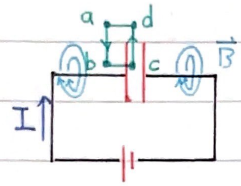
Electromagnetic Wave

Displacement and Maxwell's Equations



$$\oint \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} + \int_d^a \vec{B}\cdot d\vec{l}$$

$$\int_a^b \vec{B}\cdot d\vec{l} > 0 \quad \int_c^d \vec{B}\cdot d\vec{l} = 0 \quad \int_b^c \vec{B}\cdot d\vec{l} + \int_d^a \vec{B}\cdot d\vec{l} = 0$$

$\Rightarrow \oint \vec{B}\cdot d\vec{l} > 0$ but no current cross the loop abcd $\Rightarrow$ contradiction

$q = Cv \quad C = \frac{\epsilon_0 A}{d} \quad v = Ed$

$q = \frac{\epsilon A}{d} Ed = \epsilon E A = \epsilon \Phi_E$

conduction current $i_c = \frac{dq}{dt} = \epsilon \frac{d\Phi_E}{dt}$

displacement current

to let $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_c = 0$, we : $\oint \vec{B}\cdot d\vec{l} = \mu_0 (I_c + I_D)$ which $I_c + I_D = 0$

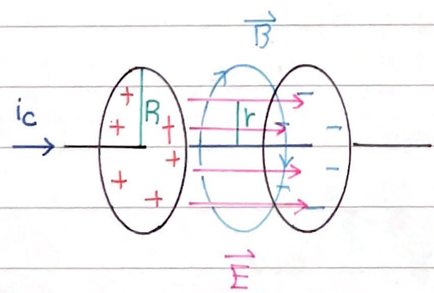
and $i_D = \epsilon \frac{d\Phi_E}{dt}$

$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 (i_c + i_D)_{encl}$ for the flat surface, $i_D = 0$, i_c

for the curved surface, $i_c = 0$, i_D

($i_c = i_D$)

$j_D = \frac{i_D}{A} = \epsilon \frac{dE}{dt}$



$$\oint \vec{B}\cdot d\vec{l} = 2\pi r B = \mu_0 \frac{i_D}{\pi R^2} \pi r^2 = \mu_0 \frac{r^2}{R^2} i_c$$

$$B = \frac{\mu_0}{2\pi} \frac{r^2}{R^2} i_c$$

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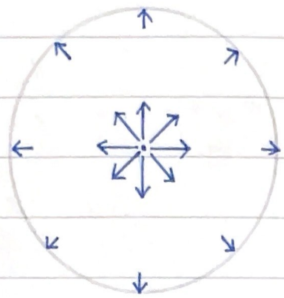


$$\vec{J}(\vec{r}) = \frac{I}{4\pi r^2}\vec{e}_{\vec{r}}$$
$$\vec{B} = \vec{0}$$

$$\oint \vec{B}\cdot d\vec{l} = 0 = \mu_0 \int \vec{J}\cdot d\vec{A} \neq 0$$ contradiction

to let $\int \vec{J}\cdot d\vec{A} = 0$, we $\int (\vec{J}+\vec{J}_{dis})\cdot d\vec{A}$ and $\vec{J}+\vec{J}_{dis} = 0$

$$\vec{J}_{dis} = -\vec{J}(\vec{r}) = -\frac{I}{4\pi r^2}\vec{e}_{\vec{r}}$$
$$= \frac{\epsilon_0(-I)}{\epsilon_0 4\pi r^2}\vec{e}_{\vec{r}}$$
$$= \epsilon_0 \frac{\frac{dQ(t)}{dt}}{4\pi\epsilon_0 r^2}\vec{e}_{\vec{r}}$$
$$= \epsilon_0 \frac{d\vec{E}(t)}{dt}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{J}\cdot d\vec{A} + \mu_0\epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$$

Maxwell's Equations

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$ Gauss's law

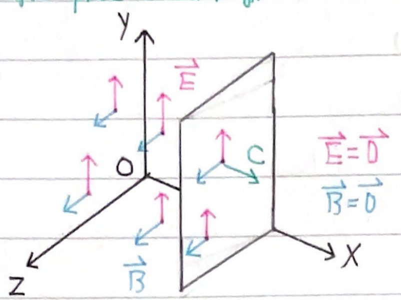
$\oint \vec{B}\cdot d\vec{A} = 0$ Gauss's law for magnetism

