UNSW SCHOOL OF COMPUTER SCIENCE AND ENGINEERING

Word Embedding and Question Answering

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INTRODUCTION

"Who is the first Prime Minister of Australia?"

Figure 1 is a screen shot of search result returning by Google. This is a question a Question Answering (QA) system should be able to respond to. QA system evaluates texts across the web or database to find answer of a particular question to return in a form of short text.

Current machine algorithms extracts answer from a short paragraph instead of a long content (e.g. an entire wiki page/a news article). When applying modern machine learning algorithms on a long content, the result can be complicated and lenthy.

This essay aims to firstly gives a mathematical heavy explanation on nowadays natural language processing (NLP) algorithms on word embedding, then propose my own heuristics based on them to predict answers from a long content based on the question being asked.

My algorithm will be tested with Google Natural Questions which contains its own private testing dataset.

GENERAL WORD EMBEDDING MODELS

Word Embedding is the collective name for techniques in NLP where words or phrases are mapped into vectors of real numbers.

Figure 1: A result returned by Google to answer "Who is the first Prime Minister of Australia?"

As per all machine learning algorithms, the general philosophy is to find the minimum of a converged function. When trying to convert words to number, its original utf code will not work well. What we need is a hard-coded map between the word itself, and its semantics. As human, we learn these semantics from daily experience which machine has no way to access, therefore, we need a mapping from words and its semantic in order for machine to decode.

First Attempt: Hot One Encoding

Given a collection of N unique words, each word is of size N. The vector is very sparse such that only one index is 1 and others are 0. This is to say each word is a **dimension**.

Figure 2 shows an example of hot encoding.

However, this model has some significant drawbacks. Firstly, it probably works fine for English as it has roughly 8000 frequent words. However, for languages like Chinese or Japanese, they are character-based, and each character will generate tons of combinations. This model will suffer from dramatic increase in dimensionalities. Secondly, all vectors in the matrix are independent to each other, while we do want them to learn the interconnections between them.

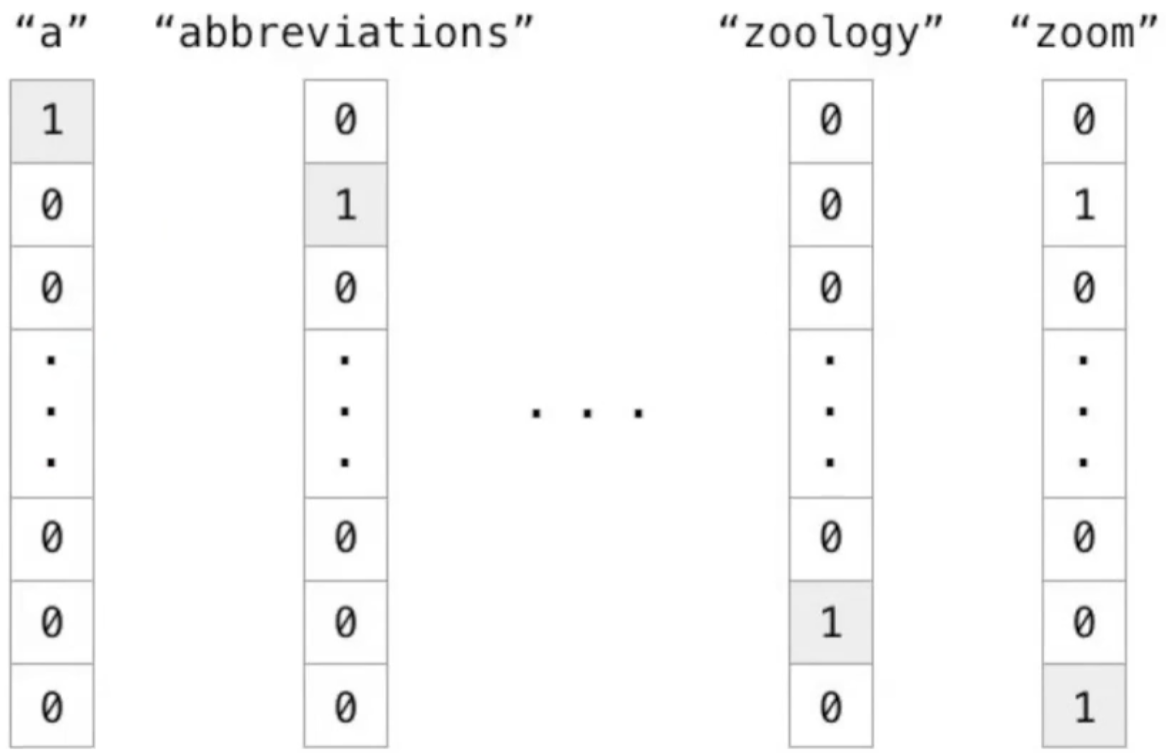

Figure 2: One Hot Encoding Example.

