Sources of Magnetic Field : Ampere's Law

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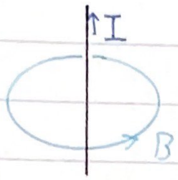
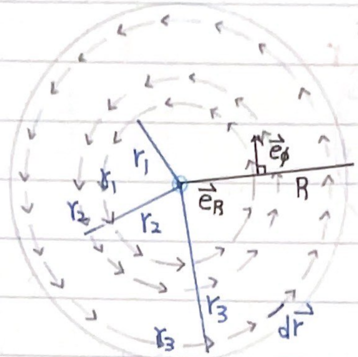
Ampere Circuital Theorem

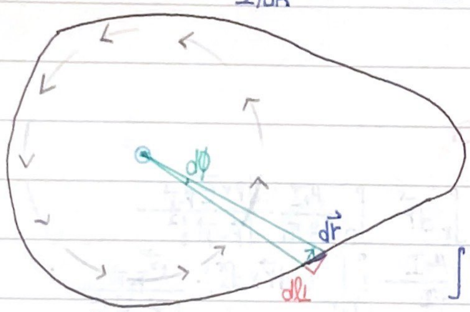




$$\vec{B} = \frac{\mu_0 I}{2\pi R}\vec{e}_\phi$$

$$\int \vec{B}\cdot d\vec{r} \underset{\because \vec{B} \parallel d\vec{r}}{=} \int |\vec{B}||d\vec{r}| = |\vec{B}|\int_r |d\vec{r}|$$

$$= \frac{\mu_0 I}{2\pi R} 2\pi R = \mu_0 I$$ prototype of Ampere's law



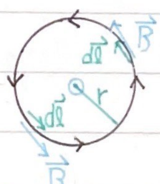
γ_4

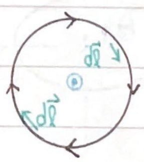
for γ_4

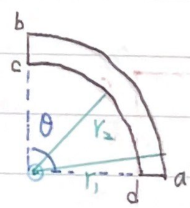
$$d\vec{r} = R\,d\phi\,\vec{e}_\phi + d\vec{l}_\perp$$

$$\int \vec{B}\cdot d\vec{r} = \int \frac{\mu_0 I}{2\pi R}\vec{e}_\phi \cdot R\,d\phi\,\vec{e}_\phi = \int \frac{\mu_0 I}{2\pi} d\phi$$

$$= \mu_0 I \int \frac{d\phi}{2\pi} = \mu_0 I$$







$$\oint \vec{B}\cdot d\vec{l} = \oint B_\parallel\, dl$$

$$= B \oint dl = \frac{\mu_0 I}{2\pi r} 2\pi r$$

$$= \mu_0 I$$

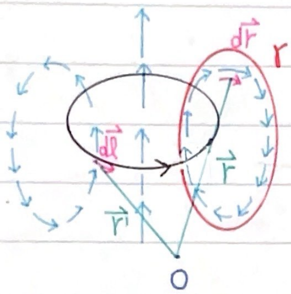
$$\oint \vec{B}\cdot d\vec{l} = -\mu_0 I$$

$$\oint \vec{B}\cdot d\vec{l} = \oint B_\parallel\, dl$$

$$= B_1 \int_a^b dl + 0\int_c^d dl + (-B_2)\int_c^d dl + 0\int_d^a dl$$

$$= \frac{\mu_0 I}{2\pi r_1} r_1\theta - \frac{\mu_0 I}{2\pi r_2} r_2\theta$$

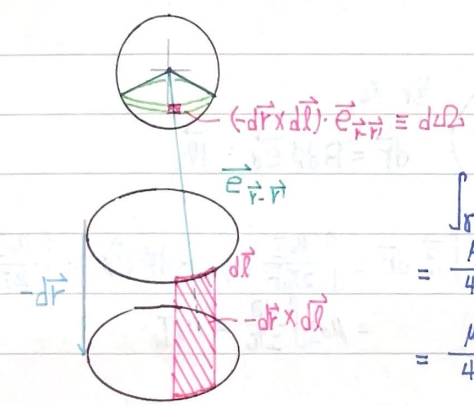
$$= 0$$



$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{e}_{\vec{r}-\vec{r}'}}{|\vec{r}-\vec{r}'|^2}, \quad B = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}\times\vec{e}_{\vec{r}-\vec{r}'}}{|\vec{r}-\vec{r}'|^2}$$

