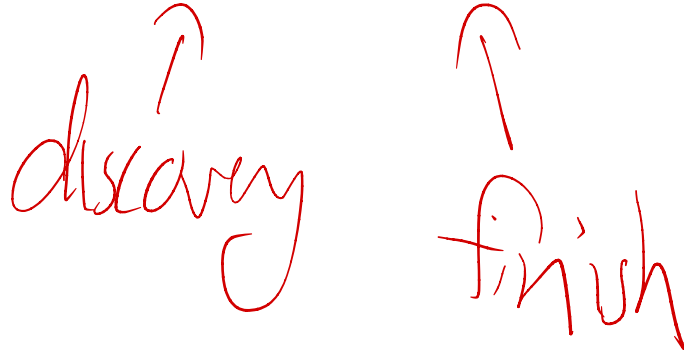


Depth First Search

- More interesting than BFS
- Works for directed and undirected graphs. Example is for directed graphs.
- Time stamp nodes with discovery and finishing times.
- Lifetime: white, $d(v)$, grey, $f(v)$, black

discovery finish



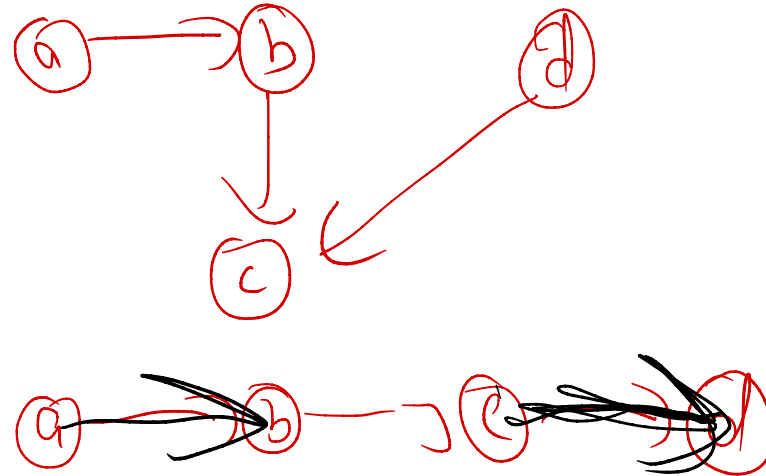