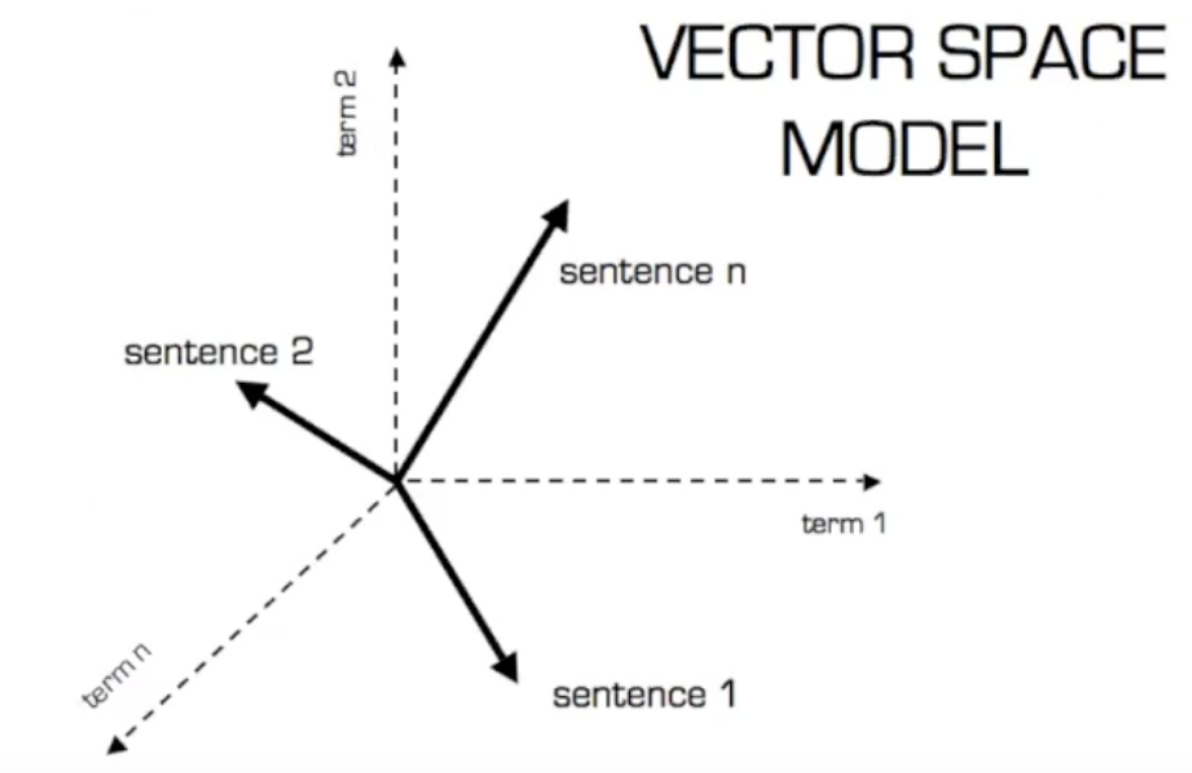

Figure 3: Bag of Words Vector Space Representation

SECOND ATTEMPT: BAG OF WORDS (BOW) MODEL

On the second attempt, we focus more on extracting information from the perspective of the whole document. Under this model, we represent words in a document as a bag (multiset) of words– we discard order and grammar, and only keep its multiplicity.

Here is an example:
Document 1: "George" "likes" "to" "play" "video" "games", "Mary" "likes" "video" "games" "too".
Document 2: "Mary" "also" "likes" "movies".

The BoW representation of the two above documentations is

BoW1: {"George" : 1, "likes : 2", "to" : 1, "play " : 2, "video" : 2, "games" : 2, "Mary" : 1, "too": 1}
BoW2: {"Mary" : 1, "also" : 1, "likes": 1, "movie" : 1}
Here, each key is the word, and the value corresponds to occurrence.
This approach could be potentially used to compare similarities between two documents. When comparing them, we construct two vectors with each index corresponding to each unique words. Each be the count of the word appeared in that specific document. The

	0	1
gender	male	female
age	child	adult

Table 1: Table 1: Represent Words in Fewer Dimensions

similarities of two documents can be measured by its Euclidean distance or its consine similarities.

