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## PHYS1241 Assignment 4

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### QUESTION 1 - PHOTOELECTRIC EXPERIMENT

#### Q1 PART A - PLANCK'S CONSTANT

If the stopping potential is  $U$ , the frequency  $f$  and the work function of sodium  $\phi$  then we know, from Einstein, that

$$U = hf - \phi$$

$$U = \frac{hc}{\lambda} - \phi$$

Let the stopping potential and wavelengths for the  $300nm$  and  $400nm$  experiments be  $U_1$ ,  $\lambda_1$ ,  $U_2$  and  $\lambda_2$  respectively. Then we can simultaneously solve for  $h$ :

$$U_1 = \frac{hc}{\lambda_1} - \phi \tag{1}$$

$$U_2 = \frac{hc}{\lambda_2} - \phi \tag{2}$$

Subtracting (2) from (1) we find:

$$\begin{aligned}
 U_1 - U_2 &= hc \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) \\
 h &= \frac{U_1 - U_2}{c \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)} \\
 &= 4.1 \times 10^{-15} \text{ eVs} \\
 &= 6.6 \times 10^{-34} \text{ Js}
 \end{aligned}$$

Where, of course, all values are rounded to 2 significant figures. This is the correct value (The accepted value is  $6.626 \times 10^{-34} \text{ eVs}$ ).

### Q1 PART B - WORK FUNCTION

We can find  $\phi$  by simply substituting the value for  $h$  in (1):

$$\begin{aligned}
 \phi &= \frac{hc}{\lambda_1} - U_1 \\
 &= 2.3 \text{ eV}
 \end{aligned}$$

This is the correct value of  $\phi$  for sodium, as the accepted value is  $2.28 \text{ eV}$  [2].

### Q1 PART C - THRESHOLD WAVELENGTH

The cutoff wavelength  $\lambda$  can now be calculated by:

$$\begin{aligned}
 0 &= \frac{hc}{\lambda} - \phi \\
 \lambda &= \frac{hc}{\phi} \\
 &= 540 \text{ nm}
 \end{aligned}$$

This is extremely close to the value if derived from previously mentioned accepted values,  $\lambda = 546.6 \text{ nm}$ .

## QUESTION 2 - SINGLY IONISED HELIUM

According to the derivation of the Bohr model, the total energy of an electron in the  $n^{\text{th}}$  shell is given by

$$E_n = -E_0 \frac{Z^2}{n^2}$$

Where  $E_0$  was  $13.6\text{eV}$ , and "0" energy is an electron not moving, infinitely far from the nucleus. Importantly,  $Z$  is the atomic number of the atom - so for Helium,  $Z = 2$ . Thus we find the energy of an electron in the  $n^{\text{th}}$  shell of a singly-ionised Helium atom as

$$E_n = -\frac{54.4}{n^2} \text{eV}$$

We can therefore calculate the frequency  $f = \frac{\Delta E}{h}$  and wavelength  $\lambda = \frac{c}{f}$  of the emission lines of singly-ionised Helium as compared to Hydrogen:

Table 1: Helium

From	To	Energy (eV)	Wavelength (nm)
4	3	2.64	469
6	4	1.89	657
7	4	2.29	542
8	4	2.55	487
9	4	2.73	455
11	5	1.73	719
12	5	1.8	690
13	5	1.85	669
$\infty$	5	2.18	570

Table 2: Hydrogen

From	To	Energy (eV)	Wavelength (nm)
3	2	1.89	657
4	2	2.55	487
5	2	2.86	434
6	2	3.02	411
7	2	3.12	397

Note that only those wavelengths in the visible range have been presented here. To see a more complete list of wavelengths, see Appendix 1.

From these values, and noting that the visible wavelength range is  $400 - 700\text{nm}$  or so, we can see that Singly-Ionised Helium would have many more visible emission lines than Hydrogen. More importantly, the series of emission lines going to  $n = 5$  approaches a visible wavelength,  $570\text{nm}$ . This would be quite faint, as very few electrons would be in shells  $n > 10$ , but if the Helium were very hot then this series of lines would visibly seem to be approaching the limit  $\lambda = 570\text{nm}$ .

### QUESTION 3 - PROPERTIES OF A FREE ELECTRON

The given wavefunction is:

$$\psi(x, t) = \sin(kx - \omega t)$$

We know that this function must be periodic in  $x$ , specifically completing a period every  $\lambda$  that  $x$  changes. This means that, with the given value of  $k$ :

$$k(x + \lambda) = kx + 2\pi$$

$$\lambda = \frac{2\pi}{k}$$

$$\lambda = 0.13\text{nm}$$

Momentum is given by DeBroglie's Equation:

$$p = \frac{h}{\lambda}$$

$$p = 5.3 \times 10^{-24} \text{kgms}^{-1}$$

Kinetic Energy is found by the usual formula, where  $p = mv$ :

$$\begin{aligned} E &= \frac{1}{2}mv^2 \\ &= \frac{1}{2} \frac{p^2}{m_e} \\ &= 1.5 \times 10^{-17} \text{J} \end{aligned}$$

Finally, speed is magnitude of velocity, and the velocity is only one-dimensional (as the wavefunction only has one space variable) so

$$\begin{aligned} |v| &= \left| \frac{p}{m_e} \right| \\ &= 5.8 \times 10^6 \text{ms}^{-1} \end{aligned}$$

## QUESTION 4 - DETECTION OF LASER LIGHT

### Q4 PART A - SEMICONDUCTOR MATERIALS

This photon would have and energy of:

$$\begin{aligned} E &= hf \\ &= \frac{hc}{\lambda} \\ &= 2.48 eV \end{aligned}$$

The photon must be able to give all of its energy to a single electron in the valence band. As this electron must jump up by the *band gap energy or more*, the band gap energy must be less than or equal to the photon's energy.