Code

DFS(*G*)

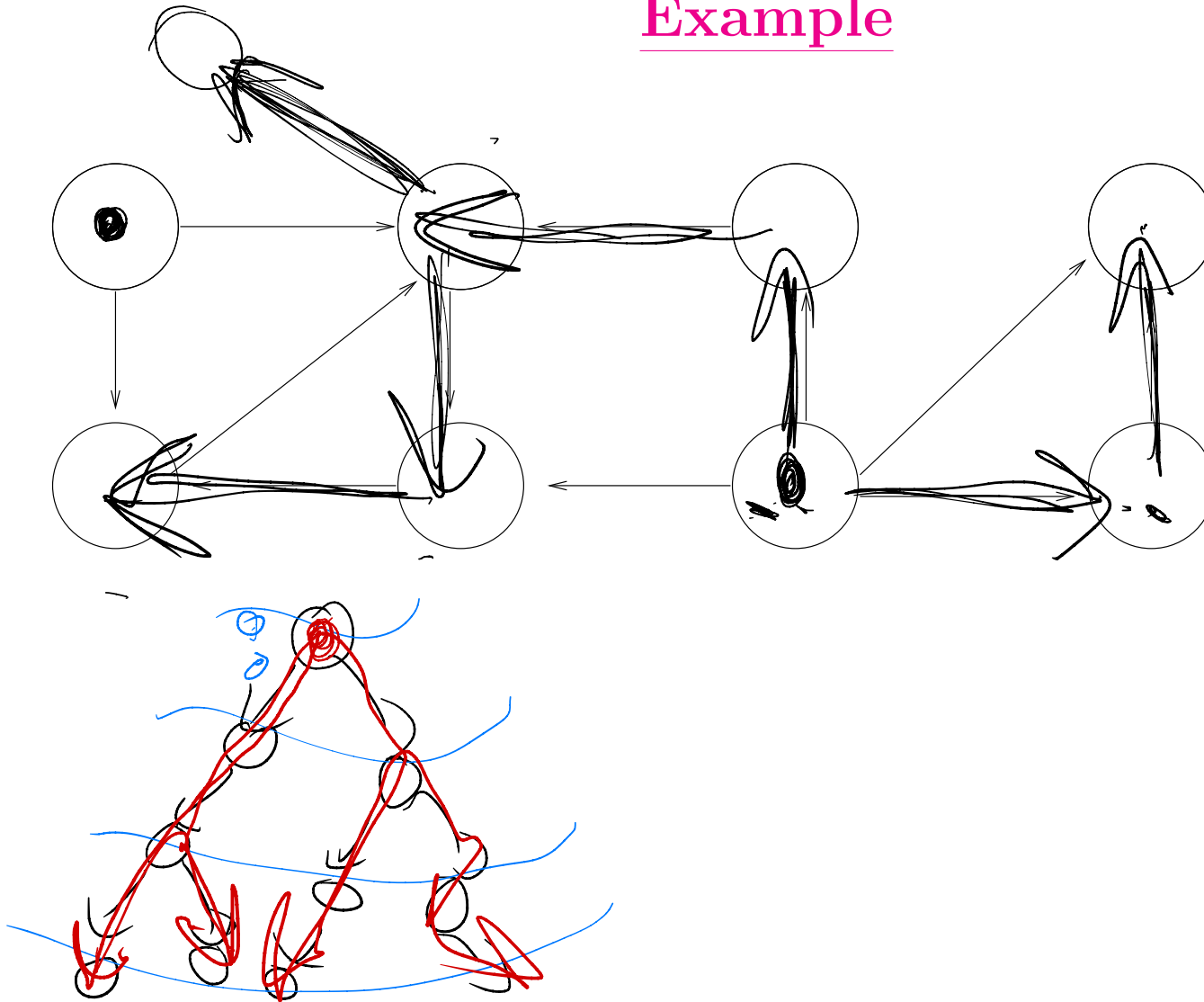
```
1  for each vertex  $u \in V[G]$ 
2      do  $color[u] \leftarrow \text{WHITE}$ 
3       $\pi[u] \leftarrow \text{NIL}$ 
4   $time \leftarrow 0$ 
5  for each vertex  $u \in V[G]$ 
6      do if  $color[u] = \text{WHITE}$ 
7          then  $\text{DFS-VISIT}(u)$ 
```

