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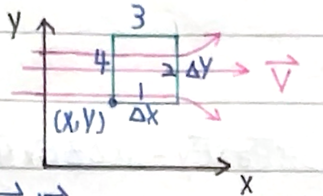
Stoke's theorem $\int_S (\nabla\times\vec{V})\cdot d\vec{a} = \oint_{\substack{\text{boundary}\\ \text{line}}} \vec{V}\cdot d\vec{l}$

(1)



$$\oint \vec{C}\cdot d\vec{s} = V_x(1)\Delta x + V_y(2)\Delta y - V_x(3)\Delta x - V_x(4)\Delta y = [V_x(1)-V_x(3)]\Delta x + [V_y(2)-V_y(4)]\Delta y$$

let $V_x(3) = V_x(1) + \frac{\partial V_x}{\partial y}\Delta y$, $[V_x(1)-V_x(3)]\Delta x = -\frac{\partial C_x}{\partial y}\Delta x\Delta y$

let $V_y(2) = V_y(4) + \frac{\partial V_y}{\partial x}\Delta x$, $[V_x(2)-V_x(4)]\Delta y = \frac{\partial C}{\partial x}\Delta x\Delta y$

or $V(x,y) - V(x,y)$

$$= V[(x+\Delta x, y)] - V(x,y) + V[(x+\Delta x, y+\Delta y)] - V[(x+\Delta x, y)] + V(x, y+\Delta y) - V(x+\Delta x, y+\Delta y) + V(x, y+\Delta y) - V(x,y)$$

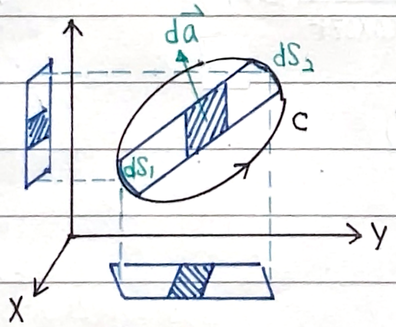
$$= \frac{\partial T}{\partial x}\Big|_y \Delta x + \frac{\partial T}{\partial y}\Big|_{x+\Delta x}\Delta y - \frac{\partial T}{\partial x}\Big|_{y+\Delta y}\Delta x - \frac{\partial T}{\partial y}\Big|_x \Delta y = -\frac{\partial}{\partial y}\frac{\partial T}{\partial x}\Delta y\Delta x + \frac{\partial}{\partial x}\frac{\partial T}{\partial y}\Delta x\Delta y$$

$$= \left(\frac{\partial}{\partial x}\frac{\partial T}{\partial y} - \frac{\partial}{\partial y}\frac{\partial T}{\partial x}\right)\vec{e}_z\cdot(\Delta x\vec{e}_x \times \Delta y\vec{e}_y)$$

$$\Rightarrow \oint \vec{C}\cdot d\vec{s} = \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\Delta x\Delta y = (\nabla\times\vec{V})\cdot\vec{n}\,\Delta a$$

(2)

$$\int_S (\nabla\times\vec{V})\cdot d\vec{a} = \int_S \Big(\frac{\partial A_x}{\partial z}\underbrace{da_y}_{-dxdz} - \frac{\partial A_x}{\partial y}\underbrace{da_z}_{dxdy}\Big) + \int_S \Big(\frac{\partial A_y}{\partial x}da_z - \frac{\partial A_y}{\partial z}da_x\Big) + \int_S \Big(\frac{\partial A_z}{\partial y}da_x - \frac{\partial A_z}{\partial x}da_y\Big)$$



$$= -\int_S dx \int_{P_1}^{P_2}\Big(\frac{\partial A_x}{\partial z}dy + \frac{\partial A_x}{\partial z}dz\Big) \qquad dA_x = \underbrace{\frac{\partial A_x}{\partial x}dx}_{0 \text{ due to } x=\text{constant}} + \frac{\partial A_x}{\partial y}dy + \frac{\partial A_x}{\partial z}dz$$

