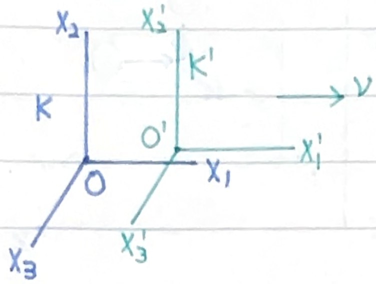
NO.

DATE

Special Theory of Relativity

Galilen Invariance



$$x_1' = x_1 - vt$$

$$x_2' = x_2$$

$$x_3' = x_3$$

$$t' = t$$

$$ds^2 = \sum_j dx_j^2 = \sum_j d(x_j')^2 = d(s')^2$$

$$F_j = m\frac{d^2x_j}{dt^2} = m\frac{d^2x_j'}{d(t')^2} = F_j'$$

$$\frac{dx_1'}{dt} = \frac{dx_1}{dt} - v$$

Lorentz Transformation

$$\sum_{j=1}^{3} x_j^2 - c^2t^2 = 0 \qquad \sum_{j=1}^{3}(x_j')^2 - c^2(t')^2 = 0$$

at $t = t' = 0$ $\quad x_1 - vt = 0 \quad x_1' = 0 \quad x_1' = x_1 - vt$

$$x_1' = \gamma(x_1 - vt)$$

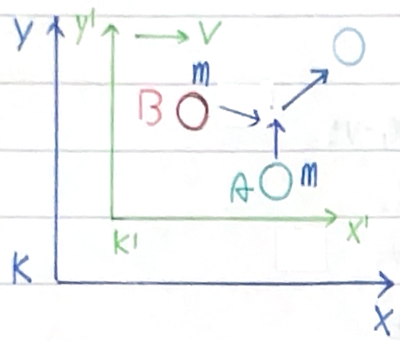
$$x_1 = \gamma'(x_1' + vt) \qquad t' = \gamma t + \frac{x_1}{\gamma v}(1-\gamma^2)$$

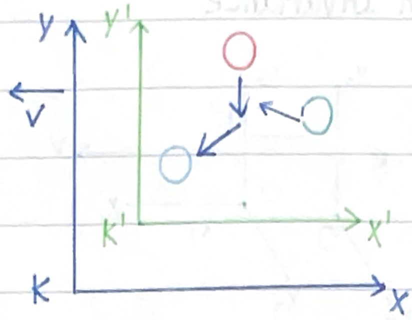
$$x_1 = ct$$

$$x_1' = ct' \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

SEASON

Relativistic Momentum





collision according to system K

$u_{Ax}=0$ $\quad P_{Ay}=mu_0$ $\quad u_{Bx}=v$ $\quad u'_{Bx}=0$

$u_{Ay}=u_0$ $\quad \Delta P_{Ay}=-2mu_0$ $\quad u_{By}=-\dfrac{u_0}{\sqrt{1-\frac{v^2}{c^2}}}$ $\quad u'_{By}=-u_0$

$$P_{By}=-mu_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\Delta P_{By}=+2mu_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\vec{P}=m\frac{d\vec{x}}{d\tau}=m\frac{d\vec{x}}{dt}\frac{dt}{d\tau}=m\frac{d\vec{x}}{dt}\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}=\frac{m\vec{u}}{\sqrt{1-\frac{u^2}{c^2}}}$$

NO.

DATE / /

Energy

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(\gamma m\vec{u})$$

$$W = \int \frac{d}{dt}(\gamma m\vec{u})\cdot\vec{u}\,dt = m\int_0^u u\,d(\gamma u)$$

$$= \gamma m u^2 - m\int_0^u \frac{u\,du}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma m u^2 + mc^2\sqrt{1-\frac{u^2}{c^2}}\Big|_0^u$$

$$= \gamma m u^2 + mc^2\sqrt{1-\frac{u^2}{c^2}} - mc^2 = \gamma mc^2 - mc^2$$

$$= \frac{mu^2}{\sqrt{1-\frac{u^2}{c^2}}} - \frac{m_0}{2}\int_0^u \frac{du^2}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{mc^2(\frac{u}{c})^2}{\sqrt{1-(\frac{u}{c})^2}} - \frac{mc^2}{2}\int_0^u \frac{d(\frac{u}{c})^2}{\sqrt{1-(\frac{u}{c})^2}}$$

$$= mc^2\frac{(\frac{u}{c})^2}{\sqrt{1-(\frac{u}{c})^2}} - mc^2\sqrt{1-(\frac{u}{c})^2}\Big|_0^u$$

$$= mc^2\left[\frac{(\frac{u}{c})^2}{\sqrt{1-(\frac{u}{c})^2}} + \sqrt{1-(\frac{u}{c})^2} - 1\right] = \frac{mc^2}{\sqrt{1-(\frac{u}{c})^2}}\left\{(\frac{u}{c})^2 + \left[1-(\frac{u}{c})^2\right] - \sqrt{1-(\frac{u}{c})^2}\right\}$$

$$= \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} - \underline{mc^2}$$ rest energy

for $u \ll c$ $\quad T = mc^2(1+\frac{1}{2}\frac{u^2}{c^2}+\cdots) - mc^2 = mc^2 + \frac{1}{2}mu^2 - mc^2 = \frac{1}{2}mu^2$

rewritten $\quad \gamma mc^2 = T + mc^2$

$$E = T + E_0$$

$p = \gamma m u$, $\quad p^2c^2 = \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^4 \underline{\frac{u^2}{c^2}}$ $\quad 1-\frac{1}{\gamma^2}$

$$= \gamma^2 m^2 c^4 (1-\frac{1}{\gamma^2})$$

$$= \gamma^2 m^2 c^4 - m^2c^4 = E^2 - E_0^2$$

$$\Rightarrow E^2 = p^2c^2 + E_0^2$$

Spacetime and Four-Vectors

consider two events separated by space and time.

in system K $\quad \Delta x_i = x_i(\text{event } 2) - x_i(\text{event } 1)$

$\Delta t = t(\text{event } 2) - t(\text{event } 1)$

$(\Delta s')^2 =$ ______

$$(\Delta s')^2 = -c^2(t')^2 + (x')^2 + (y')^2 + (z')^2 = \frac{-c^2t^2 - \frac{v^2x^2}{c^2} + 2xvt}{1-\frac{v^2}{c^2}} + \frac{x^2 + v^2t^2 - 2xvt}{1-\frac{v^2}{c^2}} + y^2 + z^2$$

$$= \frac{x^2 - \frac{v^2x^2}{c^2} - c^2\left(t^2 - \frac{v^2}{c^2}t^2\right)}{1-\frac{v^2}{c^2}} + y^2 + z^2$$

$$= -c^2t^2 + x^2 + y^2 + z^2$$

$\Rightarrow (\Delta s')^2 = \Delta s^2$

$$\Delta s^2 = \sum_{j=1}^{3}(\Delta x_j)^2 - c^2(\Delta t)^2 = (\Delta s')^2 = \sum_{j=1}^{3}(\Delta x_j')^2 - c^2(\Delta t')^2$$

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$$

consider the system K', where the particle is instantaneously at rest

$dx' = dy' = dz' = 0 \quad$ in this case $\quad dt' = d\tau$

$$-c^2d\tau^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$$

$$d\tau = \frac{dt}{\gamma}$$