Intuitively, two documents are similar if their vector points to the similar direction. We define the cosine similarities of two documents as:

$$cosim(U_k, U_i) = \frac{\langle U_k, U_i \rangle}{\|U_k\| \cdot \|U_i\|} \tag{1}$$

where $\langle U_k, U_i \rangle$ is the scalar product of U_i and U_k.

THIRD ATTEMPT: NEURAL NETWORK BASED (NNLM)

One intuitive question to ask is, Bag of Words and One Hot Encoding both treat each words as a separate dimension, but do we need so many dimensions?

The answer is probably no. Because a lot of words are similar or related. e.g.

Nouns: dog, cat, pet.
Verbs: fish, fished, fishing.
Adjective: very, great, significant.
Context: play guitar, piano, games, tennis

To not feel overwhelmed by the math at first, let's start with a easy example:
Words to be processed: boy, girl, woman, man
We can simply divide these words into two dimensions: gender and age as per Table 1 shows. This method is called **Distributed Representation**.

However, human language are so complex that we are not able to pre-define all dimensions for all words. Therefore we want to train a function **f** to get the word embedding for us. Figure 4 shows the function mapping.

To the best of my knowledge, we can do this in two ways.

AUTOENCODER (GAN)

An autoencoder is a type of neural network that learns to label dimensions in an unsupervised manner. Figure 5 shows that an autoencoder is composed of two parts: the input

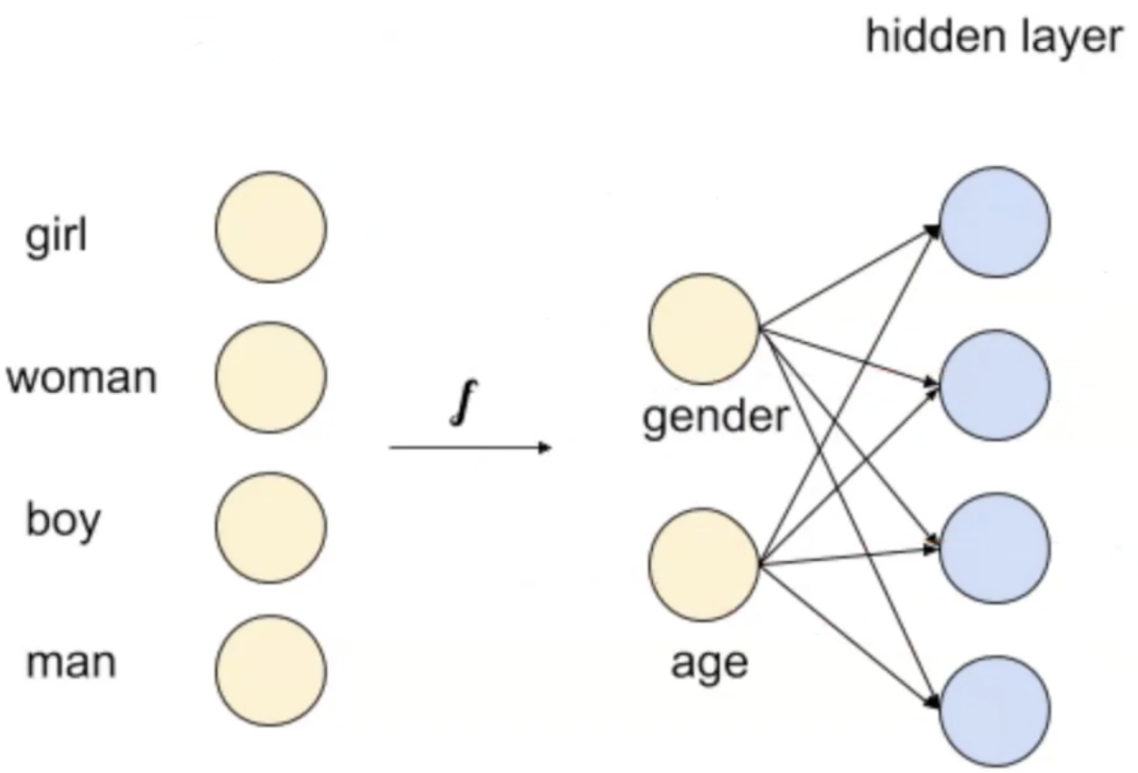

Figure 4: Figure 4: Neural Network Based Mapping

side, marked as X, and the reconstruction (output) side, marked as X'. The X tries to learn a distributed representation of the original data that aims to reduce the dimensions, and the X' side is where the autoencoder tries to generate from the reduced representation as close as possible to its original input, hence its name, reconstruction.

