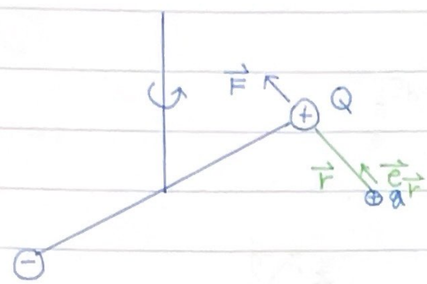
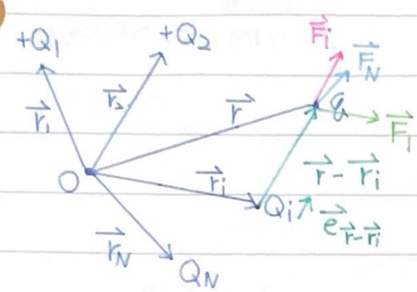
庫倫定律



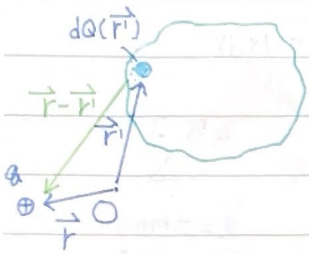
$$\vec{F} = K\frac{Qq}{|\vec{r}|^2}\vec{e}_{\vec{r}} = q\vec{E}(\vec{r})$$

$$K = \frac{1}{4\pi\epsilon_0}$$



$$\vec{F} = \sum_{i=1}^{N}\vec{F}_i = \sum K_c\frac{Q_i q}{|\vec{r}-\vec{r}_i|^2}\vec{e}_{(\vec{r}-\vec{r}_i)}$$

$$\vec{E} = \sum_{i=1}^{N} K_c Q_i\,\vec{e}_{(\vec{r}-\vec{r}_i)}/|\vec{r}-\vec{r}_i|^2$$



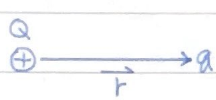
連續分佈

$$\vec{F} = \int K_c q\frac{dQ(\vec{r}')}{|\vec{r}-\vec{r}'|^2}\vec{e}_{(\vec{r}-\vec{r}')}$$

$$\vec{E} = K_c\int\frac{dQ(\vec{r}')}{|\vec{r}-\vec{r}'|^2}\vec{e}_{(\vec{r}-\vec{r}')}$$

電場

$\vec{F} \propto q$ $\lim_{q\to 0}\frac{\vec{F}}{q} = \vec{E}(\vec{r})$



$$\vec{F} = K_c\frac{Qq}{|\vec{r}-0|^2}\vec{e}_{\vec{r}} \qquad \vec{E} = \lim_{q\to 0}\frac{\vec{F}}{q} = \frac{Q}{|\vec{r}|^2}\vec{e}_{\vec{r}}$$

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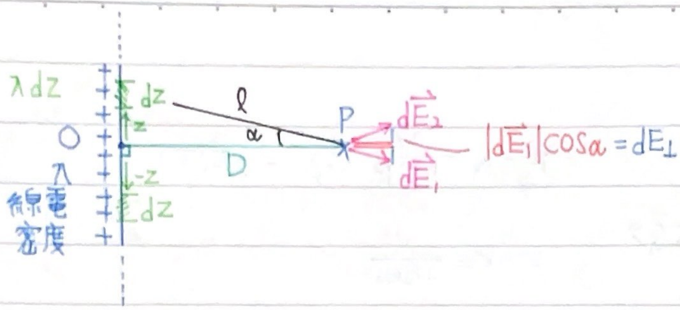
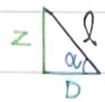




$$\vec{E}_\perp = \int d\vec{E}_\perp = K_c \int \frac{\lambda dz \cos\alpha}{l^2}$$

$$= K_c \lambda \int \frac{\cos\alpha \, D\sec^2\alpha \, d\alpha}{D^2 \sec^2\alpha}$$

