

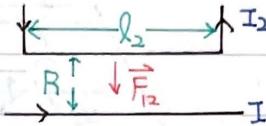
## Magnetic Field and Magnetic Forces

### Magnetic Field

Ørsted



Ampère



$$\vec{F}_{12} = C \frac{I_1 I_2}{R} \hat{l}_2 = |B_1| I_2 |\vec{l}_2|$$

$$= I_2 l_2 \times \vec{B}_1$$

$$[B_1] = \frac{[\vec{F}_{12}]}{[I_1 \cdot l]} = \frac{N}{A \cdot m} = \text{Tesla}$$

$$1 \text{ Tesla} = 10^4 \text{ Gauss}$$

if we perform an experiment by placing a particle with charge  $q$  in the magnetic field, we find the following results that are similar to those for experiment on electric forces

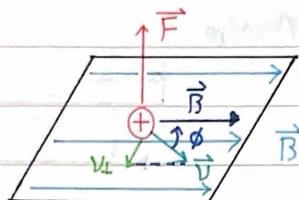
- \* the magnetic force is proportional to the charge  $q$  of the particle
- \* the magnetic force on a negative charge is directed opposite to the force on a positive charge moving in the same direction
- \* the magnetic force is proportional to the magnitude of the magnetic field vector  $\vec{B}$

we also find the following results, which are totally different from those for experiments on electric forces

- \* the magnetic force is proportional to the speed  $v$  of the particle
- \* if the velocity vector makes an angle  $\theta$  with the magnetic field, the magnitude of the magnetic force is proportional to  $\sin\theta$
- \* when a charged particle moves in a direction not parallel to the magnetic field vector, the magnetic force acts in a direction perpendicular to both  $\vec{v}$  and  $\vec{B}$ ; that is, the magnetic force is perpendicular to the plane formed by  $\vec{v}$  and  $\vec{B}$
- \* when a charged particle moves parallel to the magnetic field vector, the magnetic force on the charge is zero

$$\vec{F} = q \vec{v} \times \vec{B}, F = |q| v_L B = |q| v B \sin\phi$$

↓  
is an observation based  
on experiment



let's compare the important differences between the electric and magnetic versions of the particle in a field model

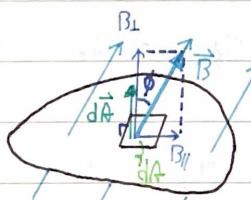
- \* the electric force vector is along the direction of the electric field, whereas the magnetic force vector is perpendicular to the magnetic field
- \* the electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion
- \* the electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement of its point of application

if a charge moving with a velocity  $\vec{v}$  in the presence of both an electric field  $\vec{E}$  and a magnetic field  $\vec{B}$  is described by two particle in a field models  
the total force (called the Lorentz force) acting on the charge is  $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

### Magnetic Flux

$$d\Phi_B = B_{\perp} dA = B \cos\phi dA = \vec{B} \cdot d\vec{A}$$

$$\Phi_B = \int B_{\perp} dA = \int B \cos\phi dA = \int \vec{B} \cdot d\vec{A} = T \cdot m^2 = \text{weber, wb}$$



the total magnetic flux through a closed surface is always zero

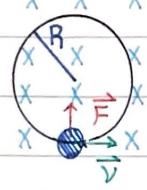
$\oint \vec{B} \cdot d\vec{A} = 0$  Gauss's law for magnetism

magnetic flux density  $B_s = \frac{d\Phi_B}{dA_{\perp}}$

### Motion of Charged Particles in a Magnetic Field

the magnetic force never has a component parallel to the particle's motion, so the magnetic force can never do work on the particle

motion of a charged particle under the action of a magnetic field along is always motion with constant speed

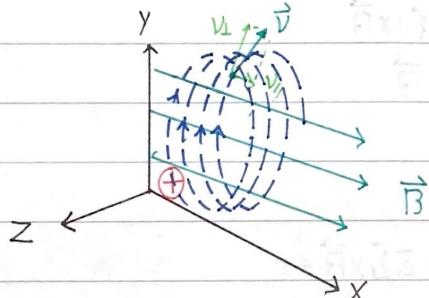
$$F_B = qvB = \frac{mv^2}{R}, R = \frac{mv}{qB} = \frac{p}{qB}$$


$$\omega = \frac{v}{R} = v \frac{|B|}{m} = \frac{|q|B}{m}$$

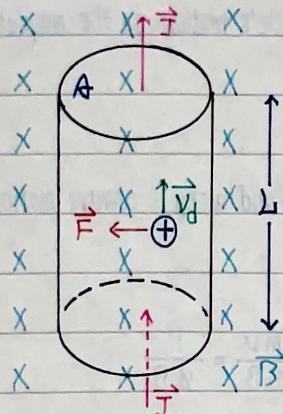
$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

the frequency  $f$  is independent of the radius  $R$  of the path, it is called the cyclotron frequency in a particle accelerator called a cyclotron

