EE 523: TAKE HOME FINAL

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a. Compare the code with the formulation given in Lecture 10. Verify that the formulation we have discussed in class can be converted to the calculations carried out in the code.
Solution:
The TM_z wave given as $\bar{E}_{inc}(\bar{r}) = e^{-j\bar{k}\cdot\bar{r}}\hat{a}_z$ can be written in polar coordinates as:

$$\bar{k} = k(\cos\phi_i, \sin\phi_i) \qquad \bar{r} = \rho(\cos\phi, \sin\phi) \tag{1}$$

$$\bar{E}_{inc}(\bar{r}) = e^{-jk\rho(\cos\phi\cos\phi_i + \sin\phi\sin\phi_i)}\hat{a}_z = e^{-jk\rho\cos(\phi-\phi_i)}\hat{a}_z \tag{2}$$

Therefore, in series form, it can be given as:

$$e^{-jk\rho\cos(\phi-\phi_i)} = \sum_{n=-\infty}^{\infty} j^{-n} J_n(k\rho) e^{jn(\phi-\phi_i)}. \tag{3}$$

Also note that $J_{-n} = (-1)^n J_n$, As a result the series solution given in equation 3 can be expressed as:

$$e^{-jk\rho\cos(\phi-\phi_i)} = J_0(k\rho) + 2\sum_{n=1}^{\infty} j^{-n} J_n(k\rho)\cos n(\phi-\phi_i). \tag{4}$$

Moreover, the scattered field can be given as:

$$\bar{E}_{scat}(\bar{r}) = a_0 H_0^{(2)}(k\rho) + \sum_{n=1}^{\infty} a_n j^{-n} H_n^{(2)}(k\rho)\cos n(\phi-\phi_i) \tag{5}$$

The sum of $\bar{E}_{inc}$ and $\bar{E}_{scat}$ must satisfy the boundary condition $\bar{E}_{scat}+\bar{E}_{inc} = 0$ at $\rho = a$, therefore,

$$a_0 H_0^{(2)}(ka) + J_0(ka) = 0, \tag{6}$$

$$a_n H_n^{(2)}(ka) + 2J_n(ka) = 0 \tag{7}$$

As result, $a_0 = -J_0(ka)/H_0^{(2)}(ka)$ and $a_n = -2J_n(ka)/H_n^{(2)}(ka)$. Finally, the total electric field can be given as:
(8)

$$\bar{E}_{tot}(\bar{r}) = J_0(k\rho) - \frac{J_0(ka)}{H_0^{(2)}(ka)} H_0^{(2)}(k\rho) + 2\sum_{n=1}^{\infty} j^{-n}\left(J_n(k\rho) - \frac{J_n(ka)}{H_n^{(2)}(ka)} H_n^{(2)}(k\rho)\right)\cos n(\phi-\phi_i)$$

It is the same electric field given in the code.

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b. In the code, the summation approximating the series expansion contains 50 terms. Perform numerical experiments to verify that for a successful approximation, the number of terms in the summation must increase as ka (i.e. the electrical size of the cross section) increases. Approximately, how many terms are required for:

(1) $ka = 0.1$
(2) $ka = 1$
(3) $ka = 10$
(4) $ka = 100$

Solution:
The convergence of the series is tested based on the relative error defined by

$$error_{relative} = \left\| \frac{\bar{E}_{tot}(\bar{a})}{\bar{E}_{inc}(\bar{a})} \right\|. \tag{9}$$

This relative error is calculate exactly at $\rho = a$ where the total electric field is supposed be zero.

