Python comments

3.0p 1 Which of the following lines is recognized by Python as a comment?

- [ ] <!-- This is a comment -->
- [ ] # This is a comment
- [ ] % This is a comment
- [ ] /* This is a comment */

Infinite loops

3.0p 2

Is it an infinite loop?

```
x = 2
while x > 0:
    y = x + 1
    print(y)
    x = y
```

- ( ) Maybe
- ( ) False
- ( ) True

Defining a function

3.0p 3 Write the function header "my_function" for a function that takes x and y as integer inputs and returns a boolean. Include type hints for the arguments and the return value. You do not need to provide a docstring.

```
1
```

Python

Complex if-statement

3.0p 4

What will be printed as the final result?

```
a = True
b = False
result = 0

if a:
    result += 1
    b = not b
if a and b:
    result += 2
else:
    result -= 1

print(result)
```

- [ ] 2
- [ ] 3
- [ ] 1
- [ ] 0

Newton-Raphson method

3.0p 5 Consider you would like to solve the following nonlinear equation

$2x^2 - 3x = 2$

using the Newton-Raphson method. Please calculate the approximate solution x_1 at the first iteration given an initial guess $x_0 = 1$.

Answer

Jacobian

3.0p 6

Which of the options is the correct Jacobian of the function $f\left(x_1, x_2, x_3\right) = \begin{pmatrix} x_1^2 \cdot x_2 + x_3^3 \\ \cos\left(x_1\right) + x_2 \\ \mathrm{e}^{x_3} \cdot x_2 \end{pmatrix}$

- ◯ $J = \begin{pmatrix} 2x_1x_2 & -\sin\left(x_1\right) & 0 \\ x_1^2 & 1 & e^{x^3} \\ 3x_3^2 & 0 & e^{x^3}x_2 \end{pmatrix}$
- ◯ $J = \begin{pmatrix} 2x_1x_2 & x_1^2 & 3x_3^3 \\ \sin\left(x_1\right) & 1 & 0 \\ 0 & \mathrm{e}^{x_3} & \mathrm{e}^{x_2} \cdot x_3 \end{pmatrix}$
- ◯ $J = \begin{pmatrix} 2x_1x_2 & x_1^2 & 3x_3^2 \\ -\sin\left(x_1\right) & 1 & 0 \\ 0 & \mathrm{e}^{x_3} & \mathrm{e}^{x_3} \cdot x_2 \end{pmatrix}$
- ◯ None of the above

Fixed-point coding

3.0p 7 Consider the nonlinear equation $x^3 - 2x - 329 = 0$. Transform this equation into a fixed-point problem in the form $g\left(x\right) = x$ and define a function g(x) in Python that returns the value of your function $g\left(x\right)$. Please also insert type hints and a suitable docstring into your function.

```

```

Python

Error definitions

3.0p 8 Consider that you applied an iterative solution method and have obtained the approximate solution $y_{n-1} = 0.711$ at iteration $n-1$ and the approximate solution $y_n = 0.705$ at iteration n. The true solution of the problem you are investigating is $y^* = 0.7$.
Compute the estimated absolute error $\overline{\epsilon}_{abs}$ you have obtained at iteration n!

Answer

Arrays

3.0p 9

What is the shape of the numpy.ndarray `mat` defined in the following way:

- [ ] (3,3,1)
- [ ] (3,3)
- [ ] (1,3)
- [ ] (1,3,3)

Jacobi MC

3.0p 10 Which of the following statements about the Jacobi method for solving a system of equations of the form $Ax = b$ are correct?

- [ ] The update rule for the Jacobi method at iteration k can be expressed as $x^{k+1} = U^{-1}\left(b - (L+D)\,x^k\right)$ considering that Matrix A can be decomposed into a lower triangular matrix L, an upper triangular matrix U, and a matrix D containing only the diagonal elements of A such that $A = D + L + U$.
- [ ] The Jacobi method can be used to solve any system of linear equations, regardless of matrix properties.
- [ ] The Jacobi method will converge for any matrix that has a non-zero determinant.
- [ ] When using the Jacobi method, each element of the solution vector is updated independently of the other elements within the same solution vector of that iteration.

Gauss-Seidel MC

3.0p 11 Which of the following statements are correct about the Gauss-Seidel method as a solution approximation method of systems of linear equations of the form $Ax = b$?

