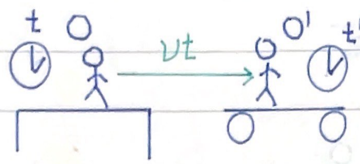
$E = mC^2$



$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

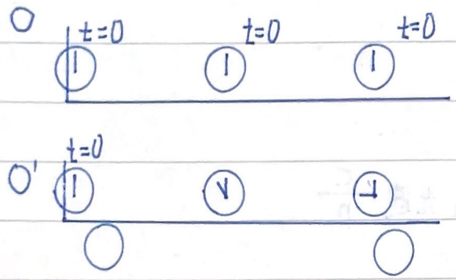
$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$t = 0 \quad x = 0$

$t' = 0 \quad x' = 0$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1 - \frac{v^2}{c^2}}}$$



事件 $O: (x, y, z, t)$

$O': (x', y', z', t')$

質點運動軌跡

$O: [x(t), y(t), z(t), t]$

$$O': \left[x'(t') = \frac{x(t) - vt}{\sqrt{1 - \frac{v^2}{c^2}}},\ y'(t') = y(t),\ z'(t') = z(t),\ t' = \frac{t - \frac{v}{c^2}x(t)}{\sqrt{1 - \frac{v^2}{c^2}}}\right]$$

在O中，速度 $u_x = \frac{dx(t)}{dt} \quad u_y = \frac{dy(t)}{dt} \quad u_z = \frac{dz(t)}{dt}$

在O'中，速度 $u_x' = \frac{dx'(t')}{dt'} = \frac{dx'(t')}{dt}\frac{dt}{dt'} = \frac{\frac{dx(t)}{dt} - v}{\sqrt{1 - \frac{v^2}{c^2}}}\frac{dt}{dt'}$, and $\frac{dt'}{dt} = \frac{1 - \frac{v}{c^2}\frac{dx(t)}{dt}}{\sqrt{1 - \frac{v^2}{c^2}}}$

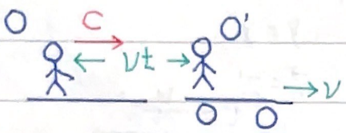
$$= \frac{u_x - v}{1 - \frac{v}{c^2}u_x} \qquad \frac{dt}{dt'} = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c^2}u_x}$$

$$u_y' = \frac{dy'(t)}{dt'} = \frac{dy(t)}{dt}\frac{dt}{dt'} = u_y\frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c^2}u_x}$$

$$u_z' = u_z\frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c^2}u_x}$$

$$u_x = \frac{u_x' + v}{1 + \frac{v}{c^2}u_x'}$$

光速在O與O'二慣性座標中 $c^2 = \frac{1}{\mu_0 \epsilon_0}$



O

$u_x = c$

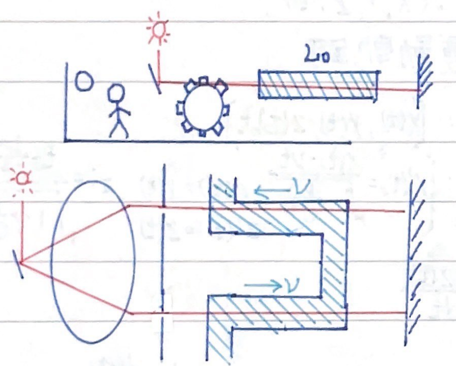
$u_y = 0$

$u_z = 0$

O'

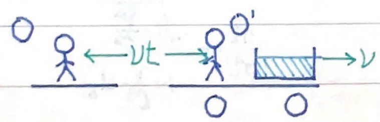
$$u_x' = \frac{c-v}{1-\frac{cv}{c^2}} = \frac{c-v}{\frac{c-v}{c}} = c$$