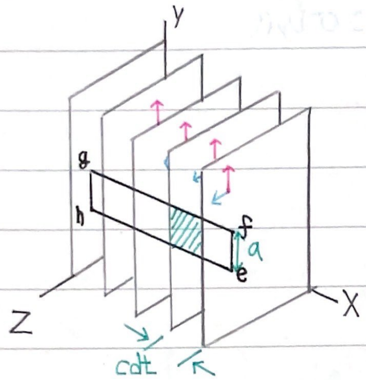
$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(i_c + \frac{d\Phi_E}{dt}\right)_{encl}$ Ampere's law

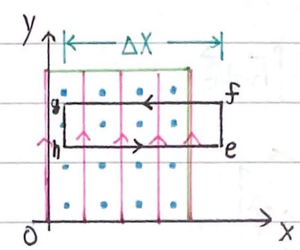
$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ Faraday's law

Plane Electromagnetic Waves

a simple plane electromagnetic wave

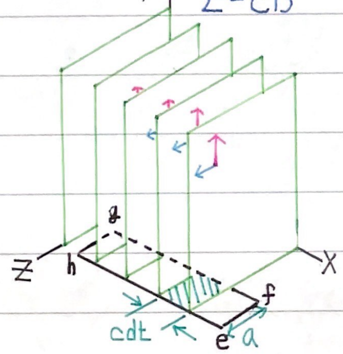


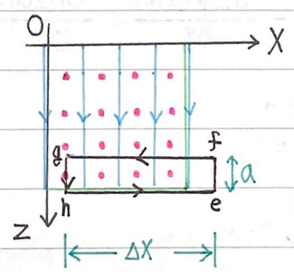




$$\oint \vec{E}\cdot d\vec{l} = -Ea = -\frac{d\Phi_B}{dt} = -\frac{dBacdt}{dt} = -Bac$$

$$E = cB$$





$$\oint \vec{B}\cdot d\vec{l} = Ba = \mu_0\epsilon_0 \frac{d\Phi_E}{dt} = \mu_0\epsilon_0 \frac{d}{dt}(Eacdt) = \mu_0\epsilon_0 Eac$$

$$B = \epsilon_0\mu_0 cE$$

$$\begin{cases} E = cB \\ E = \dfrac{1}{\epsilon_0\mu_0 c} B \end{cases}, \quad c = \frac{1}{\mu_0\epsilon_0 c}, \quad c^2 = \frac{1}{\mu_0\epsilon_0}, \quad c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

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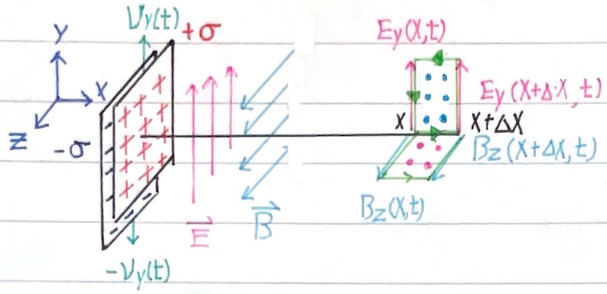
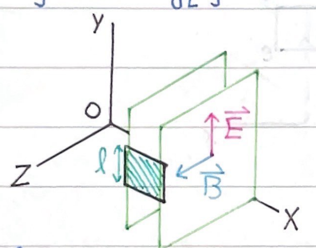
derivation of the electromagnetic wave equation



$$|\vec{B}| = \frac{\mu_0}{2} nI = \frac{\mu_0}{2} 2x \frac{1 x V_y(t)\Delta t \sigma}{\Delta t} = \frac{\mu_0}{2} 2\sigma V_y(t)$$

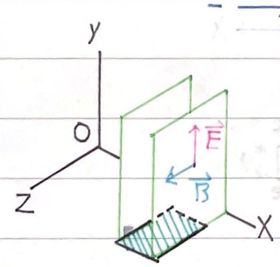
$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$



$$\oint \vec{E}\cdot d\vec{l} = -E_y(x,t)l + E_y(x+\Delta x,t)l = -\frac{d\Phi_B}{dt} = -\frac{\partial B_z(x,t)}{\partial t} l\Delta x$$

$$\frac{E_y(x+\Delta x,t) - E_y(x,t)}{\Delta x} = -\frac{\partial B_z(x,t)}{\partial t}$$

$$-\frac{\partial E_y(x,t)}{\partial x} = \frac{\partial B_z(x,t)}{\partial t}$$



$$\oint \vec{B}\cdot d\vec{l} = -B_z(x+\Delta x,t)l + B_z(x,t)l = \mu_0\epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A} = \mu_0\epsilon_0 \frac{\partial E_y(x,t)}{\partial t} l\Delta x$$

$$\frac{-B_z(x+\Delta x,t) + B_z(x,t)}{\Delta x} = \mu_0\epsilon_0 \frac{\partial E_y(x,t)}{\partial t}$$

$$-\frac{\partial B_z(x,t)}{\partial x} = \mu_0\epsilon_0 \frac{\partial E_y(x,t)}{\partial t}$$