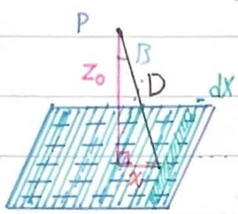
$$= \frac{K_c\lambda}{D} \int \cos\alpha \, d\alpha$$

$$= \frac{K_c\lambda}{D} \sin\alpha \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{2K_c\lambda}{D} \vec{e}_\perp$$

$\frac{z}{D} = \tan\alpha \quad \frac{D}{l} = \cos\alpha$

$l = D\sec\alpha$

$z = D\tan\alpha, \; dz = D\sec^2\alpha \, d\alpha$



σ 面電荷密度

$\lambda = \sigma \cdot 1 \cdot dx$

$$|d\vec{E}_1| = \frac{2K\lambda}{D} = \frac{2K\sigma dx}{D}$$

$$d\vec{E}_z = |d\vec{E}_1| \cos\beta$$

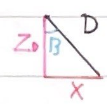
$$\vec{E}_z = \int d\vec{E}_z = \int \frac{2K\sigma}{D} \cos\beta \, dx$$

$$= 2K\sigma \int \frac{\cos\beta}{z_0 \sec\beta} z_0 \sec^2\beta \, d\beta$$

$$= 2K\sigma \int_{-\pi/2}^{\pi/2} 1 \, d\beta$$

$$= 2\pi K\sigma$$

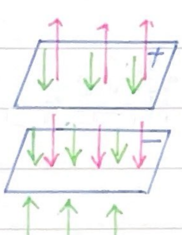
$$= \frac{\sigma}{2\epsilon_0}$$



$x = z_0 \tan\beta$

$D = z_0 \sec\beta$

$dx = z_0 \sec^2\beta \, d\beta$



$$|\vec{E}| = 2 \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

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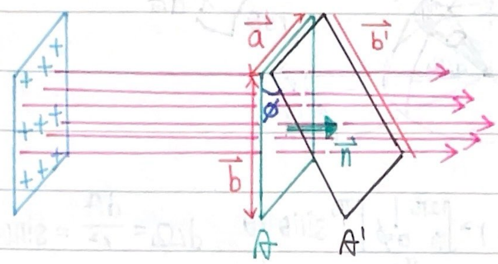
Gauss's Law

1. whether there is a net outward or inward electric flux through a closed surface depends on the sign of the enclosed charge
2. charges outside the surface do not give a net electric flux through the surface
3. the net electric flux is directly proportional to the net amount of charge enclosed with the surface but is otherwise independent of the size of the closed surface

Flux



the electric flux through the area Φ_E

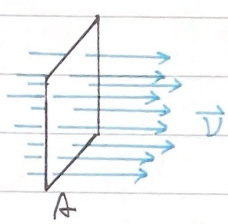
$$\Phi_{EA} = |\vec{E}|A = |\vec{E}|ab$$

$$\| $$

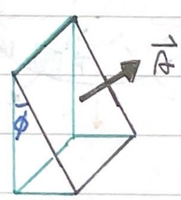
$$\Phi_{EA'} = |\vec{E}|A' = |\vec{E}|ab' = |\vec{E}|ab\cos\theta = \vec{E}\cdot\vec{A'}$$

