NO.

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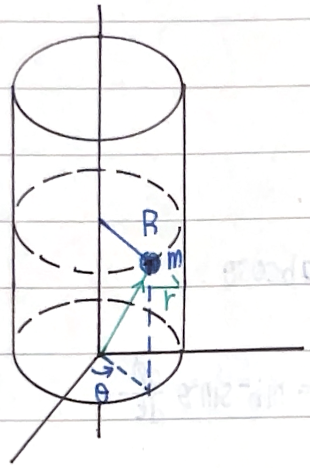
case I a particle of mass m constrained to move on the surface of a cylinder $x^2+y^2=R^2$, the force directed toward the origin and $\vec{F} = -k\vec{r}$



$$U = \frac{1}{2}kr^2 = \frac{1}{2}k(x^2+y^2+z^2) = \frac{1}{2}k(R^2+z^2)$$

$$v^2 = \left(\frac{dR}{dt}\right)^2 + R^2\left(\frac{d\theta}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 \qquad v_R = 0 \quad v_\phi = R\frac{d\phi}{dt} \quad v_z = \frac{dz}{dt}$$

$$E_K = \frac{1}{2}m\left[R^2\left(\frac{d\theta}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right]$$

$$L = E_K - U = \frac{1}{2}m\left[R^2\left(\frac{d\theta}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right] - \frac{1}{2}k(R^2+z^2)$$

$$\frac{\partial L}{\partial z} = \frac{d}{dt}\frac{\partial L}{\partial \dot{z}} \qquad \frac{\partial L}{\partial \phi} = \frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}}$$

$$-kz = m\frac{d^2z}{dt^2} \qquad 0 = \frac{d}{dt}mR^2\frac{d\phi}{dt}$$

SHM

$\vec{L}$ about the z-axis is conserved

$$P_\theta = \frac{\partial L}{\partial \dot{\theta}} = mR^2\frac{d\theta}{dt} \qquad P_z$$

$$H(z, P_\theta, P_z) = E_K + U =$$

$$\frac{dP_\theta}{dt} = -\frac{\partial H}{\partial \theta} = 0 \qquad \frac{dP_z}{dt}$$

$$P_\theta = mR^2\frac{d\theta}{dt} = \text{constant}$$

coordinates not appearing explicitly in the expressions for E_K and U are said to be cyclic

$$\frac{dq_k}{dt} = \frac{\partial H}{\partial \alpha_k} \equiv \omega_k \qquad q_k(t) = \int \omega_k\, dt \qquad -\frac{dP_k}{dt} = \frac{\partial H}{\partial q_k} = 0, \quad P_k = \alpha_k$$

NO.

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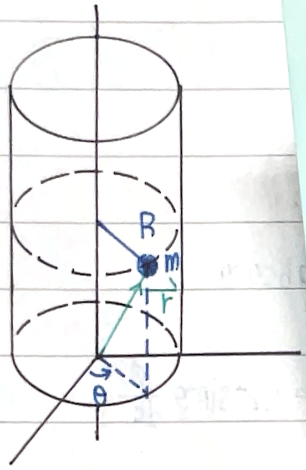
case I a particle of mass m constrained to move on the surface of a cylinder $x^2+y^2=R^2$, the force directed toward the origin



$P_\theta = \frac{\partial L}{\partial \dot\theta} = mR^2\frac{d\theta}{dt}$ $\quad P_z = \frac{\quad}{\partial \dot z} = m\frac{\quad}{dt}$

$H(Z, P_\theta, P_z) = E_k + U = \frac{P_\theta^2}{2mR^2} + \frac{P_z^2}{2m} + \frac{1}{2}kz^2$

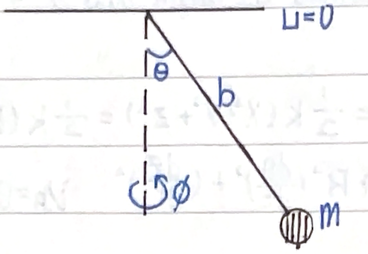
$\frac{dP_\theta}{dt} = -\frac{\partial H}{\partial \theta} = 0 \qquad \frac{dP_z}{dt} = -\frac{\partial H}{\partial Z} = -kZ \qquad \frac{d\theta}{dt} = \frac{\partial H}{\partial P_\theta} = \frac{P_\theta}{mR^2} \qquad \frac{dZ}{dt} = \frac{\partial H}{\partial P_z} = \frac{P_z}{m}$