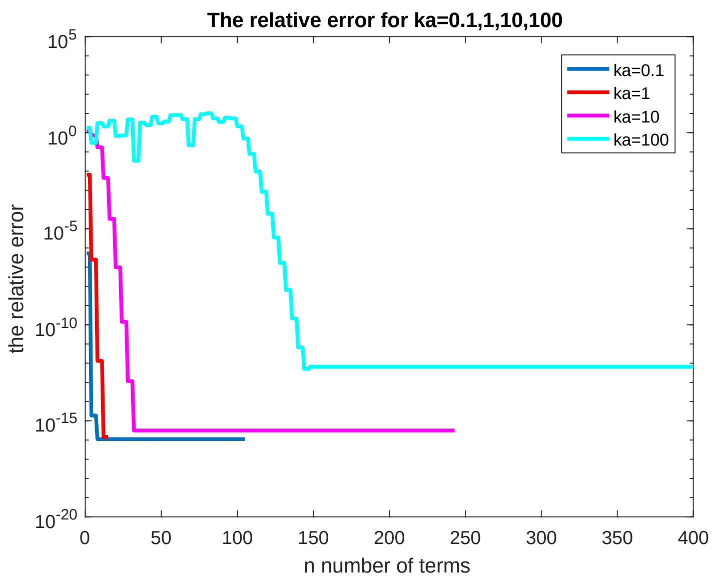


FIGURE 1. The relative error calculated at $\rho = a$ for $ka = 0.1, 1, 10, 100$ at each order of Bessel function expansion.

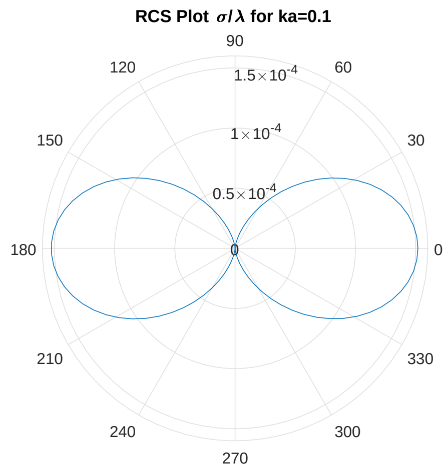
Based on the relative error plotted in figure 1, for $ka = 0.1$ and $ka = 1$ roughly 10 terms is enough to compute the scattered field with a good accuracy, for $ka = 10$ around 40 terms required and finally for $ka = 100$ around 150 terms required.

c. Plot (in MATLAB) the bistatic radar cross section of the cylinder for the four ka values given in (b). That is, plot RCS/λ (in dB) versus φ.

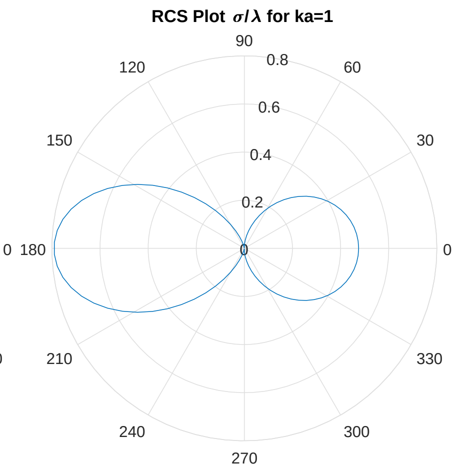
Solution:

RCS in the far field is defined as:

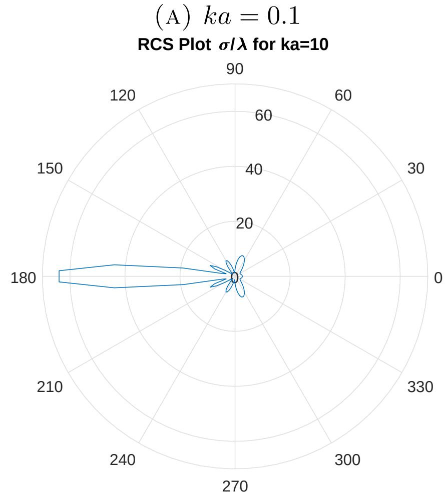
$$\sigma = lim_{k\rho\to\infty} 2\pi\rho \frac{\left|\bar{E}_{scat}\right|^2}{\left|\bar{E}_{inc}\right|^2} = \frac{16}{k}\left|\sum_{n=1}^{\infty} \frac{J_n(ka)}{H_n^{(2)}(ka)} \cos n(\phi-\phi_i)\right|^2 \tag{10}$$



