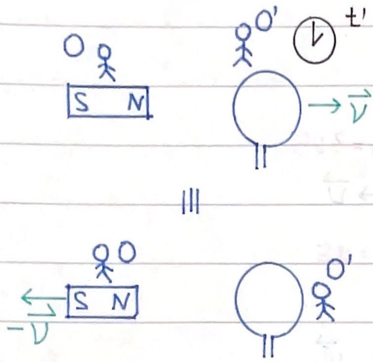
Faraday's Law

Faraday's Law

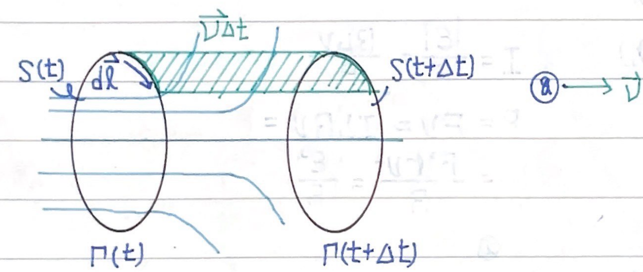


for O' $\vec{E}'\neq 0$ $\vec{B}'\neq 0$ a moving permanant magnet .

for O $\vec{E}=0$ $\vec{B}\neq 0$ permanant magnet in a rest

electric field and magnetic field is a relatively concept

for O



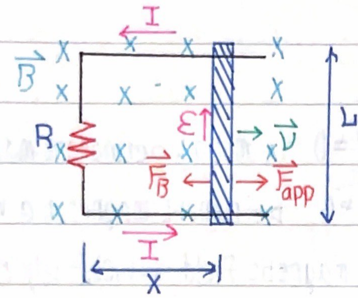
$$\text{work} = \int_{\Gamma(t)} d\vec{l}\cdot q\vec{v}\times\vec{B} = \frac{q}{\Delta t}\int_{\Gamma(t)}(d\vec{l}\times\vec{v}\Delta t)\cdot\vec{B}$$

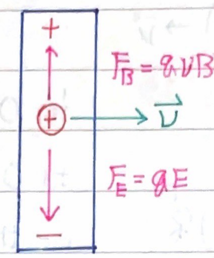
$$\frac{\text{work}}{q} = \frac{1}{\Delta t}\int(d\vec{l}\times\vec{v}\Delta t)\cdot\vec{B} = \mathcal{E},\ \text{motional emf}$$

NO.

DATE / /

Supplement





$qE = qvB$, $E = vB$ $\qquad d\mathcal{E} = (\vec{v}\times\vec{B})\cdot d\vec{l}$

$\Delta V = El = Blv$ $\qquad \mathcal{E} = \oint(\vec{v}\times\vec{B})\cdot d\vec{l}$