BASIC ARCHITECTURE OF AUTOENCODER An autoencoder consists of two parts, the encoder and the decoder, which can be defined as transitions ψ and ϕ, such that:

$$\psi : \chi \mapsto \mathfrak{F}$$
$$\phi : \mathfrak{F} \mapsto \chi$$
$$\psi, \phi = arg_{\psi,\phi} min |\chi - (\psi * \phi)\chi|$$

In the simplest form, we assume there is only one hidden layer. The encoder stage of the autoencoder takes input $x \in \mathbb{R}^d = \chi$ and maps it to $\mathbf{h} \in \mathbb{R}^p = \mathfrak{F}$.

$$\mathbf{h} = \delta(\mathbf{W}x + \mathbf{b})$$

This image $\mathbf{h}$ is the code section shown in Figure 5. δ is normally a sigmoid function defined as following:

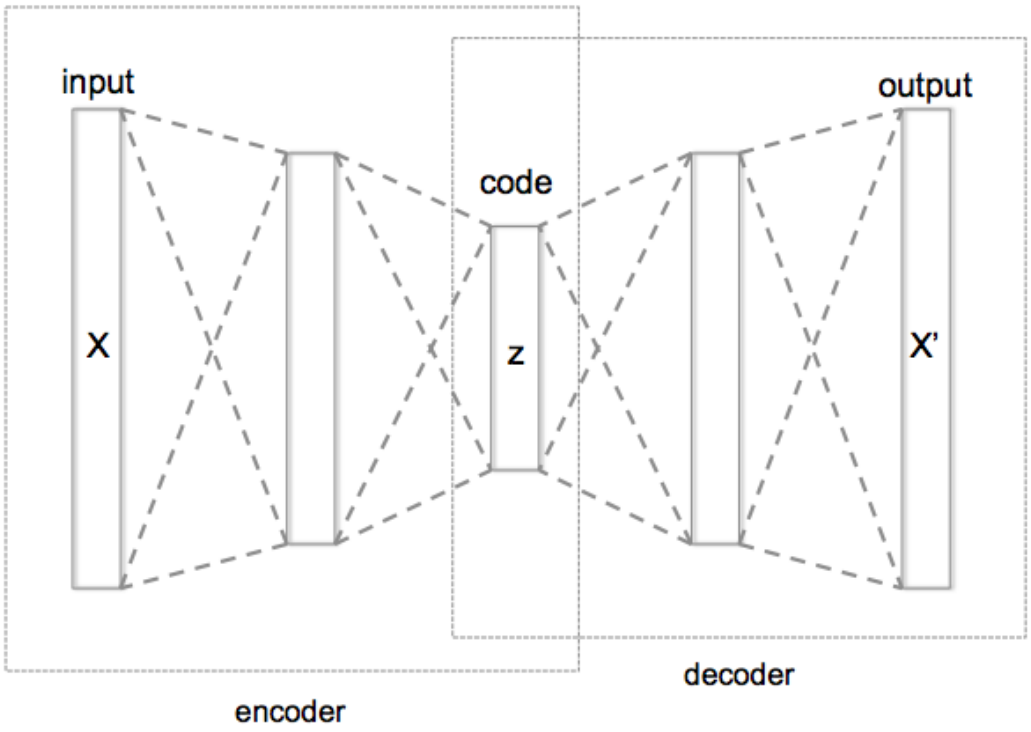

Figure 5: Schematic structure of an autoencoder with 3 fully connected hidden layers. The code (z, or h for reference in the text) is the most internal layer.

$$S(x) = \frac{1}{1+e^{-x}} = \frac{e^x}{e^x+1}$$

W is a weight matrix and **b** is a bias vector. The initializations of these two variables are random. They will be learnt iteratively through **backpropagation**.