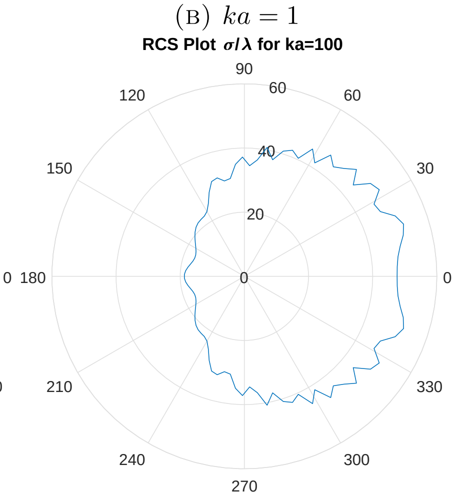
(A) $ka = 0.1$



(B) $ka = 1$



(C) $ka = 10$



(D) $ka = 100$

FIGURE 2. RCS Plots

d. For this configuration and for the same incident field $E_{inc}(x) = e^{jkx}\hat{a}_z$, evaluate the scattered far-field by the Physical Optics (PO) approach . If you are unable to evaluate the PO integral analytically, use MATLAB to evaluate the integral numerically by $-\frac{\pi}{2} < \varphi' < \frac{\pi}{2}$ (the illuminated part of the cylinder). After obtaining the far-field expression of the scattered field $E_z(\bar{r})$ and the bistatic radar cross section of the cylinder with respect to $-\frac{\pi}{2} < \varphi' < \frac{\pi}{2}$ in dB scale for the following cases:

- $ka = 10$
- $ka = 100$

Compare your results with those obtained by the analytical solution (i.e. the Mie series).

Solution:

The incident TM_x mode electric field inside the wave guide is given as:

$$\bar{E}_{inc}(\bar{r}) = e^{-jk\rho\cos(\phi-\phi_i)}\hat{a}_z \tag{11}$$

Corresponding magnetic field can be found by using the relation $\bar{\nabla} \times \bar{E} = -j\omega\bar{H}/\mu_0$, the magnetic field can be found as:

(12)

$$\bar{H}_{inc}(\bar{r}) = \frac{\omega k \sin(\phi-\phi_i)}{\mu_0} e^{-jk\rho\cos(\phi-\phi_i)}\hat{a}_\rho - \frac{\omega k \cos(\phi-\phi_i)}{\mu_0} e^{-jk\rho\cos(\phi-\phi_i)}\hat{a}_\phi$$

By using the equivalence theorem, the equivalent surface electric current can be calculated as:

$$\bar{J}_s \approx 2\hat{a}_\rho \times \bar{H}_{inc}(\bar{a}). \tag{13}$$

As a result, the equivalent electric current at PEC surface is found as:

$$\bar{J}_s \approx \frac{2\omega k \cos(\phi-\phi_i)}{\mu_0} e^{-jk\rho\cos(\phi-\phi_i)}\hat{a}_z \tag{14}$$

The scattered electric field at the far field can be computed as:

(15)

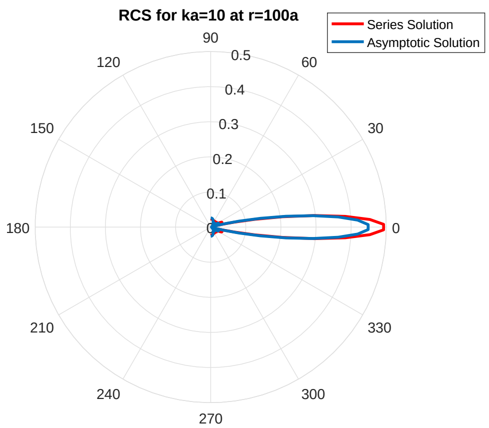
$$\bar{E}_{scat}(\bar{r}) = -\frac{k\eta}{4}\int \bar{J}_s(\bar{r}')H_0^{(2)}(k\left|\bar{r}-\bar{r}'\right|)a\mathrm{d}\phi \approx -\frac{k\eta}{4}\sqrt{\frac{2j}{\pi kr}}\int \bar{J}_s(\bar{r}')e^{-jk|\bar{r}-\bar{r}'|}a\mathrm{d}\phi.$$

