

Week 6 at a glance

Textbook reading: Chapter 3

Before Monday, Page 165-166 Introduction to Section 3.1.

Before Wednesday, Example 3.9 on page 173.

Before Friday, Page 184-185 Terminology for describing Turing machines.

For Week 7 ~~Monday~~: Introduction to Chapter 4.

We will be learning and practicing to:

- Clearly and unambiguously communicate computational ideas using appropriate formalism. Translate across levels of abstraction.
 - Use precise notation to formally define the state diagram of a Turing machine
 - Use clear English to describe computations of Turing machines informally.
 - * Motivate the definition of a Turing machine
 - * Trace the computation of a Turing machine on given input
 - * Describe the language recognized by a Turing machine
 - * Determine if a Turing machine is a decider
 - * Given an implementation-level description of a Turing machine
 - * Use high-level descriptions to define and trace Turing machines
 - * Apply dovetailing in high-level definitions of machines
 - Give examples of sets that are recognizable and decidable (and prove that they are).
 - * State the definition of the class of recognizable languages
 - * State the definition of the class of decidable languages
 - * State the definition of the class of co-recognizable languages
- Know, select and apply appropriate computing knowledge and problem-solving techniques. Reason about computation and systems.
- Describe and prove closure properties of classes of languages under certain operations.
 - Apply a general construction to create a new Turing machine from an example one.
 - Formalize a general construction from an informal description of it.
 - Use general constructions to prove closure properties of the class of decidable languages.
 - Use general constructions to prove closure properties of the class of recognizable languages.

TODO:

This week: Test 1 Attempt 1 in CBTF.

Review Quiz 5 on PrairieLearn (<http://us.prairielearn.com>), due 2/12/2025

Mid quarter feedback survey on Canvas.

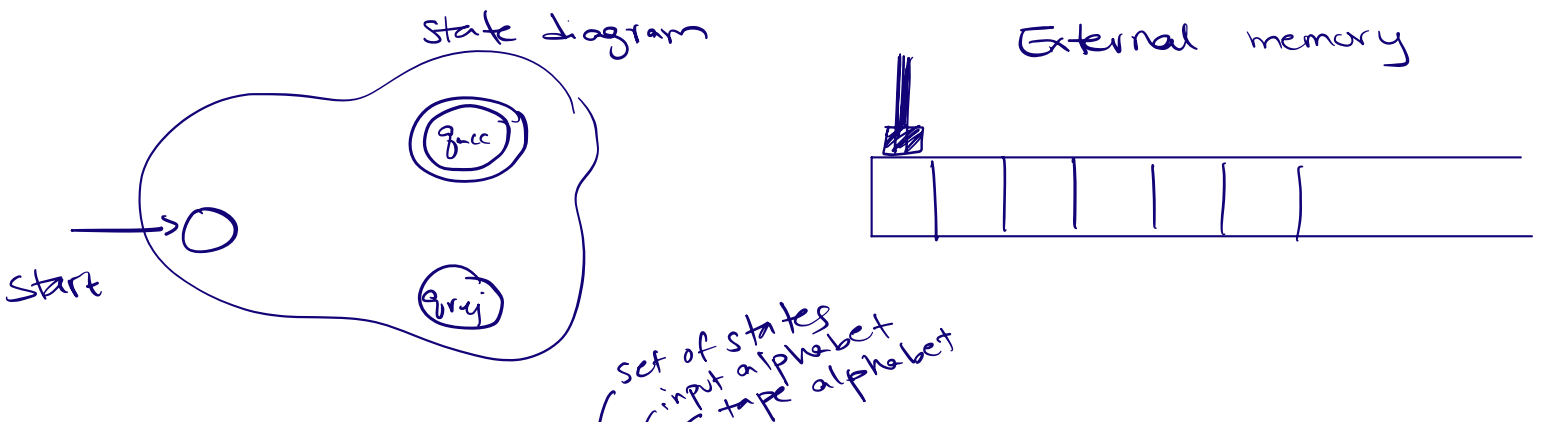
Monday: Descriptions of Turing machines

We are ready to introduce a formal model that will capture a notion of general purpose computation.

- *Similar to DFA, NFA, PDA*: input will be an arbitrary string over a fixed alphabet.
- *Different from NFA, PDA*: machine is deterministic.
- *Different from DFA, NFA, PDA*: read-write head can move both to the left and to the right, and can extend to the right past the original input.
- *Similar to DFA, NFA, PDA*: transition function drives computation one step at a time by moving within a finite set of states, always starting at designated start state.
- *Different from DFA, NFA, PDA*: the special states for rejecting and accepting take effect immediately.

