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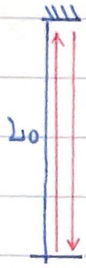
時間與距離的測量、同時性

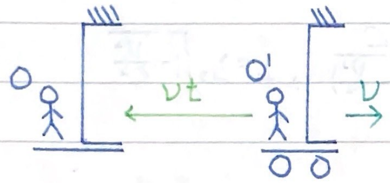
時間不是絕對的，是與座標系有關

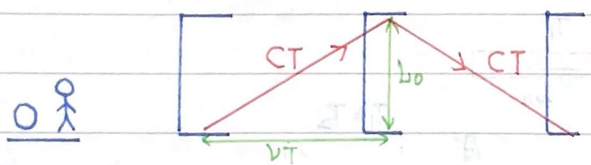
同時性不是絕對的，是相對的

運動的時鐘變慢



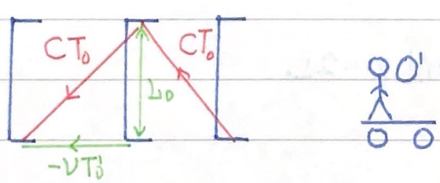
$$2T_0 = \frac{2L_0}{c}$$





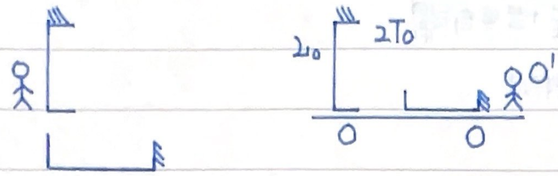
$$c^2T^2 - v^2T^2 = L_0^2,\quad T = \frac{L_0}{\sqrt{1-\frac{v^2}{c^2}}\,c} \qquad 2T = \frac{2L_0}{c\sqrt{1-\frac{v^2}{c^2}}} = \frac{2T_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

for O' $\quad 2T' = 2T_0$



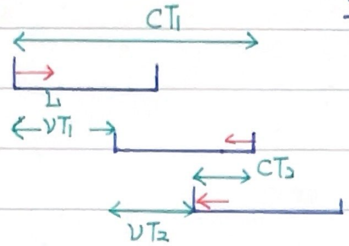
$$c^2T_0^2 - v^2T_0^2 = L_0^2,\quad T_0^2 = \frac{L_0^2}{c^2-v^2},\quad T_0 = \frac{L_0}{c\sqrt{1-\frac{v^2}{c^2}}}$$

沿運動方向運動的尺縮短



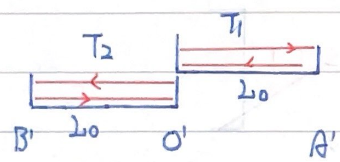
$$2T=\frac{2T_0}{C\sqrt{1-\frac{v^2}{c^2}}}$$

for O



$$T_1=\frac{L}{c-v}\qquad T_2=\frac{L}{c+v}$$

$$\frac{2L_0}{c\sqrt{1-\frac{v^2}{c^2}}}=T_1+T_2=\frac{L}{c-v}+\frac{L}{c+v}=\frac{2Lc}{c^2-v^2}=\frac{2Lc}{c^2(1-\frac{v^2}{c^2})}\ ,\ L=L_0\sqrt{1-\frac{v^2}{c^2}}$$



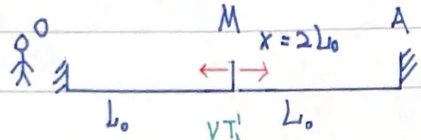
$T_1=T_2$

O $T_2<T_1$ ⇒ 在O'裡是同時，在O中不同時

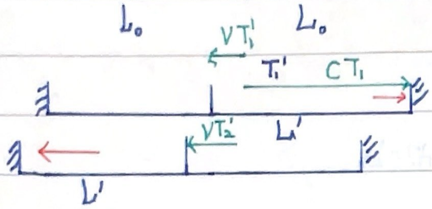
$$T_1-T_2=\frac{L}{c-v}-\frac{L}{c+v}$$

$$=\frac{2Lv}{c^2-v^2}=\frac{2L\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}}$$

$$=\frac{2L_0\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\quad \text{and } x'=2L_0$$



在O中,光同時到達O與A

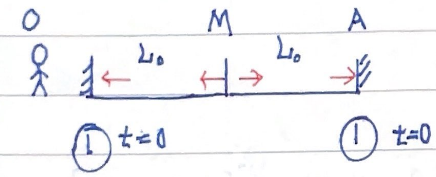


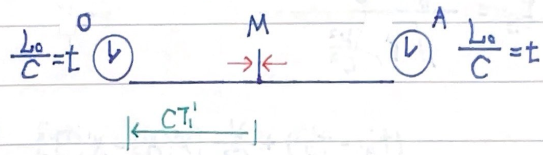
$L' = L_0\sqrt{1-\frac{v}{c}}$ $\quad (C+V)T_1' = L' \quad T_1' = \frac{L'}{C+V}$