The integral in equation 15 can be explicitly given as:

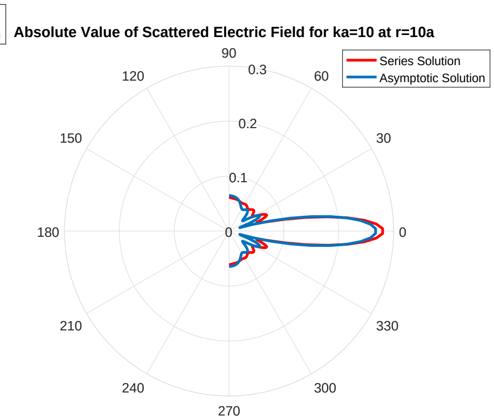
(16)

$$\bar{E}_{scat}(r,\phi) \approx -\frac{a\omega k^2\eta}{2\mu_0}\sqrt{\frac{2j}{\pi kr}}\int_{-\pi/2}^{\pi/2}\cos(\phi'-\phi_i)e^{-jka\cos(\phi'-\phi_i)}e^{-jk\sqrt{(r\cos\phi-a\cos\phi')^2+(r\sin\phi-a\sin\phi')^2}}\mathrm{d}\phi'.$$

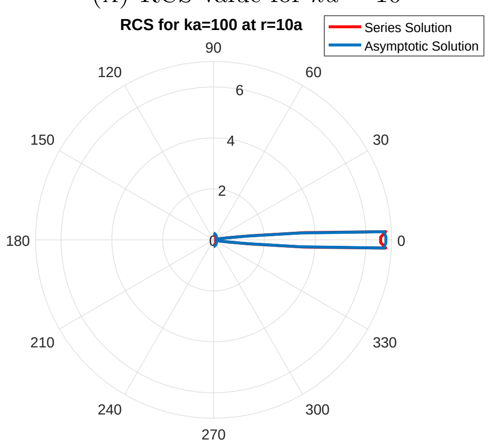
The integral 16 is taken numerically by trapezoidal rule for $-\pi/2 < \phi' < \pi/2$. The resultant electric field is compared with the electric field obtained by the series solution.



