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A Theorem Concerning the Kinetic Energy

if the kinetic energy is expressed in fixed, rectangular coordinates

$$E_k = \frac{1}{2}\sum_{\alpha=1}^{n}\sum_{i=1}^{3} m_\alpha \left(\frac{dx_{\alpha i}}{dt}\right)^2$$

for many particles

$$x_{\alpha,i} = x_{\alpha i}(q_j, t) \qquad j = 1, 2 \cdots s$$

$$\frac{dx_{\alpha i}}{dt} = \sum_{j=1}^{s} \frac{\partial x_{\alpha i}}{\partial q_j}\frac{dq_j}{dt} + \frac{\partial x_{\alpha i}}{\partial t}$$

$$\left(\frac{dx_{\alpha i}}{dt}\right)^2 = \sum_{jk} \frac{\partial x_{\alpha i}}{\partial q_j}\frac{\partial x_{\alpha i}}{\partial q_k}\frac{dq_j}{dt}\frac{dq_k}{dt} + 2\sum_{j}\frac{\partial x_{\alpha i}}{\partial q_j}\frac{\partial x_{\alpha i}}{\partial t}\frac{dq_j}{dt} + \left(\frac{\partial x_{\alpha i}}{\partial t}\right)^2$$

$$E_k = \sum_{\alpha}\sum_{ijk}\frac{1}{2}m_\alpha \frac{\partial x_{\alpha i}}{\partial q_j}\frac{\partial x_{\alpha i}}{\partial q_k}\frac{dq_j}{dt}\frac{dq_k}{dt} + \sum_{\alpha}\sum_{ij} m_\alpha \frac{\partial x_{\alpha i}}{\partial q_j}\frac{\partial x_{\alpha i}}{\partial t}\frac{dq_j}{dt} + \sum_{\alpha}\sum_{i}\frac{1}{2}m_\alpha\left(\frac{\partial x_{\alpha i}}{\partial t}\right)^2$$

$$= \sum_{jk} a_{jk}\frac{dq_j}{dt}\frac{dq_k}{dt} + \sum_j b_j \frac{dq_j}{dt} + c$$

when the system is scleronomic, $\frac{\partial x_{\alpha i}}{\partial t} = 0 \qquad b_j = 0 \qquad c = 0$

$$= \sum_{jk} a_{jk}\frac{dq_j}{dt}\frac{dq_k}{dt}$$

$$\frac{\partial E_k}{\partial q'_l} = \sum_k a_{lk}\frac{dq_k}{dt} + \sum_j a_{jl}\frac{dq_j}{dt}, \quad \sum_l \frac{dq_l}{dt}\frac{\partial E_k}{\partial q'_l} = \sum_{kl} a_{lk}\frac{dq_k}{dt}\frac{dq_l}{dt} + \sum_{jl} a_{jl}\frac{dq_j}{dt}\frac{dq_l}{dt}.$$

$$\text{and } \sum_l \frac{dq_l}{dt}\frac{\partial E_k}{\partial q'_l} = 2\sum_{jk} a_{jk}\frac{dq_j}{dt}\frac{dq_k}{dt} = 2E_k$$

$$\sum_k y_k \frac{\partial f}{\partial y_k} = nf$$

$$\frac{d L(q_1 \cdots q_n, \dot q_1 \cdots \dot q_n, t)}{dt} = \sum_i \underbrace{\frac{\partial L}{\partial q_i}}_{\frac{d}{dt}\frac{\partial L}{\partial \dot q_i} = \frac{d}{dt}p_i} \frac{dq_i}{dt} + \sum_i \frac{\partial L}{\partial \dot q_i}\frac{d^2 q_i}{dt^2} + \frac{\partial L}{\partial t}$$

$$= \sum_i \left(\frac{dp_i}{dt}\frac{dq_i}{dt} + p_i \frac{d^2 q_i}{dt^2}\right) + \frac{\partial L}{\partial t} = \frac{d}{dt}\sum_i \left(p_i \frac{dq_i}{dt}\right) + \frac{\partial L}{\partial t}$$

$=0$

$$\Rightarrow \sum_{i=1}^{n} p_i \frac{dq_i}{dt} - L = H \quad , \quad H = E_k + U$$

$$\vec r_\alpha = \vec r_\alpha(q_1, \cdots q_n), \quad \frac{dr_\alpha}{dt} = \sum_{i=1}^{n} \frac{\partial r_\alpha}{\partial q_i}\frac{\partial q_i}{\partial t}, \quad \left(\frac{dr_\alpha}{dt}\right)^2 = \sum_j \cdots \sum_k$$

$$E_k = \frac{1}{2}\sum_\alpha m_\alpha \left(\frac{dr_\alpha}{dt}\right)^2 = \frac{1}{2}\sum_{jk} A_{jk}\frac{dq_j}{dt}\frac{dq_k}{dt}$$

$$A_{jk} = A_{jk}(q_1 \cdots q_n) = \sum_\alpha m_\alpha \frac{\partial r_\alpha}{\partial q_j}\frac{\partial r_\alpha}{\partial q_k}$$

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d system cannot depend on time

and ∂q_j dt $\partial q_j'$

$$\Rightarrow \quad \frac{\partial L}{\partial t} = \sum_j \frac{dq_j}{dt}\frac{d}{dt}\frac{\partial L}{\partial q_j'} + \sum_j \frac{\partial L}{\partial q_j'}\frac{d^2 q_j}{dt^2}, \quad \frac{dL}{dt} - \sum_j \frac{d}{dt}\left(\frac{dq_j}{dt}\frac{\partial L}{\partial q_j'}\right) = 0$$

so that $\frac{d}{dt}\left(L - \sum_j \frac{dq_j}{dt}\frac{\partial L}{\partial q_j'}\right) = 0$, $L - \sum_j \frac{dq_j}{dt}\frac{\partial L}{\partial q_j'} = -H$ = constant

$$\frac{\partial L}{\partial q_j'} = \frac{\partial (E_k - U)}{\partial q_j'} = \frac{\partial E_k}{\partial q_j'} \Rightarrow (E_k - U) - \sum_j \frac{dq_j}{dt}\frac{\partial E_k}{\partial q_j'} = -H$$

$$\Rightarrow E_k - U - 2E_k = -H$$

$$\Rightarrow E_k + U = E_t = H = \text{constant}$$

Hamiltonian

$H = E_{total}$ if the following conditions are met

1. the equations of the transformation connecting the rectangular and generalized coordinates
2. the potential energy must be velocity independent