$$\mathbf{h}' = \sigma(\mathbf{W}'x + \mathbf{b}')$$

The backpropagation computes weight space **W** and bias vector b with respect of a loss function $\mathbb{C}$.

x : input (vector of features).
x' : target output, the closer to x, the better.
$\mathbb{C}$: loss function, which intuitively associate with the "cost" of certain event or representation. The aim of training is to minimize the loss function.
L: the number of layers. In the above example, L equals to 1. However, here we aims to provide a generalized form.
$W^l = W^l_{j_k}$: the weights between layer $l-1$ and l, where $W^l_{j_k}$ is the weight between the k-th node in layer l-1 and the j-th node in layer l.
f^l: sigmoid function at layer l.
The whole neural network can be represented by this function:

$$g(x) = f^L(W^L f^{L-1}(W^{L-1}...f^1(W^1 * x)...)$$

For each input x_i, there will be an output x'_i corresponding with it. the loss of the model on that pair is the cost of the difference between the predicted output $g(x_i)$ and the target output x'_i:

$$\mathbb{C}(y_i, f^L(W^L f^{L-1}(W^{L-1}...f^1(W^1 * x)...))$$

For each layer, we compute the minimum of the loss function by calculating its derivatives.

$$\frac{\partial \mathbb{C}}{\partial W^l_{j_k}}$$

Where the loss function is defined as:

$$\mathbb{C}(x, x') = |x - x'|^2 = |x - \sigma'(\mathbf{W}'(\sigma(\mathbf{W}x + b)) + b'|$$

where x is the average across all inputs.

NEURAL NETWORK LANGUAGE MODEL (NNLM)

However, the above model does not fully utilize the context of words, as in words in this category:
Context: play guitar, piano, games, tennis

More specifically, autoencoder does generate vectors that significantly reduce the dimensions compared to previous attempts but it takes into account less than 1 or two words, hence it is not so great at predicting the next word based on the previous context.
Bengio proposed the following model that makes use of the word order in a document. It can be presented by the conditional probability of the next word given all the previous ones:

$$\hat{P}(w_1^T) = \prod_{t=1}^{T} \hat{P}(w_t | w_1^{t-1})$$

where w_t is the t-th word.

In more details:

The training set is $w_1, w_2, ...w_t \in V$, where V is the vocabulary set that is roughly of size 10^5 depending on language.
We need to learn $f(w_t, ..., w_{t-n+1}) = \hat{P}(w_t | w_1^{t-1})$, in the sense it gives highest likelihood of the next word.
This is subject to constraints that:
For any choice of w_1^{t-1}

$$\sum_{i=1}^{|V|} f(i, w_t, ..., w_{t-n+1}) = 1$$
$$f > 0$$

The function $f(w_t, ..., w_{t-n+1}) = \hat{P}(w_t | w_1^{t-1})$ has two parts:
1. $\mathbb{C}: w_i \to \mathbb{C}(w_i) \in \mathbb{R}^m$ where m ranges from 30 to 100, which is significant smaller than the size of V and $w_i \in V$.
2. A probability function over words:
A function g that maps an input sequence of words $(\mathbb{C}(w_{t-n+1}), ..., \mathbb{C}(w_{t-1})$ to the next word $w_t \in V$.

$$f(i, w_t, ..., w_{t-n+1}) = g(i, \mathbb{C}(w_{t-1}, ..., C(w_{t-n+1}))$$

$\mathbb{C}$ is being shared for the whole neural network. $\mathbb{C}$ simply maps a word $w_t \in V$ to a *feature vector* of $\mathbb{R}^m$ shown as **figure 7**.

The function g might be implemented by a feed-forward or recurrent neural network with parameter ω. The overall parameter set is $\theta = (\mathbb{C}, \omega)$