$$= -\int_S dx \int_{P_1}^{P_2} dA_x = -\int_S A_x(P_2) - A_x(P_1)\, dx = -A_x(P_2)(-dS_{2x}) + A_x(P_1)(dS_{1x})$$

$$= A_{x2}\,dS_{2x} + A_{x1}\,dS_{1x} = \oint_C A_x\, dS_x$$

$$\langle(\nabla\times\vec{A})\cdot\hat{n}\rangle_p = \frac{1}{\Delta a}\oint_C \vec{A}\cdot d\vec{s} \qquad (\nabla\times\vec{A})\cdot\hat{n} = \lim_{\Delta a\to 0}\frac{1}{\Delta a}\oint_C \vec{A}\cdot d\vec{s}$$

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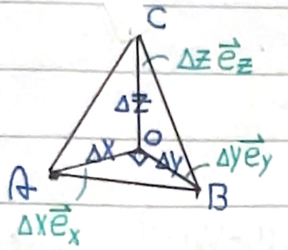
(3)



$$\overline{CB} = \Delta y\vec{e}_y - \Delta z\vec{e}_x$$

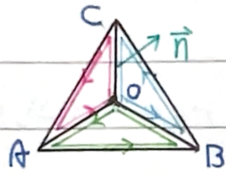
$$\overline{CA} = \Delta x\vec{e}_x - \Delta z\vec{e}_y$$

$$\overline{CA}\times\overline{CB} = \Delta x\Delta y(\vec{e}_x\times\vec{e}_y) - \Delta y\Delta z\,\vec{e}_z\times\vec{e}_y - \Delta x\Delta z(\vec{e}_x\times\vec{e}_y)$$

$$= \Delta x\Delta y\vec{e}_z + \Delta y\Delta z\,\vec{e}_x + \Delta x\Delta z\,\vec{e}_y$$

$$\text{area}(ABC)^2 = \text{area}(OAB)^2 + \text{area}(OBC)^2 + \text{area}(OAC)^2$$

$$\text{area}(ABC)^2 = \frac{1}{4}\left[(\Delta x\Delta y)^2 + (\Delta y\Delta z)^2 + (\Delta x\Delta z)^2\right]$$



$$\vec{n} = \cos\theta_x\,\vec{e}_x + \cos\theta_y\,\vec{e}_y + \cos\theta_z\,\vec{e}_z = \frac{\Delta y\Delta z\,\vec{e}_x + \Delta x\Delta z\,\vec{e}_y + \Delta x\Delta y\,\vec{e}_z}{\sqrt{(\Delta x\Delta y)^2 + (\Delta y\Delta z)^2 + (\Delta x\Delta z)^2}}$$

$$\int_{\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}} \vec{B}\cdot d\vec{l} = \int_{\overrightarrow{AB}+\overrightarrow{BO}+\overrightarrow{OA}} \vec{B}\cdot d\vec{l} + \int_{\overrightarrow{OC}+\overrightarrow{CA}+\overrightarrow{AO}} \vec{B}\cdot d\vec{l} + \int_{\overrightarrow{BC}+\overrightarrow{CO}+\overrightarrow{OB}} \vec{B}\cdot d\vec{l}$$

$$= \frac{1}{2}\Delta x\Delta y\underbrace{\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_y}{\partial y}\right)}_{(\nabla\times\vec{B})_z} + \frac{1}{2}\Delta x\Delta z\underbrace{\left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)}_{(\nabla\times\vec{B})_y} + \frac{1}{2}\Delta y\Delta z\underbrace{\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)}_{(\nabla\times B)_x}$$

$$= \nabla\times\vec{B}\;\frac{\Delta y\Delta z\,\vec{e}_x + \Delta x\Delta z\,\vec{e}_y + \Delta x\Delta y\,\vec{e}_z}{\frac{1}{2}\sqrt{(\Delta x\Delta y)^2 + (\Delta y\Delta z)^2 + (\Delta x\Delta z)^2}}\,|\Delta\vec{a}|$$

$$= (\nabla\times\vec{B})\cdot\vec{n}\,|\Delta\vec{a}| = \nabla\times\vec{B}\cdot(\Delta\vec{a})$$