if the direction of the initial velocity is not perpendicular to the field



## Magnetic Force on a Current-Carrying Conductor



the number of charges per unit volume is  $n$

$$\vec{F} = (nA\mu_0)(qv_d B) \quad \text{since } J = nqv_d, JA = I$$

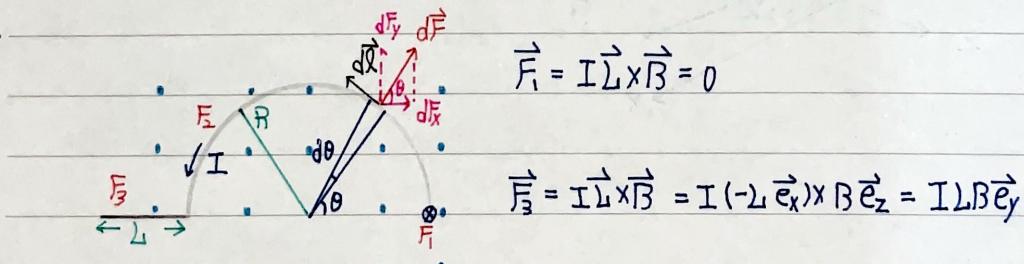
$$= ILB \quad = ILB \sin\phi$$

$$\Rightarrow \vec{F} = I\vec{l} \times \vec{B}$$

~~$$\vec{F} = \int d\vec{F} = \int I d\vec{l} \times \vec{B} \quad \text{if } \vec{B} \text{ is regular}$$~~

$$= I \left( \int d\vec{l} \right) \times \vec{B}$$

$$= I \vec{l} \times \vec{B}$$

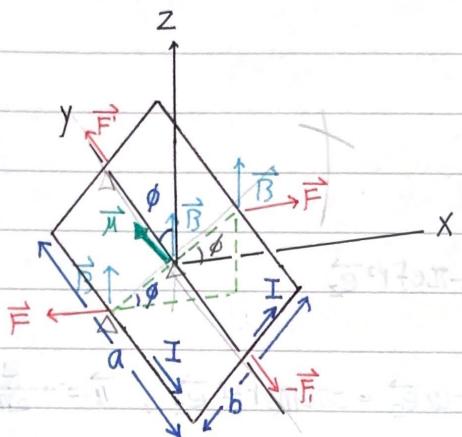


$$d\vec{F}_x = I(Rd\theta)B \cos\theta, \vec{F}_x = IRB \int_0^\pi \cos\theta d\theta = 0$$

$$d\vec{F}_y = IRd\theta B \sin\theta, \vec{F}_y = IRB \int_0^\pi \sin\theta d\theta = 2IRB$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = IB(2R+L)\vec{e}_y$$

### Force and Torque on a Current Loop



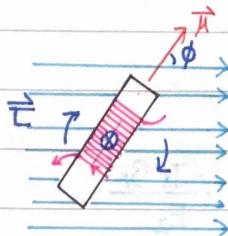
$$\vec{F} = Ia\vec{B} \quad F = Ib \sin(90^\circ - \phi) = IbB \cos\phi$$

$$I = 2F \frac{b}{2} \sin\phi$$

$$= IabB \sin\phi = IBA \sin\phi \quad \text{and magnetic dipole moment } \vec{\mu} = IA$$

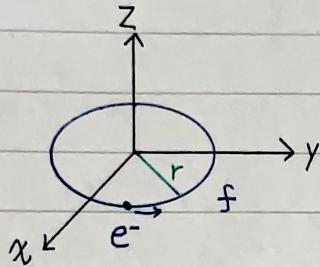
$$= \mu B \sin\phi = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\phi$$



$$T = NIAB \sin\phi$$

$$= \vec{\mu} \times \vec{B}$$

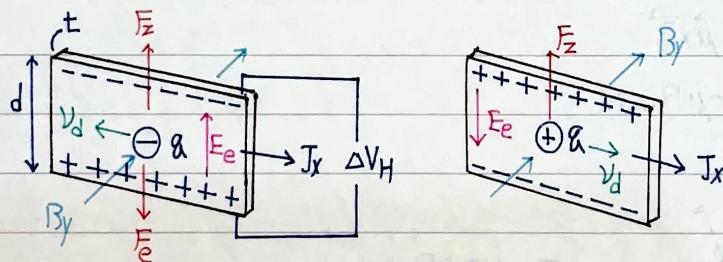


$$I = f(-e)$$

$$\vec{M} = I \vec{A} = (-ef)(\pi r^2 \vec{e}_z) = -\pi efr^2 \vec{e}_z$$

$$\vec{L} = \vec{r} \times \vec{P} = \vec{r} \times m_e \vec{v} = m_e r \omega \vec{e}_z = 2\pi m_e r^2 f \vec{e}_z, \quad \vec{\mu} = -\frac{e}{2m_e} \vec{L}$$

### The Hall Effect



$$F_z = qv_d B \quad J_x = nq v_d$$

$$qE_z + qv_d B_y = 0, \quad E_z = -v_d B_y, \quad v_d = \frac{I}{nqA}, \quad nq = \frac{-J_x B_y}{E_z}$$

$$\Delta V_H = Ed = v_d Bd = \frac{IBd}{nqA}$$

$$= \frac{IB}{nqt} = R_H \frac{IB}{t}, \quad R_H, \text{ Hall coefficient} = \frac{1}{nq}$$