define $\vec{A} = \vec{a}\times\vec{b}$

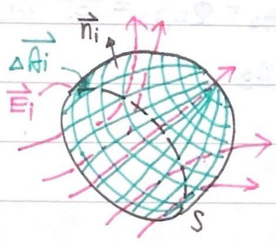
$\vec{A'} = \vec{a}\times\vec{b'}$



$$\frac{dV}{dt} = \vec{v}A$$



$$\frac{dV}{dt} = vA\cos\phi = v_\perp A = \vec{v}\cdot\vec{A}$$

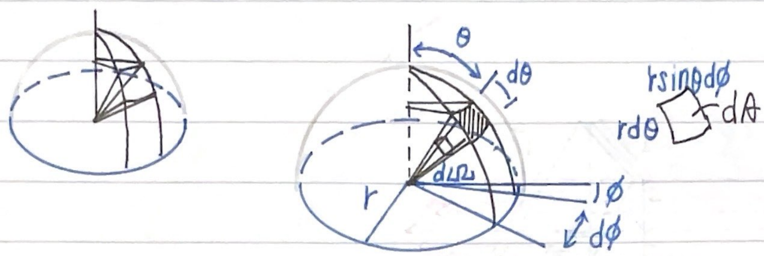


$$\lim_{\substack{N\to\infty \\ |\Delta\vec{A}_i|\to 0}} \sum_{i=1}^{N} \vec{E}_i\cdot\Delta\vec{A}_i = \Phi_{ES} = \int_S \vec{E}\cdot d\vec{A}$$

Solid Angle

in 2-D $\quad d\theta = \frac{dS}{r} \quad \Rightarrow \quad d\Omega = \frac{dA}{r^2} \;,\; \Omega = \frac{A}{r^2}$

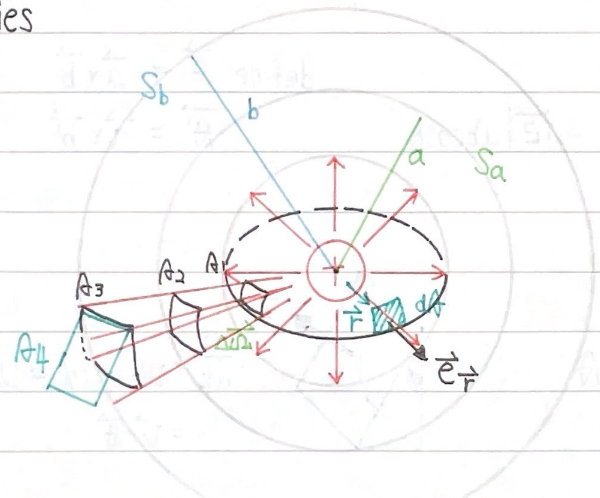
for sphere, its surface area $4\pi r^2$, so its $\Omega = 4\pi$



$dA = r\sin\theta\, d\phi\; r\,d\theta = r^2 \sin\theta\, d\theta\, d\phi$

$A = \int dA = \int_0^{2\pi}\int_0^{2\pi} r\sin\theta\, d\phi\; r\,d\theta = \underbrace{r^2\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta}_{4\pi r^2} \qquad d\Omega = \frac{dA}{r^2} = \sin\theta\, d\theta\, d\phi$

Properties



$\vec{E} \parallel d\vec{A} \parallel \vec{e}_{\vec{r}}$

$\Phi_{E.S_a} = \int \vec{E}\cdot d\vec{A} = \int |\vec{E}||d\vec{A}| = |\vec{E}| \int |d\vec{A}|$

$= \frac{Q}{4\pi\epsilon_0} \frac{1}{a^2} 4\pi a^2 = \frac{Q}{\epsilon_0}$

$\Phi_{EA_1} = \Phi_{EA_2} = \Phi_{EA_3} = \frac{Q}{\epsilon_0} \frac{|\Delta\Omega|}{4\pi}$