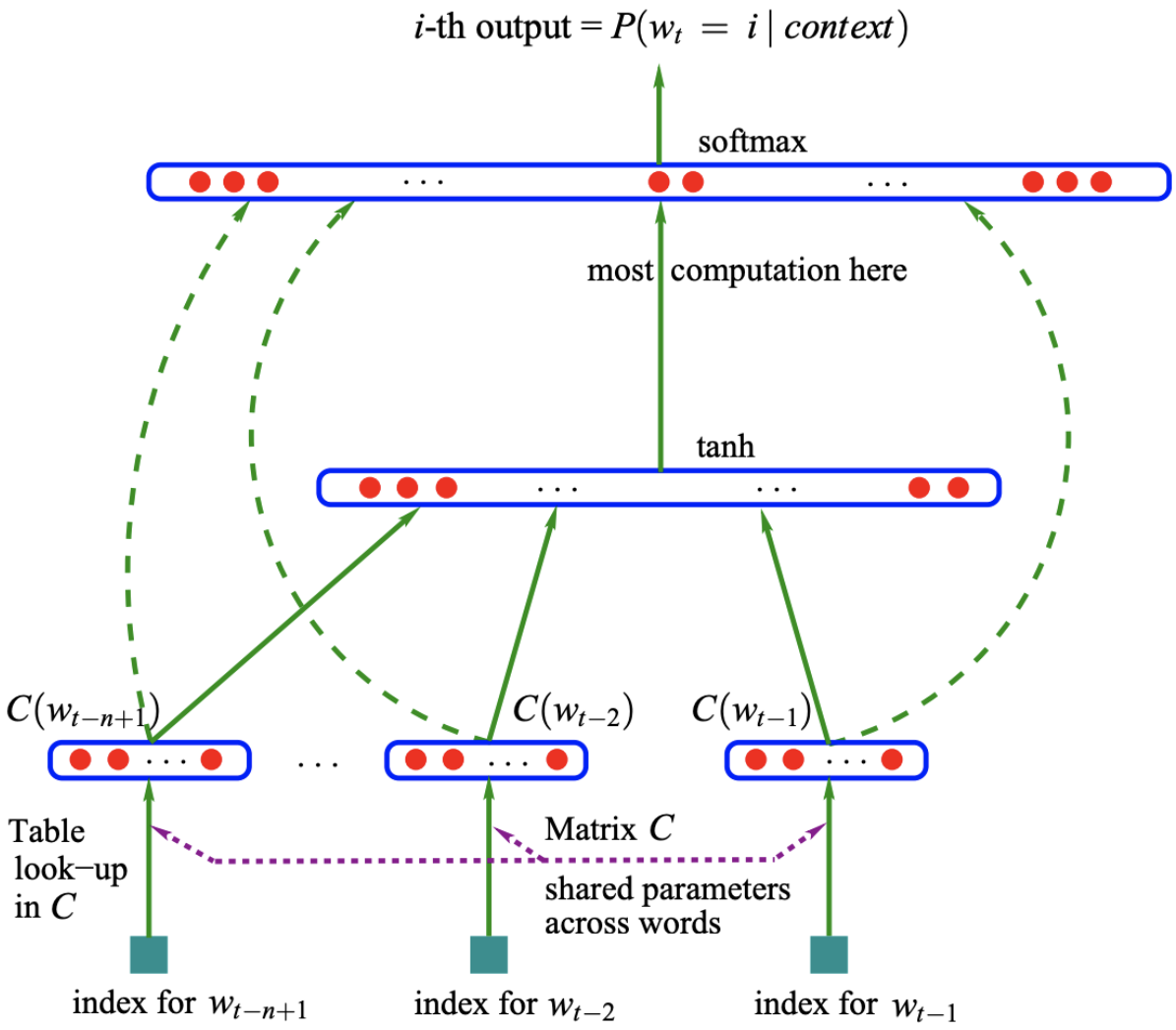

Figure 6: Neural Architecture

Training aims to maximize ω the log-likelihood.

$$L = \frac{1}{T}\sum_t log f(w_t, w_{t-1}, ..., w_{t-n+1}; \theta) + R(\theta)$$

where R is a regularization form.

The *softmax* output layer in figure 7 grants the positive probabilities summing up to 1:

$$\hat{P}(w_t|w_{t-1}, ..., w_{t-n+1}) = \frac{e^{yw_t}}{\sum_i e^{y_i}}$$

The y_i is the log probability for each possible output word $w_i \in V$, defined as follows:

$$y = b + Wx + Utanh(d + Hx)$$

as figure 7 shows, $tanh$ is applied on each word. W is the weight matrix we described in the previous attempt. x is the word feature activation vector that is a concatenation from $\mathbb{C}(w_i)$

$$x = \mathbb{C}(w_{t-1}), \mathbb{C}(w_{t-1}), ..., \mathbb{C}(w_{t-n+1})$$

b and d are biases which are addictive parameter in neural networks that only have outgoing edges but no incoming edges. Its existence is mainly to not let function passing the origin. Here is a intuitive example of training results:
`cat` and `dog` plays similar role in linguistic, syntactically and semantically. `backyard` and `frontyard` also plays similar role. `running` and `walking` is also a pair. `is` and `was` are also quite similar.
From the probability mass of them, we can simply transform:
`A cat is walking at frontyard`
to `A dog is walking at the frontyard`
to `A dog is running at the frontyard`
to `A dog was running at the frontyard`
to `A dog was running at the backyard`