$$d\vec{r}\cdot\vec{B} = \frac{\mu_0 I}{4\pi}\int d\vec{r}\cdot\frac{d\vec{l}\times\vec{e}_{\vec{r}-\vec{r}'}}{|\vec{r}-\vec{r}'|^2} \ \underset{\vec{a}\cdot(\vec{B}\times\vec{C})=(\vec{A}\times\vec{B})\cdot\vec{C}}{=}\ \frac{\mu_0 I}{4\pi}\int\frac{(d\vec{r}\times d\vec{l})\cdot\vec{e}_{\vec{r}-\vec{r}'}}{|\vec{r}-\vec{r}'|^2}$$

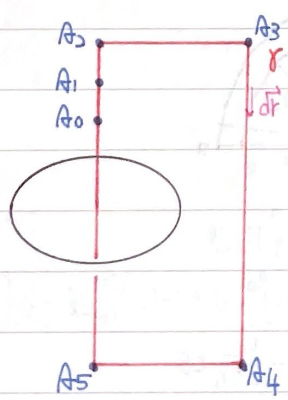
$$= \frac{\mu_0 I}{4\pi}\int\frac{(-d\vec{r}\times d\vec{l})\cdot(-\vec{e}_{\vec{r}-\vec{r}'})}{|\vec{r}-\vec{r}'|^2} = \frac{\mu_0 I}{4\pi}\int(d\Omega)$$



$$\int_\gamma d\vec{r}\cdot\int_\Gamma\frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{e}_{\vec{r}-\vec{r}'}}{|\vec{r}-\vec{r}'|^2}$$

$$= \frac{\mu_0 I}{4\pi}\int_\gamma\int_\Gamma(-d\vec{r}\times d\vec{l})\cdot\frac{\vec{e}_{\vec{r}-\vec{r}'}}{|\vec{r}-\vec{r}'|^2}$$

$$= \frac{\mu_0 I}{4\pi}\int_\gamma d\Omega$$



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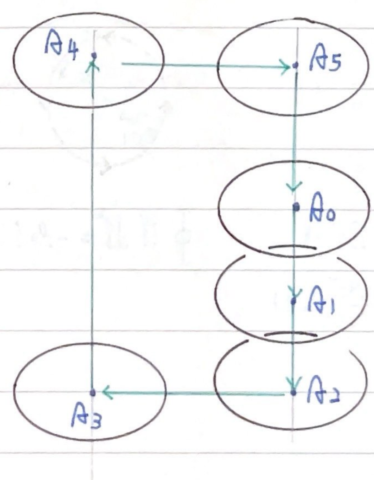

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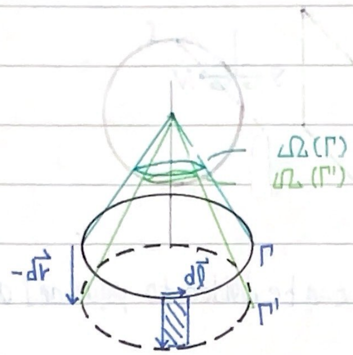
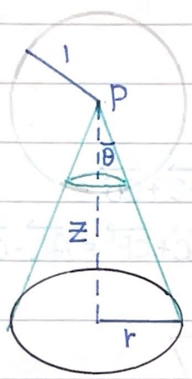
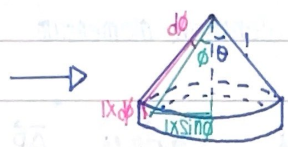
$$\int_\gamma d\vec{r}\cdot\vec{B} = \cdots = \frac{\mu_0 I}{4\pi}\int_\gamma\int_\Gamma \frac{(-d\vec{r}\times d\vec{l})\cdot \vec{e}_{\vec{r}-\vec{r}'}}{|\vec{r}-\vec{r}'|} = \frac{\mu_0 I}{4\pi}\int(-d\Omega)$$



