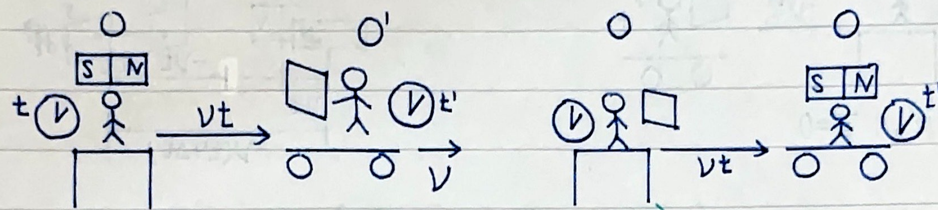
愛因斯坦特殊相對論

法拉第感應定律：磁鐵與線圈間有無相對運動

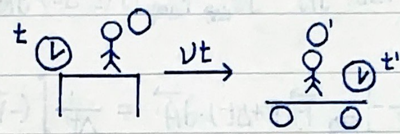


$\vec{B}\neq 0\quad \vec{E}=0\qquad \vec{B'}\neq 0\quad \vec{E'}\neq 0\qquad \vec{B}\neq 0\quad \vec{E}\neq 0$

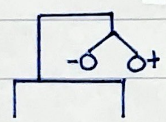
質疑　用勞羅茲力 $\vec{F}=q\vec{v}\times\vec{B}$　　$\oint_{\Gamma}\vec{E}\cdot d\vec{r}=\frac{d}{dt}\int_{S以\Gamma為框}\vec{B}\cdot d\vec{A}$

特殊相對論

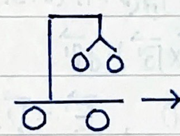
(I) 相對性原理



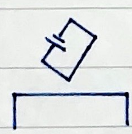
Poincare：無法以物理方法區別O或O'處於絕對靜止或絕對運動



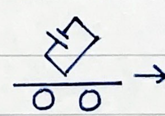
$|\vec{F}|=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$



$|\vec{F'}|=\frac{1}{4\pi\epsilon_0}\frac{q_1'q_2'}{(r')^2}$



$|\vec{B}|=\frac{\mu_0 I}{2\pi r}$



$|\vec{B'}|=\frac{\mu_0 I'}{2\pi r'}$

相對性原理在電磁現象中

O $(\vec{E},\vec{B})$　　　O' $(\vec{E'}\ \vec{B'})$

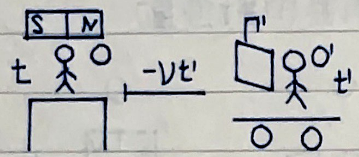
$\oint_{S封閉曲面}\vec{E}\cdot d\vec{A}=\frac{1}{\epsilon_0}$(S中之電荷總量)　　$\oint_{S'封曲}\vec{E'}\cdot d\vec{A'}=\frac{1}{\epsilon_0}(Q_{encl}\ in\ S')$

$\oint_{S封閉曲面}\vec{B}\cdot d\vec{A}=0$　　$\oint_{S'封曲}\vec{B'}\cdot d\vec{A'}=0$

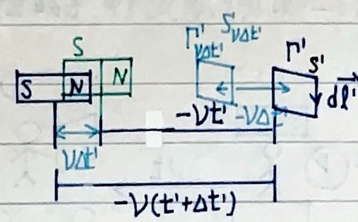
$\oint_{\Gamma}\vec{B}\cdot d\vec{l}=\mu_0\int_{S以\Gamma為邊之面}\vec{J}\cdot d\vec{A}+\mu_0\epsilon_0\frac{d}{dt}\int\vec{E}\cdot d\vec{A}$　　$\oint_{\Gamma'}\vec{B'}\cdot d\vec{l'}=\mu_0\int\vec{J'}\cdot d\vec{A'}+\mu_0\epsilon_0\frac{d}{dt'}\int\vec{E'}\cdot d\vec{A'}$