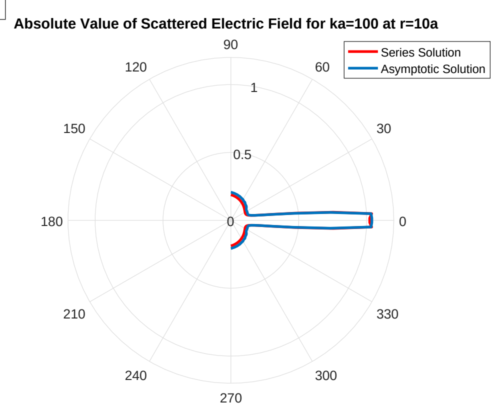
(A) RCS value for $ka = 10$



(B) Scattered Electric Field for $ka = 10$



(C) RCS value for $ka = 100$



(D) Scattered Electric Field for $ka = 100$

FIGURE 3. RCS Plots

Appendix A. Matlab Codes

Listing 1. Take Home Final Main Script

```
% created by Anil Aksu
% this routine is generated to compute the scattered ...
    electric field by
% circular PEC

clear all
format long

phii = pi/4; % the angle of incidence
phio = pi/2; % the angle of observation
freq   = 2.*pi;     % Hz, frequency
c0     = 3*1e8;        % m/sec, velocity of light in free space
lambda = c0/(freq*1e6);      % meter, wavelength
k      = 2*pi/lambda; % 1/meter, wavenumber
a=100/k;    % the radius of PEC
r=a; % the distance
%n_up= 5; % the upper limit of the series


% the error convergence
for n_up=1:400

a=0.1/k;    % the radius of PEC
% the incident field
E_inc=exp(-i*k*a*cos(phio-phii));
% the scattered electric field
[Escat_1] = fun_cylinder_PEC(freq, a, phii, phio, a,n_up);
% the percent error
err(n_up,1)=abs(real(E_inc+Escat_1)/real(E_inc));


a=10.*a; % the radius of PEC
% the incident field
E_inc=exp(-i*k*a*cos(phio-phii));
% the scattered electric field
[Escat_2] = fun_cylinder_PEC(freq, a, phii, phio, a,n_up);
% the percent error
err(n_up,2)=abs(real(E_inc+Escat_2)/real(E_inc));

a=10.*a; % the radius of PEC
% the incident field
E_inc=exp(-i*k*a*cos(phio-phii));
% the scattered electric field
[Escat_3] = fun_cylinder_PEC(freq, a, phii, phio, a,n_up);
% the percent error
err(n_up,3)=abs(real(E_inc+Escat_3)/real(E_inc));

a=10.*a; % the radius of PEC
```

```
% the incident field
E_inc=exp(-i*k*a*cos(phio-phii));
% the scattered electric field
[Escat_4] = fun_cylinder_PEC(freq, a, phii, phio, a,n_up);
% the percent error
err(n_up,4)=abs(real(E_inc+Escat_4)/real(E_inc));

end

figure(1)

semilogy(err(:,1),'LineWidth',2)
hold on
semilogy(err(:,2),'r','LineWidth',2)
hold on
semilogy(err(:,3),'m','LineWidth',2)
hold on
semilogy(err(:,4),'c','LineWidth',2)

ylabel('the relative error')
xlabel('n number of terms')
title('The relative error for ka=0.1,1,10,100')
legend('ka=0.1','ka=1','ka=10','ka=100')
```

Listing 2. Bessel Junction Series Matlab Code

```
function [Escat] = fun_cylinder_PEC(freq, a, phii, phio, ...
    r,n_up)
% ...
    *******************************************************************************
% Scattered field computation for a PEC circular cylinder ...
    (series solution)
% by Dr. Ozlem Ozgun & Dr. Mustafa Kuzuoglu
% ...
    *******************************************************************************

% INPUTS:
% freq: frequency in MHz
% a    : radius of the cylinder (m)
% phii: Angle of incidence (radian)
% phio: Angle of observation point (radian)
% r    : Radius of observation point (distance) (m)

% OUTPUT:
% Escat: scattered field

freq   = freq*1e6;     % Hz, frequency
c0     = 3*1e8;        % m/sec, velocity of light in free space
lambda = c0/freq;      % meter, wavelength
k      = 2*pi/lambda; % 1/meter, wavenumber

phii = phii-pi;
```

```

ka = k*a;
kr = k*r;

E0 = besselj(0,ka)*besselh(0,2,kr)/besselh(0,2,ka);
n = 1:n_up; % terms in the summation
E = ...
    sum(2*((-1j).^n).*cos(n*(phio-phii)).*besselj(n,ka).*besselh(n,2,kr)./besselh(n,2,ka

Escat = -(E0+E); % scattered field

end
```

Listing 3. RCS Plot Script

```
% created by Anil Aksu
% this routine is generated to RCS plot by circular PEC

clear all
format long

phii = 0; % the angle of incidence
phio = linspace(0,2*pi);; % the angle of observation
freq   = 2.*pi;    % Hz, frequency
c0     = 3*1e8;       % m/sec, velocity of light in free space
lambda = c0/(freq*1e6);     % meter, wavelength
k      = 2*pi/lambda; % 1/meter, wavenumber
a=100/k;   % the radius of PEC
r=a; % the distance
n_up=400;  % the upper limit of the series

for i=1:length(phio)

% the RCS value
RCS_1(i)= getRCS( freq, a, phii, phio(i),n_up )/lambda;

end

figure(1)
polarplot(phio,RCS_1)

title('RCS Plot \sigma/\lambda for ka=100')
```

Listing 4. RCS Matlab Code

```
function [ sigma ] = getRCS( freq, a, phii, phio,n_up )
% this function calculates RCS value at the far field, the ...
    formulation
% adopted here is based on the lecture notes of EE 523