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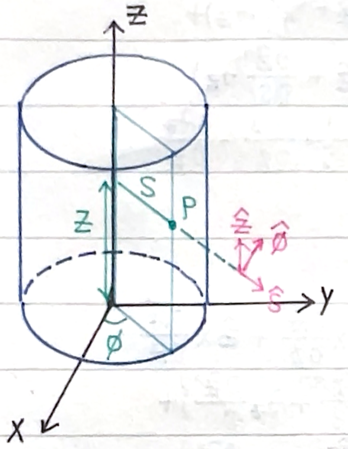
Curvilinear Coordinates

Cylindrical coordinate



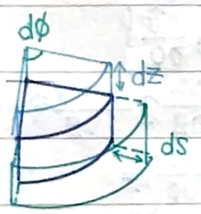
$x = s\cos\phi \qquad s = \sqrt{x^2+y^2} =$

$y = s\cdot\sin\phi \qquad \tan\phi = \frac{y}{x}$

$z = z$

$\hat{x} = \cos\phi\,\hat{s} - \sin\phi\,\hat{\phi} \qquad \hat{s} = \cos\phi\,\hat{x} + \sin\phi\,\hat{y}$

$\hat{y} = \sin\phi\,\hat{s} + \cos\phi\,\hat{\phi} \qquad \hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$



$d\tau = ds(s\,d\phi)dz$

$\vec{A} = A_s\hat{s} + A_\phi\hat{\phi} + A_z\hat{z}$

$\vec{r} = s\,\hat{s} + z\,\hat{z}$

$\frac{\partial\hat{s}}{\partial s} = 0 \qquad \frac{\partial\hat{\phi}}{\partial s} = 0 \qquad \frac{\partial\hat{z}}{\partial s} = 0$

$\frac{\partial\hat{s}}{\partial\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y} = \hat{\phi} \qquad \frac{\partial\hat{\phi}}{\partial\phi} = -\cos\phi\,\hat{x} - \sin\phi\,\hat{y} = -\hat{s} \qquad \frac{\partial\hat{z}}{\partial\phi} = 0$

$\frac{\partial\hat{s}}{\partial z} = 0 \qquad \frac{\partial\hat{\phi}}{\partial z} = 0 \qquad \frac{\partial\hat{z}}{\partial z} = 0$

$d\vec{r} = d(s\hat{s} + z\hat{z}) = \hat{s}ds + s\,d\hat{s} + \hat{z}dz + z\,d\hat{z}$

$= \hat{s}ds + s\left(\frac{\partial\hat{s}}{\partial s}ds + \frac{\partial\hat{s}}{\partial\phi}d\phi + \frac{\partial\hat{s}}{\partial z}dz\right) + \hat{z}dz + z\left(\frac{\partial\hat{z}}{\partial s}ds + \frac{\partial\hat{z}}{\partial\phi}d\phi + \frac{\partial\hat{z}}{\partial z}dz\right)$

$= \hat{s}ds + s\,d\phi\,\hat{\phi} + \hat{z}dz$

$= (\nabla u_s)ds + (\nabla u_\phi)d\phi + (\nabla u_z)dz$ $\frac{1}{s}$

$du = \nabla\cdot u\cdot d\vec{r} = \frac{\partial u}{\partial s}ds + \frac{\partial u}{\partial\phi}d\phi + \frac{\partial u}{\partial z}dz = \underbrace{\left(\frac{\partial u}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial u}{\partial\phi}\hat{\phi} + \frac{\partial u}{\partial z}\hat{z}\right)}_{\nabla\cdot u}\cdot(\hat{s}ds + s\,d\phi\,\hat{\phi} + z\,d\hat{z})$

$\nabla = \frac{\partial}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial}{\partial\phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{z}$