NO.

DATE / /

Sources of Magnetic Field : Law of Biot and Savart

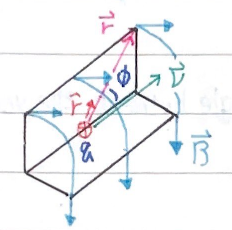
Magnetic Field of a Moving Charge

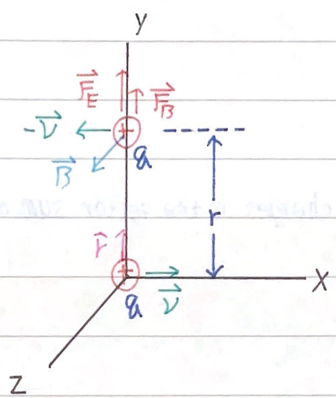
experiments show that the magnitude of $\vec{B}$

* is proportional to $|q|$ and to $\frac{1}{r^2}$
* is perpendicular to the plane containing the line from source point to field point and the particle's velocity vector $\vec{v}$

$$B = \boxed{\frac{\mu_0}{4\pi}} \frac{|q| v \sin\phi}{r^2} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{r}}{r^2}$$

proportionality constant





$$F_E = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q(v\vec{e}_x)\times\vec{e}_y}{r^2} = \frac{\mu_0}{4\pi} \frac{qv}{r^2} \vec{e}_z$$

$$\vec{F}_B = q(-\vec{v})\times\vec{B} = \frac{\mu_0}{4\pi} \frac{q^2 v^2}{r^2} \vec{e}_y$$

$$\frac{F_B}{F_E} = \frac{\mu_0 v^2}{\frac{1}{\epsilon_0}} = \epsilon\mu_0 v^2 = \frac{v^2}{c^2}$$

NO.

DATE / /

Magnetic Field of a Current Element

experiment show that

* the vector $d\vec{B}$ is perpendicular both to $d\vec{l}$ (which points in the direction of the current) and to the unit vector $\hat{r}$ directed from $d\vec{l}$ toward P
* the magnitude of $d\vec{B}$ is inversely proportional to r^2, where r is the distance from $d\vec{l}$ to P
* the magnitude of $d\vec{B}$ is proportional to the current I and to the magnitude dl of the length elements $d\vec{l}$
* the magnitude of $d\vec{B}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\vec{l}$ and $\hat{r}$

permeability of free space

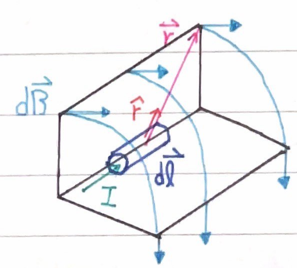
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

the total magnetic field caused by several moving charges is the vector sum of the field caused by the individual charges

$$dQ = nqA\,dl$$

$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|v_d\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{n|q|v_d A\,dl\sin\phi}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dl\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$



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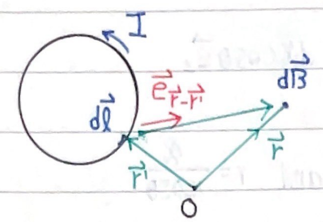
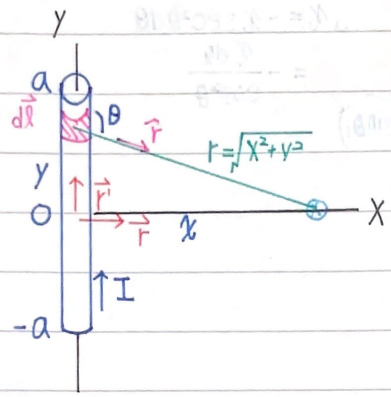
DATE / /

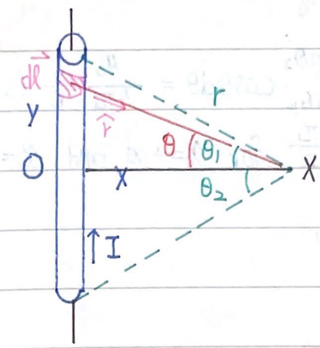
Magnetic Field of a Straight Current-Carrying Conductor