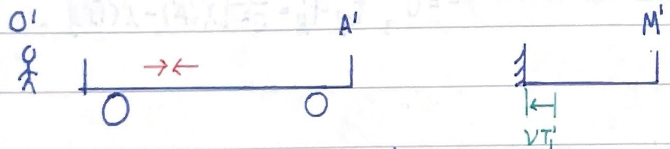
$(C-V)T_2' = L' \quad T_2' = \frac{L'}{C-V}$

$$T_1' - T_2' = -\frac{2L'V}{C^2-V^2}$$

$$= -\frac{2L'\frac{V}{C}}{1-\frac{V^2}{C^2}} = -\frac{L_0\frac{V}{C}}{\sqrt{1-\frac{V^2}{C^2}}}$$







$CT_1' + VT_1' = L'$, $T_1' = \frac{L'}{C+V}$ $\quad CT_2' - VT_2' = L'$, $T_2' = \frac{L'}{C-V}$

$$T_2' - T_1' = \frac{2L'V}{C^2-V^2} = \frac{2L_0\frac{V}{C^2}}{\sqrt{1-\frac{V^2}{C^2}}}$$

⇒ 同時性概念不是絕對,與運動中尺縮短有關

不同座標系(觀測者)時間校準系統是不一樣

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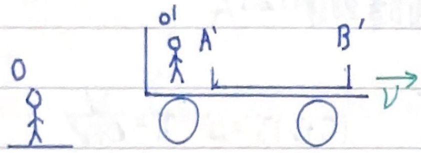

$$x'(B') - x'(A') = L_0$$

$$t_{A'} = \frac{t'_{A'} + \frac{v}{c^2} x'(A')}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\parallel$$

與同一時刻量測 A' 與 B' 位置

$$t_{B'} = \frac{t'_{B'} + \frac{v}{c^2} x'(B')}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t_{A'} - t_{B'} = \frac{(t'_{A'} - t'_{B'}) + \frac{v}{c^2}[x'(A') - x'(B')]}{\sqrt{1 - \frac{v^2}{c^2}}} = 0, \quad t'_{B'} - t'_{A'} = \frac{v}{c^2}[x'(A') - x'(B')]$$

$$x(B') - x(A') = \frac{x'(B') + v t'_{B'}}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{x'(A') + v t'_{A'}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{x'(B') - x'(A') + v(t'_{B'} - t'_{A'})}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \frac{L_0 - \frac{v^2}{c^2} L_0}{\sqrt{1 - \frac{v^2}{c^2}}} = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

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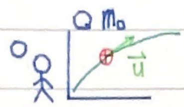
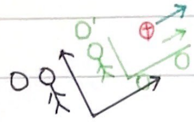
$\frac{d\vec{P}}{dt}=\vec{F}$ ，牛頓力學中 $\vec{P}=m\vec{u}$ $\vec{u}=u_x\vec{e}_x+u_y\vec{e}_y+u_z\vec{e}_z$

相對論 $\vec{P}\to\frac{m_0\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}$ $\quad \frac{d}{dt}\frac{m_0\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}=\vec{F}$





$\vec{u}=u_x\vec{e}_x+0\,\vec{e}_y+0\,\vec{e}_z$

$\nu=u_x$, $\vec{\nu}=\nu\vec{e}_x$

$$u_x'=\frac{u_x-\nu}{1-\frac{\nu u_x}{c^2}}=0 \qquad u_y'=0 \qquad u_z'=0$$

在O'中，牛頓第二定律成立

$$m_0\frac{dt}{dt'}\frac{d}{dt}\left[\frac{u_x-\nu}{1-\frac{\nu u_x}{c^2}}\right]=m_0\frac{du_x'}{dt'}=QE_x'=QE_{/\!/}'=QE_{/\!/}=QE_x$$

$$m_0\frac{du_y'}{dt'}=QE_y'=Q(\vec{E}_\perp')_y=Q\frac{(\vec{E}_\perp+\vec{\nu}\times\vec{B}_\perp)_y}{\sqrt{1-\frac{\nu^2}{c^2}}}$$

$$m_0\frac{dt}{dt'}\frac{d}{dt}\left[\frac{u_z\sqrt{1-\frac{\nu^2}{c^2}}}{1-\frac{\nu u_x}{c^2}}\right]=m_0\frac{du_z'}{dt'}=QE_z'=Q(\vec{E}_\perp')_z=Q\frac{(\vec{E}_\perp+\vec{\nu}\times\vec{B}_\perp)_z}{\sqrt{1-\frac{\nu^2}{c^2}}}$$