$P_\theta = mR^2\frac{d\theta}{dt} = \text{constant} \qquad \frac{d^2Z}{dt^2} + \omega_0^2 Z = 0, \quad \omega_0^2 \equiv \frac{k}{m}$

coordinates not appearing explicitly in the expressions for E_k and U are said to be cyclic

$\frac{dq_k}{dt} = \frac{\partial H}{\partial \alpha_k} \equiv \omega_k \qquad q_k(t) = \int \omega_k\, dt \qquad -\frac{dP_k}{dt} = \frac{\partial H}{\partial q_k} = 0, \; P_k = \alpha_k$

case II



$$E_K = \frac{1}{2} m b^2 \left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2} m b^2 \sin^2\theta \left(\frac{d\phi}{dt}\right)^2 \qquad U = -mgb\cos\theta$$

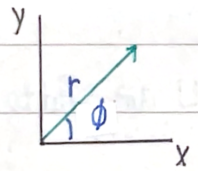
$$P_\theta = \frac{\partial L}{\partial \dot\theta} = mb^2\left(\frac{d\theta}{dt}\right) \qquad P_\phi = \frac{\partial L}{\partial \dot\phi} = mb^2 \sin^2\theta \frac{d\phi}{dt}$$

$$H = E_K + U = \frac{1}{2} mb^2 \frac{P_\theta^2}{(mb^2)^2} + \frac{1}{2} \frac{mb^2 \sin^2\theta \, P_\phi^2}{(mb^2\sin^2\theta)^2} - mgb\cos\theta$$
$$= \frac{P_\theta^2}{2mb^2} + \frac{P_\phi^2}{2mb^2\sin^2\theta} - mgb\cos\theta$$

$$\frac{d\theta}{dt} = \frac{\partial H}{\partial P_\theta} = \frac{P_\theta}{mb^2} \qquad \frac{d\phi}{dt} = \frac{\partial H}{\partial P_\phi} = \frac{P_\phi}{mb^2\sin^2\theta} \qquad \frac{dP_\theta}{dt} = -\frac{\partial H}{\partial \theta} = \frac{P_\phi^2\cos\theta}{mb^2\sin^3\theta} - mgb\sin\theta$$

$$\frac{dP_\phi}{dt} = -\frac{\partial H}{\partial \phi} = 0$$

case III one particles in two dimensions ; polar coordinate



$$v_r = \frac{dr}{dt} \qquad v_\phi = r\frac{d\phi}{dt}$$

$$E_K = \frac{1}{2} m v^2 = \frac{1}{2} m\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2\right]$$

$$L = \frac{1}{2} m\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2\right] - U(r,\phi)$$

$$\frac{\partial L}{\partial r} = \frac{d}{dt}\frac{\partial L}{\partial \dot r}$$

$$mr\left(\frac{d\phi}{dt}\right)^2 \underbrace{- \frac{\partial U}{\partial r}}_{F_r} = \frac{d}{dt}\left(m\frac{dr}{dt}\right) = m\frac{d^2r}{dt^2}$$

$$F_r = m\left[\frac{d^2r}{dt^2} - \underbrace{r\left(\frac{d\phi}{dt}\right)^2}_{\text{centripetal}}\right]$$

$$\frac{\partial L}{\partial \phi} = \frac{d}{dt}\frac{\partial L}{\partial \dot\phi}$$

$$\underbrace{-\frac{\partial U}{\partial \phi}}_{rF_\phi \text{ torque}} = \frac{d}{dt}\underbrace{\left(mr^2\frac{d\phi}{dt}\right)}_{\text{angluar momentum}} \text{ and } \nabla U = \frac{\partial U}{\partial r}\hat r + \frac{1}{r}\frac{\partial U}{\partial \phi}\hat\phi$$

$$F_\phi = -\frac{1}{r}\frac{\partial U}{\partial \phi}$$

$$\tau = \frac{d\vec{L}}{dt}$$