Tape 

(See more details: Sipser p. 166)



Formally: a Turing machine is $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ where δ is the transition function

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$$

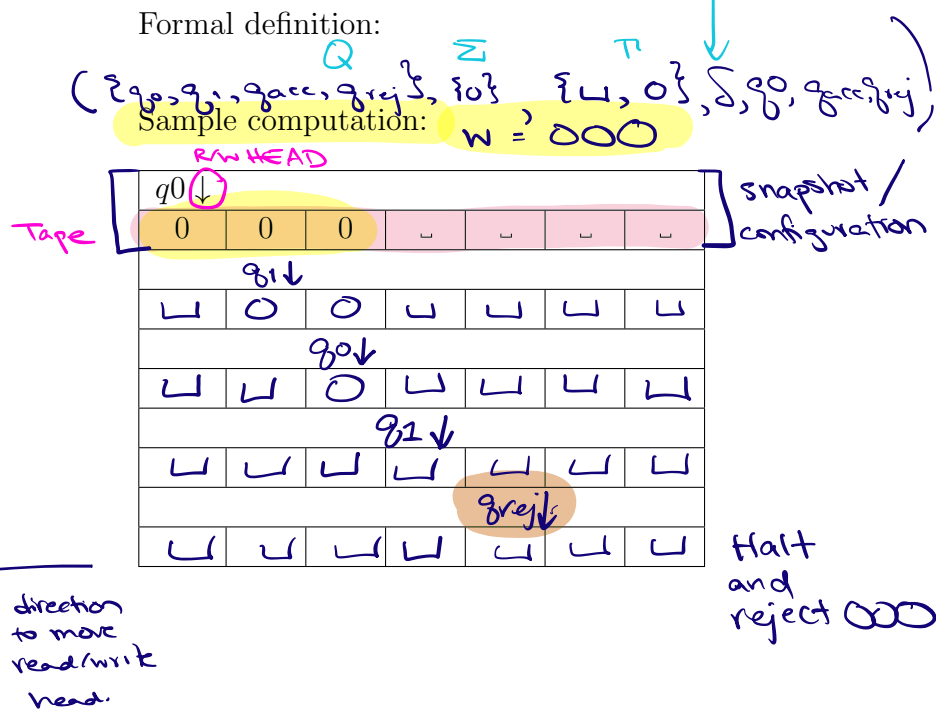
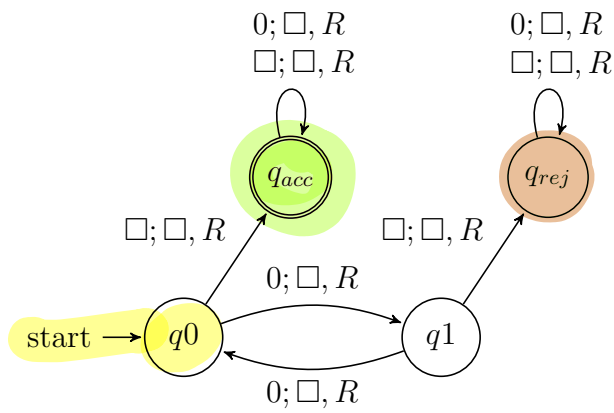
deterministic

The **computation** of M on a string w over Σ is:

- Read/write head starts at leftmost position on tape.
- Input string is written on $|w|$ -many leftmost cells of tape, rest of the tape cells have the blank symbol. **Tape alphabet** is Γ with $\sqcup \in \Gamma$ and $\Sigma \subseteq \Gamma$. The blank symbol $\sqcup \notin \Sigma$.
- Given current state of machine and current symbol being read at the tape head, the machine transitions to next state, writes a symbol to the current position of the tape head (overwriting existing symbol), and moves the tape head L or R (if possible). *if read/write head is at leftmost cell and transition function says it should move L, read/write head stays in its location*
- Computation ends **if and when** machine enters either the accept or the reject state. This is called **halting**. Note: $q_{\text{accept}} \neq q_{\text{reject}}$.

