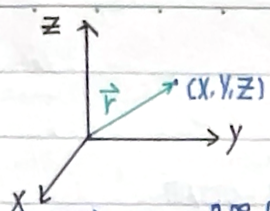


$$\vec r = x\hat x + y\hat y + z\hat z$$

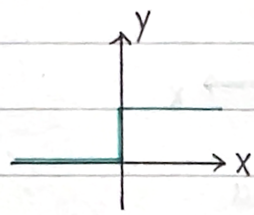
$$\delta^3(\vec r) = \delta(x)\delta(y)\delta(z)$$

$$\int \delta^3(\vec r)\,d\tau = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta(x)\delta(y)\delta(z)\,dx\,dy\,dz = 1$$

$$\int f(\vec r)\delta^3(\vec r-\vec a)\,d\tau = f(\vec a)$$

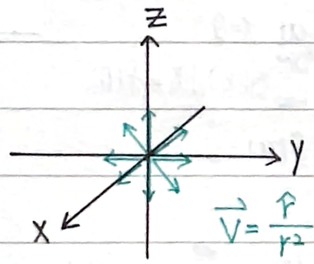
$$x\left[\frac{d}{dx}\delta(x)\right] = -\delta(x)$$

$$\int_{-\infty}^{\infty} x\,\frac{d}{dx}\delta(x)\,dx = \int_{-\infty}^{\infty} x\,d[\delta(x)] = \overset{0}{\cancel{x\,\delta(x)}}\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\overset{1}{\cancel{\delta(x)}}\,dx = -1$$



step function $H(x)\begin{cases}1 & x\ge 0\\ 0 & x\le 0\end{cases}$

$$\frac{dH(x)}{dx} = \delta(x) \qquad \int_{-\infty}^{\infty}\frac{dH(x)}{dx}\,dx = \int_{-\infty}^{\infty} dH(x) = H(x)\Big|_{-\infty}^{\infty} = 1 = \int_{-\infty}^{\infty}\delta(x)\,dx$$



$$\vec V = \frac{\hat r}{r^2}$$

$$\nabla\cdot\frac{\hat r}{r^2} = 4\pi\delta^3(\vec r) \quad \text{or} \quad \nabla\cdot\frac{\hat r-\hat r'}{(\vec r-\vec r')^2} = 4\pi\delta^3(\vec r-\vec r')$$

$$\nabla\cdot\left(\frac{1}{\vec r-\vec r'}\right) = \frac{\partial}{\partial x}\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}\hat x + \frac{\partial}{\partial y}\frac{1}{\sqrt{\cdots}}\hat y + \frac{\partial}{\partial z}\frac{1}{\sqrt{\cdots}}\hat z$$

$$= -\frac{1}{2}\frac{1}{(\cdots)^{\frac{3}{2}}}(x-x')2\hat x - \frac{1}{2}\frac{1}{\sqrt{(\cdots)^3}}2(y-y')\hat y - \frac{1}{2}\frac{1}{\sqrt{(\cdots)^3}}2(z-z')\hat z$$

$$= -\frac{1}{\sqrt{(\cdots)^3}}\left[(x-x')\hat x + (y-y')\hat y + (z-z')\hat z\right] = -\frac{1}{(\vec r-\vec r')^3}(\vec r-\vec r')$$

$$= -\frac{1}{\sqrt{(\vec r-\vec r')^2}}(\hat r-\hat r')$$

$$\nabla^2\left(\frac{1}{\vec r-\vec r'}\right) = -4\pi\delta^3(\vec r-\vec r')$$

NO.

DATE / /

proof theorem 1

(d)→(a) $\nabla \times \vec{F} = \nabla(-\nabla V) = 0$

(a)↔(c) $\oint \vec{F} \cdot d\vec{l} = \int (\nabla \times \vec{F}) \cdot da = 0$

(c)→(d) $\int_a^b \vec{F} \cdot d\vec{l} - \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b \vec{F} \cdot d\vec{l} + \int_b^a \vec{F} \cdot d\vec{l} = \oint \vec{F} \cdot d\vec{l} = 0$

$\Rightarrow \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b \vec{F} \cdot d\vec{l}$

(b)→(c) same as (c)→(d)

proof theorem 2

(d)→(a) $\nabla \cdot \vec{F} = \nabla \cdot (\nabla \times \vec{W}) = 0$

(a)↔(c) $\oint \vec{F} \cdot d\vec{a} = \int (\nabla \cdot \vec{F}) d\tau = 0$

(c)→(b) $\int_{I} \vec{F} \cdot d\vec{a} - \int_{II} \vec{F} \cdot d\vec{a} = \oint \vec{F} \cdot d\vec{a} = 0$

$\int_{I} \vec{F} \cdot d\vec{a} = \int_{II} \vec{F} \cdot d\vec{a}$

(b)→(c) same as (c)→(b)