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$$\frac{\partial}{\partial x}\left[-\frac{\partial E_y(x,t)}{\partial x}=\frac{\partial B_z(x,t)}{\partial t}\right] \Rightarrow -\frac{\partial^2 E_y(x,t)}{\partial x^2}=\frac{\partial^2 B_z(x,t)}{\partial x\,\partial t}$$

$$\frac{\partial}{\partial t}\left[-\frac{\partial B_z(x,t)}{\partial x}=\mu_0\epsilon_0\frac{\partial E_y(x,t)}{\partial t}\right] \Rightarrow -\frac{\partial^2 B_z(x,t)}{\partial x\,\partial t}=\mu_0\epsilon_0\frac{\partial^2 E_y(x,t)}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 E_y(x,t)}{\partial x^2}=\epsilon_0\mu_0\frac{\partial^2 E_y(x,t)}{\partial t^2}, \quad \frac{1}{v^2}=\epsilon_0\mu_0 \quad v=\frac{1}{\sqrt{\epsilon_0\mu_0}}$$

$$\frac{\partial^2 B_z(x,t)}{\partial x^2}=\mu_0\epsilon_0\frac{\partial B_z(x,t)}{\partial t^2}$$

let $E_y(x,t)=\phi(x+ct)+\psi(x-ct)$, $v=x+ct$ $w=x-ct$

$$\frac{\partial E_y(x,t)}{\partial x}=\frac{\partial E_y}{\partial v}\frac{\partial v}{\partial x}+\frac{\partial E_y}{\partial w}\frac{\partial w}{\partial x}=\frac{\partial E_y}{\partial v}+\frac{\partial E_y}{\partial w}$$
$$\frac{\partial E_y(x,t)}{\partial t}=\frac{\partial E_y}{\partial v}\frac{\partial v}{\partial t}+\frac{\partial E_y}{\partial w}\frac{\partial w}{\partial t}=c\left(\frac{\partial E_y}{\partial v}-\frac{\partial E_y}{\partial w}\right)$$

$$\frac{\partial^2 E_y(x,t)}{\partial x^2}=\frac{\partial^2 E_y}{\partial v^2}+\frac{\partial^2 E_y}{\partial w^2}+2\frac{\partial E_y}{\partial v}\frac{\partial E_y}{\partial w}$$
$$\frac{\partial^2 E_y(x,t)}{\partial t^2}=c^2\left(\frac{\partial^2 E_y}{\partial v^2}+\frac{\partial^2 E_y}{\partial w^2}-2\frac{\partial E_y}{\partial w}\frac{\partial E_y}{\partial v}\right)$$

$$\Rightarrow 4\frac{\partial^2 E_y}{\partial w\,\partial v}=0, \quad \frac{\partial^2 E_y}{\partial w\,\partial v}=0$$
$$\int\frac{\partial^2 E_y}{\partial w\,\partial v}\,dw=\psi(w)$$
$$E_y=\int h(v)\,dv+\psi(w)$$

$x>0$ $\quad B(0+,t)=\frac{\mu_0}{2}2\sigma v_z(t)\vec{e}_y$ $\quad x<0$ $\quad B(0-,t)=-\mu_0\sigma v_z(t)\vec{e}_y$

$x>0$

$$B_z(x,t)=\psi(w)=\psi(x-ct)=\psi\left[-c\left(t-\frac{x}{c}\right)\right]$$
$$=\tilde{\psi}\left(t-\frac{x}{c}\right)=\psi[-c(t)]=\mu_0\sigma v_z\left(t-\frac{x}{c}\right)$$
$$B_z(0+,t)=\frac{\mu_0}{2}2\sigma v_z(t)=\tilde{\psi}(t)$$
$$=\mu_0\sigma v_z(t)$$
$$-\frac{\partial B_z}{\partial x}=\mu_0\epsilon_0\frac{\partial E_y}{\partial t}=\frac{1}{c}\mu_0\sigma v_z'=\frac{1}{c^2}\frac{\partial E_y}{\partial t}$$
$$E_y=c\mu_0\sigma v_z\left(t-\frac{x}{c}\right)=cB_z(x,t)$$

