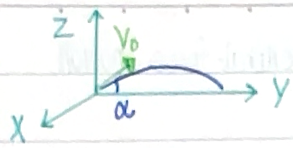
north latitude λ,



$t=0 \quad \frac{dx}{dt}=0 \quad \frac{dy}{dt}=V_0\cos\alpha \quad \frac{dz}{dt}=V_0\sin\alpha \quad x=y=z=0$

nelegect centrifugal force

$$\frac{d^2x}{dt^2}=2\frac{dy}{dt}\omega\sin\lambda \quad \frac{d^2y}{dt^2}=-2\left(\frac{dx}{dt}\omega\sin\lambda+\frac{dz}{dt}\omega\cos\lambda\right) \quad \frac{d^2z}{dt^2}=-g+2\frac{dy}{dt}\omega\cos\lambda$$

$$\Rightarrow \frac{dx}{dt}=2y\omega\sin\lambda \quad \frac{dy}{dt}=-2(x\omega\sin\lambda+z\omega\cos\lambda)+V_0\cos\alpha \quad \frac{dz}{dt}=-gt+2y\omega\cos\lambda+V_0\sin\alpha$$

let $\omega=0$

$$\frac{dx}{dt}=0 \quad \frac{dy}{dt}=V_0\cos\alpha \quad \frac{dz}{dt}=V_0\sin\alpha-gt$$

$$x=0 \quad y=V_0t\cos\alpha \quad z=V_0\sin\alpha-\frac{1}{2}gt^2$$

$$\Rightarrow \frac{dx}{dt}=2\omega V_0t\cos\alpha\sin\lambda \quad \frac{dy}{dt}=V_0\cos\alpha-2\left(V_0t\sin\alpha-\frac{1}{2}gt^2\right)\omega\cos\lambda$$

$$\frac{dz}{dt}=V_0\sin\alpha-gt+2\omega V_0t\cos\alpha\cos\lambda$$

$$\Rightarrow x=\omega V_0t^2\cos\alpha\sin\lambda \quad y=V_0t\cos\alpha-\left(V_0t^2\sin\alpha-\frac{1}{3}gt^3\right)\omega\cos\lambda$$

$$z=V_0t\sin\alpha-\frac{1}{2}gt^2+\omega V_0t^2\cos\alpha\cos\lambda$$

$$t=\frac{2V_0\sin\alpha}{g-2V_0\omega\cos\alpha\cos\lambda}$$

$$x=\frac{4V_0^3\omega\sin\lambda\cos\alpha\sin^2\alpha}{(g-2V_0\omega\cos\alpha\cos\lambda)^2}$$

NO.

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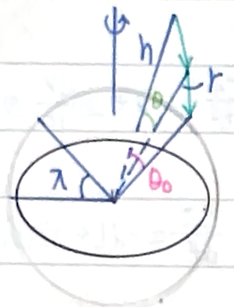
as example1 but use only the theory of central-force motion

$$v_{horizontal} = r\omega\cos\lambda = (R+h)\omega\cos\lambda$$

$$l = mrv_{hor} = m(R+h)^2\omega\cos\lambda$$



$$\frac{\alpha}{r} = 1-\varepsilon\cos\theta \qquad \frac{\alpha}{R+h} = 1-\varepsilon$$

$$r = \frac{(1-\varepsilon)(R+h)}{1-\varepsilon\cos\theta}$$

$$\frac{1}{2}r^2\frac{d\theta}{dt} = \frac{l}{2m} \qquad t = \frac{m}{l}\int_0^\theta r^2 d\theta = \frac{1}{\omega\cos\lambda}\int_0^\theta \left(\frac{1-\varepsilon}{1-\varepsilon\cos\theta}\right)^2 d\theta$$

let $\theta = \theta_0$ when $r = R$, $\frac{R}{R+h} = \frac{1-\varepsilon}{1-\varepsilon\cos\theta_0}$

$$1+\frac{h}{R} = \frac{1-\varepsilon\cos\theta_0}{1-\varepsilon} = \frac{1-\varepsilon\left[1-2\sin^2\left(\frac{\theta_0}{2}\right)\right]}{1-\varepsilon} = 1+\frac{2\varepsilon}{1-\varepsilon}\sin^2\frac{\theta_0}{2}$$

$$\frac{h}{R} = \frac{2\varepsilon}{1-\varepsilon}\sin^2\frac{\theta_0}{2} \cong \frac{\varepsilon\theta_0^2}{2(1-\varepsilon)}$$

$$t = \frac{1}{\omega\cos\lambda}\int_0^\theta \frac{d\theta}{\left[1+\frac{2\varepsilon}{1-\varepsilon}\sin^2\frac{\theta}{2}\right]^2} \cong \frac{1}{\omega\cos\lambda}\int_0^\theta \frac{d\theta}{\left[1+\frac{\varepsilon\theta^2}{2}(1-\varepsilon)\right]^2}$$

$$T \cong \frac{1}{\omega\cos\lambda}\int_0^{\theta_0}\frac{d\theta}{\left(1+\frac{h\theta^2}{R\theta_0^2}\right)^2} \cong \frac{1}{\omega\cos\lambda}\int_0^{\theta_0}\left(1-\frac{2h}{R\theta_0^2}\theta^2\right)d\theta = \frac{1}{\omega\cos\lambda}\left(1-\frac{2h}{3R}\right)\theta_0$$