$\Phi_{EA_4} = \Phi_{EA_3}$

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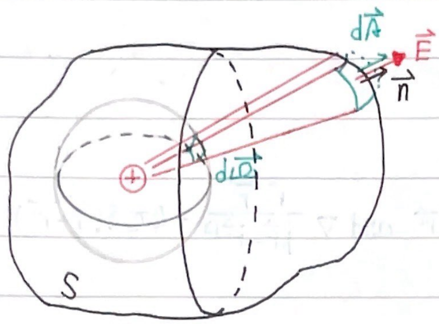
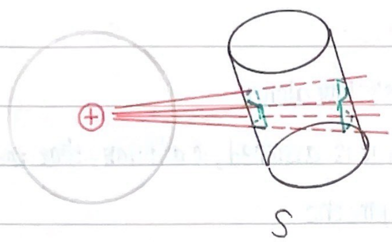
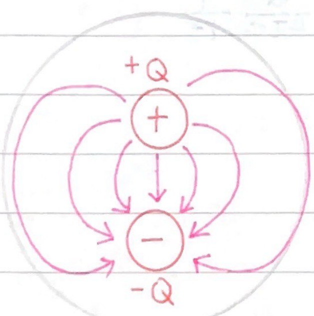


$$\vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}\frac{|d\vec{\Omega}|}{4\pi}$$

$$\oint_S \vec{E}\cdot d\vec{A} = \oint_S \frac{Q}{\epsilon_0}\frac{|d\vec{\Omega}|}{4\pi} = \frac{Q}{4\pi}\frac{1}{\epsilon_0}\oint |d\vec{\Omega}|$$
$$= \frac{Q}{\epsilon_0}$$



$$\oint \vec{E}\cdot d\vec{A} = 0$$



$$\oint_S \vec{E}\cdot d\vec{A} = \frac{1}{\epsilon_0}(+Q-Q) = 0$$

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* Gauss's law and Coulomb's law are equivalent

deriving Gauss's law from Coulomb's law

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r'})}{|\vec{r}-\vec{r'}|^3}\,\vec{r}-\vec{r'}\, d^3\vec{r'}$$

$$\nabla\cdot\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_V \rho(\vec{r'})\ \nabla\cdot\frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\, d^3\vec{r'} \quad \text{and} \quad \nabla\frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3} = 4\pi\delta(\vec{r}-\vec{r'})$$

$$= \frac{1}{\epsilon_0}\int_V \rho(\vec{r'})\,\delta(\vec{r}-\vec{r'})\, d^3\vec{r'}$$

$$= \frac{\rho(\vec{r})}{\epsilon_0}$$

deriving Coulomb's law from Gauss's law

Coulomb's law cannot be derived from Gauss's law along

however, it can be proven from Gauss's law if it is assumed, in addition, that the electric field from a point charge is spherically symmetric

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{Q}{\epsilon_0}, \quad 4\pi r^2\cdot\vec{E}(\vec{r}) = \frac{Q}{\epsilon_0}, \quad \vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}$$

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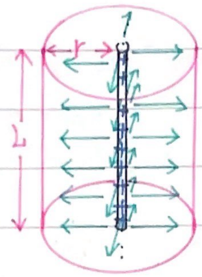
Applications of Gauss's Law

case I field of a uniform line charge

infinitely long, thin wire, the charge per unit length is λ

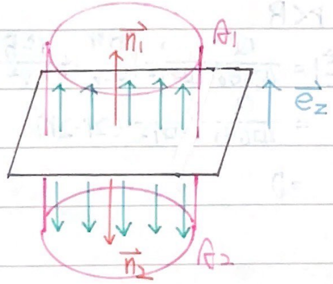


$$\int \vec{E} \cdot d\vec{A} = \int |\vec{E}||d\vec{A}| = |\vec{E}| \int |d\vec{A}| = |\vec{E}| 2\pi r L = \frac{1}{\epsilon_0} \lambda L$$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0} \frac{1}{r} \vec{e}_\perp$$

case II field of an infinite plane sheet of charge

thin, flat, infinite sheet with a uniform positive surface charge density σ



$$\int_{A_1} \vec{E} \cdot d\vec{A} + \int_{A_2} \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} \sigma A$$

$$2A |\vec{E}| = \frac{1}{\epsilon_0} \sigma A$$

$$|\vec{E}| = \frac{\sigma}{2\epsilon_0}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \vec{e}_z \quad \text{when } z>0$$