The **language recognized by the Turing machine** M , is $L(M) = \{w \in \Sigma^* \mid w \text{ is accepted by } M\}$, which is defined as

$$\{w \in \Sigma^* \mid \text{computation of } M \text{ on } w \text{ halts after entering the accept state}\}$$



$$\delta((q_0, \sqcup)) = (q_{acc}, \sqcup, R)$$

The language recognized by this machine is ...

$$\{0^{2i} \mid i \geq 0\}$$

examples of strings that are not accepted:

0
000

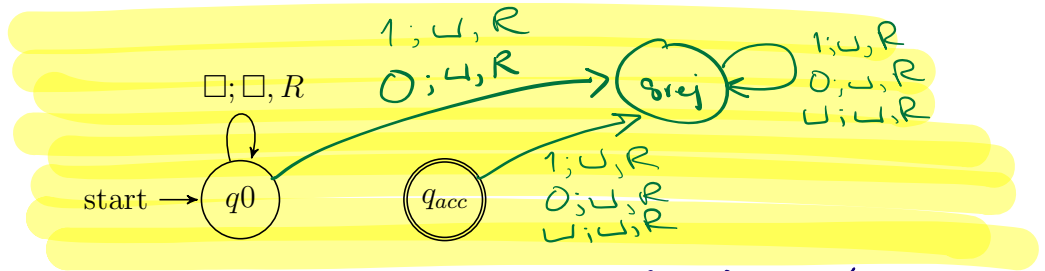
Note: this language is regular, context-free

Describing Turing machines (Sipser p. 185) To define a Turing machine, we could give a

- Formal definition:** the 7-tuple of parameters including set of states, input alphabet, tape alphabet, transition function, start state, accept state, and reject state; or,
- Implementation-level definition:** English prose that describes the Turing machine head movements relative to contents of tape, and conditions for accepting / rejecting based on those contents.
- High-level description:** description of algorithm (precise sequence of instructions), without implementation details of machine. As part of this description, can "call" and run another TM as a subroutine.

Conventions when draw diagrams: sometimes omit reject from the picture; any missing arrows directed to reject

Fix $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \sqcup\}$ for the Turing machines with the following state diagrams:



Example of string accepted:

Example of string rejected:

ϵ neither accepted nor rejected!

Implementation-level description

Scan to the right until we see the first 0 or 1, at which point we reject.

Therefore, $L(M) = \emptyset$

The language recognized by this TM is \emptyset

High-level description

On input x :

1. If $x = \epsilon$, go to step 1.
2. otherwise, reject.



Example of string accepted: None

Example of string rejected: ϵ , 0, 1, 00, 01,

This TM halts on each input.

Implementation-level description

Reject (immediately).

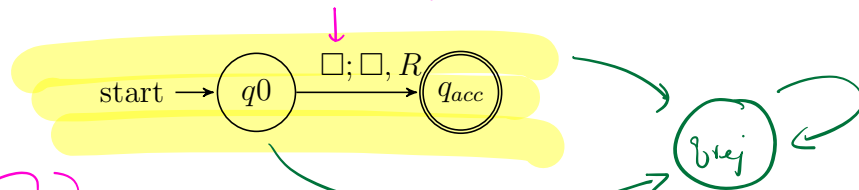
The language recognized by this TM is \emptyset

High-level description

On input x :

1. Reject.

cell in scanning is blank



Example of string accepted:

ϵ

Example of string rejected:

0, 1, 00, 01

0; \square , R
1; \square , R

The language recognized by this TM is $\{\epsilon\}$

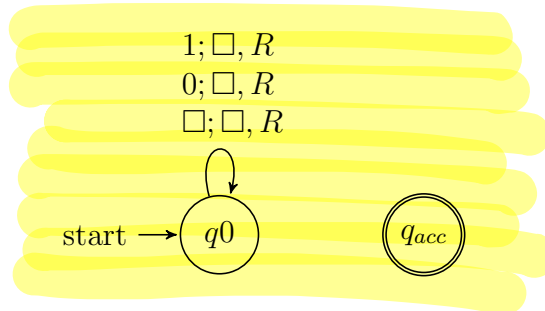
Implementation-level description

If first tape symbol is blank, accept. Otherwise, reject.

High-level description

On input x :

1. If $x = \epsilon$, accept.
2. Otherwise, reject.



Example of string accepted: None!

Example of string rejected: None!