$\oint_{\Gamma}\vec{E}\cdot d\vec{l}=-\frac{d}{dt}\int_{S以\Gamma之面}\vec{B}\cdot d\vec{A}$　　$\oint_{\Gamma'}\vec{E'}\cdot d\vec{l'}=-\frac{d}{dt'}\int_{S'以\Gamma'為面}\vec{B'}\cdot d\vec{A'}$

(I) $O(\vec{E},\vec{B})$ 與 $O'(\vec{E'},\vec{B'})$ 之間關係



$\vec{B}\neq 0 \quad \vec{E}=0$



$$\oint_{\Gamma'}\vec{E'}\cdot d\vec{l'} = -\frac{1}{\Delta t'}\left[\int_{S'}\vec{B'}(t'+\Delta t')\cdot d\vec{A'} - \int_{S'}\vec{B'}(t')\cdot d\vec{A'}\right] \qquad \int_{S'}\vec{B'}(t')\cdot d\vec{A'} = \int_{S'_{v\Delta t'}}\vec{B'}(t'+\Delta t')\cdot d\vec{A'}$$

$$\int_{S'\text{外法線}}\vec{B'}(t'+\Delta t')\cdot d\vec{A'} + \int_{S'_{v\Delta t'}\text{外法線}}\vec{B'}(t'+\Delta t')\cdot d\vec{A'} + \int_{\square}\vec{B'}(t+\Delta t')\cdot d\vec{A'} = 0$$

$$\int_{S'\ \vec{n}_{amp}}\vec{B'}(t'+\Delta t')\cdot d\vec{A'} - \int_{S'_{v\Delta t'}\ \vec{n}_{amp}}\vec{B'}(t'+\Delta t')\cdot d\vec{A'} = -\int_{\square\ \vec{n}_{amp}}\vec{B'}(t'+\Delta t)\cdot d\vec{A'}$$

$$\Rightarrow \oint_{\Gamma'}\vec{E'}\cdot d\vec{l'} = \frac{-1}{\Delta t'}\cdot -\int_{\square}\vec{B'}(t'+\Delta t')\cdot d\vec{A'} = \frac{1}{\Delta t'}\oint(-\vec{v}\Delta t'\times d\vec{l'})\cdot\underbrace{\vec{B}(t'+\Delta t')}_{\vec{A}}$$

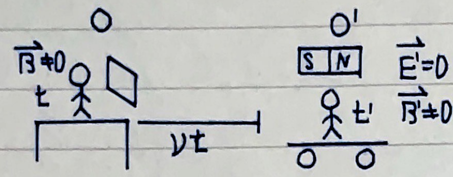
$\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C}$

$$= \frac{1}{\Delta t'}\oint_{\Gamma'}[\vec{v}\times\vec{B'}(t+\Delta t')]\cdot d\vec{l'}$$

$$\Rightarrow \ 0 = \oint_{\Gamma'}(\vec{E'}-\vec{v}\times\vec{B'})\cdot d\vec{l'}$$

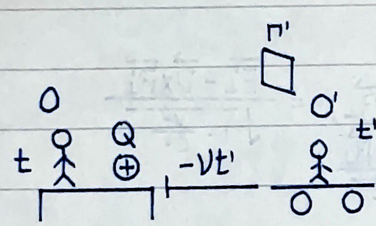
$\Rightarrow \vec{E'}-\vec{v}\times\vec{B'}=0$, $\vec{E'}=\vec{v}\times\vec{B'}$ 與 $\vec{E}=0$ 有關

(II)



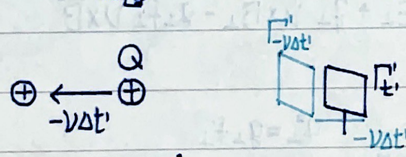
$\vec{E}=-\vec{v}\times\vec{B}$, $\vec{E}+\vec{v}\times\vec{B}=0$ 與 $\vec{E'}=0$ 有關

(III)



