NO.

DATE / /

General coordinates

$$\overset{f}{h_1}\,\overset{u}{dq_1}\,\hat{q}_1 + \overset{g}{h_2}\,\overset{v}{dq_2}\,\hat{q}_2 + \overset{h}{h_3}\,\overset{w}{dq_3}\,\hat{q}_3$$

$$d\vec{l} = h_1\,dq_1\,\hat{q}_1 + h_2\,dq_2\,\hat{q}_2 + h_3\,dq_3\,\hat{q}_3$$

	Cartesian	Cylindrical	Spherical
q_1	x	s	r
q_2	y	ϕ	θ
q_3	z	z	ϕ
h_1	1	1	1
h_2	1	s	r
h_3	1	1	$r\sin\theta$
$\hat{q}_1$	$\hat{x}$	$\hat{s}$	$\hat{r}$
$\hat{q}_2$	$\hat{y}$	$\hat{\phi}$	$\hat{\theta}$
$\hat{q}_3$	$\hat{z}$	$\hat{z}$	$\hat{\phi}$

$$\nabla f = \frac{\partial f}{\partial l_1}\hat{q}_1 + \frac{\partial f}{\partial l_2}\hat{q}_2 + \frac{\partial f}{\partial l_3}\hat{q}_3 = \frac{\partial f}{h_1\partial q_1}\hat{q}_1 + \frac{\partial f}{h_2\partial q_2}\hat{q}_2 + \frac{\partial f}{h_3\partial q_3}\hat{q}_3$$

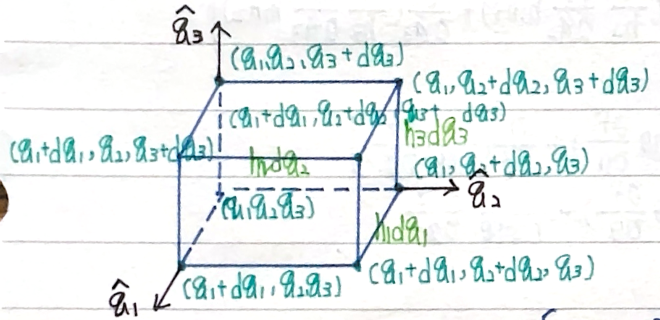
$$= \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$= \frac{\partial f}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$$

$$= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\int_V (\nabla\cdot\vec{B})\,d\tau = \oint_S \vec{B}\cdot d\vec{a}$$

$$\nabla\cdot\vec{B} = \lim_{\Delta\tau\to 0}\frac{\oint\vec{B}\cdot d\vec{a}}{\Delta\tau} \qquad \vec{B} = B_1\hat{q}_1 + B_2\hat{q}_2 + B_3\hat{q}_3 \qquad \Delta\tau = h_1h_2h_3\,dq_1\,dq_2\,dq_3$$



$$d\Phi_L = B_2(S_1)(-h_1dq_1h_3dq_3), \quad d\Phi_R = \left[B_2(S_1) + \frac{\partial B_2}{\partial q_2}dq_2\right](h_1dq_1h_3dq_3)$$

$$d\Phi_L + d\Phi_R = \frac{\partial B_2}{\partial q_2}h_1h_3\,dq_1dq_2dq_3$$

similarly, $d\Phi = dq_1dq_2dq_3\left[\frac{\partial(B_1h_2h_3)}{\partial q_1} + \frac{\partial(B_2h_1h_2)}{\partial q_2} + \frac{\partial(B_3h_1h_2)}{\partial q_3}\right]$

$$\frac{d\Phi}{d\tau} = \nabla\cdot\vec{B} = \frac{1}{h_1h_2h_3}\left[\frac{\partial(B_1h_2h_3)}{\partial q_1} + \frac{\partial(B_2h_1h_2)}{\partial q_2} - \frac{\partial(B_3h_1h_2)}{\partial q_3}\right]$$

$$= \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$

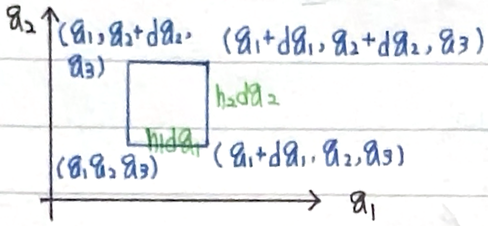
$$= \frac{1}{s}\left(\frac{\partial B_s s}{\partial s} + \frac{\partial B_\phi}{\partial \phi} + \frac{\partial B_z s}{\partial z}\right) = \frac{1}{s}\frac{\partial(sB_s)}{\partial s} + \frac{1}{s}\frac{\partial B_\phi}{\partial\phi} + \frac{\partial B_z}{\partial z}$$

$$= \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(B_r\cdot r^2\sin\theta) + \frac{\partial}{\partial\theta}(B_\theta\, r\sin\theta) + \frac{\partial}{\partial\phi}(B_\phi\, r)\right] = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2B_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta B_\theta) + \frac{1}{r\sin\theta}\frac{\partial B_\phi}{\partial\phi}$$

NO.

