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Energy of the System

the total kinetic energy of the system is equal to the sum of the kinetic energy of a particle of mass M moving with the velocity of the center of mass and the kinetic energy of motion of the individual particles relative to the center of mass

$$W_{12} = \sum_\alpha \int_1^2 \vec{F}_\alpha \cdot d\vec{r}_\alpha = \sum_\alpha \int_1^2 d\left(\tfrac{1}{2} m_\alpha \vec{v}_\alpha^{\,2}\right) = E_{k1} - E_{k2} \qquad E_k = \sum_\alpha E_{k\alpha} = \sum_\alpha \tfrac{1}{2} m_\alpha \vec{v}_\alpha^{\,2}$$

$$\frac{d\vec{r}_\alpha}{dt} = \frac{d\vec{r}_\alpha'}{dt} + \frac{d\vec{R}}{dt}$$

$$\frac{d\vec{r}_\alpha}{dt} \cdot \frac{d\vec{r}_\alpha}{dt} = v_\alpha^2 = \left(\frac{d\vec{r}_\alpha'}{dt} + \frac{d\vec{R}}{dt}\right) \cdot \left(\frac{d\vec{r}_\alpha'}{dt} + \frac{d\vec{R}}{dt}\right) = \left(\frac{d\vec{r}_\alpha'}{dt} \cdot \frac{d\vec{r}_\alpha'}{dt}\right) + 2\left(\frac{d\vec{r}_\alpha'}{dt} \cdot \frac{d\vec{R}}{dt}\right) + \left(\frac{d\vec{R}}{dt} \cdot \frac{d\vec{R}}{dt}\right)$$

$$= (v_\alpha')^2 + 2\left(\frac{d\vec{r}_\alpha'}{dt} \cdot \frac{d\vec{R}}{dt}\right) + V^2$$

$$E_k = \sum_\alpha \tfrac{1}{2} m_\alpha (v_\alpha')^2 + \sum_\alpha \tfrac{1}{2} m_\alpha V^2 + \frac{dR}{dt} \cdot \frac{d}{dt} \sum_\alpha \underbrace{m_\alpha r_\alpha'}_{=0}$$

$$= \sum \tfrac{1}{2} m_\alpha (v_\alpha')^2 + \tfrac{1}{2} M V^2$$

the total energy for a conservative system is constant

$$W_{12} = \sum_\alpha \int_1^2 F_\alpha^e \cdot dr_\alpha + \sum_{\substack{\alpha \\ \beta \neq \alpha}} \int_1^2 f_{\alpha\beta} \cdot dr_\alpha \quad \text{, if the forces are conservative} \quad \begin{aligned} F_\alpha^e &= -\nabla_\alpha U_\alpha \\ f_{\alpha\beta} &= -\nabla_\alpha U_{\alpha\beta} \end{aligned}$$

$$= -\sum_\alpha \int_1^2 \nabla_\alpha U_\alpha \cdot dr_\alpha \qquad = \sum_{\alpha<\beta} \int_1^2 f_{\alpha\beta} \cdot dr_\alpha + f_{\beta\alpha} \cdot dr_\beta$$

$$= -\sum_\alpha U_\alpha \Big|_1^2 \qquad = \sum_{\alpha<\beta} \int_1^2 f_{\alpha\beta} \cdot (dr_\alpha - dr_\beta) = \sum_{\alpha<\beta} \int_1^2 f_{\alpha\beta} \cdot dr_{\alpha\beta} = -\sum_{\alpha<\beta} \int_1^2 dU_{\alpha\beta} = -\sum_{\alpha<\beta} U_{\alpha\beta} \Big|_1^2$$

$U_{\alpha\beta}$ only of the distance between m_α and m_β

$$dU_{\alpha\beta} = \sum_i \frac{\partial U_{\alpha\beta}}{\partial x_{\alpha,i}} dx_{\alpha,i} + \frac{\partial U_{\alpha\beta}}{\partial x_{\beta i}} dx_{\beta i} = \underbrace{(\nabla_\alpha U_{\alpha\beta})}_{-f_{\alpha\beta}} \cdot dr_\alpha + \underbrace{(\nabla_\beta U_{\alpha\beta})}_{= \nabla_\beta U_{\beta\alpha} = -f_{\beta\alpha} = f_{\alpha\beta}} \cdot dr_\beta$$

$$= -f_{\alpha\beta} \cdot (dr_\alpha - dr_\beta) \qquad = -f_{\alpha\beta} \cdot dr_{\alpha\beta}$$

$$\Rightarrow W_{12} = -\sum_{\alpha} U_\alpha \Big|_1^2 - \sum_{\alpha<\beta} U_{\alpha\beta} \Big|_1^2 \qquad U = \sum_{\alpha} U_\alpha + \sum_{\alpha<\beta} U_{\alpha\beta}$$

$$= -U\Big|_1^2 = U_1 - U_2$$

$$\Rightarrow E_{k2} - E_{k1} = U_1 - U_2$$

$$E_{k1} + U_1 = E_{k2} + U_2$$

$$E_{t1} = E_{t2}$$