$\Phi_B = Blx$ and $\frac{dx}{dt} = v$

$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BLx)$

$= -BL\frac{dx}{dt} = -BLv$

$I = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R}$

$P = Fv = ILBv$

$= \frac{B^2l^2v^2}{R} = \frac{\mathcal{E}^2}{R}$

①

$F_x = ma$, $-ILB = m\frac{dv}{dt}$

$m\frac{dv}{dt} = \frac{BLv}{R}LB = -\frac{B^2L^2}{R}v$

$\frac{dv}{v} = -(\frac{B^2L^2}{mR})dt$

$\int_{v_i}^{v}\frac{dv}{v} = -\frac{B^2L^2}{mR}\int_0^t dt$

$\ln\frac{v}{v_i} = -\frac{B^2L^2}{mR}t$

$v = v_i^{-\frac{t}{\tau}}$, $\tau = \frac{mR}{B^2L^2}$

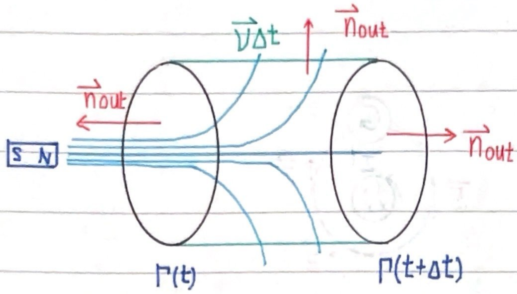
②

$P_R = -P_{bar}$

$I^2R = -\frac{d}{dt}(\frac{1}{2}mv^2)$

$\frac{B^2L^2v^2}{R} = -mv\frac{dv}{dt}$

$\frac{dv}{v} = -\frac{B^2L^2}{mR}dt$



$$\int_{\text{cyl}} \vec{B}\cdot d\vec{A} = 0 = \int_{\text{side}} (d\vec{l}\times \vec{v}\Delta t)\cdot \vec{B} + \int_{S(t+\Delta t),\ \vec{n}_{out}} d\vec{A}\cdot\vec{B} + \int_{S(t),\ \vec{n}_{out}} d\vec{A}\cdot\vec{B}$$

$$\int (d\vec{l}\times\vec{v}\Delta t)\cdot\vec{B} = -\int_{S(t+\Delta t),\ \vec{n}_{out}} d\vec{A}\cdot\vec{B} - \int_{S(t),\ \vec{n}_{out}} d\vec{A}\cdot\vec{B} = -\int_{S(t+\Delta t),\ \vec{n}_{amp}} d\vec{A}\cdot\vec{B} + \int_{S(t),\ \vec{n}_{amp}} d\vec{A}\cdot\vec{B}$$

$$= -\left[\int_{\vec{n}_{amp}} d\vec{A}\cdot\vec{B} - \int_{\vec{n}_{amp}} d\vec{A}\cdot\vec{B}\right]$$

$$\mathcal{E} = \frac{1}{\Delta t}\int (d\vec{l}\times\vec{v}\Delta t)\cdot\vec{B} = \frac{-1}{\Delta t}\Bigg[\underbrace{\int_{\vec{n}_{amp}} \vec{B}\cdot d\vec{A}}_{\phi_B(t+\Delta t)} - \underbrace{\int_{\vec{n}_{amp}} \vec{B}\cdot d\vec{A}}_{\phi_B(t)}\Bigg]$$

$$= \frac{-1}{\Delta t}\left[\phi_B(t+\Delta t) - \phi_B(t)\right] = -\frac{d\phi_B}{dt} \quad \text{Faraday's law of induction}$$

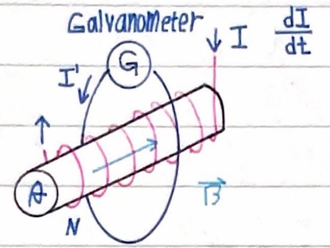
$$= -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

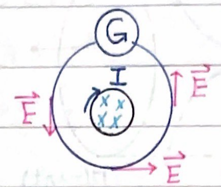
the induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop

NO.

DATE / /

Induced Electric Fields





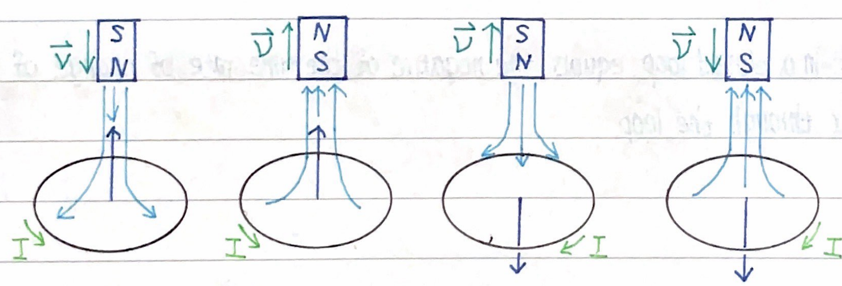
$B = \mu_0 N I$, $\Phi = BA = \mu_0 N I A$ $\quad \mathcal{E} = -\frac{d\Phi_B}{dt} = -\mu_0 N A \frac{dI}{dt}$

work $= q\,\mathcal{E} = q\,2\pi r$, $E = \frac{\mathcal{E}}{2\pi r} = -\frac{1}{2\pi r}\frac{d\Phi_B}{dt} = -\frac{r}{2}\frac{dB}{dt}$

