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3 4 | 5 2

3

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- Array $A[1 \dots n]$ – holds input
- Array $C[1 \dots k]$ – $C[j]$ holds number of elements of A less than or equal to j

Example:

| index | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|---|---|---|---|---|---|---|---|---|
| $A :$ | 2 | 9 | 1 | 8 | 6 | 5 | | | |

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$$B[C(A)] \leftarrow A$$

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| $A :$ | 2 | 9 | 1 | 8 | 6 | 5 | | | |
| $C :$ | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 | |

$B : 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1$
 $+ 2 \ 2 \ 2 \ 3 \ 4 \ 4 \ 5 \ 6 \ 5$
 $\hline 3 \ 1 \ 2 \ 5 \ 6 \ 8 \ 9$

Questions

- How do we compute C?

Counting Sort

2 3, 10^{10}

Counting-Sort(A, B, k)

```
1  for  $i \leftarrow 0$  to  $k$ 
2      do  $C[i] \leftarrow 0$ 
3  for  $j \leftarrow 1$  to  $\text{length}[A]$ 
4      do  $C[A[j]] \leftarrow C[A[j]] + 1$ 
5  ▷  $C[i]$  now contains the number of elements equal to  $i$ .
6  for  $i \leftarrow 1$  to  $k$ 
7      do  $C[i] \leftarrow C[i] + C[i - 1]$ 
8  ▷  $C[i]$  now contains the number of elements less than or equal to  $i$ .
9  for  $j \leftarrow \text{length}[A]$  downto 1
10     do  $B[C[A[j]]] \leftarrow A[j]$ 
11      $C[A[j]] \leftarrow C[A[j]] - 1$ 
```



Analysis

- Running Time $O(n + k)$
- No Comparisons
- Doesn't work on all data
- Good when k is small
- When $k = O(n)$ we have run-time $O(n + k) = O(n)$
- Examples?

Stable Sorting

- We want to sort x_1, x_2, \dots, x_n
- If $x_i > x_j$ then put x_i after x_j

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Question: Is counting sort stable?

Improving Counting Sort

Question: Should we use counting sort to sort everyone in this class by initials?

$$(26)^2 \approx 600$$

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- $n = 150$
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- Sort second initial
- Then stable sort by first initial.

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Analysis

- Sorting a single letter: $150 + 27 < 200$
- Total running time: $2(150 + 27) < 400$

Radix Sort

$\text{Radix-Sort}(A, d)$

```
1  for  $i \leftarrow 1$  to  $d$ 
2      do use a stable sort to sort array  $A$  on digit  $i$ 
```

Example

| | | | | | | |
|------------|--------------------|-----|--------------------|-----|--------------------|-----|
| 379 | STABLE SORT | 912 | STABLE SORT | 802 | STABLE SORT | 258 |
| 912 | \Rightarrow | 802 | \Rightarrow | 803 | \Rightarrow | 259 |
| 258 | | 823 | | 804 | | 269 |
| 269 | | 803 | | 912 | | 279 |
| 823 | | 804 | | 823 | | 379 |
| <u>259</u> | | 258 | | 258 | | 802 |
| 803 | | 379 | | 259 | | 803 |
| 279 | | 269 | | 269 | | 804 |
| 804 | | 359 | | 379 | | 823 |
| 802 | | 279 | | 279 | | 912 |

Radix Sort Correctness

$\text{Radix-Sort}(A, d)$

- 1 for $i \leftarrow 1$ to d
- 2 do use a stable sort to sort array A on digit i

Loop Invariant: After the i th iteration of the loop, the elements are sorted by their last i digits.

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Inductive Step:

- Assume the invariant holds after $i - 1$ iterations
- Need to prove that it holds after i iterations

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 - Numbers: $b = 10$
 - Words: $b = 27$
 - UNI (letter/number): $b = 37$

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Radix Sort Running Time:

$$O(d(n + b))$$

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Radix Sort Running Time: $O(d(n + b))$

Counting Sort Running Time: $O(n + k) = O(n + b^d)$

Example

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- $d = 7$
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