Topological Sort

- **Given a DAG, directed acylic graph**
- **Find an ordering of the vertices such that is** $(v, w) \in E$ **then** v **is before** w **in the ordering.**

Algorithm

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- **Output the nodes in order of decreasing finishing times**

Running time: $O(E)$

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a b c d e f g
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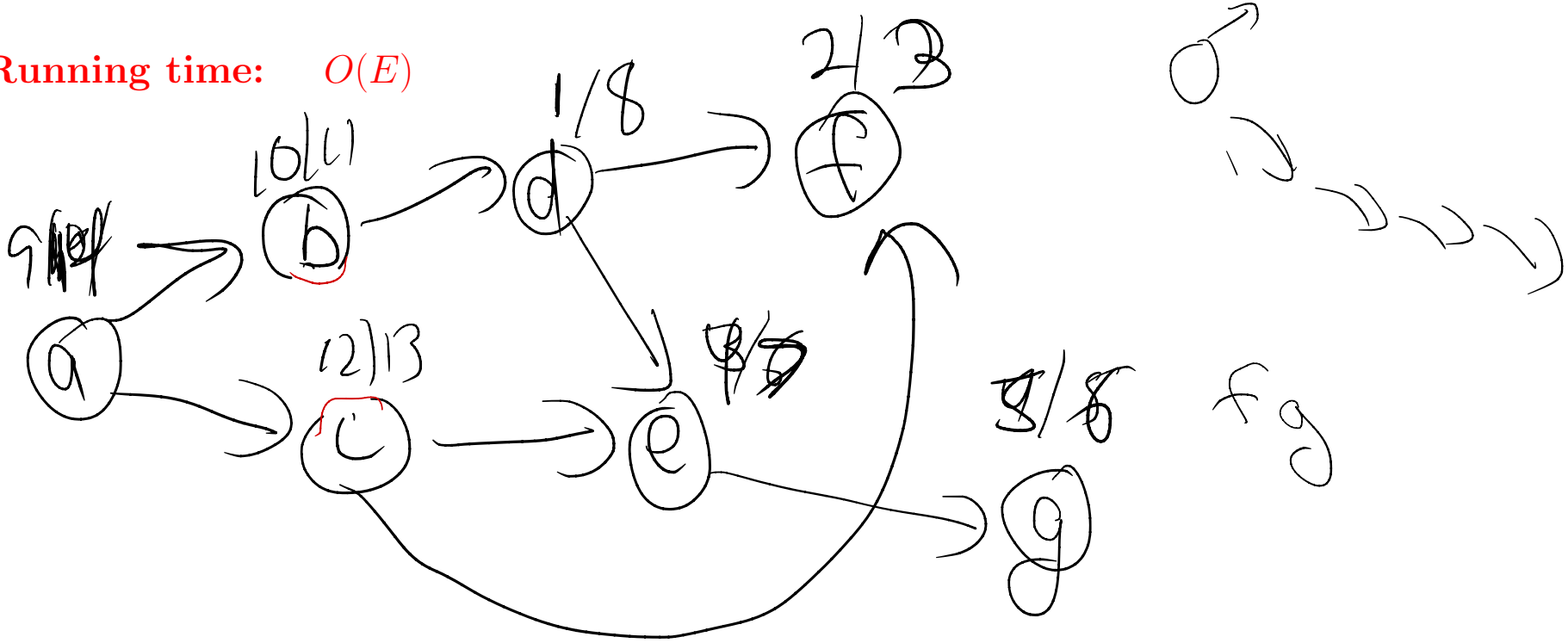
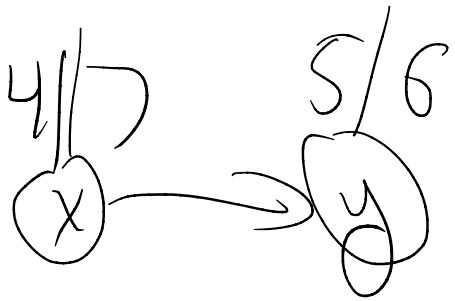


Proof of Correctness

Theorem Topological Sort Algorithm is correct, i.e. if $(x, y) \in E$ then $f(x) > f(y)$.

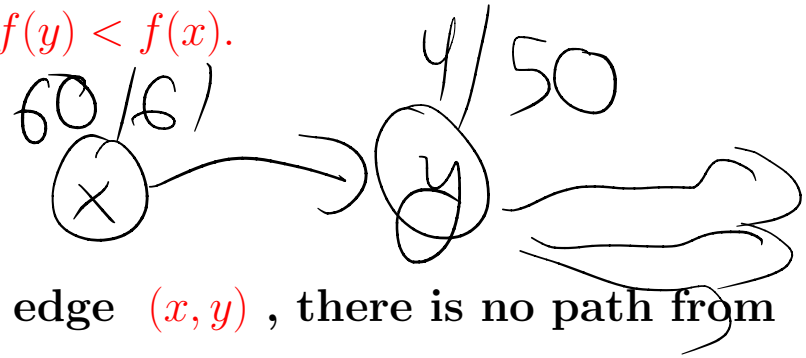


Proof

Case 1: ($d(x) < d(y)$)

- At time $d(x)$, y must be white.
- Using the parenthesis theorem,

$$d(x) < d(y) < f(y) < f(x).$$



Case 2: ($d(x) > d(y)$).

- Because G is a DAG and we have an edge (x, y) , there is no path from y to x .
- This means that $f(y) < d(x)$.
- For any vertex $d(x) < f(x)$.
- Putting these two inequalities together we get $f(y) < f(x)$.