$$\Omega(\Gamma')-\Omega(\Gamma) = -d\Omega$$





$$\int_0^\theta 2\pi \sin\phi \, d\phi = 2\pi(1-\cos\theta) = \Omega(\Gamma)$$

$$\tan\theta = \frac{r}{z}$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+\frac{r^2}{z^2}}} \qquad \Omega(\Gamma) = 2\pi\left(1-\frac{1}{\sqrt{1+\frac{r^2}{z^2}}}\right)$$

$$B_z = -\frac{\mu_0 I}{4\pi}\frac{\partial\Omega}{\partial z} = \frac{\mu_0 I}{4\pi}2\pi\frac{-\frac{1}{2}r^2(-2)z^{-3}}{\left(1+\frac{r^2}{z^2}\right)^{\frac{3}{2}}}$$

$$= \frac{\mu_0 I}{4\pi}\frac{2\pi r^2}{(z^2+r^2)^{\frac{3}{2}}} = \frac{\mu_0 I}{4\pi}\frac{2\pi r}{z^2+r^2}\frac{r}{\sqrt{z^2+r^2}}$$

$$d\vec{r}\cdot\vec{B} = -\frac{\mu_0 I}{4\pi}(-d\Omega), \quad \vec{B} = -\frac{\mu_0 I}{4\pi}\left(-\frac{d\Omega}{d\vec{r}}\right) = -\frac{\mu_0 I}{4\pi}\nabla\Omega$$

hence, $\frac{1}{4\pi}\left(-\int_\gamma d\Omega\right)$, $\oint_\gamma d\Omega = N4\pi$

$\Rightarrow \frac{\mu_0 I}{4\pi}\int(-d\Omega) = N\mu_0 I \quad N\in\mathbb{N}$ (topological invariant)

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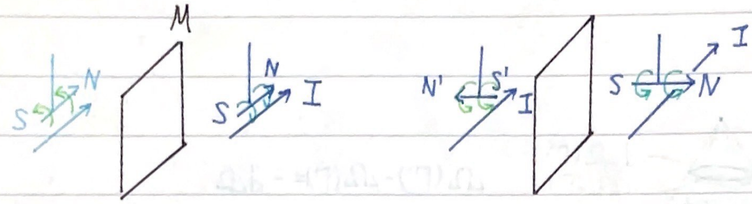
Symmetry

symmetry : physical feature of the system that is remain unchanged under some transformation

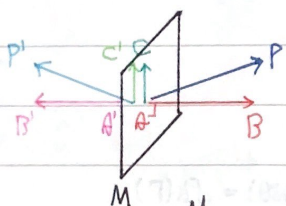


under mirror reflection transformation, vector can be divided to polar and axial vector

polar vector

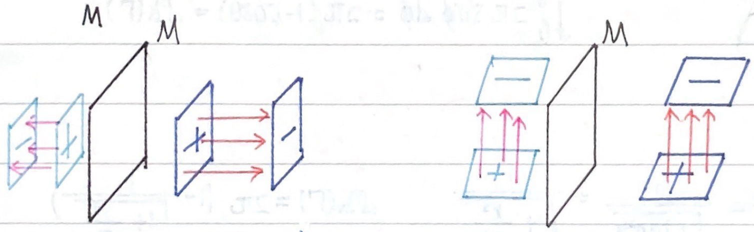
if $\vec{V} = \vec{V}_\perp + \vec{V}_\parallel$ $\quad \vec{V}' = -\vec{V}_\perp + \vec{V}_\parallel$ $\quad$ the $\vec{V}$ is polar vector

like electric field, displacement, acceleration, momentum



$\vec{AB} \perp M$ $\quad \vec{AC} \parallel M$ $\quad \vec{AP} = \vec{AC} + \vec{CP}$

