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$$\sin\zeta = \sqrt{\frac{m_1 T_1}{m_2 T_2}}\,\sin\psi \qquad \tan\psi = \frac{\sin 2\zeta}{\frac{m_1}{m_2} - \cos 2\zeta} = \frac{\sin\phi}{\frac{m_1}{m_2} - \cos\phi}$$

Inelastic Collisions

$$Q + \frac{1}{2}m_1\vec{u}_1^{\,2} + \frac{1}{2}m_2\vec{u}_2^{\,2} = \frac{1}{2}m_1\vec{v}_1^{\,2} + \frac{1}{2}m_2\vec{v}_2^{\,2}$$, Q : Q-value, enengy lose or gain

$Q=0$ elastic collision

$Q>0$ exoergic collision : kinetic eneray is gained

$Q<0$ endoergic collision : lost

$$\varepsilon = \frac{|v_2 - v_1|}{|u_2 - u_1|}$$

$\varepsilon = 1$, totally elastic collision

$\varepsilon = 0$, totally inelastic collision

example $\varepsilon = 1$ m_2 initially at rest

$$m_1\vec{u}_1 = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\frac{1}{2}m_1\vec{u}_1^{\,2} = \frac{1}{2}m_1\vec{v}_1^{\,2} + \frac{1}{2}m_2\vec{v}_2^{\,2} = \frac{1}{2}m_1\vec{v}_1^{\,2} + \frac{1}{2}m_2\left(\frac{m_1\vec{u}_1 - m_1\vec{v}_1}{m_2}\right)^2$$

$$\varepsilon = \frac{\vec{v}_2 - \vec{v}_1}{u_1} = \frac{\frac{m_1\vec{u}_1 - m_1\vec{v}_1}{m_2} - \vec{v}_1}{\vec{u}_1} = \frac{m_1}{m_2} - \frac{m_1}{m_2}\frac{\vec{v}_1}{\vec{u}_1} - \frac{\vec{v}_1}{\vec{u}_1}$$

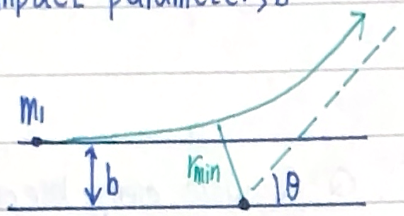
$$\Rightarrow m_1\vec{u}_1^{\,2} = m_1\vec{v}_1^{\,2} + \frac{m_1^2}{m_2}(\vec{u}_1 + \vec{v}_1 - 2\vec{u}_1\vec{v}_1), \quad 1 = x^2 + \frac{m_1}{m_2}(1 + x^2 - 2x), \quad x = \frac{\vec{v}_1}{\vec{u}_1}$$

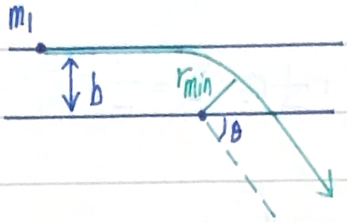
$$\left(1 + \frac{m_1}{m_2}\right)x^2 - \frac{2m_1}{m_2}x + \left(\frac{m_1}{m_2} - 1\right) = 0, \quad x = 1 = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1}$$

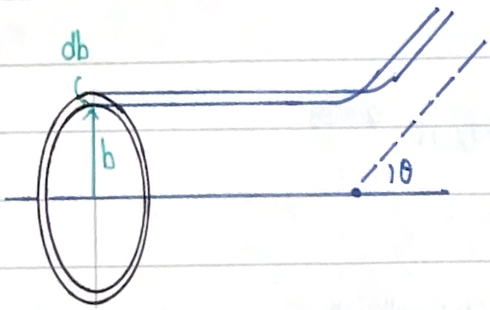
$$\Rightarrow \varepsilon = \frac{m_1}{m_2} - \frac{\frac{m_1}{m_2}\left(\frac{m_1}{m_2} - 1\right)}{\frac{m_1}{m_2} + 1} + \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} = \frac{\frac{m_1^2}{m_2^2} + \frac{m_1}{m_2} - \frac{m_1^2}{m_2^2} + \frac{m_1}{m_2} - \frac{m_1}{m_2} + 1}{\frac{m_1}{m_2} + 1} = 1$$

Scattering Cross Sections

impact parameter, b







flux density
intensity

the number of particles scattered into $d\Omega'$ per unit time, $N = 2\pi b\,|db|\,I$

$d\Omega' = 2\pi \sin\theta \, d\theta$

differential scattering cross section $\sigma(\theta) = \dfrac{\text{number of interactions per target particle that lead to scattering into } d\Omega' \text{ at the } \theta}{\text{number of incident particles per unit area}}$

$$d\Sigma = \frac{dN}{I} = 2\pi b|db| = \sigma(\theta)\, d\Omega \ , \quad \sigma(\theta) = \frac{d\Sigma}{d\Omega} = \frac{2\pi b}{2\pi \sin\theta \, d\theta} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

$$N = \int dN = I\int d\Sigma = I\int \sigma(\theta)\, d\Omega = 2\pi I \int_0^{\pi} \sigma(\theta) \sin\theta \, d\theta$$

$$\Sigma = 2\pi \int \sigma(\theta) \sin\theta \, d\theta$$

LAB system to CM system

$$\cos\theta = -X\sin^2\psi + \cos\psi\sqrt{1-X^2\sin^2\psi}\ ,\quad X=\frac{m_1}{m_2}$$