The only material that satisfies this is material A. Material B and C would not absorb the photon, as there is not enough energy to excite an electron to the conduction band.

Material B is actually an insulator, as any material with a band gap  $> 3.5 eV$  is considered an insulator [1]; in fact, it is one of the best insulators (The best known insulator is diamond with the same band gap of  $6 eV$  [1]). Material C is an unbelievably good insulator, one and a half orders of magnitude better than the best insulator we know.

### Q4 PART B - PHOTOELECTRIC EFFECT

The photoelectric effect could definitely be used to detect this light - with no backing voltage, electrons would jump off the cathode and produce a current. However, one must consider Einstein's equation for kinetic energy of the electrons  $E$ , given here in  $eV$ :

$$\begin{aligned} E &= hf - \phi \\ E &= 2.48 - \phi \end{aligned}$$

Note that for the electrons to jump,  $E > 0$  so  $\phi < 2.48 eV$ . Thus, it is important to choose a cathode with a low work-function. These materials appear to be rare compared to cathodes with higher work functions - some possible cathodes are Cesium ( $\phi = 2.1 eV$ ) or Sodium ( $\phi = 2.28 eV$ ) [2].

## REFERENCES

- [1] Prof. Mike Gal, "Physical waves", Lectures, *UNSW: PHYS1241*
- [2] R Nave, "Work Functions for Photoelectric Effect" *Hyperphysics* (2017), found at <http://hyperphysics.phy-astr.gsu.edu/hbase/Tables/photoelec.html>, accessed 26 Oct. 2017

## APPENDIX 1 - ADDITIONAL SPECTRAL LINES OF HELIUM AND HYDROGEN

These have been automatically generated. Here is the python source code, which generates data in a LaTeX table format:

```

1 import math
2
3 h = 4.1357 * 10**(-15) #eV s
4 c = 3 * 10**8 #m/s
5
6 Z = int(input('Atomic number: '))
7
8 E_0 = 13.6 #eV
9
10 def round_sigfigs(a, s):
11     most_significant = int(math.floor(math.log10(abs(a))))
12     answer = round(a, -most_significant + s - 1)
13     if -most_significant + s - 1 < 1:
14         answer = int(answer)
15     return answer
16
17 for i in range(1, 8):
18     for j in range(i+1, i+6):
19         E_i = -E_0 * Z**2 / (i**2)
20         E_j = -E_0 * Z**2 / (j**2)
21         E = E_j - E_i
22         wavelength_nm = (c*h/E) * 10**9
23         print(' {0} & {1} & {2} & {3}\\\\\\\\'.format(
24             j, i, round_sigfigs(E, 3), round_sigfigs(wavelength_nm, 3)))

```

All values are rounded to 3 significant figures, but because of how python prints decimals, trailing 0's have been omitted.

Table 3: Helium

From	To	Energy (eV)	Wavelength (nm)
2	1	40.8	30.4
3	1	48.4	25.7
4	1	51.0	24.3
5	1	52.2	23.8
6	1	52.9	23.5
3	2	7.56	164
4	2	10.2	122
5	2	11.4	109
6	2	12.1	103
7	2	12.5	99.3
4	3	2.64	469
5	3	3.87	321
6	3	4.53	274
7	3	4.93	251
8	3	5.19	239
5	4	1.22	1010
6	4	1.89	657
7	4	2.29	542
8	4	2.55	487
9	4	2.73	455
6	5	0.665	1870
7	5	1.07	1160
8	5	1.33	936
9	5	1.5	825
10	5	1.63	760
7	6	0.401	3090
8	6	0.661	1880
9	6	0.84	1480
10	6	0.967	1280
11	6	1.06	1170
8	7	0.26	4770
9	7	0.439	2830
10	7	0.566	2190
11	7	0.661	1880
12	7	0.732	1690

Table 4: Hydrogen

From	To	Energy ( $eV$ )	Wavelength ( $nm$ )
2	1	10.2	122
3	1	12.1	103
4	1	12.8	97.3
5	1	13.1	95.0
6	1	13.2	93.8
3	2	1.89	657
4	2	2.55	487
5	2	2.86	434
6	2	3.02	411
7	2	3.12	397
4	3	0.661	1880
5	3	0.967	1280
6	3	1.13	1090
7	3	1.23	1010
8	3	1.3	955
5	4	0.306	4050
6	4	0.472	2630
7	4	0.572	2170
8	4	0.637	1950
9	4	0.682	1820
6	5	0.166	7460
7	5	0.266	4660
8	5	0.332	3740
9	5	0.376	3300
10	5	0.408	3040
7	6	0.1	12400
8	6	0.165	7510
9	6	0.21	5910
10	6	0.242	5130
11	6	0.265	4680
8	7	0.0651	19100
9	7	0.11	11300
10	7	0.142	8770
11	7	0.165	7510
12	7	0.183	6780