```

```
% INPUTS:
% freq: frequency in MHz
% a    : radius of the cylinder (m)
% phii: Angle of incidence (radian)
% phio: Angle of observation point (radian)
% r    : Radius of observation point (distance) (m)

% OUTPUT:
% sigma: scattered field

freq   = freq*1e6;     % Hz, frequency
c0     = 3*1e8;        % m/sec, velocity of light in free space
lambda = c0/freq;      % meter, wavelength
k      = 2*pi/lambda; % 1/meter, wavenumber

phii = phii-pi;

ka = k*a;

E0 = besselj(0,ka)/besselh(0,2,ka);
n = 1:n_up; % terms in the summation
E = ...
    abs(sum(2*cos(n*(phio-phii)).*besselj(n,ka)./besselh(n,2,ka)));

% RCS value
sigma = (4/k)*E^2;

end
```

Listing 5. Asymptotic Electric Field Matlab Code

```
% created by Anil Aksu
% this routine is generated to compute the scattered ...
    electric field by
% circular PEC

clear all
format long

phii = pi; % the angle of incidence
phio = linspace(-0.5*pi,.5*pi); % the angle of observation
freq   = 2.*pi;     % Hz, frequency
c0     = 3*1e8;        % m/sec, velocity of light in free space
lambda = c0/(freq*1e6);      % meter, wavelength
k      = 2*pi/lambda; % 1/meter, wavenumber
a=100/k;    % the radius of PEC
r=10.*a; % the distance
n_up=200;   % the upper limit of the series
%eta=

for i=1:length(phio)

```

```
% the scattered electric field given as series
[Escat_series(i)] = fun_cylinder_PEC(freq, a, phii, ...
    phio(i), r,n_up);
[ E_asym(i) ] = getAysmptoticElectric(freq, a, phii, ...
    phio(i), r);
end

figure(1)
polarplot(phio,abs(Escat_series),'r','LineWidth',2)
hold  on
polarplot(phio,abs(E_asym),'LineWidth',2)
title('Absolute Value of Scattered Electric Field for ...
    ka=100 at r=10a')
legend('Series Solution','Asymptotic Solution')

figure(2)
polarplot(phio,2.*pi*abs(Escat_series).^2,'r','LineWidth',2)
hold  on
polarplot(phio,2.*pi*abs(E_asym).^2,'LineWidth',2)
title('RCS for ka=100 at r=10a')
legend('Series Solution','Asymptotic Solution')
```

Listing 6. Asymptotic Electric Field Integration Matlab Code

```
function [ E_asym ] = getAysmptoticElectric(freq, a, phii, ...
    phio, r)
%This function computes the asymptotic integral given in EE ...
    523 final

phid = linspace(-0.5*pi,.5*pi,100); % the angle of integral
dphi=pi/100.; % the step size of the integral
freq   = freq*1e6;     % Hz, frequency
c0     = 3*1e8;        % m/sec, velocity of light in free space
lambda = c0/freq;      % meter, wavelength
k      = 2*pi/lambda; % 1/meter, wavenumber
phii = phii-pi;

ka = k*a;
% the integral
int=0.;

for i=1:(length(phid)-1)
    f(1) = getIntFunc( phid(i), k, a, phii, phio, r);
    f(2) = getIntFunc( phid(i+1), k, a, phii, phio, r);
   % the trapezoidal rule integral
   int=int+0.5*(f(1)+f(2))*dphi;
end
% the other terms multiplication
alpha=0.82046;
% the asymptotic electric field
E_asym=-0.5*alpha*a*(k^2)*int*sqrt(2*i/(pi*k*r));
end
```

Listing 7. Function Inside Integration Matlab Code

```
function [ f ] = getIntFunc( phid, k, a, phii, phio, r)
% this function calculates the function inside the ...
    asymptotic integral

f=cos(phid-phii)*exp(-i*k*a*cos(phid-phii))*exp(-i*k*sqrt((r*cos(phio)-a*cos(phid))^2+(

end
```