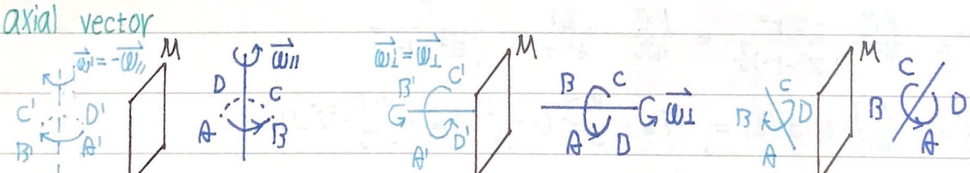
$\vec{A'B'} = -\vec{AB}$ $\quad \vec{AC} = \vec{AC'}$ $\quad \vec{AP'} = \vec{AC} + \vec{CP'} = \vec{AC} - \vec{CP}$



$\vec{E}_\perp \perp M$ $\quad \vec{E}'_\perp = \vec{E}_\perp$

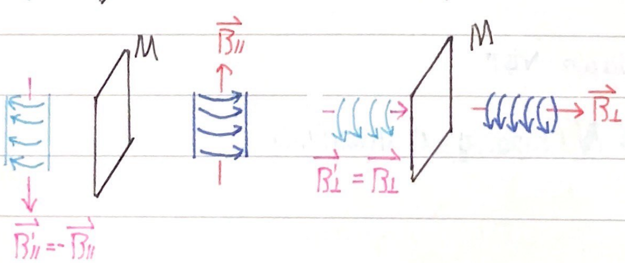
$\vec{E}_\parallel \parallel M$ $\quad \vec{E}'_\parallel = -\vec{E}_\parallel$

axial vector



$\vec{\omega}' = -\vec{\omega}_\parallel + \vec{\omega}_\perp$

$\vec{\omega} = \vec{\omega}_\parallel + \vec{\omega}_\perp$



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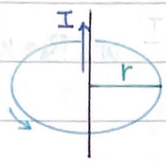
Applications of Ampere's Law

field of a long, straight, current-carrying conductor

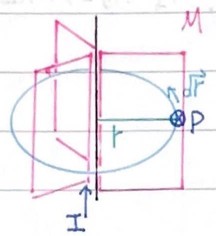
I



$$\oint \vec{B}\cdot d\vec{l} = \oint B_{\parallel}\, dl = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}\vec{e}_\phi$$

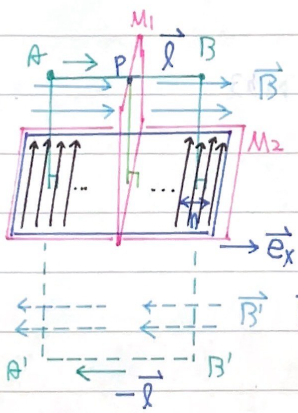
II



$\vec{B} \parallel \vec{e}_\phi \quad dr \parallel \vec{e}_\phi$

$$\begin{array}{ccl} I & \rightarrow & \vec{B} = \vec{B}_{M\parallel} + \vec{B}_{M\perp} \\ \downarrow & & \parallel \\ I' & \rightarrow & \vec{B}' = -\vec{B}_{M\parallel} + \vec{B}_{M\perp} \end{array} \Rightarrow \vec{B}_{M\parallel} = 0$$

field of a



$\vec{B} \parallel \vec{e}_x$

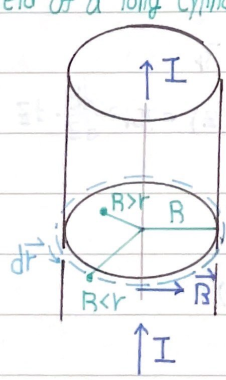
$\vec{B} = B_0\vec{e}_x \qquad \int \vec{B}\cdot d\vec{r}$

$$= B_0\vec{e}_x\cdot\vec{l} + (-B_0)\vec{e}_x\cdot(-\vec{l})$$

$$= 2|\vec{B}||\vec{l}| = \mu_0\, n l\, I$$

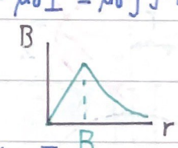
$\vec{B}' = -B_0\vec{e}_x \qquad \vec{B} = \frac{\mu_0}{2} n I$

field of a long cylindrical conductor



$\vec{B} \parallel \vec{e}_\phi \qquad \vec{J} = J_0\vec{e}_z = \frac{I}{\pi R^2}\vec{e}_z$

$$\int \vec{B}\cdot d\vec{r} = \int|\vec{B}||d\vec{r}| = |\vec{B}|\int|d\vec{r}| = |\vec{B}|2\pi r = \mu_0 I = \mu_0\int\vec{J}\cdot d\vec{A}$$