DATE / /

$$\int(\nabla\times\vec{B})\cdot d\vec{a}=\oint\vec{B}\cdot d\vec{l}$$



$$\oint\vec{B}\cdot d\vec{l}=B_{1,1}h_1dq_1+B_{2,2}h_2dq_2-B_{1,3}h_1dq_1-B_{2,4}h_2dq_2$$

$$=(B_{1,1}-B_{1,3})h_1dq_1+(B_{2,2}-B_{2,4}h_2dq_2) \qquad B_{1,3}=B_{1,1}+\frac{\partial B_1}{\partial q_2}dq_2$$

$$=-\frac{\partial B_1}{\partial q_2}h_1dq_1dq_2+\frac{\partial B_2}{\partial q_1}h_2dq_1dq_2$$

$$=\frac{1}{h_1h_2}\left(\frac{\partial B_2h_2}{\partial q_1}-\frac{\partial h_1B_1}{\partial q_2}\right)\hat{q}_3\cdot\underline{d\vec{a}} \quad h_1h_2dq_1dq_2\hat{q}_3$$

$$\nabla\times\vec{B}=\frac{1}{h_1h_2h_3}\begin{vmatrix}h_1\hat{q}_1 & h_2\hat{q}_2 & h_3\hat{q}_3\\ \frac{\partial}{\partial q_1} & \frac{\partial}{\partial q_2} & \frac{\partial}{\partial q_3}\\ h_1B_1 & h_2B_2 & h_3B_3\end{vmatrix}$$

$$=\frac{1}{h_2h_3}\left(\frac{\partial h_3B_3}{\partial q_2}-\frac{\partial h_2B_2}{\partial q_3}\right)\hat{q}_1+\frac{1}{h_1h_3}\left(\frac{\partial h_1B_1}{\partial q_3}-\frac{\partial h_3B_3}{\partial q_1}\right)\hat{q}_2+\frac{1}{h_1h_2}\left(\frac{\partial h_2B_2}{\partial q_1}-\frac{\partial h_1B_1}{\partial q_2}\right)\hat{q}_3$$

Laplacian

$$\nabla f=\frac{1}{h_1}\frac{\partial f}{\partial q_1}\hat{q}_1+\frac{1}{h_2}\frac{\partial f}{\partial q_2}\hat{q}_2+\frac{1}{h_3}\frac{\partial f}{\partial q_3}\hat{q}_3$$

$$\nabla^2 f=\frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q_1}\left(\frac{1}{h_1}\frac{\partial f}{\partial q_1}h_2h_3\right)+\frac{\partial}{\partial q_2}\left(\frac{1}{h_2}\frac{\partial f}{\partial q_2}h_1h_3\right)+\frac{\partial}{\partial q_3}\left(\frac{1}{h_3}\frac{\partial f}{\partial q_3}h_1h_2\right)\right]$$

$$=\frac{1}{s}\left(\frac{\partial}{\partial s}s\frac{\partial f}{\partial s}+\frac{1}{s^2}\frac{\partial^2 f}{\partial\phi^2}+\frac{\partial^2 f}{\partial z^2}\right)$$

$$=\frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}\left(r^2\sin\theta\frac{\partial f}{\partial r}\right)+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right)+\frac{\partial}{\partial\phi}\left(\frac{1}{\sin\theta}\frac{\partial f}{\partial\phi}\right)\right]$$

$$=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right)+\frac{1}{r^2\sin\theta}\frac{\partial^2 f}{\partial\phi^2}$$

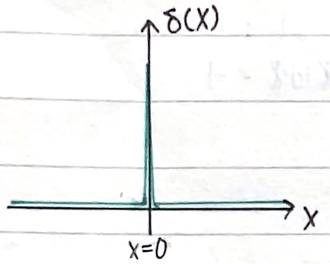
The Dirac Delta Function

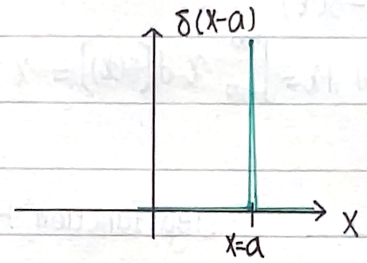
consider the vector function $\vec{V} = \frac{1}{r^2}\hat{r}$

$\nabla \cdot \vec{V} = \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial r}\left(\frac{1}{r^2} r^2 \sin\theta\right) = 0$ zero everywhere except origin

$$\oint_S \vec{V} \cdot d\vec{a} = \int (\nabla \cdot \vec{V}) d\tau = \int \frac{1}{r^2}\hat{r} \cdot r^2 \sin\theta d\theta d\phi \hat{r}$$

$$= \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi = 4\pi$$ origin





$\delta(x) \begin{cases} 0 & \text{if } x \neq 0 \\ \infty & \text{if } x = 0 \end{cases}$

$\int_{-\infty}^{\infty} \delta(x) dx = 1$

$\delta(x-a) \begin{cases} 0 & \text{if } x \neq a \\ \infty & \text{if } x = a \end{cases}$

$\int_{-\infty}^{\infty} \delta(x-a) dx = 1$

if $f(x)$ is continuous, $f(x)\delta(x)$ is zero everywhere except at $x=0$

then $f(x)\delta(x) = f(0)\delta(x)$, $\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)\int_{-\infty}^{\infty}\delta(x)dx = f(0)$

$f(x)\delta(x-a) = f(0)\delta(x-a)$, $\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a)$

two delta functions are considered equal $\int_{-\infty}^{\infty} f(x) D_1(x) dx = \int_{-\infty}^{\infty} f(x) D_2(x) dx$

example $\delta(kx) = \frac{1}{|k|}\delta(x)$ where k is any nonzero constant Similarity Relationship

$\int_{-\infty}^{\infty} f(x)\delta(kx)$ let $y = kx$, $x = \frac{y}{k}$ $dx = \frac{1}{k}dy$ Scaling law

$= \pm \int_{-\infty}^{\infty} f\left(\frac{y}{k}\right)\delta(y)\frac{dy}{k}$

$= \pm \frac{1}{k} f(0) = \frac{1}{|k|} f(0)$