Plasma Physics Term Project Presentation

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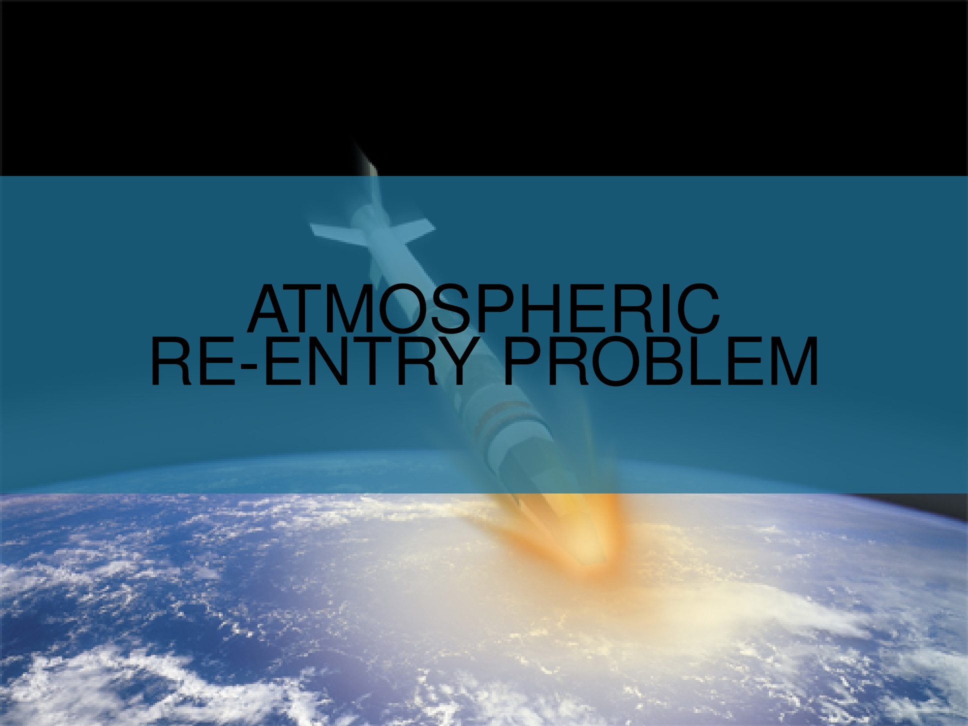
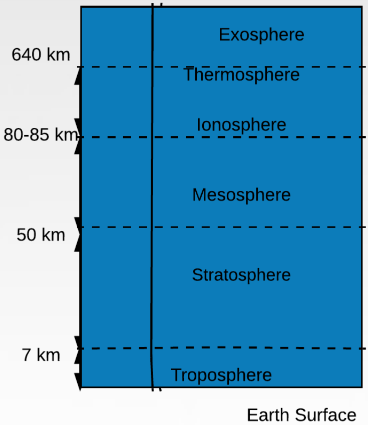
ATMOSPHERIC RE-ENTRY PROBLEM



Atmospheric Conditions

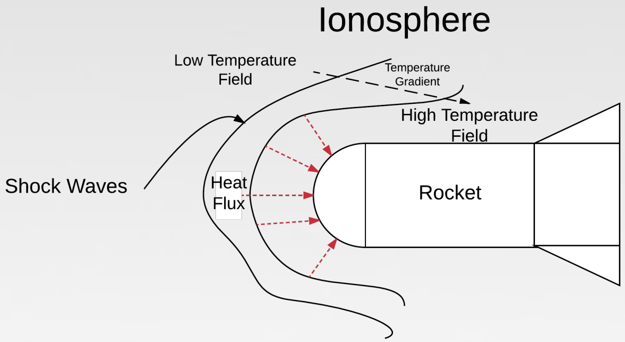
The atmosphere consists of 6 layers which are Troposphere, Stratosphere, Mesosphere, Ionosphere, Thermosphere and Exosphere.



Atmospheric Conditions

- *Troposphere:* The troposphere is the lowest part of the atmosphere and also where almost all weather takes place.
- *Stratosphere:* The second region of atmosphere extending upward from the tropopause characterized by vertical gradient in temperature which triggers internal waves within Stratosphere.
- *Mesosphere:* The mesosphere is directly above the stratosphere. Temperature decreases with height throughout the mesosphere with minimum $183K$.
- *Ionosphere:* The layer of atmosphere that is ionized by solar and cosmic radiation with temperature cycles within $200K$-$500K$.
- *Thermosphere:*The region of the atmosphere where a continuous medium assumption fails.
- *Exosphere:*The uppermost layer, where the atmosphere diminishes and merges with interplanetary space.