$x<0$

$$B_z(x,t)=\phi(x+ct)=\phi\left[c\left(t+\frac{x}{c}\right)\right]=\tilde{\phi}\left(t+\frac{x}{c}\right)$$
$$=-\mu_0\sigma v_z\left(t+\frac{x}{c}\right)$$
$$B_z(0-,t)=\phi[c(t)]=\tilde{\phi}(t)=-\mu_0\sigma v_z(t)$$
$$\frac{\partial B_z}{\partial x}=\frac{1}{c^2}\frac{\partial E_y}{\partial t}=-\frac{1}{c}\mu_0\sigma v_z'$$
$$E_y(x,t)=-c\mu_0\sigma v_z\left(t+\frac{x}{c}\right)=cB_z(x,t)$$

$$\Rightarrow |E_y(x,t)|=c\,|B_z(x,t)| \quad \vec{E}\perp\vec{B}$$

SEASON

Energy and Momentum in Electromagnetic Wave

energy density in electric field $u_E = \frac{\epsilon_0}{2}\vec{E}\cdot\vec{E} = \frac{\epsilon_0}{2}c^2\vec{B}\cdot\vec{B} = \frac{1}{2\mu_0}\vec{B}\cdot\vec{B}$

energy density in magnetic field $u_B = \frac{1}{2\mu_0}\vec{B}\cdot\vec{B}$

$\Big\}\ u_E = u_B$

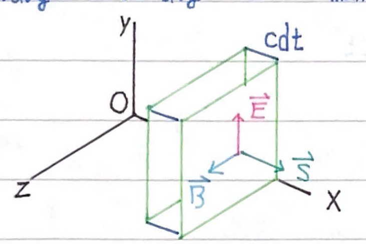
total energy density $u = u_E + u_B = 2u_E + 2u_B$

$$= \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$

$$= \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}(\sqrt{\epsilon_0\mu_0}\,E)^2$$

$$= \epsilon_0 E^2 = \frac{1}{\mu_0}B^2$$

$$u_{avg} = \epsilon_0 (E)^2_{avg} = \frac{1}{2}\epsilon_0 E^2_{max} = \frac{B^2_{max}}{2\mu_0}$$



$$dU = u\,dV = (\epsilon_0 E^2)(A\,c\,dt)$$

energy flow per unit time per unit area $S = \frac{1}{A}\frac{dU}{dt} = \epsilon_0 c|\vec{E}|^2 = \frac{|\vec{E}|^2}{\mu_0 c} = \frac{c}{\mu_0}|\vec{B}|^2$

$$= \frac{\epsilon_0}{\sqrt{\epsilon_0\mu_0}}E^2 = \sqrt{\frac{\epsilon_0}{\mu_0}}E^2 = \sqrt{\frac{\epsilon_0 c^2}{\mu_0}}|\vec{E}||\vec{B}|$$

$$= \sqrt{\frac{c^2\epsilon_0\mu_0}{\mu_0^2}}|\vec{E}||\vec{B}| = \frac{1}{\mu_0}|\vec{E}||\vec{B}|$$

in vacuum

$$[S] = \frac{1J}{s\cdot m^2} = \frac{1W}{m^2}$$

Poynting vector $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ the total energy flow per unit time, $P = \oint \vec{S}\cdot d\vec{A}$

for sinusoidal wave

$$\vec{S}(x,t) = \frac{1}{\mu_0}\vec{E}(x,t)\times\vec{B}(x,t) = \frac{1}{\mu_0}\left[E_{max}\cos(kx-\omega t)\vec{e}_y\right]\times\left[B_{max}\cos(kx-\omega t)\vec{e}_z\right]$$

$$= \frac{E_{max}B_{max}}{\mu_0}\cos^2(kx-\omega t)\vec{e}_x = \frac{E_{max}B_{max}}{2\mu_0}\left[1+\cos 2(kx-\omega t)\right]\vec{e}_x$$

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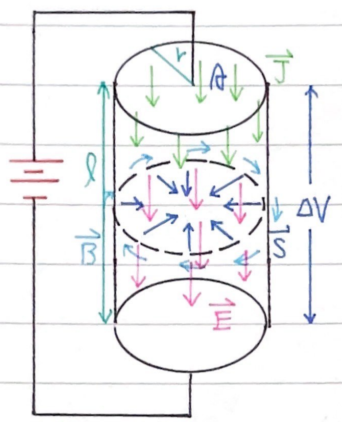
$$|\vec{S}|_{av} = \text{intensity}, I = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}\,E_{max}^2 = \frac{1}{2}\epsilon_0 c\,E_{max}^2$$

in vacuum

case I



$$\Delta V = RI \qquad |\vec{E}|\ell = \Delta V$$

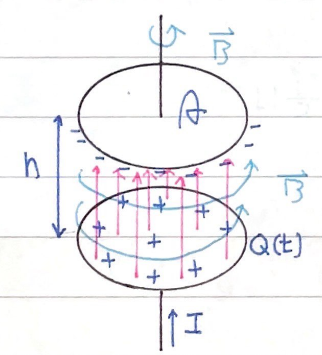
$$P = I\Delta V \qquad B\,2\pi r = \mu_0 I,\ B = \frac{\mu_0 I}{2\pi r}$$