DFS-Visit(*u*)

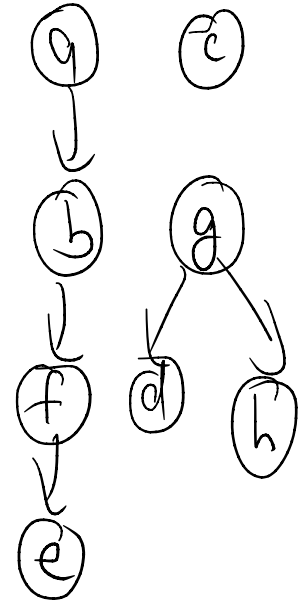
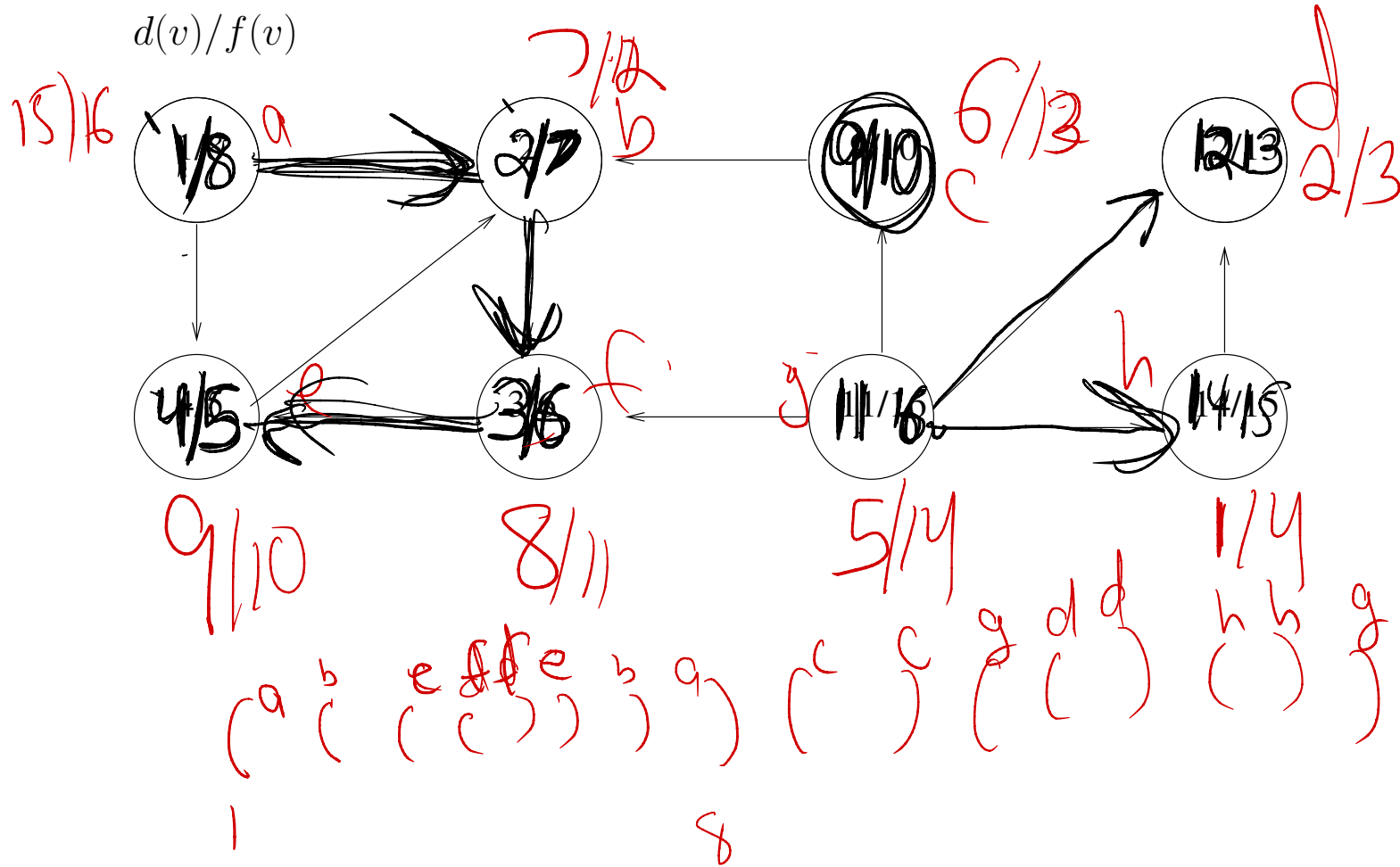
```
1   $color[u] \leftarrow \text{GRAY}$                                 ▷ White vertex  $u$  has just been discovered.
2   $time \leftarrow time + 1$ 
3   $d[u] \leftarrow time$ 
4  for each  $v \in Adj[u]$                                      ▷ Explore edge  $(u, v)$ .
5      do if  $color[v] = \text{WHITE}$ 
6          then  $\pi[v] \leftarrow u$ 
7               $\text{DFS-VISIT}(v)$ 
8   $color[u] \leftarrow \text{BLACK}$                                 ▷ Blacken  $u$ ; it is finished.
9   $f[u] \leftarrow time \leftarrow time + 1$ 
```



Example



Labeled



Structure

Parenthesization

If we represent the discovery of vertex u with a left parenthesis “(u ” and represent its finishing by a right parenthesis “ u)”, then the history of discoveries and finishings makes a well-formed expression in the sense that the parentheses are properly nested.

Parenthesis theorem In any depth-first search of a (directed or undirected) graph $G = (V, E)$, for any two vertices u and v , exactly one of the following three conditions holds:

- the intervals $[d[u], f[u]]$ and $[d[v], f[v]]$ are entirely disjoint, and neither u nor v is a descendant of the other in the depth-first forest,
- the interval $[d[u], f[u]]$ is contained entirely within the interval $[d[v], f[v]]$, and u is a descendant of v in a depth-first tree, or
- the interval $[d[v], f[v]]$ is contained entirely within the interval $[d[u], f[u]]$, and v is a descendant of u in a depth-first tree.

Nesting of descendants' intervals

Vertex v is a proper descendant of vertex u in the depth-first forest for a (directed or undirected) graph G if and only if $d[u] < d[v] < f[v] < f[u]$.