$$m_0\frac{dt}{dt'}\frac{d}{dt}\left[\frac{u_x-\nu}{1-\frac{\nu u_x}{c^2}}\right]\Bigg|_{\substack{\nu=u_x\\u_y=0\\u_z=0}}=m_0\frac{\sqrt{1-\frac{\nu^2}{c^2}}}{1-\frac{\nu u_x}{c^2}}\,\frac{\frac{du_x}{dt}}{1-\frac{\nu u_x}{c^2}}\Bigg|_{\nu=u_x}=m_0\frac{1}{\left(1-\frac{u_x^2}{c^2}\right)^{\frac{3}{2}}}\Bigg|\frac{du_x}{dt}\Bigg|_{\nu=u_x}$$

$$m_0\frac{dt}{dt'}\frac{d}{dt}\left[\frac{u_y\sqrt{1-\frac{\nu^2}{c^2}}}{1-\frac{\nu u_x}{c^2}}\right]=m_0\frac{\sqrt{1-\frac{\nu^2}{c^2}}}{1-\frac{\nu u_x}{c^2}}\,\frac{\sqrt{1-\frac{\nu^2}{c^2}}\,\frac{du_y}{dt}}{1-\frac{\nu u_x}{c^2}}\Bigg|_{\nu=u_x}=\frac{1}{1-\frac{u_x^2}{c^2}}\Bigg|\frac{du_y}{dt}\Bigg|_{\nu=u_x}$$

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$$m_0 \frac{1}{(1-\frac{u_x^2}{c^2})^{3/2}} \frac{du_x}{dt} = QE_x = Q(\vec{E} + \underbrace{\vec{u}\times\vec{B}}_{0})_x = m_0 \frac{d}{dt} \frac{u_x}{(1-\frac{u_x^2}{c^2})^{\frac{1}{2}}}$$

$$m_0 \frac{1}{(1-\frac{u_x^2}{c^2})^{\frac{1}{2}}} \frac{du_y}{dt}\bigg|_{\substack{u_y=0\\u_z=0}} = Q(\vec{E}_\perp + \vec{v}\times\vec{B}_\perp)_y = \underbrace{Q(\vec{E}+\vec{u}\times\vec{B})_y}_{\text{勞羅茲力}}$$

$$\parallel$$

$$m_0 \frac{d}{dt} \frac{u_y}{(1-\frac{u_x^2}{c^2})^{\frac{1}{2}}}$$

$$\frac{d}{dt}\frac{u_x}{(1-\frac{u_x^2}{c^2})^{\frac{1}{2}}} = \frac{\frac{du_x}{dt}}{(1-\frac{u_x^2}{c^2})^{\frac{1}{2}}} + \frac{u_x \times (-\frac{1}{2})(\frac{-2u_x}{c^2}\frac{du_x}{dt})}{(1-\frac{u_x^2}{c^2})^{3/2}} = \frac{1}{(1-\frac{u_x^2}{c^2})^{\frac{3}{2}}}\frac{du_x}{dt}$$

$$\vec{P} = \frac{m_0\vec{u}}{\sqrt{1-\frac{u\cdot u}{c^2}}} \qquad Q(\vec{E}+\vec{u}\times\vec{B}) = \frac{d}{dt}\frac{m_0\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}} = \vec{F}$$

$$\vec{u}\cdot\left[\frac{d}{dt}\frac{m_0\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\right] = \vec{u}\cdot\vec{F} = \frac{dE}{dt}$$

$$\underbrace{m_0 \frac{\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}} \cancel{\frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}} \cdot \frac{d}{dt}\frac{\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}}_{\frac{d}{dt}[\vec{g}(t)\cdot\vec{g}(t)] = 2\vec{g}(t)\cdot\frac{d\vec{g}(t)}{dt}} = \frac{\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\cdot\vec{F} = \frac{\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\frac{dE}{dt}$$

$$\frac{m_0}{2}\frac{d}{dt}\frac{\vec{u}\cdot\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}} = \frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\frac{dE}{dt}$$

$$\frac{m_0}{2}c^2\frac{d}{dt}\left[\frac{\frac{\vec{u}\cdot\vec{u}}{c^2}}{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}\right] = \frac{m_0c^2}{2}\frac{d}{dt}\left[\frac{1}{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}-1\right] = \frac{m_0c^2}{2}\frac{d}{dt}\left[\frac{1}{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}\right]$$

$$= \frac{m_0c^2}{2}\frac{d}{dt}\left[\left(\frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\right)^2\right] = m_0c^2\frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\frac{d}{dt}\frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}} = \frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\frac{dE}{dt}$$

$$\Rightarrow \frac{dE}{dt} = m_0c^2\frac{d}{dt}\frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}$$