$$|\vec{S}| = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$2\pi r\ell\,|\vec{S}| = 2\pi r\ell\,\frac{1}{\mu_0}\,\frac{\Delta V}{\ell}\,\frac{\mu_0 I}{2\pi r}$$

$$P = I\Delta V$$

case II



$$|\vec{E}| = \frac{Q}{\epsilon_0 A}, \quad u_E = \frac{\epsilon_0}{2}|\vec{E}|^2$$

$$\frac{\epsilon_0}{2}|\vec{E}|^2 A h = U_E, \quad \frac{dU_E}{dt} = \frac{\epsilon_0}{2} A h\, 2|\vec{E}|\,\frac{d|\vec{E}|}{dt}$$

$$\oint \vec{B}\cdot d\vec{\ell} = 2\pi r B = \mu_0\epsilon_0 \frac{d}{dt}|\vec{E}||\vec{A}| = \mu_0\epsilon_0\frac{d}{dt}\frac{Q(t)}{\epsilon_0} = \mu_0 I$$

$$|\vec{S}| = \frac{1}{\mu_0}|\vec{E}||\vec{B}| \qquad 2\pi r h|\vec{S}| = \frac{1}{\mu_0}\,\frac{Q(t)}{\epsilon_0 A}\,\frac{\mu_0 I}{2\pi r}\,2\pi r h = I\,\frac{Q(t)}{\epsilon_0 A}\,h$$

$$= \epsilon_0 A\,\frac{d|\vec{E}|}{dt}\cdot|\vec{E}|h = \frac{dU_E}{dt}$$

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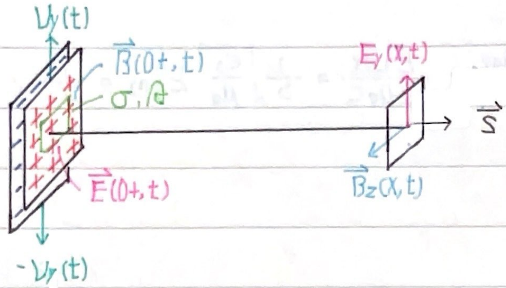


$$\vec{E}(x,t) = -c\mu_0\sigma v_y(t-\frac{x}{c})\vec{e}_y$$

$$\vec{B}(x,t) = \mu_0\sigma v_y(t-\frac{x}{c})\vec{e}_z$$

$$\frac{dW}{dt} = \vec{F}\cdot\vec{v} = [A\sigma\vec{E}(0+,t)]\cdot[v_y(t)\vec{e}_y]$$

$$= -A\sigma c\mu_0\sigma v_y^2(t) = -Ac\mu_0\sigma^2 v_y^2(t)$$

$$A|\vec{S}| = A\frac{1}{\mu_0}c\mu_0^2\sigma^2 v_y^2(t-\frac{x}{c}) = Ac\mu_0\sigma^2 v_y^2(t-\frac{x}{c})$$

$$\vec{F} = q\vec{v}\times\vec{B} = A\sigma v_y(t)\vec{e}_y\times\vec{B}(0+,t)$$

$$= -A\sigma v_y(t)\mu_0\sigma v_y(t-\frac{0+}{c})\vec{e}_x$$

$$= -A\mu_0\sigma^2 v_y^2(t)\vec{e}_x = -A|\vec{S}(0+,t)|\frac{1}{c}$$

$$= \frac{dP_x\vec{e}_x}{dt} = \frac{dW}{dt}\frac{1}{c}\vec{e}_x \quad \Rightarrow \quad |\vec{p}| = \frac{1}{c}U$$

momentum density $\frac{dp}{dV} = \frac{1}{c}(u_E+u_B) = \frac{1}{c^2}c(u_E+u_B)$

$$= \frac{EB}{\mu_0 c^2} = \frac{1}{c^2}|\vec{S}|$$

$$\frac{d\vec{p}}{dV} = \frac{1}{c^2}c\vec{S} = \mu_0\epsilon_0\frac{1}{\mu_0}\vec{E}\times\vec{B} = \epsilon_0\vec{E}\times\vec{B}$$