$$\theta_0 \cong \frac{\omega T\cos\lambda}{1-\frac{2h}{3R}} \cong \omega T\cos\lambda\left(1+\frac{2h}{3R}\right)$$

net easterly deviation $d = R\theta_0 - R\omega T\cos\lambda = \frac{2}{3}h\omega T\cos\lambda \quad T \approx \sqrt{\frac{2h}{g}}$

$$= \frac{1}{3}\omega\cos\lambda\sqrt{\frac{8h^3}{g}}$$

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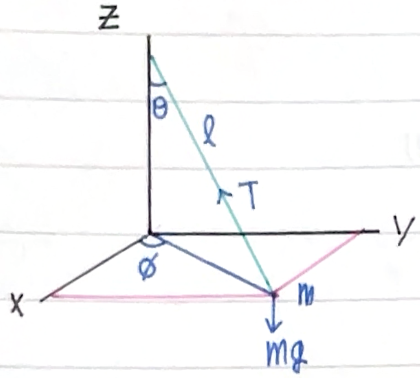
Foucault pendulum



$$\vec{a_r} = \vec{g} + \frac{\vec{T}}{m} - 2\vec{\omega}\times\vec{v_r} \qquad \vec{g} = -g\vec{e_z}$$

$$\vec{T} = -T\sin\theta\cos\phi\,\vec{e_x} - T\sin\theta\sin\phi\,\vec{e_y} + T\cos\theta\,\vec{e_z} \qquad \theta \text{ is small}$$

$$\simeq -T\frac{x}{l}\vec{e_x} - T\frac{y}{l}\vec{e_y} + T\vec{e_z}$$

$$\vec{\omega} = -\omega\cos\lambda\,\vec{e_x} + \omega\sin\lambda\,\vec{e_z} \qquad \vec{v_r} = \frac{dx}{dt}\vec{e_x} + \frac{dy}{dt}\vec{e_y}$$

$$\vec{\omega}\times\vec{v_r} = -\frac{dy}{dt}\omega\sin\lambda\,\vec{e_x} + \frac{dx}{dt}\omega\sin\lambda\,\vec{e_y} - \frac{dy}{dt}\omega\cos\lambda\,\vec{e_z}$$

$$\frac{d^2x}{dt^2} = -\frac{T}{m}\frac{x}{l} + 2\frac{dy}{dt}\omega\sin\lambda \qquad \frac{d^2y}{dt^2} = -\frac{T}{m}\frac{y}{l} + 2\frac{dx}{dt}\omega\sin\lambda$$

let $T \approx mg$ $\quad \alpha^2 = \frac{T}{ml} \approx \frac{g}{l} \quad \Omega = \omega\sin\lambda$

$$\frac{d^2x}{dt^2} + \alpha^2 x = 2\Omega\frac{dy}{dt} \quad -(1) \qquad \frac{d^2y}{dt^2} + \alpha^2 y = -2\Omega\frac{dx}{dt} \quad -(2)$$

(2) × i + (1) :

$$\left(\frac{d^2x}{dt^2} + \frac{dy}{dt}i\right) + \alpha^2(x+iy) = -2\Omega\left(i\frac{dx}{dt} - \frac{dy}{dt}\right) = -2\Omega i\left(\frac{dx}{dt} + \frac{dy}{dt}i\right)$$

let $s = x + iy$

$$\frac{d^2s}{dt^2} + 2i\Omega\frac{ds}{dt} + \alpha^2 s = 0$$

$$s(t) = e^{-i\Omega t}\left(Ae^{\sqrt{-\omega_z^2-\alpha^2}\,t} + Be^{-\sqrt{-\omega_z^2-\alpha^2}\,t}\right)$$

when $\omega = 0$, $\Omega = 0$ $\qquad s(t) = Ae^{i\alpha t} + Be^{-i\alpha t} = A'\cos\alpha t + B'\sin\alpha t$

$$T = \frac{2\pi}{\alpha} = 2\pi\sqrt{\frac{l}{g}} \qquad \alpha \gg \Omega$$

$$\Rightarrow S(t) \approx e^{-\Omega t i}\left(Ae^{i\alpha t} + Be^{-i\alpha t}\right) = e^{-i\Omega t} S'(t) \quad \text{— (3)}$$

$$t = T_p = \frac{2\pi}{\Omega} = \frac{2\pi}{\omega \sin\lambda}, \quad e^{-i\Omega T_p} = e^{-i2\pi} = 1$$

let $S = X + iY \quad S' = X' + iY'$ into (3)

$$X(t) + iY(t) = (\cos\Omega t - i\sin\Omega t)\left[X'(t) + iY'(t)\right]$$
$$= (X'\cos\Omega t + Y'\sin\Omega t) + i(-X'\sin\Omega t + Y'\cos\Omega t)$$

$$\begin{cases} X = X'\cos\Omega t + Y'\sin\Omega t \\ Y = -X'\sin\Omega t + Y'\cos\Omega t \end{cases}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\Omega t & \sin\Omega t \\ -\sin\Omega t & \cos\Omega t \end{bmatrix}}_{R} \begin{bmatrix} X' \\ Y' \end{bmatrix}$$