Overview of lecture topics

- Topic 1: Course introduction
- Topic 2: Introduction to frequency domain and Laplace transformation
- Topic 3: Dynamic Behavior of Linear systems
- Topic 4: Frequency Response Analysis and Bode plots
- Topic 5: Mathematical Description of Chemical Systems
- Topic 6: Nonlinear ODE’s, Linearity, Linearization Feedback, Stability, Root Locus
- Topic 7: Feedback Controller Design and Bode stability
- Topic 8: Advanced (Enhanced) Process Control

EINDHOVEN UNIVERSITY OF TECHNOLOGY

Dynamics and Control of Processes

Lecture 4: Frequency Response Analysis and Bode plots

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Course 6E8X0

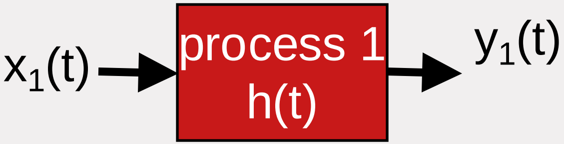
Outline Lecture 4

- Frequency Response Analysis of Typical Linear Systems

- Bode diagrams

Frequency Response Analysis

General Linear Differential Equation



$$y_1(t) = h(t) \otimes x_1(t) = \int_0^\infty h(\tau)x_1(t-\tau)d\tau$$

$$x_1(t) = A\sin\omega t$$

$\Updownarrow$ Laplace transform

$$Y_1(s) = H(s)\cdot X_1(s)$$

$$X_s(s) = \frac{A\omega}{s^2+\omega^2}$$

Frequency Response Analysis

First Order Systems:

Response to sinusoidal input :

$$u(t) = A\sin(\omega t) \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\Longleftrightarrow}} U(s) = \frac{A\omega}{s^2+\omega^2}$$

$$Y(s) = \frac{K}{(\tau s+1)}\frac{A\omega}{(s^2+\omega^2)} = \frac{c_1}{\tau s+1} + \frac{c_2}{s+j\omega} + \frac{c_3}{s-j\omega}$$

$$c_1 = (\tau s+1)Y(s)\big|_{s=\frac{-1}{\tau}} = \frac{KA\omega\tau}{(1+\omega^2\tau^2)}$$

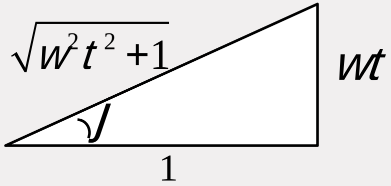
$$c_2 = (s+j\omega)Y(s)\big|_{s=-j\omega} = \frac{-KA\omega\tau}{2(\omega^2\tau^2+j\omega\tau)} \qquad c_3 = (s-j\omega)Y(s)\big|_{s=j\omega} = \frac{-KA\omega\tau}{2(\omega^2\tau^2-j\omega\tau)}$$

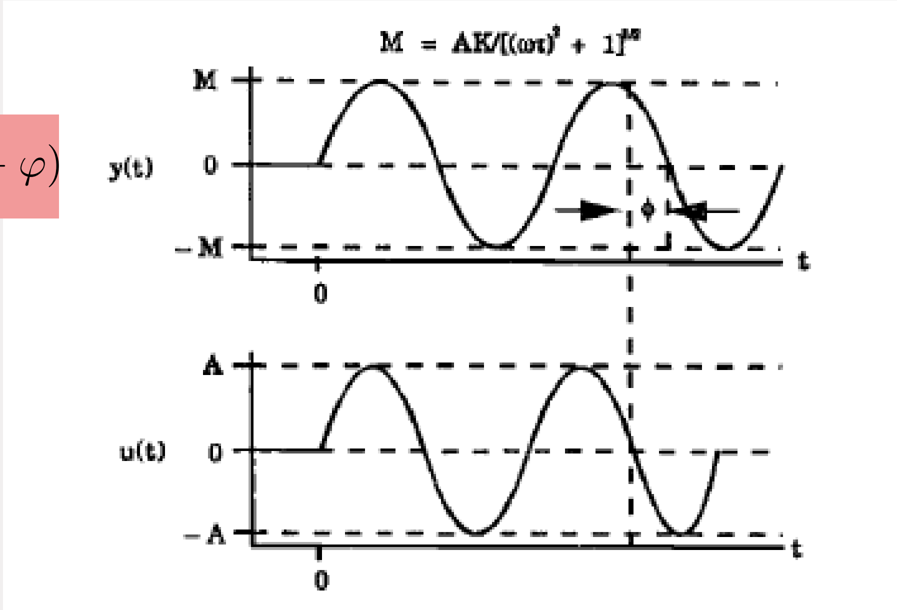
Frequency Response Analysis

First Order Systems:

