

$$P = \frac{\partial L}{\partial \dot{X}} = \frac{\partial E_K}{\partial \dot{X}} = (m_1 + m_2) \frac{dX}{dt}, \quad \frac{dX}{dt} = \frac{P}{m_1 + m_2}$$

NO.

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$$H = \frac{P^2}{2(m_1 + m_2)} - (m_1 + m_2)gX$$

$$\frac{dX}{dt} = \frac{\partial H}{\partial P} = \frac{H}{m_1 + m_2} \quad \frac{dP}{dt} = -\frac{\partial H}{\partial X} = (m_1 - m_2)g$$

$$\frac{d^2X}{dt^2} = -\frac{m_1 - m_2}{m_1 + m_2} g$$

$$J^2 + m_1 g X + m_2 g Y$$

$$\frac{dy}{dt} = -\frac{d^2X}{dt^2}$$

$$L = m_1 \frac{d^2X}{dt^2}$$

$$= m_2 \frac{d^2Y}{dt^2}$$

$$x + y + \pi LR = l, \quad y = -x$$

$$E_K = \frac{1}{2} m_1 \left(\frac{dx}{dt} \right)^2 + \frac{1}{2} m_2 \left(\frac{dy}{dt} \right)^2$$

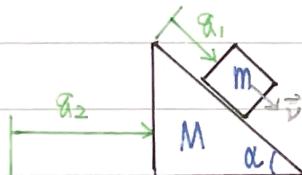
$$U = -m_1 g x - m_2 g y = -(m_1 - m_2) g X + \text{constant}$$

$$L = E_K - U = \frac{1}{2} (m_1 + m_2) \left(\frac{dx}{dt} \right)^2 + (m_1 - m_2) g X$$

$$\frac{\partial L}{\partial X} = \frac{d}{dt} \frac{\partial L}{\partial \dot{X}}, \quad (m_1 - m_2)g = (m_1 + m_2) \frac{dX^2}{dt^2}, \quad \frac{dX^2}{dt^2} = \frac{m_1 - m_2}{m_1 + m_2} g \quad \text{easily measure for } g$$

case II a block sliding on a wedge

$$\left(\frac{d\theta_1}{dt} \right)^2 \cos^2 \alpha + \left(\frac{d\theta_2}{dt} \right)^2 + 2 \frac{d\theta_1}{dt} \cos \alpha \frac{d\theta_2}{dt} + \left(\frac{d\theta_2}{dt} \right)^2 \sin^2 \alpha$$



$$T_M = \frac{1}{2} M \frac{d\theta_2}{dt} \quad \vec{v} = (v_x, v_y) = \left(\frac{d\theta_1}{dt} \cos \alpha + \frac{d\theta_2}{dt}, \frac{d\theta_1}{dt} \sin \alpha \right)$$

$$T_m = \frac{1}{2} m (v_x^2 + v_y^2) = \frac{1}{2} m \left[\left(\frac{d\theta_1}{dt} \right)^2 + \left(\frac{d\theta_2}{dt} \right)^2 + 2 \frac{d\theta_1}{dt} \frac{d\theta_2}{dt} \cos \alpha \right]$$

$$T = T_m + T_M = \frac{1}{2} (M+m) \left(\frac{d\theta_2}{dt} \right)^2 + \frac{1}{2} m \left[\left(\frac{d\theta_1}{dt} \right)^2 + 2 \frac{d\theta_1}{dt} \frac{d\theta_2}{dt} \cos \alpha \right]$$

$$U = -mg \alpha \sin \alpha$$

$$L = E_K - U = \frac{1}{2} (M+m) \left(\frac{d\theta_2}{dt} \right)^2 + \frac{1}{2} m \left[\left(\frac{d\theta_1}{dt} \right)^2 + 2 \frac{d\theta_1}{dt} \frac{d\theta_2}{dt} \cos \alpha \right] + mg \alpha \sin \alpha$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_2}$$

$$M \frac{d\dot{\theta}_2}{dt} + m \left(\frac{d\dot{\theta}_2}{dt} + \frac{d\dot{\theta}_1}{dt} \cos \alpha \right) = \text{constant}$$

$$\frac{d^2\theta_2}{dt^2} = -\frac{m}{M+m} \frac{d^2\theta_1}{dt^2} \cos \alpha$$

$$\frac{d^2\theta_1}{dt^2} = \frac{g \sin \alpha}{1 - \frac{m \cos^2 \alpha}{M+m}}$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_1}$$

$$mg \sin \alpha = \frac{d}{dt} \left[m \left(\frac{d\dot{\theta}_1}{dt} + \frac{d\dot{\theta}_2}{dt} \cos \alpha \right) \right]$$