$\vec{E}\neq 0$ $\vec{B}=0$ $\vec{E'}\neq 0$ $\vec{B'}\neq 0$

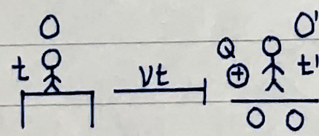
$$\int_{\Gamma'}\vec{B'}\cdot d\vec{l'}=\mu_0\underbrace{\int_{S'\perp x\Gamma'}}_{\text{為蓋}}\vec{J'}\cdot d\vec{A'}+\mu_0\epsilon_0\frac{d}{dt'}\underbrace{\int_{S'\perp x\Gamma'}}_{\text{為 side}}\vec{E'}\cdot d\vec{A'}$$



$$\int_{\square}\vec{E'}(t'+\Delta t')\cdot d\vec{A'}=0$$

$\vec{B'}+\frac{\vec{v}}{c^2}\times\vec{E'}=0$ 與 $\vec{B}=0$ 有關

(IV)



$\vec{E}\neq 0$ $\vec{B}\neq 0$ $\vec{E'}\neq 0$ $\vec{B'}=0$

$\vec{B}-\frac{\vec{v}}{c^2}\times\vec{E}=0$ 與 $\vec{B'}=0$ 有關

$$O: \vec{E} = \vec{E}_{\parallel} + \vec{E}_{\perp} \qquad \vec{B} = \vec{B}_{\parallel} + \vec{B}_{\perp}$$

$$O': \vec{E'} = \vec{E'}_{\parallel} + \vec{E'}_{\perp} \qquad \vec{B'} = \vec{B'}_{\parallel} + \vec{B'}_{\perp}$$

$$\vec{E}_{\parallel} = -k'_{\parallel}(\vec{E'} + \vec{v} \times \vec{B'})_{\parallel} = k_{\parallel} \vec{E'}_{\parallel}$$

$$\vec{E'}_{\parallel} = k_{\parallel}(\vec{E} - \vec{v} \times \vec{B})_{\parallel} = k_{\parallel} \vec{E}_{\parallel} \qquad \vec{E}_{\parallel} = \vec{E'}_{\parallel}$$

$$\vec{E'}_{\parallel} = k_{\parallel} \vec{E}_{\parallel} = k_{\parallel} k_{\parallel} \vec{E'}_{\parallel} \qquad k_{\parallel} = +1$$

$$\vec{E}_{\perp} = g_{\perp}(\vec{E'} - v \times \vec{B'})_{\perp} = g_{\perp}(\vec{E'}_{\perp} - \vec{v} \times \vec{B'}_{\perp}) = \frac{\vec{E'}_{\perp} - \vec{v} \times \vec{B'}_{\perp}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\vec{E'}_{\perp} = g_{\perp}(\vec{E}_{\perp} + \vec{v} \times \vec{B}_{\perp}) = \frac{\vec{E}_{\perp} + \vec{v} \times \vec{B}_{\perp}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\vec{B}_{\perp} = f_{\perp}\left(\vec{B'}_{\perp} + \frac{\vec{v}}{c^2} \times \vec{E'}_{\perp}\right)$$

$$\vec{B'}_{\perp} = f_{\perp}\left(\vec{B}_{\perp} - \frac{\vec{v}}{c^2} \times \vec{E}_{\perp}\right)$$

$$\vec{E}_{\perp} = g_{\perp}\left[g_{\perp}(\vec{E}_{\perp} + \vec{v} \times \vec{B}_{\perp}) - g_{\perp}\vec{v} \times f_{\perp}\left(\vec{B}_{\perp} - \frac{\vec{v}}{c^2} \times \vec{E}_{\perp}\right)\right)$$

$$= g_{\perp}\vec{E}_{\perp} - g_{\perp} f_{\perp} \frac{v^2}{c^2} \vec{E}_{\perp} + g_{\perp}^2 \vec{v} \times \vec{B}_{\perp} - g_{\perp} f_{\perp} \vec{v} \times \vec{B}$$

$$= g_{\perp}^2 \left(1 - \frac{v^2}{c^2}\right) \vec{E}_{\perp}$$

$$g_{\perp}^2 = g_{\perp} f_{\perp}$$

$$g_{\perp} = f_{\perp} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma$$

$$g_{\perp}^2 \left(1 - \frac{v^2}{c^2}\right) = 1$$