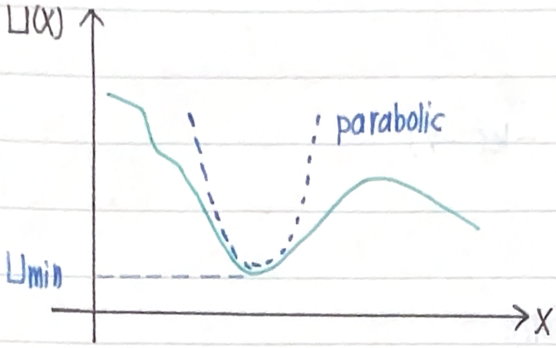
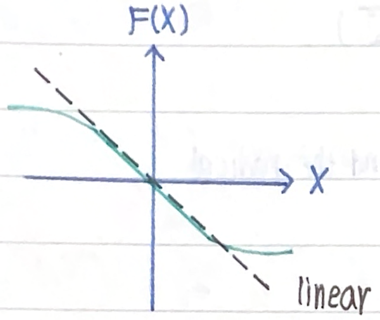
Nonlinear Oscillations and Chaos

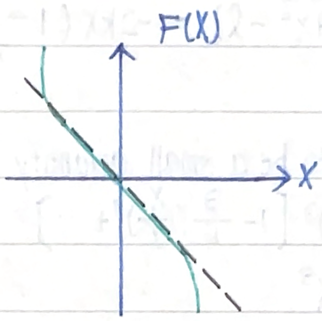
Nonlinear Oscillations



in a symmetric situation $F(x) \cong -kx + \epsilon x^3$

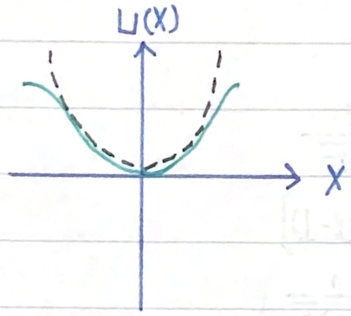
$$U(x) = \frac{1}{2}kx^2 - \frac{1}{4}\epsilon x^4$$

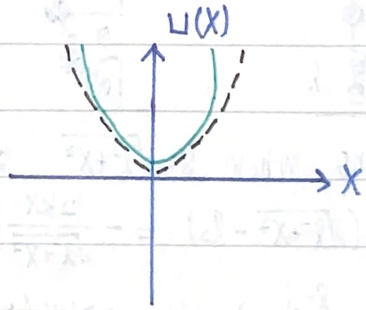




$\epsilon > 0$ | $\epsilon < 0$

the force is less than linear term : soft | hard





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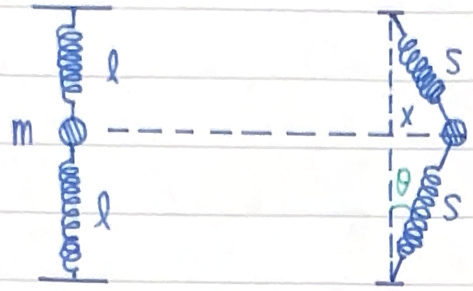


$-k(s-\ell)$

$F = -2k(s-\ell)\sin\theta$ net horizontal force

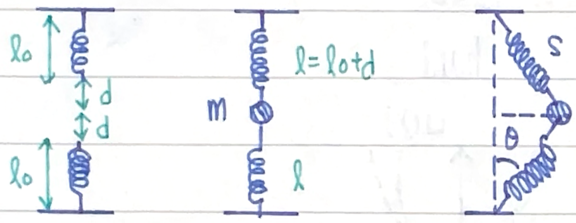
$s = \sqrt{\ell^2+x^2}$ $\qquad \sin\theta = \frac{x}{s} = \frac{x}{\sqrt{\ell^2+x^2}}$

$$F = -\frac{2kx}{\sqrt{\ell^2+x^2}}\left(\sqrt{\ell^2+x^2}-\ell\right) = -2kx\left(1-\frac{1}{\sqrt{1+(\frac{x}{\ell})^2}}\right)$$