$$-\frac{\sigma}{2\epsilon_0} \vec{e}_z \quad \text{when } z<0$$

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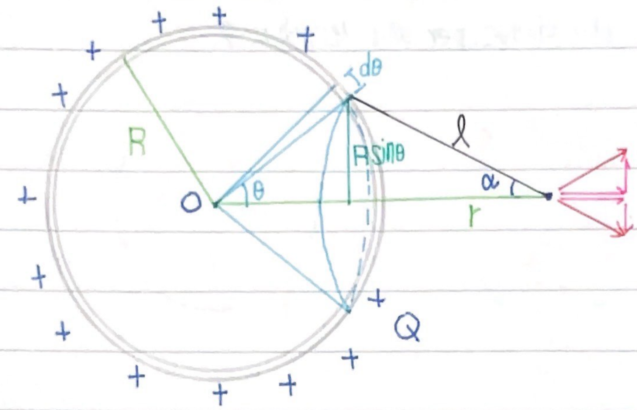
case Ⅲ charge on a hollow sphere

thin-walled, hollow sphere



by Coulomb's law

$$\int Q \frac{2\pi R\sin\theta R\, d\theta}{\epsilon 4\pi R^2}\frac{\cos\alpha}{l^2} = \frac{Q}{4\pi\epsilon_0}\int \frac{\sin\theta}{2}\frac{\cos\alpha}{l^2}\, d\theta$$

$$= \frac{Q}{4\pi\epsilon_0}\int \frac{1}{2}\frac{l}{Rr}\frac{1}{l^2}\frac{l^2+r^2-R^2}{2lr}\, dl$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{1}{4Rr^2}\int 1+\frac{r^2-R^2}{l^2}\, dl$$

$l^2 = R^2+r^2-2Rr\cos\theta$

$2l\, dl = 2Rr\sin\theta\, d\theta$

$R^2 = l^2+r^2-2lr\cos\alpha$

$r>R$

$$|\vec{E}| = \frac{Q}{4\pi\epsilon_0}\frac{1}{4Rr^2}\int_{r-R}^{r+R} 1+\frac{r^2-R^2}{l^2}\, dl$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{1}{4Rr^2}4R$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}$$

$r<R$

$$|\vec{E}| = \frac{Q}{4\pi\epsilon_0}\frac{1}{4Rr^2}\int_{R-r}^{R+r} 1+\frac{r^2-R^2}{l^2}\, dl$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{1}{4Rr^2}(2r-2r)$$

$$= 0$$

by Gauss's law

$$\int \vec{E}\cdot d\vec{A} = \int |\vec{E}||d\vec{A}| = |\vec{E}|\int |d\vec{A}| = |\vec{E}|4\pi r^2 = \frac{Q}{\epsilon_0}$$

$\vec{E}\cdot d\vec{A} = \frac{1}{\epsilon_0}$ (total charge enclosed by the surface, Q_{enel})

