EE 523 Electromagnetic Wave Theory

Take-Home Exam

(30% of the Final Exam)

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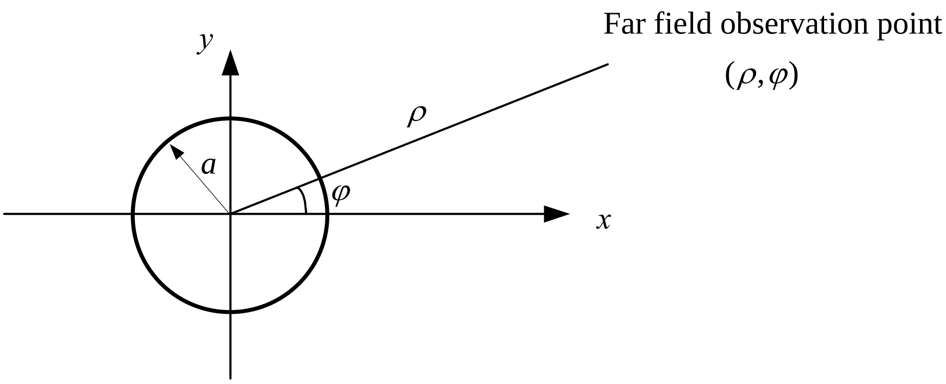
23:55 on Tuesday, January 23, 2018

Consider the following infinitely-long PEC cylinder with circular cross section:



The problem of scattering of a TM_z plane wave by an infinite circular PEC cylinder has been given in Lecture 10 (pp. 48-53). In the MATLAB function ***fun_cylinder_PEC.m***, the scattered field $E_z(\rho,\varphi)$ is evaluated by means of a truncated series expansion. It is assumed that the incident wave is a plane wave of the form $\overline{E}^{inc}(\overline{r}) = e^{-j\overline{k}\cdot\overline{r}}\hat{a}_z$ with the angle of incidence φ_i ("phii" in the code). The radius of the cylinder is denoted by a.

a. Compare the code with the formulation given in Lecture 10. Verify that the formulation we have discussed in class can be converted to the calculations carried out in the code.
   Hints:
   1. Note that the circular cylinder is invariant under rotations with respect to φ.
   2. Consult a reference book (or sources in the internet) to express Bessel functions of negative order $-n$ in terms of Bessel functions of positive order n.
b. In the code, the summation approximating the series expansion contains 50 terms. Perform numerical experiments to verify that for a successful approximation, the number of terms in the summation must increase as ka (i.e. the *electrical size* of the cross section) increases. Approximately, how many terms are required for:
   i. $ka = 0.1$
   ii. $ka = 1$
   iii. $ka = 10$
   iv. $ka = 100$

c. Plot (in MATLAB) the *bistatic* radar cross section of the cylinder for the four ka values given in (b). That is, plot RCS/λ (in dB) versus φ. **Hint:** Because of rotational symmetry, φ_i may be chosen as zero.

d. For this configuration and for the same incident field $\bar{E}^{\text{inc}}(x) = \mathrm{e}^{jkx}\hat{a}_z$, evaluate the scattered far-field by the Physical Optics (PO) approach . If you are unable to evaluate the PO integral analytically, use MATLAB to evaluate the integral numerically by discretizing the interval $-\frac{\pi}{2} < \varphi' < \frac{\pi}{2}$, (the illuminated part of the cylinder). After obtaining the far-field expression of the scattered field $E_z(\bar{r})$, plot (by using MATLAB), the normalized magnitude of $E_z(\bar{r})$ and the *bistatic* radar cross section of the cylinder with respect to $-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$ in dB scale for the following cases:

   i. $ka = 10$
   ii. $ka = 100$

   Compare your results with those obtained by the analytical solution (i.e. the Mie series).

   Hints:

   1. If $\bar{J}(x, y) = J_z(x, y)\hat{a}_z$, the only non-zero component of the scattered E-field is $E_z(\bar{r})$, which is given by the expression:

      $$E_z(\bar{r}) = -\frac{k\eta}{4}\int J_z(a\cos\varphi', a\sin\varphi')H_0^{(2)}\left(k\left|\bar{r} - \bar{r}'\right|\right)a\,d\varphi',$$

      where $\bar{r}' = a\cos\varphi'\hat{a}_x + a\cos\varphi'\hat{a}_y$.

   2. The asymptotic approximation for $H_0^{(2)}\left(k\left|\bar{r} - \bar{r}'\right|\right)$ is

      $$H_0^{(2)}\left(k\left|\bar{r} - \bar{r}'\right|\right) \approx \sqrt{\frac{2j}{\pi k\left|\bar{r} - \bar{r}'\right|}}\mathrm{e}^{-jk\left|\bar{r} - \bar{r}'\right|}$$, for large values of $k\left|\bar{r} - \bar{r}'\right|$.