The language recognized by this TM is \emptyset

Implementation-level description

Scan tape left to right, erasing each cell in turn.
write blank symbol

High-level description

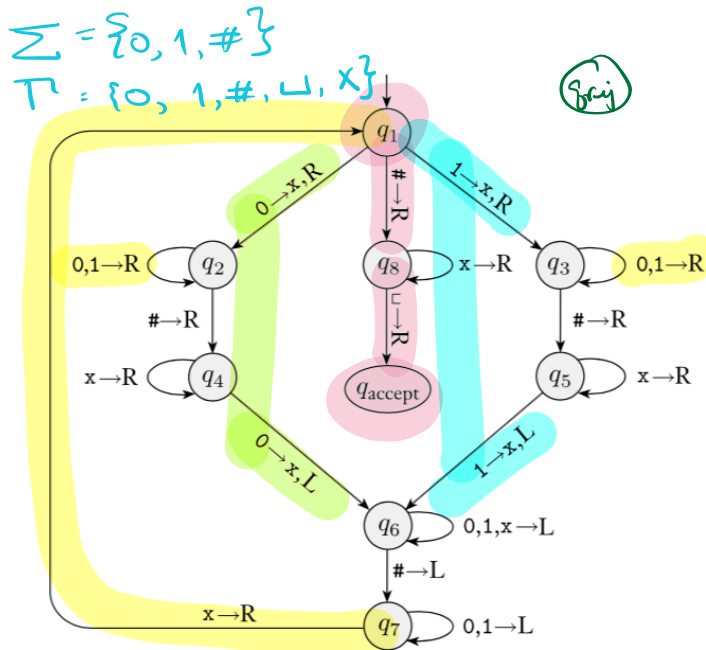
On input x :

1. Go to step 1

Wednesday: Recognizable and decidable languages

Sipser Figure 3.10

Conventions in state diagram of TM: $b \rightarrow R$ label means $b \rightarrow b, R$ and all arrows missing from diagram represent transitions with output (q_{reject}, \sqcup, R)



10 states

Implementation level description of this machine:

Zig-zag across tape to corresponding positions on either side of # to check whether the characters in these positions agree. If they do not, or if there is no #, reject. If they do, cross them off.

Once all symbols to the left of the # are crossed off, check for any un-crossed-off symbols to the right of #; if there are any, reject; if there aren't, accept.

The language recognized by this machine is

$$\{w\#w \mid w \in \{0, 1\}^*\}$$

Computation on input string 01#01

$q_1 \downarrow$	0	1	#	0	1	\sqcup	\sqcup
$q_2 \downarrow$	X	1	#	0	1	\sqcup	\sqcup
$q_2 \downarrow$	X	1	#	0	1	\sqcup	\sqcup
$q_4 \downarrow$	X	1	#	0	1	\sqcup	\sqcup
$q_6 \downarrow$	X	1	#	X	1	\sqcup	\sqcup
$q_7 \downarrow$	X	1	#	X	1	\sqcup	\sqcup
$q_7 \downarrow$	X	1	#	X	1	\sqcup	\sqcup
$q_1 \downarrow$	X	1	#	X	1	\sqcup	\sqcup
$q_3 \downarrow$	X	X	#	X	1	\sqcup	\sqcup
$q_5 \downarrow$	X	X	#	X	1	\sqcup	\sqcup
$q_6 \downarrow$	X	X	#	X	X	\sqcup	\sqcup
$q_7 \downarrow$	X	X	#	X	X	\sqcup	\sqcup
$q_1 \downarrow$	X	X	#	X	X	\sqcup	\sqcup
$q_8 \downarrow$	X	X	#	X	X	\sqcup	\sqcup
$q_8 \downarrow$	X	X	#	X	X	\sqcup	\sqcup
$q_8 \downarrow$	X	X	#	X	X	\sqcup	\sqcup
$q_{accept} \downarrow$	X	X	#	X	X	\sqcup	\sqcup

Notice 01#1— is rejected.

But 01#01 is accepted!

High-level description of this machine is

Extra practice

Computation on input string 01#1

[illegible]

Recall: High-level descriptions of Turing machine algorithms are written as indented text within quotation marks. Stages of the algorithm are typically numbered consecutively. The first line specifies the input to the machine, which must be a string.

Motivated by our warmup examples:

A language L is **recognized** by a Turing machine M means

$L = \{ w \mid M \text{ accepts } w \}$.
 i.e. for each string in L , M accepts the string
 for each string not in L , M rejects or doesn't halt on the string

A Turing machine M **recognizes** a language L means

$L = \{ w \mid M \text{ accepts } w \}$.
 i.e. for each string, if M accepts string, then string is in L .
 if M rejects string or doesn't halt on string, then string is not in L .