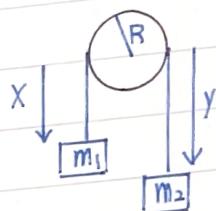
$$= M \left(\frac{d^2\theta_1}{dt^2} + \frac{d^2\theta_2}{dt^2} \cos \alpha \right)$$

case I Atwood's machine

$$L = \frac{1}{2} m_1 \left(\frac{dx}{dt} \right)^2 + \frac{1}{2} m_2 \left(\frac{dy}{dt} \right)^2 + m_1 g x + m_2 g y$$

$$f(x, y) = x + y = \text{constant}$$

$$\frac{d^2y}{dt^2} = -\frac{d^2x}{dt^2}$$



inextensible

$$\frac{\partial L}{\partial x} + \lambda \frac{\partial f}{\partial x} = \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} \quad m_1 \ddot{x} + \lambda = m_1 \frac{d^2x}{dt^2}$$

$$\frac{\partial L}{\partial y} + \lambda \frac{\partial f}{\partial y} = \frac{d}{dt} \frac{\partial L}{\partial \dot{y}} \quad m_2 \ddot{y} + \lambda = m_2 \frac{d^2y}{dt^2}$$

$$\frac{d^2x}{dt^2} = \frac{(m_1 - m_2)g}{m_1 + m_2} \quad \lambda = -\frac{d^2y}{dt^2}$$

$$x + y + \pi R = l, \quad y = -x$$

$$E_k = \frac{1}{2} m_1 \left(\frac{dx}{dt} \right)^2 + \frac{1}{2} m_2 \left(\frac{dy}{dt} \right)^2$$

$$U = -m_1 g x - m_2 g y$$

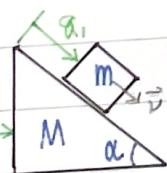
$$L = E_k - U$$

$$= \frac{1}{2} (m_1 + m_2) \left(\frac{dx}{dt} \right)^2 + (m_1 - m_2) g x$$

$$\frac{\partial L}{\partial x} = \frac{d}{dt} \frac{\partial L}{\partial \dot{x}}, \quad (m_1 - m_2) g = (m_1 + m_2) \frac{d\dot{x}^2}{dt^2}, \quad \frac{d\dot{x}^2}{dt^2} = \frac{m_1 - m_2}{m_1 + m_2} g \quad \text{easily measure for } g$$

case II a block sliding on a wedge

$$\left(\frac{d\theta_1}{dt} \right)^2 \cos^2 \alpha + \left(\frac{d\theta_2}{dt} \right)^2 + 2 \frac{d\theta_1}{dt} \frac{d\theta_2}{dt} \cos \alpha + \left(\frac{d\theta_2}{dt} \right)^2 \sin^2 \alpha$$



$$T_M = \frac{1}{2} M \frac{d\theta_2}{dt}$$

$$\vec{v} = (v_x, v_y) = \left(\frac{d\theta_1}{dt} \cos \alpha + \frac{d\theta_2}{dt}, \frac{d\theta_1}{dt} \sin \alpha \right)$$

$$T_m = \frac{1}{2} m (v_x^2 + v_y^2) = \frac{1}{2} m \left[\left(\frac{d\theta_1}{dt} \right)^2 + \left(\frac{d\theta_2}{dt} \right)^2 + 2 \frac{d\theta_1}{dt} \frac{d\theta_2}{dt} \cos \alpha \right]$$

$$T = T_m + T_M = \frac{1}{2} (M+m) \left(\frac{d\theta_2}{dt} \right)^2 + \frac{1}{2} m \left[\left(\frac{d\theta_1}{dt} \right)^2 + 2 \frac{d\theta_1}{dt} \frac{d\theta_2}{dt} \cos \alpha \right]$$

$$U = -mg a_1 \sin \alpha$$

$$L = E_k - U = \frac{1}{2} (M+m) \left(\frac{d\theta_2}{dt} \right)^2 + \frac{1}{2} m \left[\left(\frac{d\theta_1}{dt} \right)^2 + 2 \frac{d\theta_1}{dt} \frac{d\theta_2}{dt} \cos \alpha \right] + mg a_1 \sin \alpha$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = \frac{d}{dt} \frac{\partial L}{\partial \theta_2}$$

$$M \frac{d\dot{\theta}_2}{dt} + m \left(\frac{d\theta_2}{dt} + \frac{d\theta_1}{dt} \cos \alpha \right) = \text{constant}$$

$$\frac{d^2\theta_2}{dt^2} = -\frac{m}{M+m} \frac{d^2\theta_1}{dt^2} \cos \alpha$$

$$\frac{d^2\theta_1}{dt^2} = \frac{g \sin \alpha}{1 - \frac{m \cos^2 \alpha}{M+m}}$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = \frac{d}{dt} \frac{\partial L}{\partial \theta_1}$$

$$m g \sin \alpha = \frac{d}{dt} \left[M \left(\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt} \cos \alpha \right) \right]$$

$$= M \left(\frac{d^2\theta_1}{dt^2} + \frac{d^2\theta_2}{dt^2} \cos \alpha \right)$$