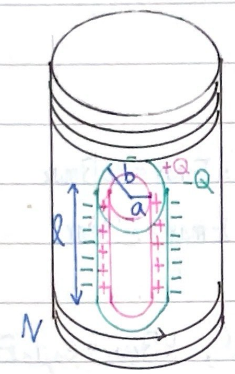
momentum flow rate per unit area $\frac{1}{A}\frac{dp}{dt} = \frac{S}{c} = \frac{EB}{\mu_0 c}$ = pressure, P_{rad}

radiation pressure $P_{rad} = \frac{F}{A} = -\mu_0\sigma^2 v_y^2(t)\vec{e}_x = -|\vec{S}(0+,t)|\frac{1}{c}\vec{e}_x$

electromagnetic wave $\longrightarrow$ | totally absorbed $\quad P_{rad} = \frac{S_{av}}{c} = \frac{I}{c}$

electromagnetic wave $\rightleftarrows$ | totally reflected $\quad P_{rad} = \frac{2S_{av}}{c} = \frac{2I}{c}$

case II



$$\vec{E} = \frac{1}{2\pi\epsilon_0}\frac{Q}{\ell}\frac{1}{r}\vec{e}_\perp \qquad \vec{B} = \mu_0 n I(t)$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = nI(t)\frac{1}{2\pi\epsilon_0}\frac{Q}{\ell}\frac{1}{r}(-\vec{e}_\phi)$$

$$\frac{d\vec{P}}{dV} = \frac{1}{c^2}\vec{S} = \frac{1}{c^2}nI(t)\frac{1}{2\pi\epsilon_0}\frac{Q}{\ell}\frac{1}{r}(-\vec{e}_\phi)$$

$$\frac{d\vec{L}}{dV} = \vec{R}\times\frac{d\vec{P}}{dV} = \frac{-1}{c^2}nI(t)\frac{1}{2\pi\epsilon_0}\frac{Q}{\ell}$$

$$\vec{L} = -\frac{1}{c^2}nI(t)\frac{1}{2\pi\epsilon_0}\frac{Q}{\ell}\ell\pi(b^2-a^2) = -\frac{1}{c^2}nI(t)\frac{1}{2\epsilon_0}Q\pi(b^2-a^2)$$

$$= -\mu_0 nI(t)\frac{Q}{2}\pi(b^2-a^2)$$

$I(t): I_0 \rightarrow 0$

$$2\pi r|\vec{E}_{ind}| = -\frac{d}{dt}(\pi r^2\mu_0 nI), \quad \vec{E}_{ind} = \frac{r}{2}\mu_0 n\frac{dI(t)}{dt}$$

$$\tau = aQ|\vec{E}_{ind}|_{r=a} - bQ|\vec{E}_{ind}|_{r=b} = \frac{d\vec{L}}{dt} = \left(\frac{a^2}{2}-\frac{b^2}{2}\right)\mu_0 n\frac{dI(t)}{dt}Q$$

$$\Delta\vec{L} = \int\vec{\tau}\,dt = \int|\vec{\tau}|\,dt$$

$$= \int\left(\frac{a^2}{2}-\frac{b^2}{2}\right)\mu_0 nQ\frac{dI(t)}{dt}dt = \left(\frac{a^2}{2}-\frac{b^2}{2}\right)\mu_0 nI\Big|_{I_i}^{I_f}\cdot Q$$

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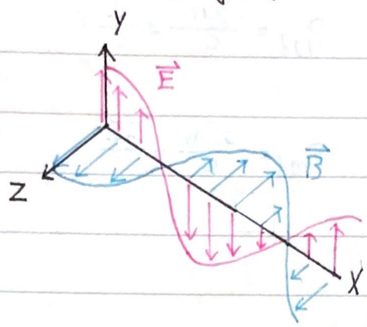
Sinusodial Electromagnetic Waves



$$\vec{E}(x,t) = E_{max}\cos(kx-\omega t)\,\vec{e}_y$$

$$\vec{B}(x,t) = B_{max}\cos(kx-\omega t)\,\vec{e}_z$$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} = kE_{max}\sin(kx-\omega t) = \omega B_{max}\sin(kx-\omega t) \Rightarrow kE_{max} = \omega B_{max}$$

$$\Rightarrow E_{max} = \frac{\omega}{k}B_{max} = \frac{2\pi f}{\frac{2\pi}{\lambda}}B_{max} = f\lambda B_{max} = cB_{max} \quad \Rightarrow E_{max} = cB_{max}$$

$$-\frac{\partial B_z}{\partial x} = \epsilon_0\mu_0\frac{\partial E_y}{\partial t} = kB_{max}\sin(kx-\omega t) = \mu_0\epsilon_0\omega E_{max}\sin(kx-\omega t), \quad kB_{max} = \epsilon_0\mu_0 E_{max}$$

$$B_{max} = \frac{\epsilon_0\mu_0\omega}{k}E_{max} = \frac{2\pi f}{\frac{2\pi c^2}{f}}E_{max} = \frac{f\lambda}{c^2}E_{max} = \frac{1}{c}E_{max} \Rightarrow E_{max} = cB_{max}$$

