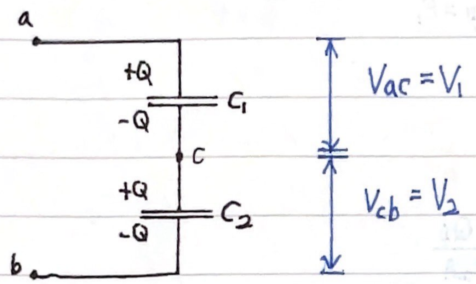
RC Circuits

Capacitors in Series and Parallel

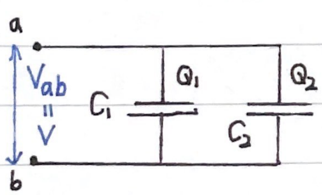
series



$$V_{ac} = V_1 = \frac{Q}{C_1} \qquad V_{cb} = V_2 = \frac{Q}{C_2}$$

$$V_{ab} = V_1 + V_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right), \quad \frac{V_{ab}}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

parallel



$$Q_1 = C_1 V \qquad Q_2 = C_2 V$$

$$Q = Q_1 + Q_2 = (C_1 + C_2)V$$

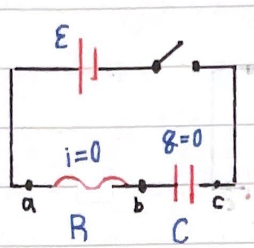
$$\frac{Q}{V} = C_1 + C_2$$

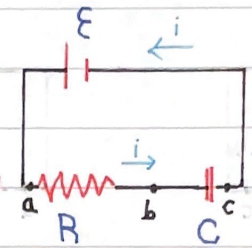
NO.

DATE / /

RC Circuits

charging





$V_{ab} = iR \quad V_{bc} = \frac{q}{C} \quad \varepsilon - iR - \frac{q}{C} = 0 \quad , \; i = \frac{\varepsilon}{R} - \frac{q}{RC}$

$\frac{\varepsilon}{R} = \frac{Q_f}{RC} \; , \; Q_f = C\varepsilon$

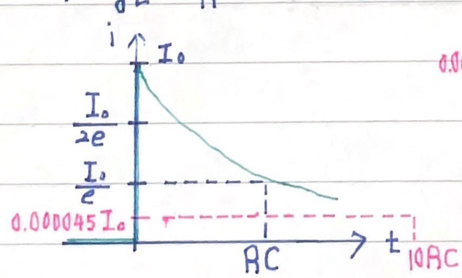
$I_0(t=0) = \frac{\varepsilon}{R}$ due to $q=0$ at $t=0$

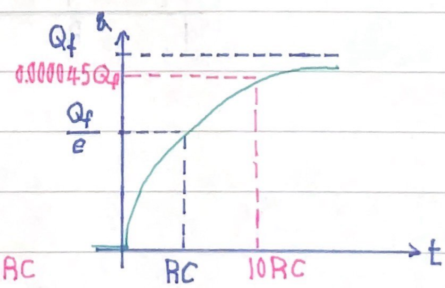
$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC} = -\frac{1}{RC}(q - C\varepsilon) \; , \quad \frac{dq}{q - C\varepsilon} = -\frac{dt}{RC}$

$\int_0^q \frac{dq}{q - C\varepsilon} = -\int_0^t \frac{dt}{RC} \; , \; \ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC} \; , \quad \frac{q - C\varepsilon}{-C\varepsilon} = e^{-\frac{t}{RC}}$

$q = C\varepsilon(1 - e^{-\frac{t}{RC}}) = Q_f(1 - e^{-\frac{t}{RC}}) \qquad -C\varepsilon e^{-\frac{t}{RC}} \left(-\frac{1}{RC}\right)$

$i = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$



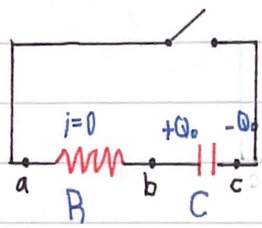


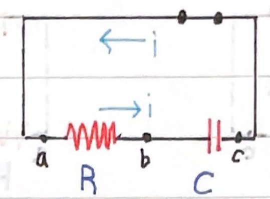
time constant

when $t = RC$, $q = Q_f(1 - e^{-1}) \approx 0.632 Q_f$

and the time , $\tau = RC$, we called relaxation time

discharging





$$i = \frac{dq}{dt} = -\frac{q}{RC} \quad \text{with } \mathcal{E}=0$$

when $t=0$, $q=Q_0$ $\quad I_0 = -\frac{Q_0}{RC}$

$$\int_{Q_0}^{q} \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt, \quad \ln\frac{q}{Q_0} = -\frac{t}{RC}, \quad q = Q_0 e^{-\frac{t}{RC}}$$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$