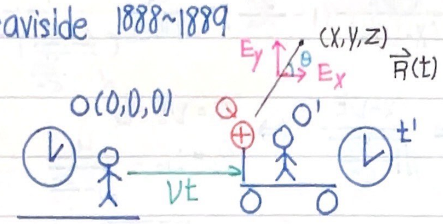
三種勞羅茲座標轉換

一個以速度ν沿x軸作等速直線運動產生的電場與磁場

Heaviside 1888~1889



電荷在t時刻位置 $\vec{r}(t) = \nu t\vec{e}_x + 0\vec{e}_y + 0\vec{e}_z$

$\vec{E}(x,y,z) = E_x\vec{e}_x + E_y\vec{e}_y + E_z\vec{e}_z$

$$E_x = \frac{Q}{4\pi\epsilon_0}\frac{x-\nu t}{[(x-\nu t)^2+y^2+z^2]^{3/2}}\frac{1-\frac{\nu^2}{c^2}}{\left[1-\frac{\nu^2}{c^2}\sin^2\theta\right]^{\frac{3}{2}}}$$

$$E_y = \frac{Q}{4\pi\epsilon_0}\frac{y}{[(x-\nu t)^2+y^2+z^2]^{3/2}}\frac{1-\frac{\nu^2}{c^2}}{\left[1-\frac{\nu^2}{c^2}\sin^2\theta\right]^{\frac{3}{2}}}$$

$$E_z = \frac{Q}{4\pi\epsilon_0}\frac{z}{[(x-\nu t)^2+y^2+z^2]^{3/2}}\frac{1-\frac{\nu^2}{c^2}}{\left[1-\frac{\nu^2}{c^2}\sin^2\theta\right]^{\frac{3}{2}}}$$

$$\vec{B} = \frac{\vec{\nu}}{c^2}\times\vec{E}$$

$\nu \ll c$

$$\vec{B} = \mu_0\epsilon_0\vec{\nu}\times\frac{Q}{4\pi\epsilon_0}\frac{\vec{e}_R}{|\vec{R}(t)|^2}$$

$E'_{\parallel} = E_{\parallel}$ $\quad B'_{\parallel} = B_{\parallel}$ $\qquad E_{\parallel} = E'_{\parallel}$ $\quad B_{\parallel} = B'_{\parallel}$

$$\vec{E}'_{\perp} = \frac{\vec{E}_{\perp} + \vec{\nu}\times\vec{B}_{\perp}}{\sqrt{1-\frac{\nu^2}{c^2}}} \qquad \vec{E}_{\perp} = \frac{\vec{E}'_{\perp} - \vec{\nu}\times\vec{B}'_{\perp}}{\sqrt{1-\frac{\nu^2}{c^2}}}$$

$$\vec{B}'_{\perp} = \frac{\vec{B}_{\perp} - \frac{\vec{\nu}}{c^2}\times\vec{E}_{\perp}}{\sqrt{1-\frac{\nu^2}{c^2}}} \qquad \vec{B}_{\perp} = \frac{\vec{B}'_{\perp} + \frac{\vec{\nu}}{c^2}\times\vec{E}'_{\perp}}{\sqrt{1-\frac{\nu^2}{c^2}}}$$

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取 $x = L + vt \quad y=0 \quad z=0 \quad \theta=0,\pi \quad \sin\theta=0$

$$E_x(L+vt=x,\ y=0,\ z=0,\ t) = \frac{Q}{4\pi\epsilon_0}\frac{x-vt}{(x-vt)^3}\left(1-\frac{v^2}{c^2}\right) = \frac{Q}{4\pi\epsilon_0}\frac{1}{\left(\frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2}$$

$$\parallel$$

$$E_{\parallel} = E'_{\parallel} = \frac{Q}{4\pi\epsilon_0}\frac{1}{(x')^2} \quad \Rightarrow x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \quad \Leftrightarrow \quad \begin{aligned} x &= vt+L \\ x' &= L'_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}} \end{aligned}$$