Re-entry Problem



- *Abrupt changes in state quantities such as temperature and density.*
- *Consequently, destructive thermo-mechanical effects on a rocket or a space vehicle [4].*
- *Possible destructive electromagnetic effects [3, 4] as well.*

Objective

- *Understanding the behaviour of shocks in Ionosphere:* How the magnetic field and the electric field induced by ions and electrons affect the shock surface.
- *How the presence of shock affects the electric field and the magnetic field:*If there exist and abrupt change in the electric and the magnetic fields before and after shock layer.
- *How the geometric design and the material selection of a rocket can be modified to overcome the disruptive effects of a shock wave:*There is a sudden increase in temperature field after shock surface around the rocket. These may be overcome by improving the design of the rocket.

Methods of Analysis

- *Single Fluid Magneto-Hydrodynamic Model:* It is derived under continuum assumption by summing of momentum and energy equations of each species.
- *Simplified 1-D Analytical Gas Dynamic Model:* It is a basis for estimating and validating more advanced analysis tools.
- *2-D Spectral Element MHD Numerical Model:* It is a state of art numerical scheme used to produce more realistic results for the atmospheric re-entry problem.

Magneto-hydrodynamic Flows

the continuity is given as:

$$m_i\left[\frac{\partial n_i}{\partial t}+\frac{\partial n_i u_j}{\partial x_j}\right]=0. \tag{1}$$

The momentum equations for each species are given as:

$$\sum_i m_i n_i\left[\frac{\partial \rho \mathbf{u}_i}{\partial t}+u_i^j\frac{\partial \mathbf{u}_i}{\partial x_j}\right]=-\nabla p_i+n_i q_i\mathbf{E}+\mu\nabla^2\mathbf{u}_i+R_{ik}(\mathbf{u}_k-\mathbf{u}_i), \tag{2}$$

The energy equation[1] is given as:

$$\sum_i m_i n_i c_{p_i}\left(\frac{\partial T_i}{\partial t}+u_i^j\frac{\partial T_i}{\partial x_j}\right)=2\mu\mathbf{D}_i^{mn}\mathbf{D}_i^{mn}+\kappa\nabla^2 T_i. \tag{3}$$

where c_{p_i} is heat capacitance and

$$\mathbf{D}_i^{mn}=\begin{bmatrix}\frac{\partial u_i^x}{\partial x} & \frac{1}{2}\left(\frac{\partial u_i^x}{\partial y}+\frac{\partial u_i^y}{\partial x}\right)\\ \frac{1}{2}\left(\frac{\partial u_i^x}{\partial y}+\frac{\partial u_i^y}{\partial x}\right) & \frac{\partial u_i^y}{\partial y}\end{bmatrix} \tag{4}$$

Magneto-hydrodynamic Flows: Continued

The electric field **E** can be formulated in terms of a potential field ϕ.

$$\mathbf{E} = -\nabla\phi. \tag{5}$$

and also

$$\epsilon_0 \nabla^2 \phi = e(n_e - Zn_i). \tag{6}$$

Finally, the pressure term can be found by utilizing the state equation:

$$p_i = n_i \gamma T_i \tag{7}$$

Shocks In Compressible Flows: 1-D Model

The mass continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0, \tag{8}$$

The momentum equation:

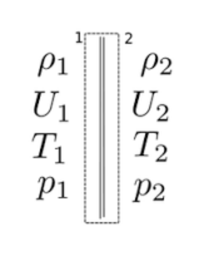
$$\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x}\left(\rho u^2 + p - \frac{4\mu}{3}\frac{\partial u}{\partial x}\right) = 0, \tag{9}$$

The energy equation:

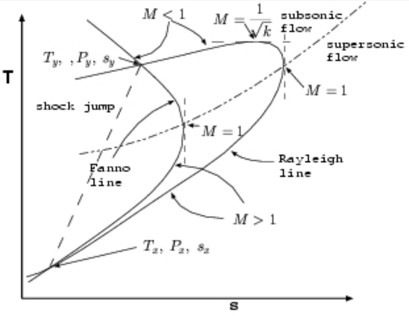
$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2 + \rho e\right) + \frac{\partial}{\partial x}\left(\rho u(\frac{u^2}{2} + h) - \frac{4\mu}{3}u\frac{\partial u}{\partial x} - \kappa\frac{\partial T}{\partial x}\right) = 0. \tag{10}$$

where μ is viscosity and κ is conductivity.

Shocks Conditions In Compressible Flows



the difference between the quantities $[f(\rho)] = f(\rho^+) - f(\rho^-)$ and $[\rho] = \rho^+ - \rho^-$ are given in bracket notation.



by Green's theorem around infinitely thin shock surface:

$$\oint_C f(\rho)\mathrm{d}t - \rho\mathrm{d}x = 0. \qquad (11)$$

It results in:

$$[\rho]\mathrm{d}x = [f(\rho)]\mathrm{d}t. \qquad (12)$$

Numerical Method: Spectral Element Method

Any quantity can be given as:

$$u = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \alpha_{ij}(t) P_i(x) P_j(y), \tag{13}$$

Legendre polynomial are orthogonal set of functions on their mother interval $x \in [-1, 1]$, therefore:

$$< P_n(x) P_m(x) > \int_{-1}^{1} P_n(x) P_m(x) \mathrm{d}x = \frac{2}{2+n} \delta_{mn}. \tag{14}$$

In case of weak formulation, the governing equations are multiplied with $P_i(x)P_j(y)$ and integrated over whole domain.