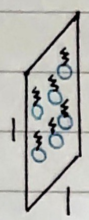
electromagnetic wave in matter

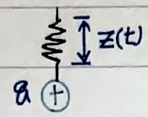
$E = vB \qquad B = c\mu v E \qquad v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{KK_m}}\frac{1}{\sqrt{\mu_0\epsilon_0}} = \frac{c}{\sqrt{KK_m}}$

$K_m \simeq 1 \quad = \frac{1}{\sqrt{K}}\frac{1}{\sqrt{\mu_0\epsilon_0}} = \frac{c}{\sqrt{K}}$

$$\frac{c}{v} = n = \sqrt{KK_m} \simeq \sqrt{K}$$

$\sigma = Nq$



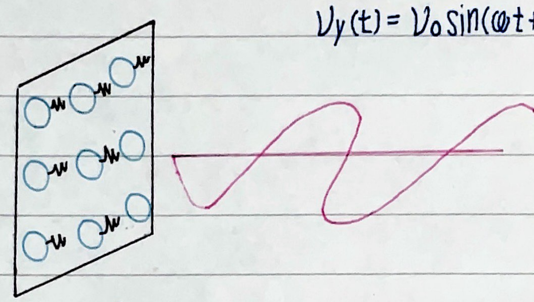


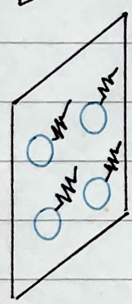
$$m\frac{d^2z(t)}{dt^2} = -kz(t)$$

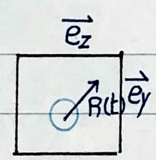
$$v_z(t) = \frac{dz(t)}{dt} = v_0 \sin(\omega t+\alpha) = v_0\cos(\omega t+\beta)$$

N atoms

$$v_y(t) = v_0\sin(\omega t+\alpha) = v_0\cos(\omega t+\beta)$$







$$\vec{R}(t) = y(t)\vec{e}_y + z(t)\vec{e}_z \qquad \vec{v}(t) = \frac{dy(t)}{dt}\vec{e}_y + \frac{dz(t)}{dt}\vec{e}_z$$

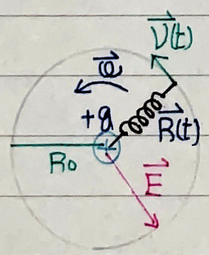
$$m\frac{d^2y(t)}{dt^2} = ky(t)$$

$$m\frac{d^2z(t)}{dt^2} = kz(t)$$

$$y(t) = R_0\cos(\omega t) \qquad v_y(t) = -R_0\omega\sin(\omega t)$$

$$z(t) = R_0\sin(\omega t) \qquad v_z(t) = R_0\omega\cos(\omega t) = -R_0\omega\sin(\omega t+\tfrac{\pi}{2})$$

$$\vec{E}(x,t) = -c\mu_0\frac{\sigma}{2}\vec{v}\left(t-\frac{x}{c}\right)$$



$$\vec{v}(t)\cdot q\vec{E}(0^+,t) = \frac{dW}{dt}$$

$$\underset{[\vec{\omega}\times\vec{R}(t)]}{\parallel}\cdot[q\vec{E}(0^+,t)]$$

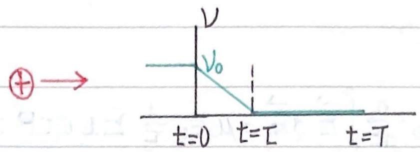
$$= \vec{\omega}\cdot[\vec{R}(t)\times q\vec{E}(0^+,t)]$$

$\vec{\omega} = \omega\vec{e}_x$ $\quad = \vec{\omega}\cdot\vec{\tau} = \vec{\omega}\cdot\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{\omega}\cdot\vec{L})$ if 等角速度