if we consider $\frac{x}{\ell}$ to be a small quantity and expand the radical

$$\therefore F = -k\ell\left(\frac{x}{\ell}\right)^3\left[1-\frac{3}{4}\left(\frac{x}{\ell}\right)^3+\cdots\right]$$

$$\cong -\left(\frac{k}{\ell^2}\right)x^3$$



$F = -2k(s-\ell_0)\sin\theta$ where $s = \sqrt{\ell^2+x^2}$ $\quad \sin\theta = \frac{x}{\sqrt{\ell^2+x^2}}$

$$F(x) = -\frac{2kx}{\sqrt{\ell^2+x^2}}\left(\sqrt{\ell^2+x^2}-\ell_0\right) = -\frac{2kx}{\sqrt{\ell^2+x^2}}\left[\sqrt{\ell^2+x^2}-(\ell-d)\right]$$

$$= -2kx\left(1-\frac{\ell-d}{\sqrt{\ell^2+x^2}}\right) = -2kx\left(1-\frac{\ell-d}{\ell}\frac{1}{\sqrt{1+\frac{x^2}{\ell^2}}}\right)$$

$$\cong -2kx\left[1-\frac{\ell-d}{\ell}\left(1-\frac{1}{2}\frac{x^2}{\ell^2}\right)\right] = -2kx\left[1-\left(1-\frac{d}{\ell}\right)+\frac{1}{2}\frac{\ell-d}{\ell}\frac{x^2}{\ell^2}\right]$$

$$= -\frac{2kd}{\ell}x - \frac{k(\ell-d)}{\ell^3}x^3$$

$U(x) = -\int F(x)\,dx$ $\qquad \varepsilon' = -\frac{k(\ell-d)}{\ell^3} < 0 \Rightarrow$ hard

$$= \frac{kd}{\ell}x^2 + \frac{k(\ell-d)}{4\ell^3}x^4$$

NO.

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if we have a driving force $F_0\cos(\omega t)$, the equation of motion for the stretched spring

$$m\frac{d^2x}{dt^2} = -\frac{2kd}{l}x - \frac{k(l-d)}{l^3}x^3 + F_0\cos(\omega t)$$

let $\varepsilon = \frac{\varepsilon'}{m}$ $\quad a = \frac{2kd}{ml}$ $\quad G = \frac{F_0}{m}$

then $\frac{d^2x}{dt^2} = -ax + \varepsilon x^3 + G\cos(\omega t)$

try a solution $x_1 = A\cos(\omega t)$

$$\Rightarrow \frac{d^2x_2}{dt^2} = -aA\cos(\omega t) + \varepsilon A^3\cos^3(\omega t) + G\cos(\omega t)$$

$$\cos^3\omega t = \frac{3}{4}\cos\omega t + \frac{1}{4}\cos 3\omega t$$

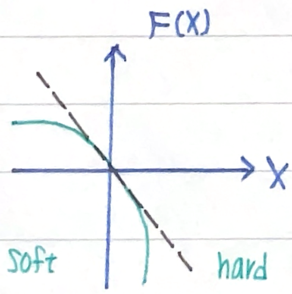
$$\frac{d^2x_2}{dt^2} = -\left(aA - \frac{3}{4}\varepsilon A^3 - G\right)\cos\omega t - \frac{\varepsilon A^3}{4}\cos 3\omega t$$

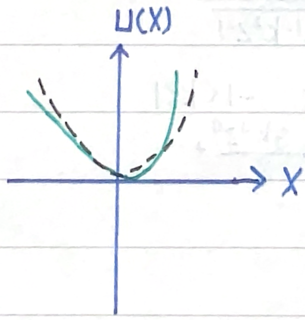
integrating twice $\downarrow$

$$x_2 = \frac{1}{\omega^2}\left(aA - \frac{3}{4}\varepsilon A^3 - G\right)\cos\omega t - \frac{\varepsilon A^3}{36\omega^2}\cos 3\omega t$$

in real physical situations, we are often concerned with symmetric forces and potentials, but some cases have asymmetric forms, for example

$$F(x) = -kx + \lambda x^2$$
$$U(x) = \frac{1}{2}kx^2 - \frac{1}{3}\lambda x^3$$





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