Regular Expressions

CSE 105 Week 1 Discussion

Deadlines and Logistics

- Make sure you have access to the class website, canvas, piazza and gradescope
- Make sure you can see all our OH in the class calendar
- Make sure you have access to PrairieLearn and PrairieTest.
- Read the grading scheme on the class website !
- Schedule your tests asap on PrairieTest !
- HW1 due next week on 8th (Tuesday) at 5 PM
- Link for the slides : https://docs.google.com/presentation/d/1rYhfMTnWjxi5GmIOC0Oj13gXquWENJHr5Azs4E_DpOA/edit?usp=sharing

What can you expect from the discussion section

- Recap* of key concepts from lectures
- Lots of informal interaction
- Practice problems from notes, review quizzes, Sipser and HW
- Ask us any questions and get your doubts answered !

PS - All screenshots taken either from Sipser or class notes unless specified otherwise

*See next slide for the extent of the recap

What NOT to expect from the discussion section

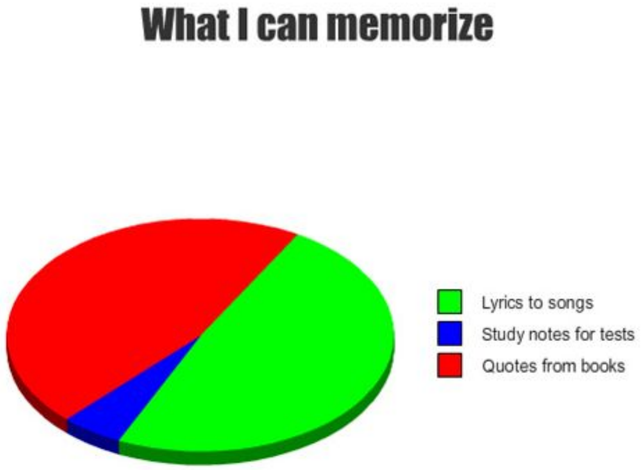
- A self contained “lecture” covering CSE 105. The discussion does NOT replace M/W/F lectures
- Answers to “Graded for correctness” HW questions
- Self contained notes to study from

Are the discussion sections podcasted/online/recorded ?

Ask me why

General Information

Do not memorize anything for the sake of completion of HWs, exams etc without first understanding underlying concepts !



Motivation

1. What are the capabilities and limitations of computers ?
2. Can we answer how “difficult” (or if even possible) a certain computation task is ?
3. Can we mathematically model computational problems ?

Important notation

Alphabet	A non-empty finite set, usually denoted as Σ
Symbol	An element of the alphabet
String over Σ	A finite list of symbols from Σ
Language over Σ	A set of strings over Σ
Σ*	Set of all possible strings formed from symbols in Σ

Key takeaways

- A language is a **SET** of strings
- Any time you see a language contain anything BUT strings, alarm bells should go off.
- You can have an empty language denoted by Ø. This is a language with NO strings

Is this statement correct, given Language L = {w|w is a string over {0} such that |w| = 1 }

L = 0 ?

Key takeaways

- A language is a **SET** of strings
- Any time you see a language contain anything BUT strings, alarm bells should go off.
- You can have an empty language denoted by Ø. This is a language with NO strings

Is this statement correct, given Language L = {w|w is a string over {0} such that |w| = 1 }

L = 0 ?

No : A language cant be just a string, it can has to be a set of (0 or more) strings!
L = {0} is a correct notation

Review Quiz Question

Consider the language $\{w \mid w \textbf{ is a string over } \{0,1\} \textbf{ and } |w| \textbf{ is an integer multiple of } 3\}$. Which of the following are elements of this language? (Select all and only that apply)

- [ ] 000
- [ ] 0
- [ ] The empty set
- [ ] $(1,0,1)$
- [ ] $\{000\}$
- [ ] The empty string

Select all possible options that apply. ?

What option(s) can you eliminate right away by virtue of the definition of strings and languages ?

Review Quiz Question

Consider the language $\{w \mid w \text{ is a string over } \{0,1\} \text{ and } |w| \text{ is an integer multiple of } 3\}$. Which of the following are elements of this language? (Select all and only that apply)

- [ ] 000
- [ ] 0
- [ ] ~~The empty set~~
- [ ] ~~(1, 0, 1)~~
- [ ] ~~{000}~~
- [ ] The empty string

Select all possible options that apply.