FOURTH ATTEMPT: WORD2VEC

The above attempt only considers "context" before certain word. However, in 2013, a paper published by Mikolov and Jeff Dean proposed a famous model, Word2Vec, that considers bidirectional context, i.e. context before and after the predicting word.

It has a very similar structure as the attempt we elaborated above, but it is more focusing on **Word Embedding**, i.e. representing a word in vector form, instead of predicting the word based on preceding context.

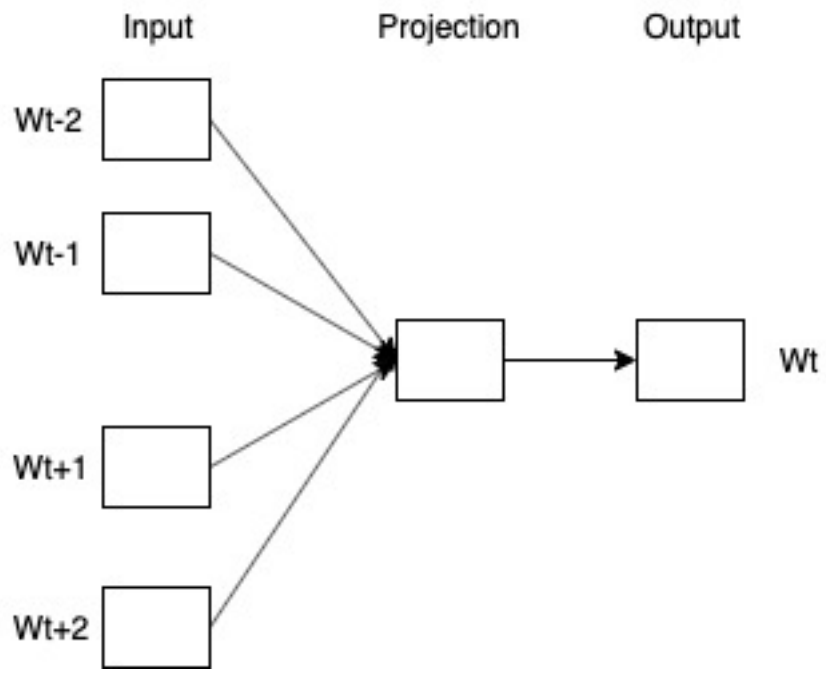

Figure 7: Continuous Bag-of-Words

Word2Vec is a two-layer neural net that takes a input word and output it as a feature vector. The core purpose is to gather similar word into the same vector space by calculatingthe probabilities –the likelihood that they will co-occur.

It has the following two approaches.

CONTINUOUS BAG-OF-WORD (CBOW)

The aim of CBOW is the predict the middle word given the words surrounding it.
For example, *{I ?? video games} → {play}*

As figure 7 shows, this is quite similar to the feed-forward neural network described above. It removes all the hidden layers, and its projection layer is shared among all words, where the projection layer is either sum or average of all the vectors of the words. Note that despite its name being "Bag-of-words", it bears no relevance to the first attempt we mentioned above.

CONTINUOUS SKIP-GRAM(SKIP GRAM)

Opposite to CBOW model, the objective of *skip gram* is to predict surrounding words from a single word, as shown in figure 8.

Given a sequence of words $w_1, w_2, w_3, ..., w_n$, similar to the maximization of log-likelihood described in the third attempt, the goal of the Skip-gram model is to maximize the average