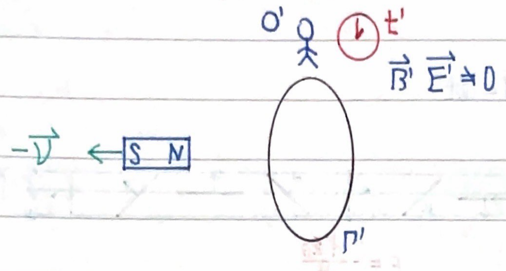
$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Lenz's Law

the direction of any magnetic induction effect is such as to oppose the cause of the effect

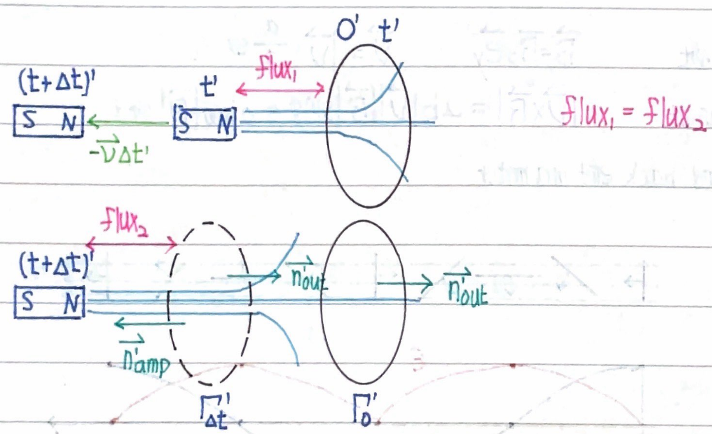


Principle of Relativity



O: $\vec{B}$ $\quad\vec{E}=0$ $\qquad\qquad \oint d\vec{l}'\cdot\vec{E}' = -\frac{d}{dt'}\int_S \vec{B}'(t')\cdot d\vec{A}'$

O': $\vec{B}'$ $\quad\vec{E}'\neq 0$



$$\oint \vec{E}'\cdot d\vec{l}' = \mathcal{E}'_{ind} = \frac{-1}{\Delta t'}\left[\int_{\Gamma'_0}\vec{B}'(t+\Delta t)'\cdot d\vec{A}' - \int_{\Gamma'_0}\vec{B}'(t')\cdot d\vec{A}'\right]$$

$$= \frac{-1}{\Delta t'}\left[\int_{\Gamma'_0,\ \vec{n}_{amp}}\vec{B}'(t+\Delta t)'\cdot d\vec{A}' - \int_{\Gamma'_{\Delta t},\ \vec{n}_{amp}}\vec{B}'(t+\Delta t)'\cdot d\vec{A}'\right]$$

$$\int_{\Gamma'_{\Delta t'},\ \vec{n}_{out}}\vec{B}'(t+\Delta t)'\cdot d\vec{A}' + \int_{\Gamma'_0,\ \vec{n}_{out}}\vec{B}'(t+\Delta t)'\cdot d\vec{A}' + \int_{\boxminus,\ \vec{n}_{out}}\vec{B}'(t+\Delta t)'\cdot d\vec{A}' = 0$$

$$-\int_{\Gamma'_{\Delta t'},\ \vec{n}_{amp}}\vec{B}'(t+\Delta t)'\cdot d\vec{A}' + \int_{\Gamma'_0,\ \vec{n}_{amp}}\vec{B}'(t+\Delta t)'\cdot d\vec{A}' = -\int_{\boxminus}\vec{B}'(t+\Delta t)'\cdot d\vec{A}' = \int(-\vec{v}\Delta t\times d\vec{l}')\cdot\vec{B}'(t+\Delta t)'$$

$$= -\int d\vec{l}'\cdot(\vec{v}'\Delta t')\times\vec{B}'(t'+\Delta t')$$

$$\Rightarrow \frac{-1}{\Delta t'}\int d\vec{l}'\cdot\left[(\vec{v}\Delta t')\times\vec{B}'(t'+\Delta t')\right] = \int_{\Gamma'_0}\vec{v}\times\vec{B}'(t')\cdot d\vec{l}'$$