Numerical Method: Operator Splitting

The time integration is performed in two steps:

First Step: Integration of non-linear, electro-magnetic and pressure terms are performed explicitly,

$$[\mathbf{M}]\frac{u^{*}-u^{n}}{\Delta t}=N^{n}_{term}+E^{n}_{term}+P^{n}_{term} \tag{15}$$

where [**M**] is the mass matrix generated by weak formulation.

Second Step: Integration of diffusive terms are performed implicitly,

$$[\mathbf{M}]\frac{u^{n+1}-u^{*}}{\Delta t}=[\mathbf{K}]u^{n+1} \tag{16}$$

where [**K**] is the matrix generated by weak formulation of diffusive terms.

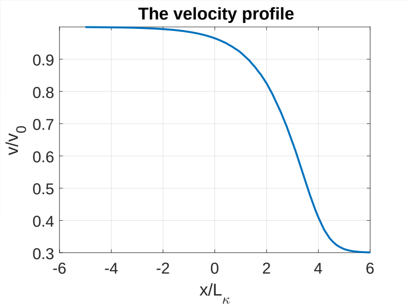
Notice: Implicit formulation is preferred for the stability of the time integration.

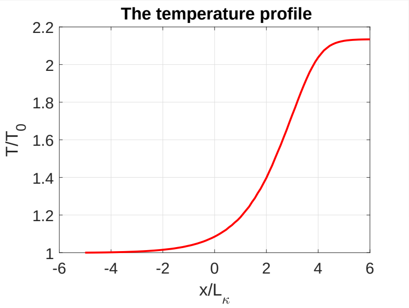
Results: 1-D Analytical Shock Solution In Compressible Flows

For large and small Prandtl number which is defined as $Pr = \mu Cp/\kappa$, the steady-state solution to the set of equations[2] above is given as:

$$x = \frac{2L_k}{\gamma + 1} \log \left[(v_0 - v)^{v_0/(v_0 - v_1)} (v - v_1)^{-v_1/(v_0 - v_1)} \right]. \qquad (17)$$

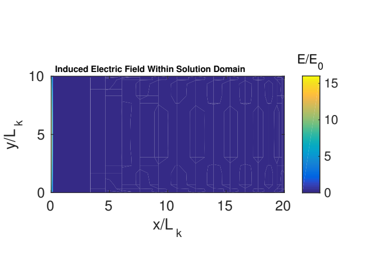
where $L_k = \kappa_0 / m_0 Cv$.

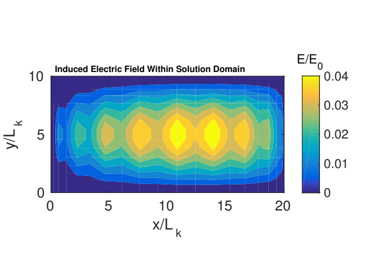


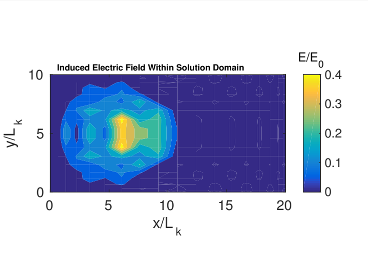


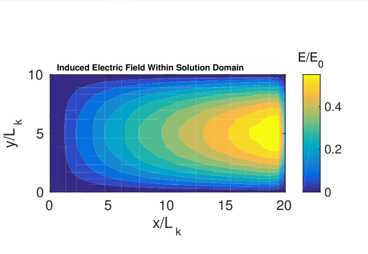
Results: 2-D Numerical MHD Solution

Induced Electric Field in various time steps









Conclusion

The effects of compressibility is significant in the atmospheric re-entry problem:

- *The presence of shock leads to a dramatic temperature increase around a re-entry vehicle or rocket.*
- *Since the charged particles accumulates in the region after shock, there is also a significant increase in electric field after shock layer.*

2-D Spectral Element Method is an accurate numerical algorithm to analyze MHD problem with shock layers:

- *It gives high resolution in space and time.*
- *A finer mesh around the shock layer enables to get more detailed description of flow conditions.*

Future Work

- *Implementation of Statistical Model:* Vlasov Equations describe the motion of plasma more accurately.
- *3-D Spectral Element MHD Numerical Model:* The more realistic case would occur in 3-D model, possibly more complex dynamics would take place.
- *Extension to Non-rectangular Geometries:* These methodologies are supposed to be applied to space vehicles or rockets, therefore it is supposed to be extrapolated to more realistic geometries.

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THANK YOU


Practice makes perfect, but nobody's perfect, so why practice?
_Kurt Cobain