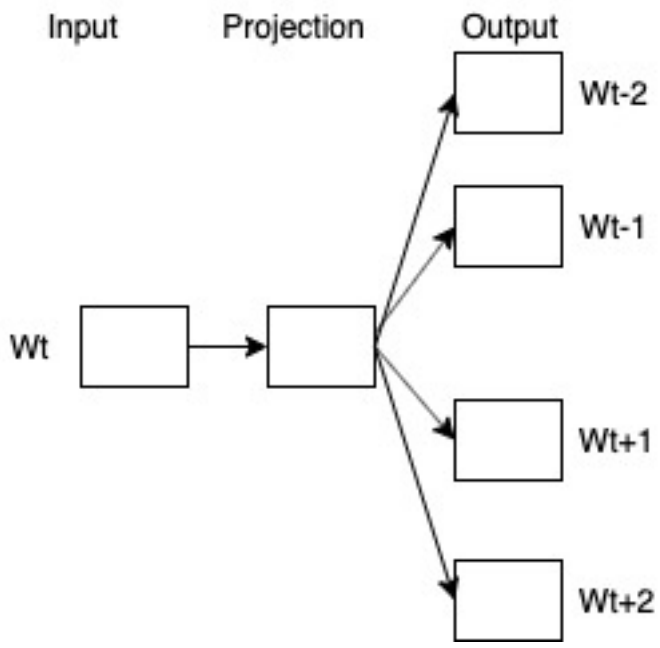

Figure 8: Continuous Skip-Gram

log probability:

$$\frac{1}{T}\sum_{t=1}^{T}\sum_{-c\leq j\leq c, j\neq 0} logp(w_{t+j}|w_t)$$

where c is the training size. Obviously, larger c will lead to higher accuracy, in the cost of trading run time. We define $p(w_{t+j}|w_t)$ using the *softmax* function.

$$p(w_O|w_I) = \frac{exp(x_{w_O}^T v_{w_I})}{\sum_{w=1}^{W} exp(x_{w_O}^T v_{w_I})}$$

where x_w and v_w are input and output shown in **figure 8**. W is the size of vocabulary in the language we trained in.

Refinement of Word2Vec

Word2Vec is a quite expensive algorithm just because of the significant time it takes to run it.
The training complexity of the architecture is proportional to

$$Q = C \times (D + D \times log_2(V))$$

where C is the maximum distance of the words.

However. there are several ways we can improve its run-time.

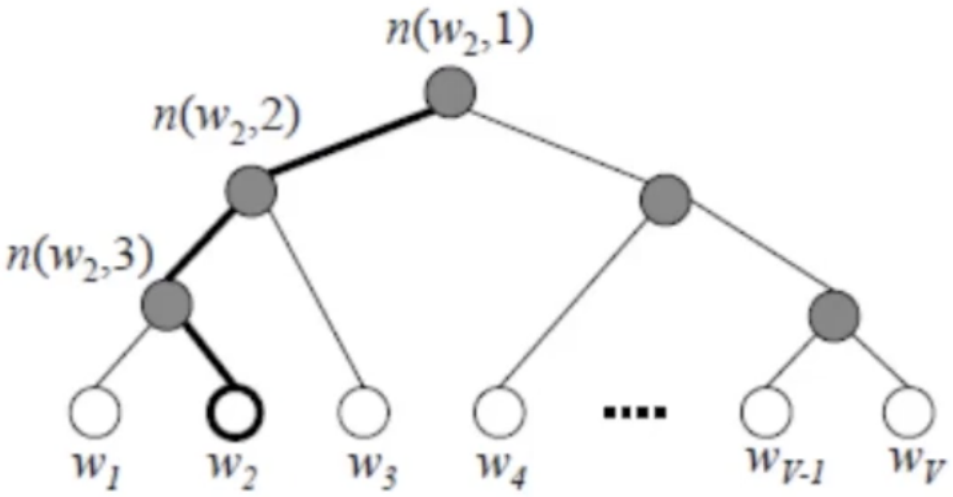

Figure 9: Hierarchical Softmax Model

HIERARCHICAL SOFTMAX This model in natural probabilistic language was firstly proposed by Morin and Bengio. The main advantage is that this model improves searching W to $log_2 W$.

The **Hierarchical Softmax** uses a binary tree representation that represents W words. For each node, its child nodes' probabilities are explicitly represented.

More intuitively, each node can be reached by traversing from the root of the tree. Nodes that are further down the tree have less probability of being selected as output.

Let $n(w, j)$ be the j-th node on the path from root w, and let $L(w)$ be the length of the path. therefore,

$$n(w,1) = root$$
$$n(w,L(w)) = w$$

NEGATIVE SAMPLING

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[2] R Nave, ”Work Functions for Photoelectric Effect” *Hyperphysics* (2017), found at <http://hyperphysics.phy-astr.gsu.edu/hbase/Tables/photoelec.html>, accessed 26 Oct. 2017