$$\int_{\Gamma'_0}\vec{E}'\cdot d\vec{l}' = \int_{\Gamma'_0}(\vec{v}'\times\vec{B}')\cdot d\vec{l}', \quad \int_{\Gamma'_0}\left[\vec{E}'(t')-\vec{v}'\times\vec{B}'(t')\right]\cdot d\vec{l}' = 0, \quad \vec{E}'=\vec{v}'\times\vec{B}'(t')$$

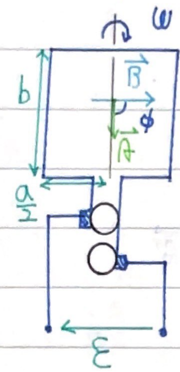
$$\vec{E}'-\vec{v}'\times\vec{B}'=\vec{0}'$$

NO.

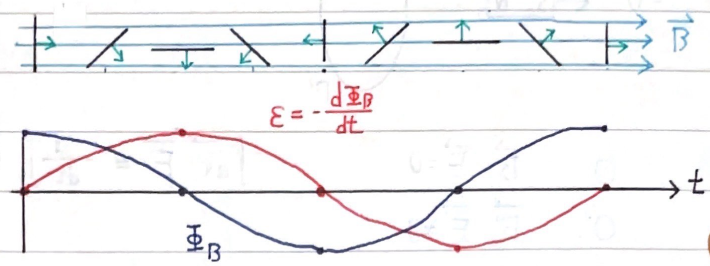
DATE / /

Applications of Faraday's law

case I: a simple alternator



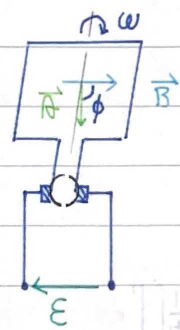
at $t=0$, $\phi=0$ $\quad \phi=\omega t$

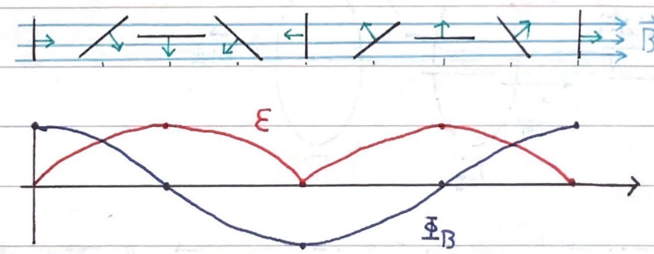


$\Phi_B = BA\cos\phi = BA\cos\omega t$ $\qquad \vec{B}=B_0\vec{e}_y$ $\qquad \vec{v}=|\vec{v}|\frac{a}{2}\omega$

$\mathcal{E} = -\frac{d\Phi_B}{dt} = \omega BA\sin\omega t$ $\qquad \mathcal{E} = 2b|\vec{v}||\vec{B}|\sin\phi = ab\omega|\vec{B}|\sin\phi$

case II a DC generator and back emf in a motor



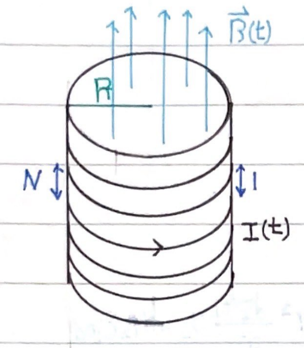


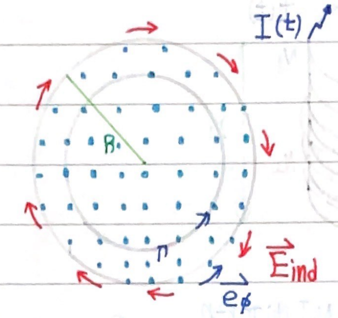
$|\mathcal{E}| = N\omega BA|\sin\omega t|$ $\qquad (|\sin\omega t|)_{av} = \frac{\int_0^{\frac{\pi}{\omega}}\sin\omega t\,dt}{\frac{\pi}{\omega}} = \frac{2}{\pi}$ $\qquad \mathcal{E}_{av} = \frac{2N\omega BA}{\pi}$

NO.