Fizeau experiment



在介質中之光速 $\frac{c}{n}$

$\frac{c}{n} \pm v(1-\frac{1}{n^2})$ if $n \to 1$, 真空 $= c$



$$u_x' = \frac{c}{n} \quad u_x = \frac{\frac{c}{n}+v}{1+\frac{v}{c^2}\frac{c}{n}} \underset{\text{if } c \gg v}{\approx} \frac{\frac{c}{n}+v}{1+\frac{v}{cn}} \approx \left(\frac{c}{n}+v\right)\left(1-\frac{v}{cn}+\cdots\right)$$

$$= \frac{c}{n}+v-\frac{v}{n^2} = \frac{c}{n}+v\left(1-\frac{1}{n^2}\right)$$

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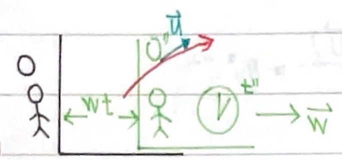
$$E=\frac{m_0c^2}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}+A \qquad \text{if } \vec{u}=0,\ E=m_0c^2 \Rightarrow A=0$$

$$=m(\vec{u})\,c^2 \qquad m(\vec{u})=\frac{m_0}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}$$



$$\frac{d}{dt}\frac{m_0\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}=\vec{F} \qquad \vec{F}=Q(\vec{E}+\vec{u}\times\vec{B})$$

$$\frac{d}{dt''}\frac{m_0\vec{u''}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}=\vec{F''} \qquad \vec{F''}=Q(\vec{E''}+\vec{u''}\times\vec{B''})$$

$$\vec{E''_{\parallel}}=\vec{E_{\parallel}} \qquad \vec{B''_{\parallel}}=\vec{B_{\parallel}} \qquad \vec{E''_{\perp}}=\frac{\vec{E_{\perp}}+\vec{W}\times\vec{B_{\perp}}}{\sqrt{1-\frac{W^2}{c^2}}} \qquad \vec{B''_{\perp}}=\frac{B_{\perp}-\frac{\vec{W}}{c^2}\times\vec{E_{\perp}}}{\sqrt{1-\frac{W^2}{c^2}}}$$

$$\vec{u}\cdot\vec{F}=\vec{u}\cdot\frac{d}{dt}\frac{m_0\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}=\frac{dE}{dt}$$ energy

$$\Big\downarrow \times\frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}$$

$$\Rightarrow \frac{1}{2}\frac{d}{dt}\left(\frac{\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\cdot\frac{\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\right)$$

$$\Rightarrow \frac{m_0}{2}c^2\frac{d}{dt}\left(\frac{\frac{1}{c^2}\vec{u}\cdot\vec{u}}{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}\right)=\frac{m_0}{2}c^2\frac{d}{dt}\left[\frac{1}{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}-1\right]=\frac{m_0}{2}c^2\frac{d}{dt}\frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}$$

$$=m_0c^2\frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\frac{d}{dt}\left(\frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\right)$$

$$\Rightarrow \vec{P}=\frac{m_0\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}} \qquad E=\frac{m_0c^2}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}$$

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$$\frac{m_0 c^2}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}} - m_0 c^2 = E_k \, ,$$

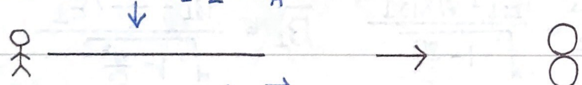
$$m_0 c^2 \left[1-\frac{\vec{u}\cdot\vec{u}}{c^2}\right]^{-\frac{1}{2}} = m_0 c^2 \left[1 - \frac{1}{2}\left(-\frac{\vec{u}\cdot\vec{u}}{c^2}\right) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}\left(\frac{\vec{u}\cdot\vec{u}}{c^2}\right)^2 + \cdots\right]$$
$$= m_0 c^2 + \frac{1}{2} m_0 \vec{u}\cdot\vec{u} + \frac{3}{8} m_0 \frac{(\vec{u}\cdot\vec{u})^2}{c^2}$$

$$\left(\frac{E}{c}\right)^2 - \vec{p}\cdot\vec{p} = \frac{m_0^2 c^2}{1-\frac{\vec{u}\cdot\vec{u}}{c^2}} - \frac{m_0^2\, \vec{u}\cdot\vec{u}}{1-\frac{\vec{u}\cdot\vec{u}}{c^2}} = \frac{m_0^2 (c^2 - \vec{u}\cdot\vec{u})}{\frac{c^2-\vec{u}\cdot\vec{u}}{c^2}} = m_0^2 c^2$$

$$E' = \frac{m_0 c^2}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}} \qquad \vec{p}\,' = \frac{m_0 \vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}} \qquad \left(\frac{E}{c}\right)^2 - \vec{p}\cdot\vec{p} = m_0 c^2$$

for light, $\frac{E}{c} = |\vec{p}|$ $\quad \left(\frac{E}{c}\right)^2 - \vec{p}\cdot\vec{p} = 0$, $m_0 = 0$