Can eliminate all non string options ! (since only strings can be elements of a language

What option(s) can you eliminate right away by virtue of the definition of strings and languages ?

Regular Expressions

Remember that at the end of the day, Languages are sets. How can we define a set?

1. List out all the elements
2. Use set builder notation and describe membership condition

Or

3. Use recursive definitions (Regular Expressions !)

The language described by a regular expression R is $L(R)$

Recap

Let A and B be languages. We define the regular operations ***union***, ***concatenation***, and ***star*** as follows:

- **Union:** $A \cup B = \{x | x \in A \text{ or } x \in B\}$.
- **Concatenation:** $A \circ B = \{xy | x \in A \text{ and } y \in B\}$.
- **Star:** $A^* = \{x_1 x_2 \ldots x_k | k \geq 0 \text{ and each } x_i \in A\}$.

Pop quiz : If $\Sigma = \{0,1\}$ and language $A = \{0, 01, 1\}$ and $B = \{\varepsilon, 1\}$ What is

- $A \cup B$
- $A \circ B$
- B^*
- Is $(A^*)^* = A^*$?

Recap

Let A and B be languages. We define the regular operations ***union***, ***concatenation***, and ***star*** as follows:

- **Union:** $A \cup B = \{x | x \in A \text{ or } x \in B\}$.
- **Concatenation:** $A \circ B = \{xy | x \in A \text{ and } y \in B\}$.
- **Star:** $A^* = \{x_1 x_2 \ldots x_k | k \geq 0 \text{ and each } x_i \in A\}$.

Pop quiz : If ∑ = {0,1} and language A = {0, 01, 1} and B = {ε, 1} What is

- A U B : {ε, 0, 1, 01}
- A○B : {0, 01, 1, 011, 11}
- B* : {1, 11, 111 ....} or {$1_0 \ldots 1_k$| k is a non negative integer}
- Is (A*)* = A* ? : Yes ! Both expressions produce the set of all strings that can be formed by concatenating strings in A with one another as many times as we want

R is a regular expression over the alphabet Σ

1. $R = a$, where $a \in \Sigma$
2. $R = \varepsilon$
3. $R = \emptyset$
4. $R = (R_1 \cup R_2)$, where R_1, R_2 are themselves regular expressions
5. $R = (R_1 \circ R_2)$, where R_1, R_2 are themselves regular expressions
6. (R_1^*), where R_1 is a regular expression.

Context is super important !

For the following examples assume the alphabet is $\Sigma_1 = \{0, 1\}$:

The language described by the regular expression 0 is $L(0) = \{0\}$

The language described by the regular expression 1 is $L(1) = \{1\}$

The language described by the regular expression ε is $L(\varepsilon) = \{\varepsilon\}$

The language described by the regular expression $\emptyset$ is $L(\emptyset) = \emptyset$

The language described by the regular expression $(\Sigma_1\Sigma_1\Sigma_1)^*$ is $L(\ (\Sigma_1\Sigma_1\Sigma_1)^*\ ) =$

Do both the Σ_1 refer to the same thing ?

Context is super important !

Refers to the alphabet set Σ_1 containing symbols 1 and 0 !

For the following examples assume the alphabet is $\Sigma_1 = \{0, 1\}$:

The language described by the regular expression 0 is $L(0) = \{0\}$

The language described by the regular expression 1 is $L(1) = \{1\}$

The language described by the regular expression ε is $L(\varepsilon) = \{\varepsilon\}$

The language described by the regular expression $\emptyset$ is $L(\emptyset) = \emptyset$

The language described by the regular expression $(\Sigma_1\Sigma_1\Sigma_1)^*$ is $L(\ (\Sigma_1\Sigma_1\Sigma_1)^*\ ) =$

Refers to any one occurrence of any symbol (0 or 1 in this case) from Σ_1.

Do both the Σ_1 refer to the same thing ? : NO !

Other conventions

Assuming Σ is the alphabet, we use the following conventions

Σ	regular expression describing language consisting of all strings of length 1 over Σ
$*$ then $\circ$ then $\cup$	precedence order, unless parentheses are used to change it
$R_1 R_2$	shorthand for $R_1 \circ R_2$ (concatenation symbol is implicit)
R^+	shorthand for $R^* \circ R$
R^k	shorthand for R concatenated with itself k times, where k is a (specific) natural number

Remember

A regular expression should describe a SET of strings over an alphabet since it is descriptive of a Language, which is a SET of strings over an alphabet.