$r>R$

$Q_{enel} = Q$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\vec{e}_r$$

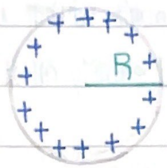
$r<R$

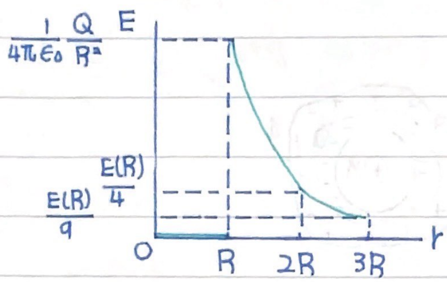
$Q_{enel} = 0$

$\vec{E} = 0$

case IV field of a charged conducting sphere

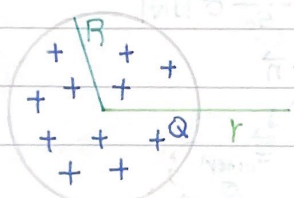
same as the case III





case V field of a uniformly charged sphere

uniformly throughout the volume of an insulating sphere with radius R

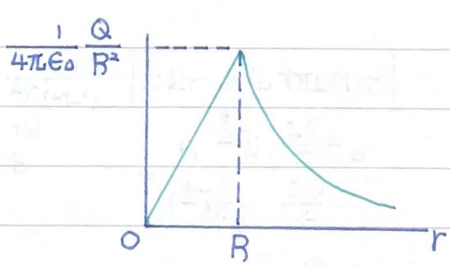


if $r>R$, $\int_S \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$, $|\vec{E}|4\pi r^2 = \frac{Q}{\epsilon_0}$, $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$

if $r<R$, $Q_{encl} \neq Q$

$Q_{encl} = \frac{4}{3}\pi r^3 \frac{Q}{\frac{4}{3}\pi R^3} = \frac{r^3}{R^3}Q$

$\int_S \vec{E}\cdot d\vec{A} = \frac{r^3}{R^3}Q\frac{1}{\epsilon_0}$, $|\vec{E}|4\pi r^2 = \frac{1}{\epsilon_0}\frac{r^3}{R^3}Q$, $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}r$



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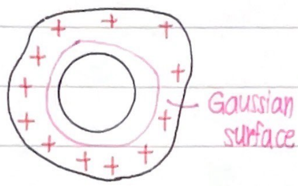
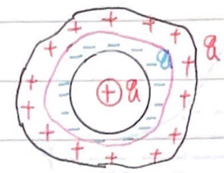
Charges on Conductors

in an electrostatic situation the electric field at every point within a conductor is zero and that any excess charge on a solid conductor is located entirely on it surface

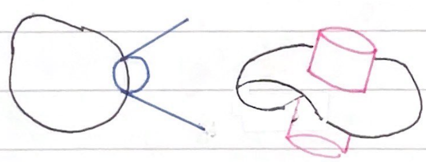
if there is a cavity inside the conductor





$$\int_S \vec{E}\cdot d\vec{A} = 0 = \frac{Q_{encl}}{\epsilon_0}$$

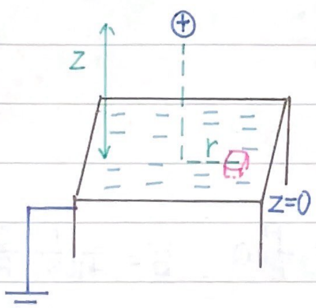
$$\int_S \vec{E}\cdot d\vec{A} = 0$$



$$\int \vec{E}\cdot d\vec{A} = \frac{1}{\epsilon_0}\sigma|d\vec{A}|$$

$$\vec{E} = \frac{\sigma}{\epsilon_0}\vec{n}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\vec{n} \qquad \vec{E}_{other} = \frac{\sigma}{2\epsilon_0}\vec{n}$$



$$\vec{E} = \frac{\sigma}{\epsilon_0}\vec{n} = \vec{E} + \vec{E}$$

$$\frac{\sigma}{2\epsilon_0}\vec{n}\cdot\vec{n} = -\frac{Q}{4\pi\epsilon_0}\frac{1}{(z^2+r^2)}\frac{z}{\sqrt{z^2+r^2}}$$

$$\sigma(r) = -\frac{Q}{2\pi}\frac{z}{(z^2+r^2)^{\frac{3}{2}}}$$

$$\int\sigma(r)\,2\pi r\,dr = -Qz\int\frac{r\,dr}{(z^2+r^2)^{3/2}}$$

$$= -\frac{Qz}{2}\int u^{-\frac{3}{2}}du$$

let $u = z^2+r^2$

$du = 2r\,dr$

$$= -\frac{Qz}{2}\left(-2u^{-\frac{1}{2}}\right)\Big|_0^\infty$$

$$= -Q$$