DATE / /

case III





$r < R$

$$\int_{\Gamma} \vec{E}_{ind} \cdot d\vec{l} = -\int \frac{d\vec{B}(t)}{dt} \cdot d\vec{A} = -\mu_0 N \frac{dI(t)}{dt} \pi r^2$$

$$= -2\pi r \left|\vec{E}_{ind}\right|$$

$$\Rightarrow \left|\vec{E}_{ind}\right| = \frac{r}{2} \mu_0 n \frac{dI(t)}{dt}$$

$$\vec{E}_{ind} = -\frac{r}{2} \mu_0 N \frac{dI(t)}{dt} \vec{e}_\phi$$

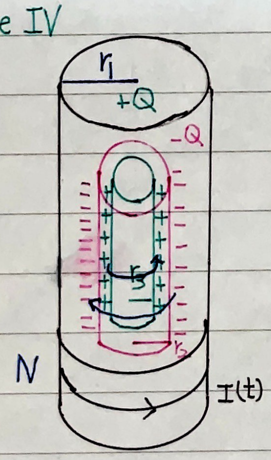
$r > R$

$$-2\pi r \left|\vec{E}_{ind}\right| = -\mu_0 N \frac{dI(t)}{dt} \pi R^2$$

$$\left|\vec{E}_{ind}\right| = \frac{R^2}{2r} \mu_0 N \frac{dI(t)}{dt}$$

$$\vec{E}_{ind} = -\frac{R^2}{2r} \mu_0 N \frac{dI(t)}{dt} \vec{e}_\phi$$

case IV



$I(0) = I_0 > 0 \quad I(t) \to 0$

$$r = r_3 \quad \tau_3 = +Q\left|\vec{E}_{ind}\right|_{r_3} = -Q \frac{r_3}{2} \mu_0 N \frac{dI(t)}{dt} r_3$$

$$= -Q \frac{r_3^2}{2} \mu_0 N \frac{dI(t)}{dt}$$

$$r = r_2 \quad \tau_2 = -Q\left|\vec{E}_{ind}\right|_{r_2} = +Q \frac{r_2^2}{2} \mu_0 N \frac{dI(t)}{dt}$$

$$\tau_{total} = \frac{Q}{2}(r_3^2 - r_2^2) \mu_0 N \frac{dI(t)}{dt} = \frac{d\vec{L}}{dt}$$

$$\vec{L} = \frac{Q}{2}(r_3^2 - r_2^2) \mu_0 N \int_0^\infty \frac{dI(t)}{dt} dt$$

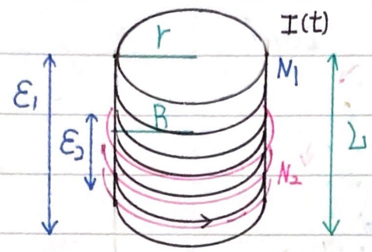
$$L(\infty) - L(0) = \frac{Q}{2}(r_3^2 - r_2^2) \mu_0 N I_0$$

angular momentum conservation

NO.

DATE / /

case V



$$\varepsilon_{ind \cdot N_1} = -\frac{d}{dt}\int \mu_0 N_1 I(t) \pi r^2 n$$

$$= -\mu_0 n \pi r^2 \frac{dI(t)}{dt}$$

$$\varepsilon_{ind \cdot N_2} = -\mu_0 n \pi r^2 \frac{dI(t)}{dt} N_2 = N_2 \varepsilon_{ind}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{-\mu_0 n \frac{dI(t)}{dt} \pi r^2 N_1}{-\mu_0 n \frac{dI(t)}{dt} \pi r^2 N_2} = \frac{N_1}{N_2}$$

$$\varepsilon_1 = -\mu_0 n \frac{dI(t)}{dt} \pi r^2 \ell n , \quad N_1 = \ell n$$