Ⓐ $u_z \vec{e}_z = \vec{u}_A$



$-u_z \vec{e}_z = \vec{u}_B$ Ⓑ

$m_{A_0} = m_{B_0}$

$$\vec{p}_A = \frac{m_0 u_z}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}} \qquad E_A = \frac{m_0 c^2}{\sqrt{1-\frac{u_z^2}{c^2}}} \qquad \vec{p}_A + \vec{p}_B = 0$$

$$\vec{p}_B = \frac{-m_0 u_z}{\sqrt{1-\frac{(-u_z)^2}{c^2}}} \qquad E_B = \frac{m_0 c^2}{\sqrt{1-\frac{(-u_z)^2}{c^2}}} \qquad E_A + E_B = \frac{2 m_0 c^2}{\sqrt{1-\frac{u_z^2}{c^2}}}$$

$$m_{0_{A+B}} = \frac{2 m_0}{\sqrt{1-\frac{u_z^2}{c^2}}}$$

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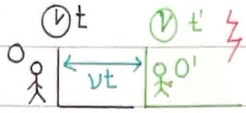


$$O \qquad (ct, x, y, z) \qquad (0,0,0,0)$$

$$O' \qquad (ct', x', y', z') \qquad (0,0,0,0)$$

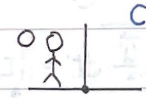
$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \qquad y = y' \qquad z = z'$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}} \quad , \quad ct' = \frac{ct - \frac{v}{c}x}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$c^2(t')^2 - (x')^2 = \frac{c^2t^2 + \frac{v^2}{c^2}x^2 - 2ct\frac{v}{c}x - x^2 - v^2t^2 + 2xvt}{1-\frac{v^2}{c^2}}$$

$$= \frac{(c^2t^2 - x^2)(1-\frac{v^2}{c^2})}{1-\frac{v^2}{c^2}} = c^2t^2 - x^2$$

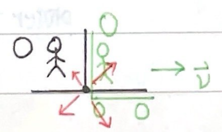
$$c^2(t')^2 - (x')^2 - (y')^2 - (z')^2 = c^2t^2 - x^2 - y^2 - z^2$$



$c^2t^2 - x^2 - y^2 - z^2 > 0$ 類時　$c^2t^2 - x^2 - y^2 - z^2 = 0$ 類光

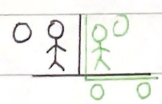
$$\parallel$$

$$c^2(t')^2 - (x')^2 - (y')^2 - (z')^2$$



$c^2t^2 - x^2 - y^2 - z^2 < 0$ 類空

同時性只能類空時間定義



$$\left(\frac{E}{c}\right)^2 - \vec{P}\cdot\vec{P} = m_0^2c^2 = \left(\frac{E'}{c}\right)^2 - \vec{P'}\cdot\vec{P'}$$

$$P_\mu = \left(\frac{E}{c}, \vec{P}\right) = \left(\frac{E}{c}, P_x, P_y, P_z\right)$$

$$\vec{r} = (x, y, z) \qquad \vec{r}\cdot\vec{r} = x^2 + y^2 + z^2$$

$$x_\mu = (ct, x, y, z) \qquad x_\mu x_\mu = c^2t^2 - x^2 - y^2 - z^2$$

$$P_\mu P_\mu = \left(\frac{E}{c}\right)^2 - P_x^2 - P_y^2 - P_z^2 = m_0^2c^2 = \left(\frac{E'}{c}\right)^2 - (P_x')^2 - (P_y')^2 - (P_z')^2$$

