NO.

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$$= 4\pi\left\{\frac{a^3}{32r_1}e^{-\frac{4r_1}{a}} + \left[-\frac{ar_1}{4}-\frac{a^2}{8}-\frac{a^3}{32r_1}+\frac{ar_1}{4}-\frac{a^2}{16}\right]e^{-\frac{8r_1}{a}}\right\}$$

$$= \frac{\pi a^2}{8}\left[\frac{a}{r_1}e^{-\frac{4r_1}{a}} - (2+\frac{a}{r_1})e^{-\frac{8r_1}{a}}\right]$$

$$\Rightarrow \left\langle\frac{1}{|r_1-r_2|}\right\rangle = \frac{8}{\pi a^4}4\pi\int_0^\infty\left[\frac{a}{r_1}e^{-\frac{4r_1}{a}}-(2+\frac{a}{r_1})e^{-\frac{8r_1}{a}}\right]r_1^2dr_1 \qquad = \frac{e^2}{4\pi\epsilon_0}\frac{8}{\pi a^3}\int\left[1-(1+\frac{2r_1}{a})e^{-\frac{4r_1}{a}}\right]e^{-\frac{4r_1}{a}} r_1\sin\theta dr_1 d\theta_1 d\phi_1$$

$$= \frac{32}{a^4}\left[a\int_0^\infty r_1e^{-\frac{4r_1}{a}}dr_1 - 2\int_0^\infty r_1^2e^{-\frac{8r_1}{a}}dr_1 - a\int_0^\infty r_1e^{-\frac{8r_1}{a}}dr_1\right]$$

$$= \frac{32}{a^4}\left[a(\frac{a}{4})^2-2\cdot2(\frac{a}{8})^3-a\cdot(\frac{a}{8})^2\right] = \frac{32}{a}(\frac{1}{16}-\frac{1}{128}-\frac{1}{64}) = \frac{5}{4a}$$

$$V_{ee} \approx \frac{e^2}{4\pi\epsilon_0}\left\langle\frac{1}{|r_1-r_2|}\right\rangle = \frac{5}{4}\frac{e^2}{4\pi\epsilon_0}\frac{1}{a} = \frac{5}{4}\frac{m}{\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2$$

$$= \frac{5}{2}(-E_1) = 34\text{ eV}$$

$\Rightarrow E = -108.8 + 34 = -74.8\text{ eV}$ (error 5.3%)

effective nuclear charge

use a trial function $\psi_1(\vec{r}_1,\vec{r}_2) = \left(\frac{Z}{a_0}\right)^3\frac{1}{\pi}e^{-\frac{Zr_1}{a_0}}e^{-\frac{Zr_2}{a_0}}$

$$H = -\frac{\hbar^2}{2m}(\nabla_1^2+\nabla_2^2) - \frac{e^2}{4\pi\epsilon_0}\left(\frac{Z}{r_1}+\frac{Z}{r_2}\right) + \frac{e^2}{4\pi\epsilon_0}\left(\frac{Z-2}{r_1}+\frac{Z-2}{r_2}+\frac{1}{|r_1-r_2|}\right)$$

$$\langle H\rangle = \langle\psi|\hat{H}_1|\psi\rangle + \langle\psi|\hat{H}_2|\psi\rangle + \langle\psi|\hat{H}_{12}|\psi\rangle = 2\langle\psi|\hat{H}_1|\psi\rangle + \langle\psi|\hat{H}_{12}|\psi\rangle$$

$$= 2Z^2E_1 + 2(Z-2)\frac{e^2}{4\pi\epsilon_0}\underbrace{\left\langle\frac{1}{r}\right\rangle}_{\frac{Z}{a}} + \underbrace{\langle V_{ee}\rangle}_{\frac{5Z}{8}\frac{e^2}{4\pi\epsilon_0} = -\frac{5Z}{4}E_1}$$

$$= \left(-2Z^2+\frac{27}{4}Z\right)E_1$$

$$\frac{d\langle H\rangle}{dZ} = \left(-4Z+\frac{27}{4}\right)E_1 = 0,\quad Z = \frac{27}{16},\quad \langle H\rangle = -77.5\text{ eV (error 1.9\%)}$$

In general

$$\psi_0 = \frac{1}{\pi}\left(\frac{Z_0}{a_0}\right)^3 e^{-\frac{Z_0r_1}{a_0}}e^{-\frac{Z_0r_2}{a_0}}$$

$$\langle H\rangle = 2Z_0^2E_1 - \frac{5}{4}Z_0E_1 - 4Z(Z-Z_0)E_1 = \left(-2Z^2+4ZZ_0-\frac{5}{4}Z\right)E_1$$

$$\frac{\partial\langle H\rangle}{\partial Z} = \left(-4Z+4Z_0-\frac{5}{4}\right)E_1 = 0,\quad Z = Z_0-\frac{5}{16}$$

$$\langle H_{min}\rangle = \frac{(16Z_0-5)^2}{128}E_1$$

$Z_0=1$ (H^-) $\quad E_{min} = -12.9\text{ eV}$

$Z_0=2$ (He) $\quad E_{min} = -77.5\text{ eV}$

$Z_0=3$ (Li^+) $\quad E_{min} = -196\text{ eV}$