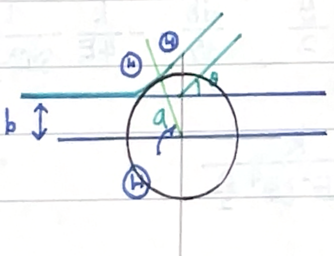
$$\frac{d\cos\theta}{d\psi} = \frac{-\sin\psi\left(\sqrt{1-X^2\sin^2\psi}+X\cos\psi\right)^2}{\sqrt{1-X^2\sin^2\psi}}$$

$$\sigma(\theta)\,d\Omega' = \sigma(\psi)\,d\Omega\ ,\quad \sigma(\theta)\cdot 2\pi\sin\theta\,d\theta = \sigma(\psi)\,2\pi\sin\psi\,d\psi\ ,\quad \sigma(\psi)=\sigma(\theta)\frac{\sin\theta\,d\theta}{\sin\psi\,d\psi} = \frac{\sigma(\theta)}{\sin\psi}\,\frac{-d\cos\theta}{d\psi}$$

$$\Rightarrow \sigma(\psi) = \sigma(\theta)\frac{\left(X\cos\psi+\sqrt{1-X^2\sin^2\psi}\right)^2}{\sqrt{1-X^2\sin^2\psi}}$$

When $m_1=m_2$, $X=1$, $\theta=2\psi$ $\quad \sigma(\psi)=4\cos\psi\,\sigma(\theta)\big|_{\theta=2\psi}$

when $m_1\ll m_2$, $X\approx 0$, $\theta\approx\psi$ $\quad \sigma(\psi)\cong\sigma(\theta)\big|_{\theta=\psi}$



$$U(r)\begin{cases}0 & r>a\\ \infty & r<a\end{cases}$$

$$E=\frac{1}{2}\mu(\vec{u}')^2,\quad \vec{u}'=\sqrt{\frac{2E}{\mu}}$$

$$\vec{l} = b\mu\vec{u}' = b\mu\sqrt{\frac{2E}{\mu}} = b\sqrt{2\mu E}$$

$$\int d\Theta = \int\frac{l\,dr}{r^2\sqrt{2\mu\left(E-U-\frac{l^2}{2\mu r^2}\right)}}\ ,\quad \int_{\Theta}^{2\Theta} d\Theta = \int_a^\infty\frac{\sqrt{2\mu E}\,b\,dr}{r^2\sqrt{2\mu E-\frac{2\mu E b^2}{r^2}}} = \int_a^\infty\frac{b\,dr}{r\sqrt{r^2-b^2}}$$

$$\Rightarrow \Theta = \sin^{-1}\left(-\frac{b}{r}\right)\Big|_a^\infty = \sin^{-1}\frac{b}{a}\ ,\quad \sin\Theta=\frac{b}{a}$$

$$2\Theta+\theta=\pi \qquad b=a\sin\Theta = a\sin\left(\frac{\pi-\theta}{2}\right) = a\cos\frac{\theta}{2}$$

$$\sigma(\theta) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{a\cos\frac{\theta}{2}}{\sin\theta}\ \frac{a}{2}\sin\frac{\theta}{2} = \frac{a^2}{4}$$

$$\Sigma = \int_0^\pi \sigma(\theta)\cdot 2\pi\sin\theta\,d\theta = \pi a^2$$

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rutherford scattering formula

$$\int d\Theta = \int \frac{\ell\, dr}{r^2\sqrt{2\mu\, E - U - \frac{\ell^2}{2\mu r^2}}}$$

$$\Theta = \int_{r_{min}}^{\infty} \frac{b\, dr}{r^2\sqrt{2\mu(E - \frac{k}{r} - \frac{\ell^2}{2\mu r^2})}} \qquad \text{let } z = \frac{1}{r},\ dz = -\frac{dr}{r^2}$$

$$= \int_0^{z_0} \frac{b\, dz}{\sqrt{1 - \frac{k}{E}z - b^2 z^2}} \qquad z_0 = -\frac{k}{2Eb^2} + \frac{1}{b^2}\sqrt{(\frac{k}{2E})^2 + b^2}$$

$$\Rightarrow \Theta = -\sin^{-1}\left[\frac{2b^2 z + \frac{k}{E}}{\sqrt{(\frac{k}{E})^2 + 4b^2}}\right]\Bigg|_0^{z_0} = -\frac{\pi}{2} + \sin^{-1}\left[\frac{\frac{k}{2Eb}}{\sqrt{1 + (\frac{k}{2Eb})^2}}\right]$$

$$\Rightarrow \sin(\frac{\pi}{2} + \Theta) = \frac{\frac{k}{2Eb}}{\sqrt{1 + (\frac{k}{2Eb})^2}} = \cos\Theta, \quad \tan\Theta = \frac{2Eb}{k}$$

$$b = \frac{k}{2E}\tan\Theta = \frac{k}{2E}\tan(\frac{\pi - \theta}{2}) = \frac{k}{2E}\cot\frac{\theta}{2}, \quad \frac{db}{d\theta} = -\frac{k}{4E}\frac{1}{\sin^2\frac{\theta}{2}}$$

$$\sigma(\theta) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{k^2}{8E^2}\frac{\cot\frac{\theta}{2}}{\sin\theta\sin^2\frac{\theta}{2}} = \frac{k^2}{16E^2\sin^4\frac{\theta}{2}}$$