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}}{|\vec r-\vec r'|^2}\,\vec{e}_{\vec r-\vec r'}$$

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\vec{e}_{\vec r-\vec r'}}{|\vec r-\vec r'|^2}$$





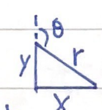
I

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{e}_{\vec r-\vec r'}}{r^2}$$

$$= \frac{\mu_0 I}{4\pi}\frac{|d\vec{l}||\vec{e}_{\vec r-\vec r'}|\sin\theta}{r^2}\,\vec{e}_\phi$$

$$d\vec{l} = dy\,\vec{e}_y$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{\sin\theta}{r^2}dy\,\vec{e}_\phi$$



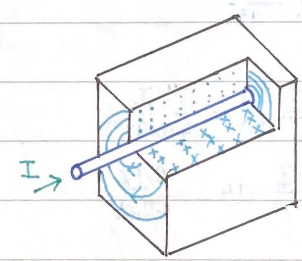
$$\frac{y}{x} = \cot(\pi-\theta) \qquad \frac{r}{x} = \csc(\pi-\theta)$$

$$dy = x\csc^2(\pi-\theta)\,d\theta$$

$$= \frac{\mu_0 I}{4\pi}\int_0^{\pi}\frac{\sin\theta}{x^2\csc^2\theta}\,x\csc^2\theta\,d\theta$$

$$= \frac{\mu_0 I}{4\pi}\frac{1}{x}\int_0^{\pi}\sin\theta\,d\theta$$

$$= \frac{\mu_0 I}{2\pi}\frac{1}{x}\,\vec{e}_\phi$$



II

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dy}{(x^2+y^2)}\frac{x}{\sqrt{x^2+y^2}}$$

$$= \frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}}$$

when $a \gg x$, in limit $a\to\infty$, $B = \frac{\mu_0 I}{2\pi}\frac{1}{x}$

NO.

DATE / /

III

$$d\vec{l}\times\hat{r} = -\left|d\vec{l}\times\hat{r}\right|\vec{e}_z = -\left[dy\sin(\tfrac{\pi}{2}-\theta)\right]\vec{e}_z = -dx\cos\theta\,\vec{e}_z$$

$$d\vec{B} = -(dB)\vec{e}_z = -\frac{\mu_0 I}{4\pi}\frac{dy\cos\theta}{r^2}\vec{e}_z \quad \text{and} \quad r = \frac{x}{\cos\theta}$$

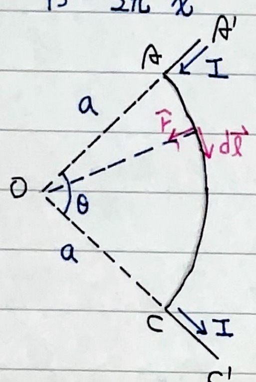
$$= \frac{\mu_0 I}{4\pi}\frac{x\,d\theta}{\cos^2\theta}\frac{\cos^2\theta}{x^2}\cos\theta \qquad y = -x\tan\theta$$

$$= \frac{\mu_0 I}{4\pi}\frac{\cos\theta}{x}d\theta \qquad dy = -x\sec^2\theta\,d\theta = -\frac{x\,d\theta}{\cos^2\theta}$$

$$B = \frac{\mu_0 I}{4\pi}\frac{1}{x}\int_{\theta_2}^{\theta_1}\cos\theta\,d\theta = \frac{\mu_0}{4\pi}\frac{1}{x}(\sin\theta_1-\sin\theta_2)$$

if $\theta_1 = \frac{\pi}{2}$ $\theta_2 = -\frac{\pi}{2}$ for $x = +\infty$ and $x = -\infty$

$$B = \frac{\mu_0 I}{2\pi}\frac{1}{x}$$



AA' and CC' $d\vec{l}\times\hat{r} = 0$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,ds}{a^2}$$