All “operations” and “conventions” you see in a regular expression boil down to some fundamental operation(s) on set(s).

What operations and what sets ?

Remember

A regular expression should describe a SET of strings over an alphabet since it is descriptive of a Language, which is a SET of strings over an alphabet.

All “operations” and “conventions” you see in a regular expression boil down to some fundamental operation(s) on set(s).

What operations and what sets ?

U, * and ○

As a basis step :

- Element ∈ alphabet or
- ε
- Ø

Inductively:

On any regular expression(s)

When you see... The language described by the regular expression $1^* \circ 1$ is $L(1^* \circ 1) =$

You need to think : sets and operations !

Solve :

When you see... The language described by the regular expression $1^* \circ 1$ is $L(1^* \circ 1) =$

You need to think : sets and operations !

Solve :

$L(1^* \circ 1)$
$= L(1^*) \circ L(1)$
$= L(1)^* \circ L(1)$
$= \{1\}^* \circ \{1\}$
$= \{1^k | k>=0\} \circ \{1\}$
$= \{1^k 1 | k>=0\}$ (sufficient to leave it at this step)
$= \{1^k | k>=1\}$

From Regex to Language

The language over $\Sigma_1 = \{0,1\}$ described by the regular expression Σ_1^*1 is $L(\Sigma_1^*1) =$ BLANK

Describe this set in (a) Simple english and (b) Set builder notation

(a) Set of all strings over Σ_1 ending in a 1
(b) $\{x^k1 | k>=0, x \in \Sigma_1\}$

Describe the language generated by these Regex(over Σ = {0,1}) : Basics

1. (0U1)
2. (0U1)*
3. Σ*
4. (0)U(1)
5. (01) U (1)
6. (01)* U (1)
7. (01 U 1)*

No, Seriously, try them yourself first !

Describe the language generated by these Regex(over $\Sigma = \{0,1\}$) : Basics

1. (0U1) : {0,1}
2. (0U1)* : $\{x^k \mid k>=0, x \in \Sigma\}$
3. Σ^* : Set of all strings that can be created from elements in Σ (Notice similarity to (2)
4. (0)U(1) : {0, 1}
5. (01) U (1): {01, 1}
6. (01)* U (1): $\{(01)^k \mid k>=0\}$ U {1}
7. (01 U 1)* : $\{x^k \mid k>=0, x \in \{01,1\}\}$

Describe the language generated by these Regex(over $\Sigma = \{0,1\}$) : Level up

1. $(01)(01)^*(01)$
2. $(01)^+(01)$
3. $1 \cup 11 \cup 111 \cup 1111 \cup 11111^*$
4. $\varepsilon^*\Sigma\Sigma\Sigma\varepsilon^*$
5. $(\varepsilon \cup \Sigma)(\varepsilon \cup \Sigma)(\varepsilon \cup \Sigma)$
6. $\Sigma^*1\Sigma^*0\Sigma^* \cup \Sigma^*0\Sigma^*1\Sigma^*$
7. $(1\Sigma^*1) \cup (0\Sigma^*0) \cup (\varepsilon) \cup (1) \cup (0)$

No, Seriously, try them yourself first !

Describe the language generated by these Regex(over $\Sigma = \{0,1\}$) : Level up

1. $(01)(01)^*(01)$: Set of strings containing repeating units of (01) with at least 2 repeats
2. $(01)^+(01)$: Set of strings containing repeating units of (01) with at least 2 repeats (notice similarity to (1))
3. $1 \cup 11 \cup 111 \cup 1111 \cup 11111^*$: Same language as 11^* or 1^+
4. $\varepsilon^*\Sigma\Sigma\Sigma\varepsilon^*$: All strings of size 3
5. $(\varepsilon \cup \Sigma)(\varepsilon \cup \Sigma)(\varepsilon \cup \Sigma)$: All strings of size at most 3
6. $\Sigma^*1\Sigma^*0\Sigma^* \cup \Sigma^*0\Sigma^*1\Sigma^*$: All strings containing at least one 1 and one 0
7. $(1\Sigma^*1) \cup (0\Sigma^*0) \cup (\varepsilon) \cup (1) \cup (0)$: All strings starting and ending with the same symbol

Sipser 1.18 practice (pg 86)