A Turing machine M is a **decider** means for each strings the computation of the M on this string halts (in finitely many steps).

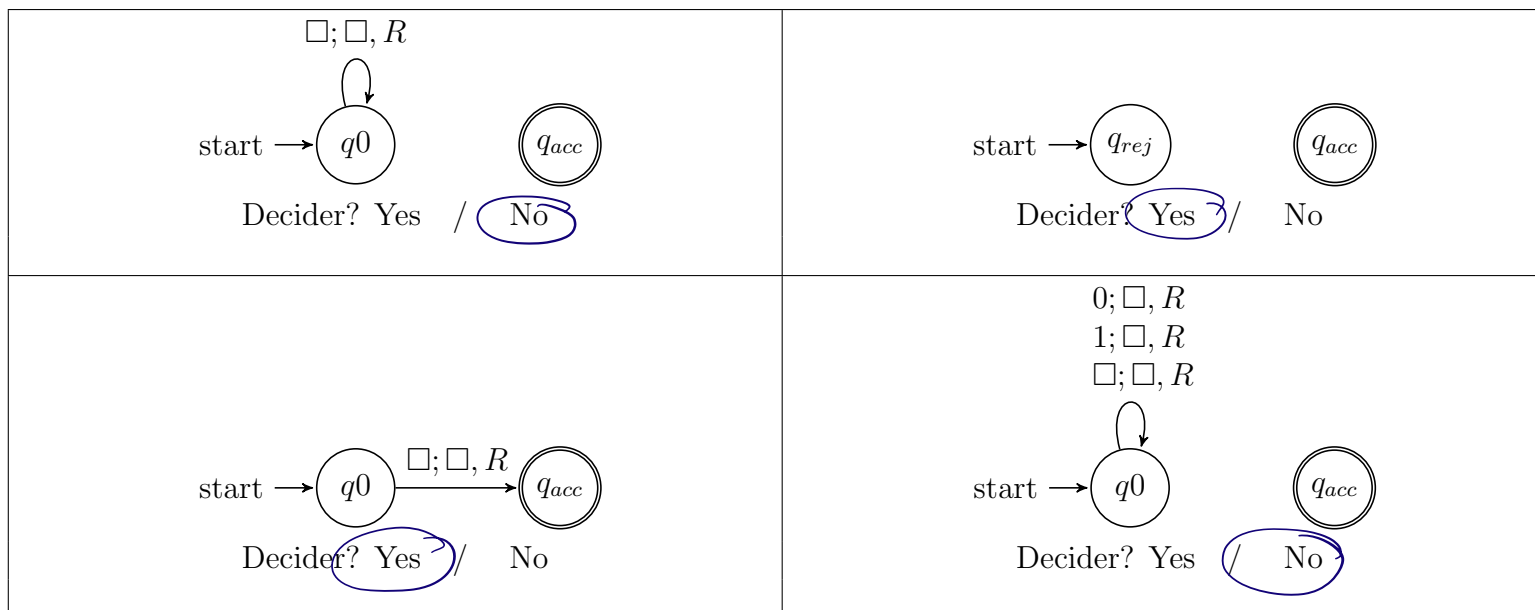
A language L is **decided** by a Turing machine M means M is a decider and M recognizes L .

i.e. for each string in L , M accepts the string
 for each string not in L , M rejects the string.

A Turing machine M **decides** a language L means

M is a decider and M recognizes L .

Fix $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \sqcup\}$ for the Turing machines with the following state diagrams:



Friday: Closure for the classes of recognizable and decidable languages

A **Turing-recognizable** language is a set of strings that is the language recognized by some Turing machine. We also say that such languages are recognizable.

A **Turing-decidable** language is a set of strings that is the language recognized by some decider. We also say that such languages are decidable.

An **unrecognizable** language is a language that is not Turing-recognizable.

An **undecidable** language is a language that is not Turing-decidable.

True or False: Any decidable language is also recognizable.

True or False: Any recognizable language is also decidable.

True or False: Any undecidable language is also unrecognizable.

True or False: Any unrecognizable language is also undecidable.

True or False: The class of Turing-decidable languages is closed under complementation.

Using formal definition:

Using high-level description:

Church-Turing Thesis (Sipser p. 183): The informal notion of algorithm is formalized completely and correctly by the formal definition of a Turing machine. In other words: all reasonably expressive models of computation are equally expressive with the standard Turing machine.