$$L'_0\sqrt{1-\frac{v^2}{c^2}} = L$$

Lorentz-Fitzgerald $\quad L = L'_0\sqrt{1-\frac{v^2}{c^2}}$

取 $x=vt \quad y=0 \quad z=L_0$

$$E_z(x=vt,\ y=0,\ z=L,\ t) = \frac{Q}{4\pi\epsilon_0}\frac{1}{z^2}\frac{1-\frac{v^2}{c^2}}{\left[1-\frac{v^2}{c^2}\sin^2\frac{\pi}{2}\right]^{3/2}} = \frac{Q}{4\pi\epsilon_0}\frac{1}{z^2}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$B_y(x=vt,\ y=0,\ z=L_0,\ t) = -\frac{v}{c^2}E_z \qquad \vec{B}_{\perp} = B_y\vec{e}_y$$

$$E'_{z'}\vec{e}_z = \vec{E}'_{\perp} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(\vec{E}_{\perp} + \vec{v}\times\vec{B}_{\perp}\right)$$

$$= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\frac{v^2}{c^2}\right)E_z = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\frac{v^2}{c^2}\right)\frac{Q}{4\pi\epsilon_0}\frac{1}{z^2}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= E'_{z'}(x'=0,\ y'=0,\ z') = \frac{Q}{4\pi\epsilon_0}\frac{1}{(z')^2} = \frac{Q}{4\pi\epsilon_0}\frac{1}{z^2}$$

$$\Rightarrow \begin{aligned} z &= L_0 \\ z' &= L_0 \end{aligned}$$

$$x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \qquad y=y' \qquad z=z'$$

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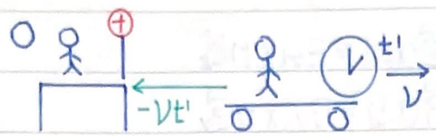

$$\vec{E}'(x',y',z',t') = E'_{x'}\vec{e}_{x'} + E'_{y'}\vec{e}_{y'} + E'_{z'}\vec{e}_{z'}$$

$$E'_{x'} = \frac{Q}{4\pi\epsilon_0}\frac{x'+vt'}{[(x'+vt')^2+(y')^2+(z')^2]^{3/2}}\frac{1-\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2}\sin^2\theta')^{\frac{3}{2}}}$$

$$E'_{y'} = \frac{Q}{4\pi\epsilon_0}\frac{x'+vt'}{[(x'+vt')^2+(y')^2+(z')^2]^{3/2}}\frac{1-\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2}\sin^2\theta')^{\frac{3}{2}}}$$

$$x = \frac{x'+vt}{\sqrt{1-\frac{v^2}{c^2}}} \qquad y=y' \quad z=z'$$

now, we have

$$\begin{cases} x' = \dfrac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \\ x = \dfrac{x+vt}{\sqrt{1+\frac{v^2}{c^2}}} \end{cases} \qquad \text{let } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \begin{matrix} x' = (x-vt)\gamma \\ x = (x'+vt')\gamma \end{matrix}$$

$$x = [(x-vt)\gamma + vt']\gamma = \frac{x-vt}{1-\frac{v^2}{c^2}} + \gamma v t'$$

$$\frac{x(1-\frac{v^2}{c^2})-x-vt}{1-\frac{v^2}{c^2}} = \gamma v t', \quad (-\frac{v^2}{c^2}x+vt)\gamma^2 = \gamma v t$$

$$\Big\downarrow \times \frac{1}{\gamma v}$$

$$t' = \gamma(t-\frac{v}{c^2}x) = \frac{t-\frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \gamma(t+\frac{v}{c^2}x') = \frac{t'+\frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}}$$

