$$\frac{d}{d\dot{q}_i}\sum_{jk} A_{jk}\frac{dq_j}{dt}\frac{dq_k}{dt} = \sum_{jk} A_{jk}\left(\frac{d\dot{q}_j}{d\dot{q}_i}\frac{dq_k}{dt} + \frac{dq_j}{dt}\frac{d\dot{q}_k}{d\dot{q}_i}\right)$$

$$= \sum_{jk} A_{jk}\,\delta_{ji}\frac{dq_k}{dt} + \frac{dq_j}{dt}\delta_{ki} = \sum_k A_{ik}\frac{dq_k}{dt} + \sum_j A_{ji}\frac{dq_j}{dt}$$

$$= 2\sum_j A_{ij}\frac{dq_j}{dt}$$

$$\frac{\partial T}{\partial \dot{q}_i} = \frac{\partial L}{\partial \dot{q}_i} = p_i = \sum_j A_{ij}\frac{dq_j}{dt}$$

$$\sum p_i \frac{dq_i}{dt} = \sum_i\left(\sum_j A_{ij}\frac{dq_j}{dt}\right)\frac{dq_i}{dt} = \sum_{ij} A_{ij}\frac{dq_i}{dt}\frac{dq_j}{dt} = 2E_k$$

NO.

DATE

Conservation Theorems Revisited

conservation of energy

time is homogenous within an inertial reference frame

therefore, the Lagrangian that describes a closed system cannot depe

$$\frac{\partial L}{\partial t} = 0 = \sum_j \frac{\partial L}{\partial q_j}\frac{dq_j}{dt} + \sum_j \frac{\partial L}{\partial q_j'}\frac{d^2 q_j}{dt^2}$$

and $\frac{\partial L}{\partial q_j} = \frac{d}{dt}\frac{\partial L}{\partial q_j'}$

$$\Rightarrow \quad \frac{\partial L}{\partial t} = \sum_j \frac{dq_j}{dt}\frac{d}{dt}\frac{\partial L}{\partial q_j'} + \sum_j \frac{\partial L}{\partial q_j'}\frac{d^2 q_j}{dt^2}, \quad \frac{dL}{dt} - \sum_j \frac{d}{dt}\left(\frac{dq_j}{dt}\frac{\partial L}{\partial q_j'}\right) =$$

so that $\frac{d}{dt}\left(L - \sum_j \frac{dq_j}{dt}\frac{\partial L}{\partial q_j'}\right) = 0$, $\quad L - \sum_j \frac{dq_j}{dt}\frac{\partial L}{\partial q_j'} = -H = \text{constant}$

$$\frac{\partial L}{\partial q_j'} = \frac{\partial (E_k - U)}{\partial q_j'} = \frac{\partial E_k}{\partial q_j'} \quad \Rightarrow \quad (E_k - U) - \sum_j \frac{dq_j}{dt}\frac{\partial E_k}{\partial q_j'} = -H$$

$$\Rightarrow \quad E_k - U - 2E_k = -H$$

$$\Rightarrow \quad E_k + U = E_t = H = \text{constant}$$

Hamiltonian

H = Etotal if the following conditions are

1. the equations of the transforma
coordinates

2. th