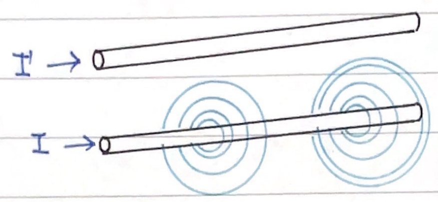
$$B = \frac{\mu_0 I}{4\pi a^2}\int dl = \frac{\mu_0 I}{4\pi a^2}l \qquad \text{when } \theta = 2\pi,\ B = \frac{\mu_0 I}{2a}$$

$$= \frac{\mu_0 I}{4\pi a^2}a\theta = \frac{\mu_0 I\theta}{4\pi a}$$

NO.

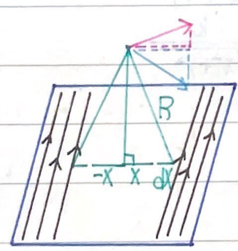
DATE / /

Force Between Parallel Conductors



$B = \frac{\mu_0 I}{2\pi r}$ $\quad F = I'LB = \frac{\mu_0 I I' L}{2\pi r}$ $\quad \frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$

one ampere is that unvarying current that, if present in each of two parallel conductors of infinite length and one meter apart in empty space, causes each conductors to experience a force of exactly 2×10^{-7} newtons per meter of length



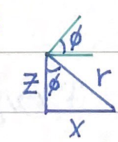
$|d\vec{B}| = \frac{\mu_0}{2\pi}\frac{(ndx)I}{R}$, $dB_x = \frac{\mu_0}{2\pi}\frac{ndxI}{R}\cos\phi$

$B_x = \int dB_x = \frac{\mu_0 I}{2\pi}\int\frac{\cos\phi}{R}dx$

$= \frac{\mu_0 I R}{2\pi}\int\frac{\cos\phi}{z\sec\phi} z\sec^2\phi \, d\phi$

$= \frac{\mu_0 I R}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi$

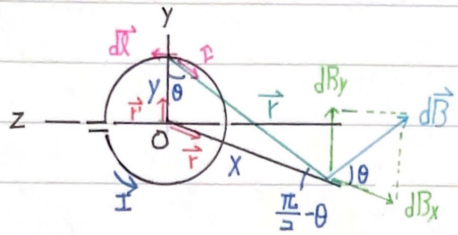
$= \begin{cases} \frac{\mu_0 I R}{2} & z>0 \\ -\frac{\mu_0 I R}{2} & z<0 \end{cases}$



$\frac{r}{z} = \sec\phi$ $\quad \frac{x}{z} = \tan\phi$

$dx = z\sec^2\phi \, d\phi$

Magnetic Field of a Circular Current Loop



I

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{e}_{\vec{r}-\vec{r'}}}{r^2} \qquad |d\vec{B}| = \frac{\mu_0 I}{4\pi} \frac{|d\vec{l}|}{r^2} \qquad dB_x = \frac{\mu_0 I}{4\pi} \frac{|d\vec{l}|}{r^2} \cos\theta$$

$$B_x = \frac{\mu_0 I}{4\pi} \frac{1}{x^2+y^2} \frac{y}{\sqrt{x^2+y^2}} \int d\vec{l}$$

$$= \frac{\mu_0 I}{4\pi} \frac{y}{(x^2+y^2)^{3/2}} 2\pi y$$

$$= \frac{\mu_0 I}{2} \frac{\pi y^2}{(x^2+y^2)^{\frac{3}{2}}}$$

II

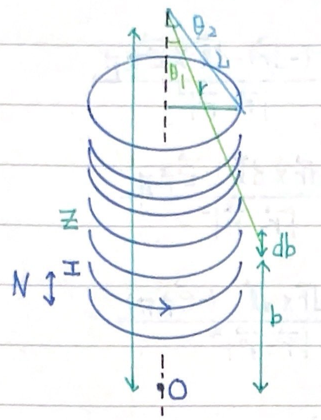
$$dB = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2+y^2)} \qquad dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi} \frac{dl}{x^2+y^2} \frac{y}{\sqrt{x^2+y^2}}$$

