

Selection

Select(A,i,n) – Given a set of n numbers A, find the i th smallest.

Example

3 18 1 8 4 47 45 10 23

- $i = 1$ – 1 minimum
- $i = 9$ – 47 maximum
- $i = 5$ – 10 median
- $i = 7$ – 23

What is a straightforward algorithm for Selection?

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- Sort the numbers and return $A[i]$.
- $O(n \log n)$ time.

Selection

Question: Is selection (median) harder than sorting? Is it necessary to sort to find the median?

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Answer: No. We can find the median in $O(n)$ time.

A recursive strategy

- Pick an element to be the pivot p .
- Split the items into 2 sets
 - L – the set of items less than or equal to the pivot
 - H – the set of items greater than the pivot.
- Recurse on the appropriate set.

Deterministic Selection

SELECT(A,i,n)

1 if ($n = 1$)

2 then return $A[1]$

3 $p = \text{MEDIAN}(A)$

4

5

6 $L = \{x \in A : x \leq p\}$

$H = \{x \in A : x > p\}$

7 if $i \leq |L|$

8 then SELECT($L, i, |L|$)

9 else SELECT($H, i - |L|, |H|$)

Analysis

- Let $M(n)$ be the time to find the median
- Let $T(n)$ be the time for selection
- Splitting the items into L and H takes $O(n)$ time.

$$T(n) = T(n/2) + M(n) + O(n).$$

- If $M(n) = O(n)$, then $T(n) = O(n)$
- But why should $M(n) = O(n)$, and isn't the reasoning circular?

Modified Goal

New Goal: Suppose that, in $O(n)$ time, we find a number “close” to the median, then what happens.

- Question 1: What if, in $O(n)$ time, we found a number in the middle half?
- Question 2: What if we used our selection algorithm recursively to help find a number in the middle half?

Modified Goal

New Goal: Suppose that, in $O(n)$ time, we find a number “close” to the median, then what happens.

- Question 1: What if, in $O(n)$ time, we found a number in the middle half?
- Question 2: What if we used our selection algorithm recursively to help find a number in the middle half?

Answer 1: If we found a number in the middle half, we would have the recurrence

$$T(n) \leq T(3n/4) + O(n)$$

which by the master method solves to $O(n)$.

Deterministic Selection (2)

SELECT(A,i,n)

1 if $(n = 1)$

2 then return A

3 Split the items into $\lfloor n/5 \rfloor$ groups 5 (and one more group).

Call these groups $G_1, G_2, \dots, G_{\lfloor n/5 \rfloor}$

4 Find the median m_i of each G_i

5 Recursively compute the median of medians,

$p = \text{SELECT}(\{m_1, \dots, m_{\lfloor n/5 \rfloor}\}, \lfloor n/10 \rfloor, \lfloor n/5 \rfloor)$

6 $L = \{x \in A : x \leq p\}$

$H = \{x \in A : x > p\}$

7 if $i \leq |L|$

8 then SELECT($L, i, |L|$)

9 else SELECT($H, i - |L|, |H|$)

Proof

Correctness Need to prove that lines 3-6 return an item in the middle half.

Proof

Running time

$$T(n) \leq T(3n/4) + T(n/5) + O(n)$$