Give the regular expression generating the following languages over $\Sigma = \{0,1\}$

a. $\{w \mid w$ begins with a 1 and ends with a 0$\}$
b. $\{w \mid w$ contains at least three 1s$\}$
c. $\{w \mid w$ contains the substring 0101 (i.e., $w = x0101y$ for some x and y)$\}$
d. $\{w \mid w$ has length at least 3 and its third symbol is a 0$\}$
e. $\{w \mid w$ starts with 0 and has odd length, or starts with 1 and has even length$\}$
f. $\{w \mid w$ doesn't contain the substring 110$\}$
g. $\{w \mid$ the length of w is at most 5$\}$
h. $\{w \mid w$ is any string except 11 and 111$\}$
i. $\{w \mid$ every odd position of w is a 1$\}$
j. $\{w \mid w$ contains at least two 0s and at most one 1$\}$
k. $\{\varepsilon, 0\}$
l. $\{w \mid w$ contains an even number of 0s, or contains exactly two 1s$\}$
m. The empty set
n. All strings except the empty string

No, Seriously, try them yourself first !

Sipser 1.18 practice (pg 86)

Give the regular expression generating the following languages over $\Sigma = \{0,1\}$: These are not the only regular expressions that generate the language !

a. $\{w \mid w$ begins with a 1 and ends with a 0$\}$
b. $\{w \mid w$ contains at least three 1s$\}$
c. $\{w \mid w$ contains the substring 0101 (i.e., $w = x0101y$ for some x and y)$\}$
d. $\{w \mid w$ has length at least 3 and its third symbol is a 0$\}$
e. $\{w \mid w$ starts with 0 and has odd length, or starts with 1 and has even length$\}$
f. $\{w \mid w$ doesn't contain the substring 110$\}$
g. $\{w \mid$ the length of w is at most 5$\}$
h. $\{w \mid w$ is any string except 11 and 111$\}$
i. $\{w \mid$ every odd position of w is a 1$\}$
j. $\{w \mid w$ contains at least two 0s and at most one 1$\}$
k. $\{\varepsilon, 0\}$
l. $\{w \mid w$ contains an even number of 0s, or contains exactly two 1s$\}$
m. The empty set
n. All strings except the empty string

a. $1\Sigma^*0$
b. $\Sigma^*1\Sigma^*1\Sigma^*1\Sigma^*$
c. $\Sigma^*0101\Sigma^*$ (How to modify if 0101 is a *proper substring* ?)
d. $\Sigma\Sigma0$
e. $0(\Sigma\Sigma)^* \cup 1(\Sigma)(\Sigma\Sigma)^*$
f. $(0\cup10)^*1^*$
g. $(\varepsilon\cup\Sigma)^5$
h. $\varepsilon \cup \Sigma \cup \Sigma0 \cup 10 \cup \Sigma\Sigma0 \cup 0\Sigma1 \cup \Sigma01 \cup \Sigma^4\Sigma^*$
i. $1(\Sigma1)^*$
j. $(0^*00^*00^*10^*) \cup (0^*00^*10^*00^*) \cup (0^*10^*00^*00^*)$
k. $\varepsilon\cup0$
l. $(00)^* \cup 11$
m. $\emptyset$
n. Σ^+ or $\Sigma\Sigma^*$ or $\Sigma^*\Sigma$

HW Problem (2b)

(b) (*Graded for completeness*) A friend tells you that each regular expression that has a Kleene star (*) describes an infinite language. Are they right? Either help them justify their claim or give a counterexample to disprove it and explain your counterexample.

HW Problem (2b)

(b) (*Graded for completeness*) A friend tells you that each regular expression that has a Kleene star (*) describes an infinite language. Are they right? Either help them justify their claim or give a counterexample to disprove it and explain your counterexample.

Ans : Think about what is the result of the following regular expression : $\emptyset^*$

Describe the language expressed by these Regex over ∑

Assume R is some arbitrary Regex over ∑

1. $\emptyset^*$
2. $R \circ \varepsilon$
3. $R \circ \emptyset$
4. $R \cup \emptyset$
5. $R \cup \varepsilon$

Describe the language expressed by these Regex over ∑

Assume R is some arbitrary Regex over ∑

1. Ø* : {ε}
2. R○ε : R
3. R○Ø : Ø
4. R U Ø : R
5. R U ε : equals R if R contained ε. Else simply add ε to the language generated by R

Questions?

Good luck for HW 1 !