- [ ] The update rule of the Gauss-Seidel method at iteration k can be expressed as $x^{k+1} = D^{-1}\left(b - Lx^{k+1} - Ux^k\right)$ considering that matrix A can be decomposed into a lower triangular matrix L, an upper triangular matrix U, and a diagonal matrix D containing the diagonal elements of A, such that $A = D + L + U$.
- [ ] The Gauss-Seidel method is an iterative method that provides an exact solution in a finite number of steps for any linear system.
- [ ] The Gauss-Seidel method converges more slowly than the Jacobi method for diagonally dominant matrices.

Linear system of equation

3.0p 12 Consider the following set of linear equations:$(1)\ 5x_2 + x_1 + 6x_3 - 7 = 0$

$(2)\ 5x_1 + 4x_2 + x_3 = 5$

$(3)\ 7x_3 + 2x_1 + 3x_2 - 1 = 0$

Transform this set of linear equations into the form $Ax = b$. Then, implement A and b in Python as arrays of type numpy.ndarray.

```

```

Python

Integration built-in functions

3.0p 13 Which Python library provides built-in functions for numerical integration?

- ☐ SciPy
- ☐ SymPy
- ☐ Matplotlib
- ☐ NumPy

Simpson 1/3: Features

3.0p 14 Which of the following is a characteristic of the composite Simpson's 1/3 rule method?

- ☐ It is a second-order accurate method, meaning that the error decreases quadratically with the step length
- ☐ It has a bounded error proportional to h^4, where h is the step length
- ☐ It is derived from quadratic interpolation
- ☐ It requires an even number of intervals

Integration: Composite Simpson 1/3

3.0p **15** If you have a 1D domain grid with 11 discretization points, how many integration steps are performed when using the Composite Simpson 1/3 method?

- ☐ 10
- ☐ 9
- ☐ 4
- ☐ 5

Differentiation: 1st order central difference scheme

3.0p **16** To derive the first-order central difference scheme used to compute the derivative in a point x_i you can start by summing the Taylor approximation in x_{i+1} and x_{i-1}.

- ○ False
- ○ True

Error types

3.0p **17** Explain the difference between local truncation error and global truncation error in the context of numerical solution methods for ordinary differential equations (ODEs).

Stability of ODE methods

3.0p **18** Which methods for solving ordinary differential equations (ODEs) are only conditionally stable depending on the step size?

- ☐

- [ ] Runge-Kutta
- [ ] Heun's method
- [ ] Backward Euler
- [ ] Forward Euler

Number of BCs

3.0p **19** For a boundary value problem (BVP), how many boundary conditions are typically needed for a third-order ordinary differential equation?

- ○ 2
- ○ 4
- ○ 1
- ○ 3

Sufficient boundary conditions

3.0p **20** Given is a boundary value problem (BVP) with one second-order ordinary differential equation (ODE) and one Neumann boundary condition. How many solutions exist for the ODE?

- ○ 2
- ○ Infinitely many
- ○ 1
- ○ 0

BVP with complex geometries

3.0p **21** Which numerical method is most suitable to solve boundary value problems (BVPs) with irregular grids or complex geometries?

- ○ Shooting method

- Runge-Kutta method
- Finite differences method
- Finite element method

Equation system for ODE 4

3.0p 22 Consider the ODE $\frac{d^2y}{dx^2} + \beta\frac{dy}{dx} + \gamma y = f\left(x\right)$, with boundary conditions $y\left(0\right) = y_a$ and $y\left(1\right) = y_b$. Discretize the equation using central differences for both the first and second derivatives and express the system in matrix form for **internal** points $Ay = b$. Provide the expression for the first point $A\left[0\right]y = b\left[0\right]$.

New equation

Update guess for shooting method

3.0p 23 In the shooting method, which numerical techniques are commonly used to update the initial condition guesses?

- [ ] Heun's method
- [ ] Euler's method
- [ ] Linear interpolation
- [ ] Secant method

Secant method

3.0p 24 Describe how the secant method is used in the shooting method to improve the guess for the initial condition.

Shooting method for stiff problems

3.0p 25 Explain the drawback of the shooting method for stiff boundary value problems.

ODE classification

3.0p 26 What is the order of the following ordinary differential equation (ODE)? $\frac{d^2y}{\mathrm{d}t^2} + y^3 + t = 0$

- 3rd order
- 2nd order
- 1st order

Explicit vs. implicit

3.0p 27 Explain the difference between explicit and implicit methods in solving ordinary differential equations (ODEs) and discuss the resulting characteristics of explicit and implicit methods.

Shooting method definition

3.0p 28 Explain the basic principle of the shooting method in the context of solving boundary value problems (BVPs).