Response to sinusoidal input :

$$y(t) = KA\left(\frac{\omega\tau}{\omega^2\tau^2+1}\right)e^{\frac{-t}{\tau}} + \frac{KA}{\sqrt{\omega^2\tau^2+1}}\sin\left(\omega t - \varphi\right)$$



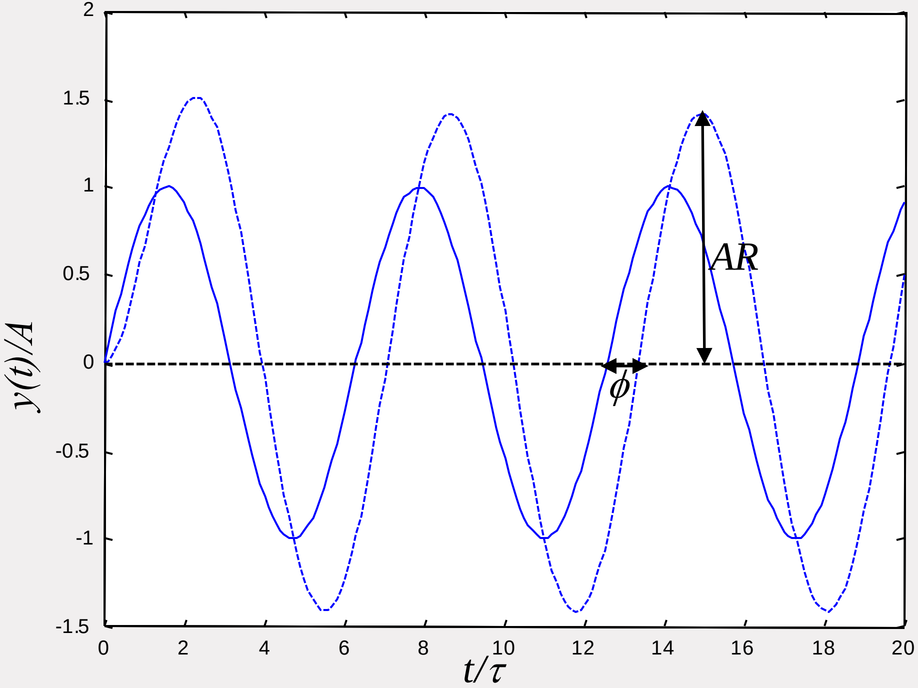


$$\lim_{t\to\infty} y(t) = \frac{KA}{\sqrt{\omega^2\tau^2+1}}\sin\left(\omega t - \varphi\right)$$

Frequency Response Analysis

First Order Systems:

$$\lim_{t\to\infty} y(t) = \frac{KA}{\sqrt{\omega^2\tau^2+1}}\sin(\omega t-\varphi)$$



Sinusoidal input A·sinωt results in sinusoidal output characterized by AR and ϕ

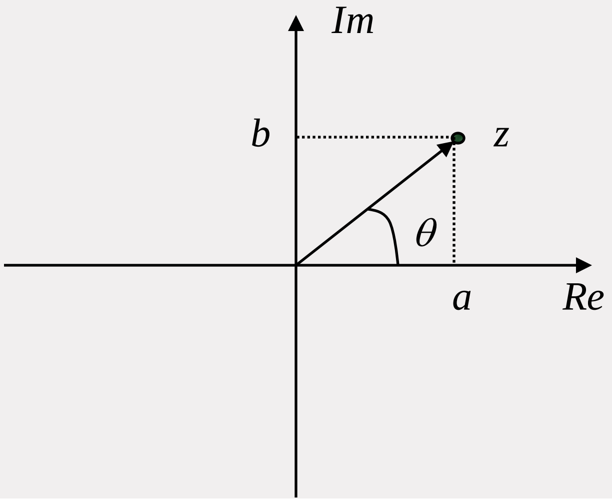
$$\mathrm{AR} = \frac{K}{\sqrt{(\omega\tau)^2+1}}, \quad \varphi = \tan^{-1}(-\omega\tau)$$

Question:

Do we have to take the inverse Laplace transform to calculate the Amplitude Response (AR) and phase shift ϕ of any process?

Recall Complex Numbers (Self Study 1)

$$z = a + bj,\ a = Re(z),\ b = Im(z)$$



$$a = |z| \cos\theta, \quad b = |z| \sin\theta$$

$$|z| = \sqrt{Re(z)^2 + Im(z)^2}$$

$$arg(z) = \theta = \tan^{-1}\left(\frac{Im(z)}{Re(z)}\right), \quad z = |z|e^{j\theta}$$

