NO.

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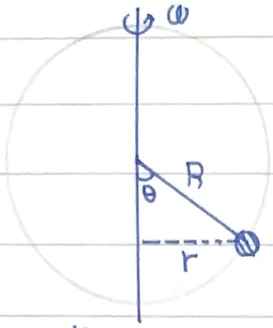
case III a bead on a spinning wire hoop



$v = R\frac{d\theta}{dt}$ $r\omega = R\sin\theta\,\omega$

$E_k = \frac{1}{2}mv^2 = \frac{1}{2}mR^2\left[\left(\frac{d\theta}{dt}\right)^2 + \omega^2\sin^2\theta\right]$, $U = mgR(1-\cos\theta)$

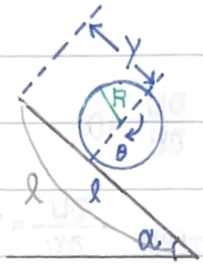
$L = \frac{1}{2}mR^2\left[\left(\frac{d\theta}{dt}\right)^2 + \omega^2\sin^2\theta\right] - mgR(1-\cos\theta)$

$\frac{\partial L}{\partial\theta} = \frac{d}{dt}\frac{\partial L}{\partial\dot\theta}$, $mR^2\omega^2\sin\theta\cos\theta - mgR\sin\theta = mR^2\frac{d^2\theta}{dt^2}$, $\frac{d^2\theta}{dt^2} = (\omega^2\cos\theta - \frac{g}{R})\sin\theta$

if $\frac{d^2\theta}{dt^2} = 0$, $(\omega^2\cos\theta - \frac{g}{R})\sin\theta = 0$, $\cos\theta = \frac{g}{\omega^2 R}$ $\theta_0 = \pm\cos^{-1}\frac{g}{\omega^2 R}$

SEASON

Lagrange's Equations with Undetermined Multipliers



$$E_k = \frac{1}{2}M\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2 = \frac{1}{2}M\left(\frac{dy}{dt}\right)^2 + \frac{1}{4}MR^2\left(\frac{d\theta}{dt}\right)^2$$

$$U = Mg(\ell - y)\sin\alpha$$

$$L = E_k - U = \frac{1}{2}M\left(\frac{dy}{dt}\right)^2 + \frac{1}{4}MR^2\left(\frac{d\theta}{dt}\right)^2 + Mg(y-\ell)\sin\alpha$$

constraint $f(y,\theta) = y - R\theta = 0$

$$\frac{\partial L}{\partial y} - \frac{d}{dt}\frac{\partial L}{\partial y'} + \lambda\frac{\partial f}{\partial y} = 0$$
$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\frac{\partial L}{\partial \theta'} + \lambda\frac{\partial f}{\partial \theta} = 0$$

, $Mg\sin\alpha - M\frac{d^2y}{dt^2} + \lambda = 0$

$-\frac{1}{2}MR^2\frac{d^2\theta}{dt^2} - \lambda R = 0$

and $y = R\theta$, $\frac{d^2\theta}{dt^2} = \frac{1}{R}\frac{d^2y}{dt^2}$

$$\Rightarrow \lambda = -\frac{1}{2}M\frac{d^2y}{dt^2} \Rightarrow \frac{d^2y}{dt^2} = \frac{2}{3}g\sin\alpha, \quad \lambda = -\frac{1}{3}Mg\sin\alpha$$

$$\frac{d^2\theta}{dt^2} = \frac{2}{3}\frac{g}{R}\sin\alpha$$

$$Q_y = \lambda\frac{\partial f}{\partial y} = \lambda = -\frac{Mg\sin\alpha}{3} \quad : \text{force}$$

$$Q_\theta = \lambda\frac{\partial f}{\partial \theta} = -\lambda R = \frac{MgR\sin\alpha}{3} \quad : \text{torque}$$