$r>R,\ \int\vec{J}\cdot d\vec{A} = |\vec{J}|\pi R^2 = I$

$$|\vec{B}|2\pi r = \mu_0\frac{I}{\pi R^2}\pi R^2,\quad \vec{B} = \frac{\mu_0}{2\pi}\frac{I}{r}$$

$$r<R \quad |\vec{B}|2\pi r = \mu_0\frac{I}{\pi R^2}\pi r^2,\quad \vec{B} = \frac{\mu_0 I}{2\pi}\frac{r}{R^2}$$

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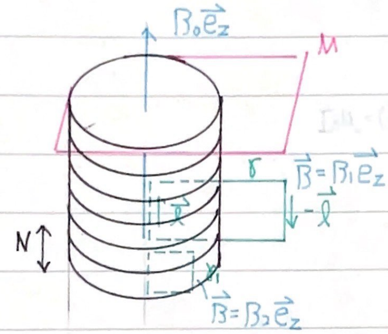
field of a solenoid



for γ

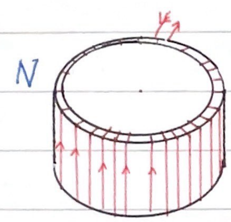
$$\int_\gamma \vec{B}\cdot d\vec{r} = B_0 l - B_1 l = \mu_0|\vec{l}|NI \quad \text{and } B_0=\mu_0 NI$$

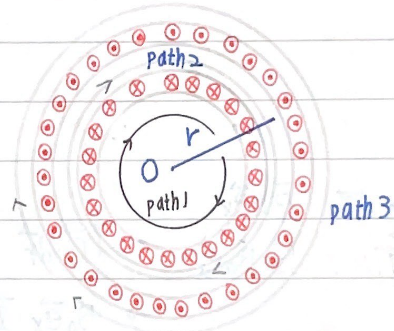
$$\Rightarrow B_1 = 0$$

for γ'

$$\int_{\gamma'} \vec{B}\cdot d\vec{r} = B_0 l - B_2 l = 0 \;,\; B_2 = B_0 = \mu_0 N I$$

field of a torodial solenoid





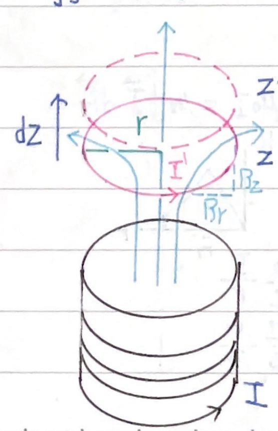
path 1 $\oint \vec{B}\cdot d\vec{l} = \mu_0 \cdot 0 = 0 \;,\; \vec{B}=0$

path 3 $\oint \vec{B}\cdot d\vec{l} = \mu_0(I+(-I)) = 0 \;,\; \vec{B}=0$

path 2 $\oint \vec{B}\cdot d\vec{l} = B\,2\pi r = \mu_0 N I \;,\; B = \frac{\mu_0 N I}{2\pi r}$

Gauss's Law in Magnetism

$$\oint_S \vec{B}\cdot d\vec{a} = 0$$



$$\pi r^2 B_z(z) = \pi r^2 B_z(z+dz) + 2\pi r\,dz\,B_r$$

$$-2\pi r B_r\,dz = \pi r^2 B_z(z+dz) - \pi r^2 B_z(z) = \pi r^2 \frac{dB}{dz}dz$$

$$B_r = -\frac{r}{2}\frac{dB}{dz}$$

$$F_z = -2\pi r I' B_r = -2\pi r I'\left(-\frac{r}{2}\frac{dB}{dz}\right)$$

$$= \pi r^2 I' \frac{dB}{dz} = \mu_b \frac{dB}{dz}$$