$$dB_y = dB\sin\theta = \frac{\mu_0 I}{4\pi} \frac{dl}{x^2+y^2} \frac{x}{\sqrt{x^2+y^2}}$$

$$B_x = \int \frac{\mu_0 I}{4\pi} \frac{y\,dl}{(x^2+y^2)^{\frac{3}{2}}} = \frac{\mu_0 I}{4\pi} \frac{y}{(x^2+y^2)^{3/2}} \int dl$$

$$= \frac{\mu_0 I}{2} \frac{y^2}{(x^2+y^2)^{\frac{3}{2}}}$$

if $x=0$, $B_x = \frac{\mu_0 I}{2y}$ $\qquad$ if $x \gg y$, $B_x \approx \frac{\mu_0 I y^2}{2x^3} = \frac{\mu_0}{2\pi} \frac{\mu}{x^3}$ $\qquad \mu = I\pi y^2$



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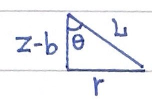
$$dB_z = \frac{\mu_0}{2\pi}\frac{I n dz\, \pi r^2}{L^3}$$

$$B_z = \int dB_z = \frac{\mu_0 I n \pi r^2}{2\pi}\int \frac{dz}{L^3}$$



$$= \frac{\mu_0 I n \pi r^2}{2\pi}\int \frac{r\csc^2\theta}{r^3\csc^3\theta}\,d\theta$$

$$= \frac{\mu_0 I n}{2\pi}\int \sin\theta\, d\theta$$

$\frac{r}{L} = \sin\theta$, $L = r\csc\theta$

$z-b = r\cot\theta$, $db = r\csc\theta\, d\theta$

$$= \frac{\mu_0 I n}{2}(-\cos\theta)\Big|_{\theta_1}^{\theta_2}$$

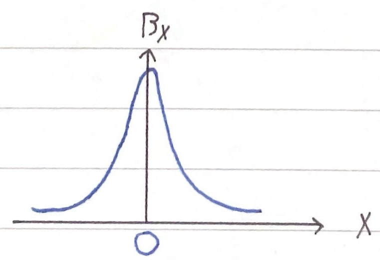
$$= \frac{\mu_0 I n}{2}(\cos\theta_1 - \cos\theta_2)$$

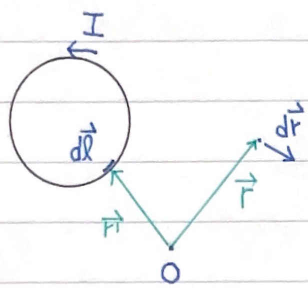
when $\theta_1 \to 0$, $\theta_2 \to 2\pi$ $B_z = \mu_0 I N$ infinite solenoid

II

$$B_x = \frac{\mu_0 N I y^2}{2(x^2+y^2)^{3/2}}$$

the maximum value of the field is at $x=0$, $B_x = \frac{\mu_0 N I}{2y}$





$$\vec{B}\cdot(-d\vec{r}) = \frac{\mu_0 I}{4\pi}\int \frac{(-d\vec{r})\cdot d\vec{l} \times \vec{e}_{\vec{r}-\vec{r'}}}{|\vec{r}-\vec{r'}|^2}$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{(-d\vec{r} \times d\vec{l})\cdot \vec{e}_{\vec{r}-\vec{r'}}}{|\vec{r}-\vec{r'}|^2}$$

$$= -\frac{\mu_0 I}{4\pi}\int \frac{-d\vec{r} \times d\vec{l} \cdot (-\vec{e}_{\vec{r}-\vec{r'}})}{|\vec{r}-\vec{r'}|^2}$$

$$= -\frac{\mu_0 I}{4\pi}\, d\Omega$$