

## To Reviewer # 1:

We appreciate the careful reviewing and in-depth observations made by the reviewer. The comments are very helpful and we have revised accordingly.

### To Comment # 1:

Concern: *In Remark 1.1, it is stated that “without loss of generality”, the general elliptical operator is considered. Since the problem statement started with elliptical PDEs, why is it necessary to mention this?*

Response: Yes, thank you for raising this point. The original intention was to emphasize the subordinate relationship between the problem definition and the example illustration:

Type of problem of interest	Potential applications
$\mathcal{L}_s(\Theta(s), u(s)) = f(s) \quad \text{in } \mathcal{S}$	Permeability study in hydrology $2^{nd}$ order elliptic problem
$u = g^{(D)} \quad \text{at } \partial\Gamma_D$	Steady-state heat conduction in thermology $2^{nd}$ order elliptic problem
$\nabla u \cdot \hat{n} = g^{(N)} \quad \text{at } \partial\Gamma_N$	Structural analysis in civil engineering $4^{th}$ order elliptic problem

The proposed model by no means is restricted to problems of structural analysis, rather it is capable of efficiently dealing with high-dimensional elliptic systems of a general form. To eliminate the confusion on this restatement, Remark 1.1 has been rewritten.

### To Comment # 2:

Concern: *In Section 2.2, it is mentioned that “f” is “implicit” in general. However, in the illustrative example, it is known explicitly.*

Response: Thank you so much for catching this point. The word “Implicit” in Section 2.2 means the quantity of interest (QoI) is an implicit function of the input variables. In the illustrative example, the governing equation is given. However, the QoI such as Von Mises stress can not be formulated as an explicit function of the random variables. The concept of explicit/implicit function used in this paper is based on the following material:

1. Krantz, S.G. and Parks, H.R., 2012. The implicit function theorem: history, theory, and applications. Springer Science & Business Media.
2. Rudin, W., 1964. Principles of mathematical analysis (Vol. 3). New York: McGraw-hill.
3. Mordukhovich, B.S., 2006. Variational analysis and generalized differentiation I: Basic theory (Vol. 330). Springer Science & Business Media.

**To Comment # 3:**

Concern: *Some of the standard materials should be removed. For example, most of the content in Section 2.3 and 3.3.*

Response: Yes, we agree with the comment made by the reviewer. Therefore, Section 2.3 and 3.3 have been removed. Crucial contents regarding the surrogate modeling and uncertainty quantification have been moved to the case study, i.e. Section 4.

**To Comment # 4:**

Concern: *Some of the references from CACAIE regarding uncertainty quantification should be introduced to reflect the connection of the present paper with the journal.*

Response: Yes, we have added some references from CACAIE.

**To Comment # 5:**

Concern: *The illustrative example regards to a simple plate. A real application will be important to illustrate the method.*

Response: Yes, we understand the importance of model effectiveness and generalizability in the development of surrogate models for uncertainty quantification and propagation. To extend our current work, we have further carried out studies including the Poisson's equation, Darcy's law, and nonlinear geometry analysis. Enclosed please find the supplementary file (file name: supplementary.pdf).