}-\vec{r_2}|})$

if we ignore e-e repulsion

$\Rightarrow \psi_0(\vec{r_1},\vec{r_2}) \equiv \psi_{100}(\vec{r_1})\psi_{100}(\vec{r_2}) = \frac{8}{\pi a^3} e^{-\frac{2(r_1+r_2)}{a}}$

$E = \langle\psi|\hat{H}|\psi\rangle = 2E_{1s}(Z=2) = 2(-13.6)\times 2^2 = -108.8\,eV$ (Experiment : $-78.975\,eV$) 37.77%

to improve accuracy $\quad e^{-\frac{4(r_1+r_2)}{a}} = \int \frac{e^{-\frac{4r_2}{a}}}{\sqrt{r_1^2+r_2^2-2r_1r_2\cos\theta}} r_2^2 \sin\theta_2\, dr_2\, d\theta_2\, d\phi_2$

$\langle \frac{1}{|r_1-r_2|}\rangle = (\frac{8}{\pi a^3})^2 \iint \frac{e^{-\frac{4(r_1+r_2)}{a}}}{\sqrt{r_1^2+r_2^2-2r_1r_2\cos\theta_2}} d^3r_2\, d^3r_1 \qquad$ assume r_1 fixed

$\bullet = 2\pi \int_0^{\infty} e^{-\frac{4(r_1+r_2)}{a}} \left[\int_0^{\pi} \frac{\sin\theta_2}{\sqrt{r_1^2+r_2^2-2r_1r_2\cos\theta_2}} d\theta_2\right] r_2^2 dr_2 = 2\pi\int_0^{\infty} e^{-\frac{4r_2}{a}} \frac{1}{r_1r_2}[(r_1+r_2)-|r_1-r_2|] r_2^2 dr_2$

$d\phi_2$

$\blacksquare = \frac{1}{r_1r_2}\sqrt{r_1^2+r_2^2-2r_1r_2\cos\theta_2}\Big|_0^{\pi} = \frac{1}{r_1r_2}(\sqrt{r_1^2+r_2^2+2r_1r_2} - \sqrt{r_1^2+r_2^2-2r_1r_2})$

$= 4\pi(\frac{1}{r_1}\int_0^{r_1} e^{-0} r_2^2 dr_2 + \int_{r_1}^{\infty} e^{-0} r_2 dr_2)$

$= \frac{1}{r_1r_2}[(r_1+r_2)-|r_1-r_2|] = \frac{2}{r_1},\ r_2<r_1$

$\frac{2}{r_2},\ r_2>r_1$

$= \frac{\pi a^3}{8r_1}[1-(1+\frac{2r_1}{a})e^{-\frac{4r_1}{a}}]$

$\Rightarrow \bullet = 4\pi e^{-\frac{4r_1}{a}}(\frac{1}{r_1}\int_0^{r_1} r_2^2 e^{-\frac{4r_2}{a}} dr_2 + \int_{r_1}^{\infty} r_2 e^{-\frac{4r_2}{a}} dr_2) \quad \frac{1}{r_1}\int_0^{r_1} r_2^2 e^{-\frac{4r_2}{a}} dr_2$

$= \frac{1}{r_1}\left[-\frac{a}{4}r_2^2 e^{-\frac{4r_2}{a}} + \frac{a}{2}(\frac{a}{4})^2 e^{-\frac{4r_2}{a}}(-\frac{4r_2}{a}-1)\right]\Big|_0^{r_1}$

$= -\frac{a}{4r_1}(r_1^2 e^{-\frac{4r_1}{a}} + \frac{ar_1}{2}e^{-\frac{4r_1}{a}} + \frac{a^2}{8}e^{-\frac{4r_1}{a}} - \frac{a^2}{8})$