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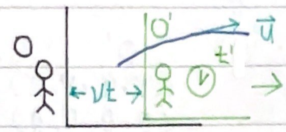


$$\vec{P'} = \frac{m_0 \vec{u'}}{\sqrt{1 - \frac{\vec{u'}\cdot\vec{u'}}{c^2}}} \qquad U'_x = \frac{U_x - v}{1 - \frac{vU_x}{c^2}} \qquad U'_{y,z} = \frac{U_{y,z}\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{vU_x}{c^2}}$$

$$P'_x = \frac{P_x - \frac{v}{c}\cdot\frac{E}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \frac{E'}{c} = \frac{\frac{E}{c} - \frac{v}{c}P_x}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad E' = \frac{m_0 c^2}{\sqrt{1 - \frac{\vec{u'}\cdot\vec{u'}}{c^2}}}$$

$$P'_y = P_y \qquad P'_z = P_z$$

$$1 - \frac{(U'_x)^2 + (U'_y)^2 + (U'_z)^2}{c^2} = \frac{1 - \frac{v^2}{c^2}}{\left(1 - \frac{vU_x}{c^2}\right)^2} = 1 - \frac{U_x^2 + U_y^2 + U_z^2}{c^2}$$

$$\frac{dt}{dt'} = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{vU_x}{c^2}} = \frac{\sqrt{1 - \frac{\vec{u'}\cdot\vec{u'}}{c^2}}}{\sqrt{1 - \frac{\vec{u}\cdot\vec{u}}{c^2}}}$$

$$\sqrt{1 - \frac{\vec{u}\cdot\vec{u}}{c^2}}\, dt = \sqrt{1 - \frac{\vec{u'}\cdot\vec{u'}}{c^2}}\, dt' = d\tau$$

proter time

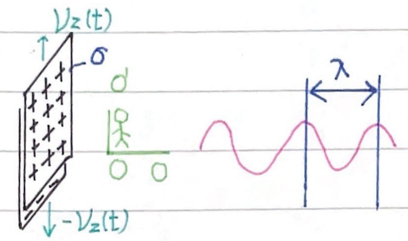
$$\frac{E'}{c} = \frac{m_0 c}{\sqrt{1 - \frac{\vec{u'}\cdot\vec{u'}}{c^2}}} = \frac{m_0 c}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{1 - \frac{vU_x}{c^2}}{\sqrt{1 - \frac{\vec{u}\cdot\vec{u}}{c^2}}} = \frac{\frac{E}{c}\left(1 - \frac{vU_x}{c^2}\right)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{\frac{E}{c} - \frac{v}{c}P_x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$P'_y = \frac{m_0 u'_y}{\sqrt{1-\frac{\vec{u}'\cdot\vec{u}'}{c^2}}} = \frac{1-\frac{v u_x}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\,\frac{m_0}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\,\frac{u_y\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v u_x}{c^2}} = P_y$$

$A_\mu = (A_0, A_x, A_y, A_z)$ $\quad A_\mu B_\mu = A_0B_0 - A_xB_x - A_yB_y - A_zB_z$

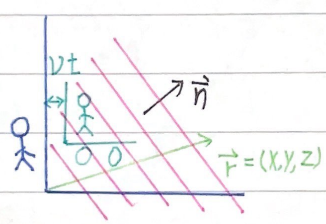
$B_\mu = (B_0, B_x, B_y, B_z)$



$$E_z(x,t) = -c\mu_0\sigma v_z\left(t-\frac{x}{c}\right) \qquad v_z(t) = v_0\sin(\omega t)$$

$$= -c\mu_0\sigma v_0 \sin\left[\omega\left(t-\frac{x}{c}\right)\right]$$

$$= E_0\sin(\omega t - k_x x) \quad , k_x\lambda = 2\pi, \quad k_x = \frac{2\pi}{\lambda}$$



$$|\vec{k}| = \frac{2\pi}{\lambda}$$

$$\frac{2\pi}{\lambda}\vec{n} = \vec{k} = (k_x, k_y, k_z)$$

$$\frac{2\pi}{\lambda}(\vec{n}\cdot\vec{r}) = \vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z$$

$O: (\omega t - k_x x - k_y y - k_z z)$ = 相位 $\qquad x' = \dfrac{x - \frac{v}{c}(ct)}{\sqrt{1-\frac{v^2}{c^2}}} \qquad y'=y \quad z'=z$