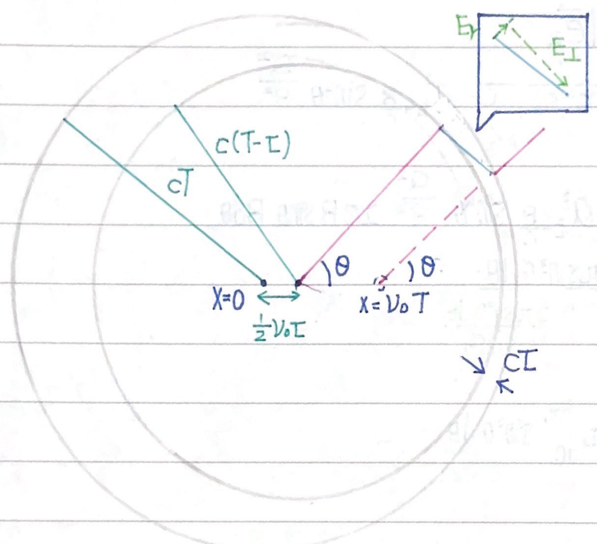
$$\Delta W = \vec{\omega}\cdot\Delta\vec{L} = |\vec{\omega}|\cdot\frac{\vec{\omega}}{|\vec{\omega}|}\cdot\Delta\vec{L} = \omega\vec{e}_x\cdot\Delta\vec{L}$$

$$\Delta L_x = \frac{\hbar\omega}{\omega} = \hbar$$

Radiation by an Accelerated Charge



assume that $v_0 \ll c$ $\quad a=\frac{v_0}{\tau}$



$\frac{E_\perp}{E_r}=\frac{v_0T\sin\theta}{c\tau}$, and $E_r=\frac{1}{4\pi\epsilon_0}\frac{q}{R^2}=\frac{1}{4\pi\epsilon_0}\frac{q}{c^2T^2}$

$\Rightarrow E_\perp=\frac{v_0T\sin\theta}{c\tau}E_r=\frac{qv_0\sin\theta}{4\pi\epsilon_0c^3T\tau}$

$=\frac{qa\sin\theta}{4\pi\epsilon_0c^2R}$, is proportional to $\frac{1}{R}$, not to $\frac{1}{R^2}$

as time goes on and R increases, the $E_\perp$ will become very much stronger than E_r

energy density $\frac{\epsilon_0E_\perp^2}{2}=\frac{q^2a^2\sin^2\theta}{32\pi^2\epsilon_0R^2c^4}$, energy $\frac{q^2a^2}{32\pi^2\epsilon_0R^2c^4}\overset{\langle\sin^2\theta\rangle}{\underline{\frac{2}{3}}}\overset{\text{volume of shell}}{\underline{4\pi R^2c\tau}}=\frac{q^2a^2\tau}{12\pi\epsilon_0c^3}$

$\Rightarrow$ total energy in transverse electromagnetic field $\frac{q^2a^2\tau}{6\pi\epsilon_0c^3}$ = power other derivation

$$\oint \vec{B}\cdot d\vec{l} = 2\pi R' \sin\theta \,|\vec{B}| = \mu_0 \epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A} = \mu_0\epsilon_0 \frac{1}{\tau} E_\perp c\tau R' 2\pi \sin\theta$$

$$\Rightarrow |\vec{B}| = \frac{1}{c^2} c\,|\vec{E}_\perp| = \frac{1}{c}|\vec{E}_\perp|$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}_\perp \times \vec{B} = \frac{1}{\mu_0}|\vec{E}_\perp||\vec{B}|\vec{e}_R$$

$$= \frac{1}{\mu_0}\frac{1}{c}|\vec{E}_\perp|^2 \vec{e}_R = \frac{1}{\mu_0 c}\,\frac{1}{16\pi^2\epsilon_0^2}\,\frac{1}{c^4}\,a^2_{t-\frac{R}{c}} \sin^2\theta \,\frac{Q^2}{R^2}$$

$$\oint \vec{S}\cdot d\vec{A} = \frac{1}{\mu_0 c}\,\frac{1}{16\pi^2\epsilon_0^2}\,\frac{1}{c^4}\,a^2_{t-\frac{R}{c}} \sin^2\theta\,\frac{Q^2}{R^2}\,2\pi R\sin\theta\, R\,d\theta$$

$$= \frac{1}{\mu_0 c}\,\frac{1}{16\pi^2\epsilon_0^2}\,\frac{Q^2a^2}{c^4}\,\underline{2\pi\sin^3\theta\, d\theta}$$

$$\sin^2\theta \,\underbrace{2\pi\sin\theta\, d\theta}_{d\Omega}$$

$$\oint \vec{S}\cdot d\vec{A} = \frac{1}{\mu_0 c}\,\frac{1}{16\pi^2\epsilon_0^2}\,\frac{Q^2a^2}{c^4}\,2\pi\int_0^\pi \sin^3\theta\, d\theta$$

$$= \frac{Q^2a^2}{\mu_0 c^5\epsilon_0^2}\,\frac{2\pi}{16\pi^2}\,\frac{4}{3}$$

$$= \frac{Q^2a^2}{6\pi c^3\epsilon_0} = \frac{dE}{dt} \quad \text{lamor formula}$$