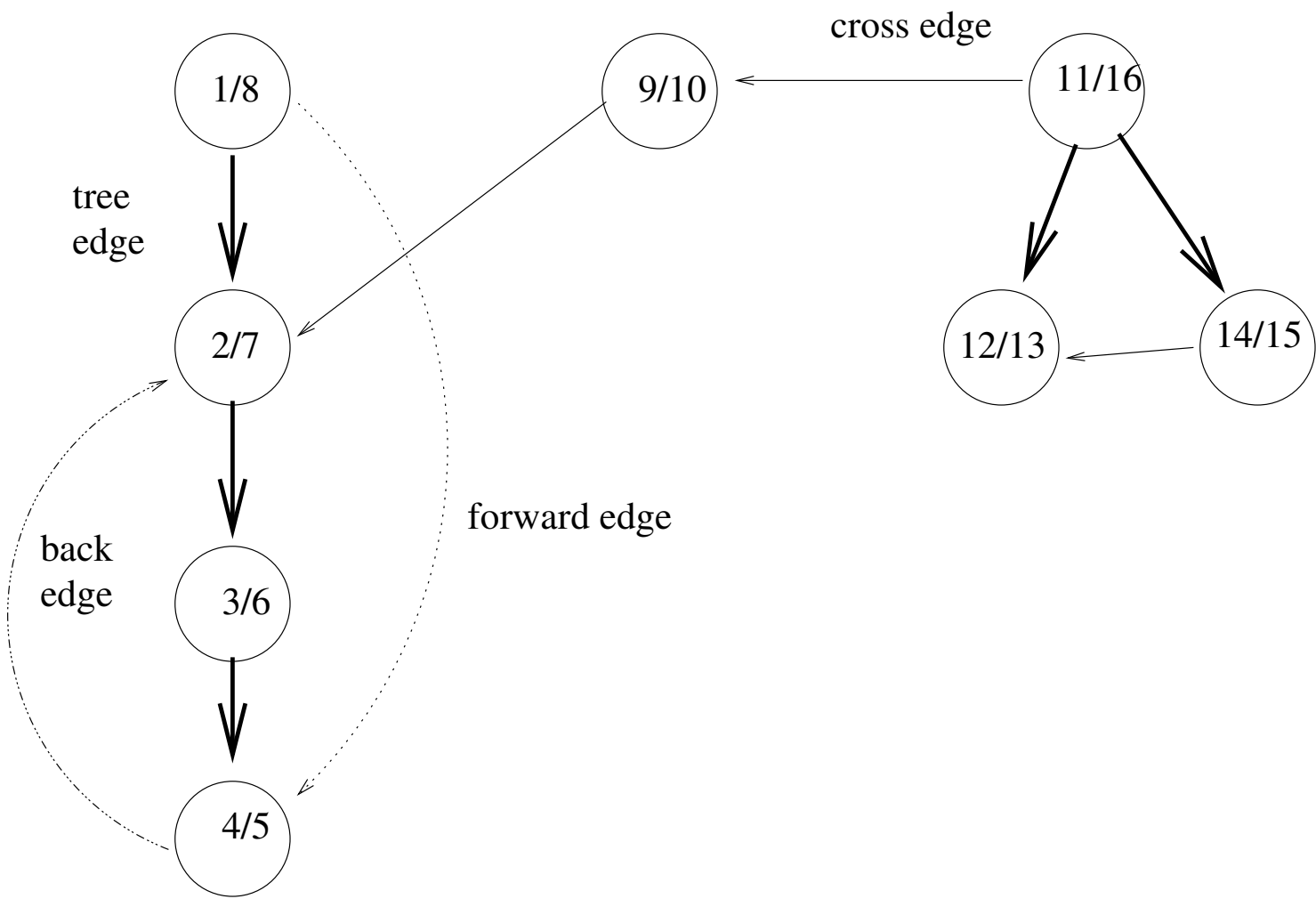
More Structure

White-path theorem

In a depth-first forest of a (directed or undirected) graph $G = (V, E)$, vertex v is a descendant of vertex u if and only if at the time $d[u]$ that the search discovers u , vertex v can be reached from u along a path consisting entirely of white vertices

Edge classification

1. **Tree edges** are edges in the depth-first forest G_π . Edge (u, v) is a tree edge if v was first discovered by exploring edge (u, v) .
2. **Back edges** are those edges (u, v) connecting a vertex u to an ancestor v in a depth-first tree. Self-loops, which may occur in directed graphs, are considered to be back edges.
3. **Forward edges** are those nontree edges (u, v) connecting a vertex u to a descendant v in a depth-first tree.
4. **Cross edges** are all other edges. They can go between vertices in the same depth-first tree, as long as one vertex is not an ancestor of the other, or they can go between vertices in different depth-first trees.



Running Time:

```
1  for each  $u \in V$ 
2      do for each  $v \in \text{Adj}(v)$ 
3          do Something  $O(1)$  time
```

Each edge and vertex is processed once:

$$O(E + V) = O(E)$$