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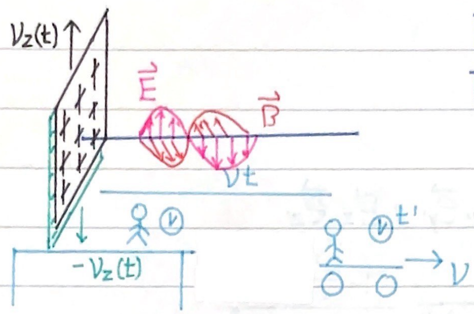
Voigt 1890



$$\vec{E}(x,t) = E_z(x,t)\vec{e}_z$$
$$\vec{B}(x,t) = B_y(x,t)\vec{e}_y$$

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial}{\partial t^2}\right)\vec{E}_z(x,t) = 0$$

$$\left(\frac{\partial}{\partial x} - \frac{1}{c}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}\right)E_z(x,t) = 0$$

$$\vec{E}'_\perp = \frac{\vec{E}_\perp + \vec{v}\times\vec{B}_\perp}{\sqrt{1-\frac{v^2}{c^2}}} = \vec{E}'_{z'}\vec{e}_{z'} \quad , \quad E'_{z'} = \frac{1-\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}E_z$$

$$\left(\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial (t')^2}\right)E'_{z'}(x',t') = 0 \quad , \quad \left(\frac{\partial}{\partial x'} - \frac{1}{c}\frac{\partial}{\partial t'}\right)\left(\frac{\partial}{\partial x'} + \frac{1}{c}\frac{\partial}{\partial t'}\right)E'_{z'}(x',t') = 0$$

let $x' = k(x - vt) \qquad t' = at - \frac{b}{c}x$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial}{\partial t'}\frac{\partial t'}{\partial x} = k\frac{\partial}{\partial x'} - \frac{b}{c}\frac{\partial}{\partial t'}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial}{\partial t'}\frac{\partial t'}{\partial x} = -kv\frac{\partial}{\partial x'} + a\frac{\partial}{\partial t'}$$

$$\Rightarrow \left[k\frac{\partial}{\partial x'} - \frac{c}{b}\frac{\partial}{\partial t'} + k\frac{v}{c}\frac{\partial}{\partial x'} - \frac{a}{c}\frac{\partial}{\partial t'}\right]\left[k\frac{\partial}{\partial x'} - \frac{b}{c}\frac{\partial}{\partial t'} - k\frac{v}{c}\frac{\partial}{\partial x'} + \frac{a}{c}\frac{\partial}{\partial t'}\right]E_z(x,t) = 0$$

$$\left[k\left(1+\frac{v}{c}\right)\frac{\partial}{\partial x'} - \frac{a+b}{c}\frac{\partial}{\partial t'}\right]\left[k\left(1-\frac{v}{c}\right)\frac{\partial}{\partial x'} + \frac{a-b}{c}\frac{\partial}{\partial t'}\right]E'_{z'}(\quad) = 0$$

$$k\left(1+\frac{v}{c}\right)k\left(1-\frac{v}{c}\right)\left[\frac{\partial}{\partial x'} - \frac{1}{c}\underbrace{\frac{a+b}{k(1+\frac{v}{c})}}_{1}\frac{\partial}{\partial t'}\right]\left[\frac{\partial}{\partial x'} + \frac{1}{c}\underbrace{\frac{a-b}{k(1-\frac{v}{c})}}_{1}\frac{\partial}{\partial t'}\right]E'_{z'} = 0$$

$$a+b = k\left(1+\frac{v}{c}\right) \qquad a = k \qquad k^2\left(1-\frac{v^2}{c^2}\right) = 1 \qquad k = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$a-b = k\left(1-\frac{v}{c}\right) \qquad b = k\frac{v}{c}$$

$$\Rightarrow x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \qquad t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}}$$

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用光速C在O與O'看是 $C^2=\frac{1}{\mu_0\varepsilon_0}$

$x'=k(x-vt)$ —— (1)

$x=k(x'+vt')$ —— (2)

在 $t=0=t'$ 沿X方向射出光

$x'=ct'$ $x=ct$ 代入 (1)(2)

$$\begin{cases} ct'=k(ct-vt) \\ ct=k(ct'+vt') \end{cases}$$

$$c^2tt'=k^2(c-v)(c+v)tt'$$

$$k^2\left(1-\frac{v^2}{c^2}\right)=1$$

$$k=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow x'=\frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \qquad x=\frac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}}$$