$O': (\omega' t' - k'_x x' - k'_y y' - k'_z z')$

$$ct' = \frac{ct - \frac{v}{c}x}{\sqrt{1-\frac{v^2}{c^2}}}$$

$k_y = k'_y \quad k_z = k'_z \qquad \dfrac{\omega'}{c} = \dfrac{\frac{\omega}{c} - \frac{v}{c}k_x}{\sqrt{1-\frac{v^2}{c^2}}} \qquad k'_x = \dfrac{k_x - \frac{v}{c}\frac{\omega}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$

$k_x = |\vec{k}| = \dfrac{\omega}{c} \qquad = \dfrac{\frac{\omega}{c}\left(1-\frac{v}{c}\right)}{\sqrt{1-\frac{v^2}{c^2}}} = \dfrac{\frac{\omega}{c}\sqrt{\left(1-\frac{v}{c}\right)^2}}{\sqrt{1-\frac{v}{c}}\sqrt{1+\frac{v}{c}}} = \dfrac{\omega}{c}\,\dfrac{\sqrt{1-\frac{v}{c}}}{\sqrt{1+\frac{v}{c}}}$ 都卜勒

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$$E = hf = \frac{h}{2\pi} 2\pi f = \hbar\omega \qquad |\vec{P}| = \frac{E}{C} = \frac{\hbar\omega}{c} = \hbar|\vec{k}|$$

$$\vec{P} = \hbar\vec{k} \qquad \frac{E}{c} = \frac{\hbar\omega}{c}$$

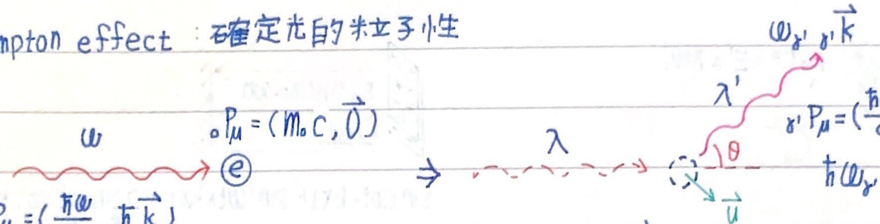
$$\left(\frac{E}{c}, \vec{P}\right) = \hbar\left(\frac{\omega}{c}, \vec{k}\right)$$

$$P_\mu\left(\frac{E}{c}, P_x, P_y, P_z\right) \qquad P_\mu P_\mu = \left(\frac{E}{c}\right)^2 - (P_x^2$$

SEASON

$$P_\mu = \left(\frac{E}{c}, P_x, P_y, P_z\right) \qquad P_\mu P_\mu = \left(\frac{E}{c}\right)^2 - (P_x^2 + P_y^2 + P_z^2) = m_0^2 c^2$$

Compton effect：確定光的粒子性



$${}_0P_\mu = (m_0 c, \vec{0})$$

$${}_{\gamma'}P_\mu = \left(\frac{\hbar\omega_{\gamma'}}{c}, \hbar_{\gamma'}\vec{k}\right)$$

$$\hbar\omega_{\gamma'} = {}_{\gamma'}E$$

$${}_{\gamma}P_\mu = \left(\frac{\hbar\omega}{c}, \hbar\vec{k}\right)$$

$${}_e\vec{P} = \frac{m_0\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}} \qquad {}_eE = \frac{m_0 c^2}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}$$

$${}_{\gamma}E + m_0c^2 = {}_{\gamma'}E + {}_eE, \quad \hbar\omega_\gamma + m_0c^2 = \hbar\omega_{\gamma'} + \frac{m_0 c^2}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}$$ 能量守恆

$$\hbar_\gamma\vec{k} + \vec{0} = \hbar_{\gamma'}\vec{k} + \frac{m_0\vec{u}}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}$$ 動量守恆

$${}_{\gamma}P_\mu + {}_0P_\mu = {}_{\gamma'}P_\mu + {}_eP_\mu$$ 四維動量

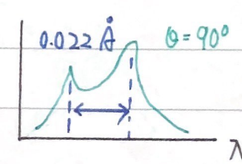
$${}_{\gamma}P_\mu + {}_0P_\mu - {}_{\gamma'}P_\mu = {}_eP_\mu$$

$$\Rightarrow ({}_{\gamma}P_\mu + {}_0P_\mu - {}_{\gamma'}P_\mu)({}_{\gamma}P_\mu + {}_0P_\mu - {}_{\gamma'}P_\mu) = {}_eP_\mu \cdot {}_eP_\mu$$

$$\underbrace{{}_{\gamma}P_\mu\,{}_{\gamma}P_\mu}_{0} + \underbrace{{}_0P_\mu\,{}_0P_\mu}_{m_0^2c^2} + \underbrace{{}_{\gamma'}P_\mu\,{}_{\gamma'}P_\mu}_{0} + \underbrace{2\,{}_0P_\mu\,{}_{\gamma}P_\mu}_{2m_0c\frac{\hbar\omega_\gamma}{c}} - \underbrace{2\,{}_0P_\mu\,{}_{\gamma'}P_\mu}_{2m_0c\frac{\hbar\omega_{\gamma'}}{c}} - 2\underbrace{{}_{\gamma}P_\mu\,{}_{\gamma'}P_\mu}_{\frac{\hbar\omega_\gamma}{c}\frac{\hbar\omega_{\gamma'}}{c} - \hbar_\gamma\vec{k}\cdot\hbar_{\gamma'}\vec{k} \,=\, \frac{\hbar\omega_\gamma}{c}\frac{\hbar\omega_{\gamma'}}{c}(1-\cos\theta)} = m_0^2c^2$$

$$2m_0c\frac{\hbar\omega_\gamma}{c} - 2m_0c\frac{\hbar\omega_{\gamma'}}{c} = 2\frac{\hbar\omega_\gamma}{c}\frac{\hbar\omega_{\gamma'}}{c}(1-\cos\theta)$$

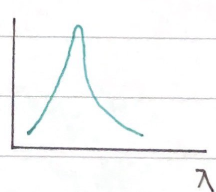
$$\times \frac{1}{\omega_\gamma\omega_{\gamma'}}\frac{1}{2m_0} \downarrow$$



$$\frac{1}{\omega_{k'}} - \frac{1}{\omega_\gamma} = \frac{\hbar}{m_0c^2}(1-\cos\theta), \quad \frac{c}{\omega_{k'}} - \frac{c}{\omega_k} = \frac{h}{2\pi m_0 c}(1-\cos\theta)$$

$$\frac{2\pi c}{\omega_{\gamma'}} - \frac{2\pi c}{\omega_\gamma} = \frac{h}{m_0c}(1-\cos\theta) = \lambda' - \lambda$$

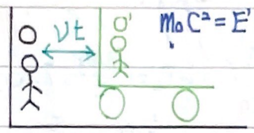
$$\frac{h}{m_0c} = 0.024\times10^{-10}\,m \text{ for } e^-$$



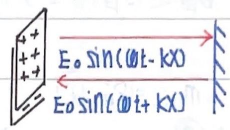
De Broglie 物質波

for light $E = hf = \hbar\omega$

$$|\vec{P}| = \frac{E}{c} = \frac{\hbar\omega}{c} = \hbar|\vec{k}|$$



$m_0c^2 = E' = \hbar\omega_0'$



$$\sin(\omega t - kx) + \sin(\omega t + kx) = 2\sin\omega t\cos kx$$

駐波

O' $\sin(\omega_0' t')$

O $$\omega_0' t' = \frac{m_0c^2}{\hbar}\frac{t - \frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\hbar}\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}t - \frac{m_0vx}{\sqrt{1-\frac{v^2}{c^2}}}\frac{1}{\hbar}$$

$$\frac{E}{\hbar}t - \frac{P_x}{\hbar}x = \omega t - k_x x,\quad k_x = \frac{P_x}{\hbar}\quad |\vec{k}| = \frac{|\vec{P}|}{\hbar}$$

$$|\vec{P}| = \frac{h}{\lambda},\quad \vec{P} = \hbar\vec{k}$$