$$z_1 = a + bj, \quad z_2 = a - bj$$

$$|z_1| = |z_2|, \arg(z_1) = -arg(z_2)$$

Frequency Response Analysis

Let us consider first order system

$$H(s) = \frac{Y(s)}{U(s)} = \frac{K}{(\tau s + 1)}$$

Let us substitute $s = jw$ in $H(s)$

Complex number

$$H(j\omega) = \frac{K}{(j\omega\tau + 1)} = \frac{K}{(j\omega\tau + 1)}\frac{(1 - j\omega\tau)}{(1 - j\omega\tau)} = \frac{K}{1 + \omega^2\tau^2} - j\frac{K\omega\tau}{1 + \omega^2\tau^2}$$

$$|H(j\omega)| = \frac{K}{\sqrt{(1 + \omega^2\tau^2)}} = \mathrm{AR} \qquad \arg\left(H(j\omega)\right) = \tan^{-1}\left(\frac{\frac{-K\omega\tau}{(1+\omega^2\tau^2)}}{\frac{K}{(1+\omega^2\tau^2)}}\right) = \tan^{-1}(-\omega\tau)$$

Frequency Response Analysis

For a linear process with general transfer function

$$H(s) = \frac{(s - z_1)(s - z_2)\cdots(s - z_m)}{(s - p_1)(s - p_2)\cdots(s - p_n)} e^{\theta s}$$

We can express the frequency response of the systems for a given input frequency ω

$$H(j\omega) = \underbrace{|H(j\omega)|}_{\text{modulus}} e^{j\varphi}$$

argument

Frequency Response Analysis

Main Outcome:

- The stationary state response of any linear process to a sinusoidal input is sinusoidal.

- The amplitude ratio of the resulting signal is given by the Modulus of the transfer function model expressed in the frequency domain: $|H(j\omega)|$.

- The Phase Shift ϕ is given by the argument of the transfer function model in the frequency domain: $\arg(H(j\omega))$

Frequency Response Analysis

Take Home Message:

For any linear process we can calculate the amplitude ratio and phase shift by:

- Substitute s=jω in the transfer function H(s)
- H(jω) is a complex number. Its modulus is the amplitude ratio of the process and its argument is the phase shift.
- The effect of the frequency –ω- on the process is called the frequency response of the process

Frequency Response Analysis

Integrating Systems:

$$H(s) = \frac{K}{s} \Rightarrow \qquad H(j\omega) = \frac{K}{j\omega}\left(\frac{-j\omega}{-j\omega}\right)$$

$$AR = \frac{K}{\omega}, \quad \varphi = \tan^{-1}\left(\frac{-\frac{K}{\omega}}{0}\right) = -\frac{\pi}{2}$$

Time Delay (Deadtime):

$$H(s) = e^{-\theta s} \Rightarrow \qquad H(j\omega) = e^{-j\theta\omega}$$

$$\mathrm{AR} = 1, \quad \varphi = -\theta\omega$$

Frequency Response Analysis

Examples:

N Processes in Series

$$H(s) = H_1(s)H_2(s)\cdots H_n(s)$$

$$\begin{aligned} H(j\omega) &= H_1(j\omega)H_2(j\omega)\cdots H_n(j\omega) \\ &= |H_1(j\omega)|e^{j\varphi_1}|H_2(j\omega)|e^{j\varphi_2}\cdots|H_n(j\omega)|e^{j\varphi_n} \end{aligned}$$

$$\mathrm{AR} = |H_1(j\omega)||H_2(j\omega)|\cdots|H_n(j\omega)| = \prod_{i=1}^{n}|H_j(j\omega)|$$

$$\varphi = \arg\left(H(j\omega)\right) = \sum_{i=1}^{n}\arg\left(H_i(j\omega)\right) = \sum_{i=1}^{n}\varphi_i$$

Frequency Response Analysis

Examples:

N First Order Processes in Series

$$H(s) = \frac{K_1}{(\tau_1 s + 1)} \frac{K_2}{(\tau_2 s + 1)} \cdots \frac{K_n}{(\tau_n s + 1)}$$

$$\mathrm{AR} = |H_1(j\omega)||H_2(j\omega)| \cdots |H_n(j\omega)| = \prod_{i=1}^{n} |H_j(j\omega)|$$

$$\varphi = \arg\left(H(j\omega)\right) = \sum_{i=1}^{n} \arg\left(H_i(j\omega)\right) = \sum_{i=1}^{n} \varphi_i$$

$$\mathrm{AR} = \frac{K_1}{\sqrt{(\omega^2\tau_1^2 + 1)}} \frac{K_2}{\sqrt{(\omega^2\tau_2^2 + 1)}} \cdots \frac{K_n}{\sqrt{(\omega^2\tau_n^2 + 1)}}$$

$$\begin{aligned}\varphi = & -\tan^{-1}(\omega\tau_1) - \tan^{-1}(\omega\tau_2) \cdots \\ & - \tan^{-1}(\omega\tau_n)\end{aligned}$$

Frequency Response Analysis

Examples:

First Order Process with Deadtime

$$H(s) = \frac{K}{(\tau s + 1)} e^{-\theta s} \qquad H_1(s) = \frac{K}{(\tau s + 1)}, \quad H_2(s) = e^{-\theta s}$$

$$\mathrm{AR} = |H_1(j\omega)||H_2(j\omega)|$$

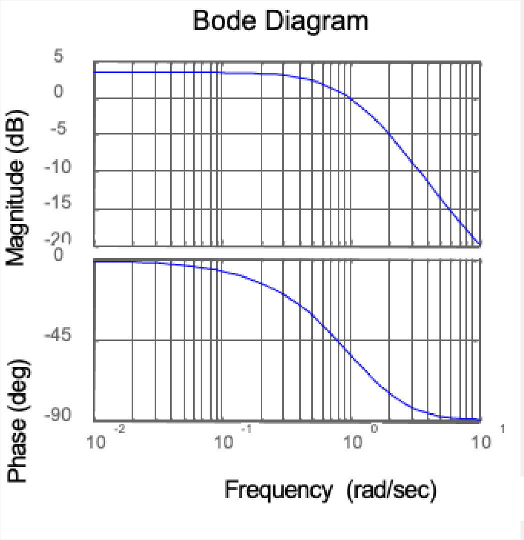
$$\varphi = \arg\left(H(j\omega)\right) = \sum_{i=1}^{2} \arg\left(H_i(j\omega)\right) = \sum_{i}^{2} \varphi_i$$

$$\begin{aligned}\mathrm{AR} &= |H_1(j\omega)||H_2(j\omega)| \\ &= \frac{K}{\sqrt{(\omega^2\tau^2 + 1)}} \cdot 1\end{aligned}$$

$$= -\tan^{-1}(\omega\tau_1) - \theta\omega$$

Frequency Response Analysis

To study the effect of ω on the frequency response analysis we use graphical representations: Bode Diagrams



Bode diagrams

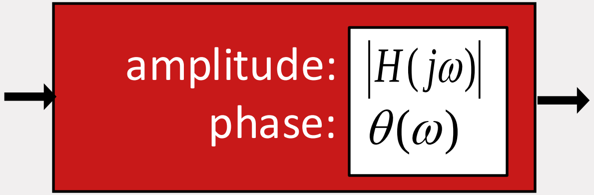
$$Y(s) = H(s)X(s) \quad \text{with: } s = j\omega$$

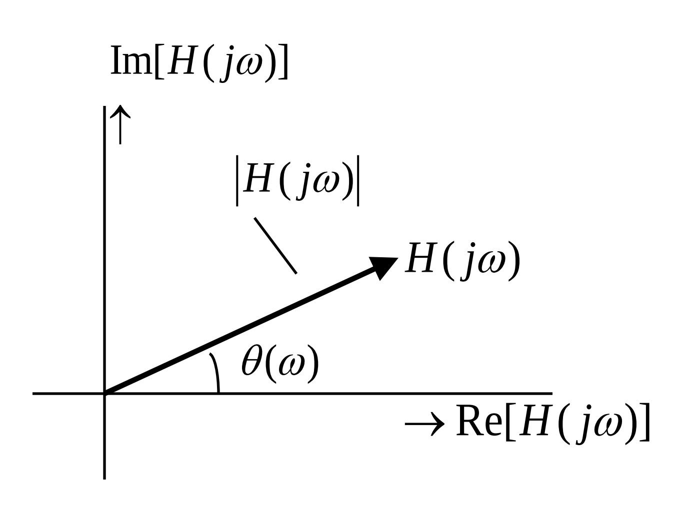
$$Y(j\omega) = H(j\omega)X(j\omega)$$

$$H(j\omega) = \mathrm{Re}[H(j\omega)] + \mathrm{Im}[H(j\omega)]$$

in polar coordinates:

$$H(j\omega) = |H(j\omega)|e^{j\theta(\omega)}$$





Bode diagrams

$$\sin \omega t \longrightarrow \boxed{\left|H(j\omega)\right| e^{j\theta(\omega)}} \longrightarrow \underbrace{\left|H(j\omega)\right|}_{\text{amplitude}} \sin(\omega t + \underbrace{\theta(\omega)}_{\text{phase shift}})$$

Bode Diagram:

graphical representation of H(jω)

- plot 1: amplitude against ω
- plot 2: phase against ω

Bode diagrams

Example: $H(s) = \frac{10}{s+10}$ $x(t) = \sin(\omega_0 t)$

$$Y(s) = H(s)X(s) = \frac{10}{s+10}.\frac{\omega_0}{s^2+\omega_0^2} = \frac{10\omega_0}{(s+10)(s+j\omega_0)(s-j\omega_0)} = \frac{A}{s+10} + \frac{B}{s+j\omega_0} + \frac{C}{s-j\omega_0}$$

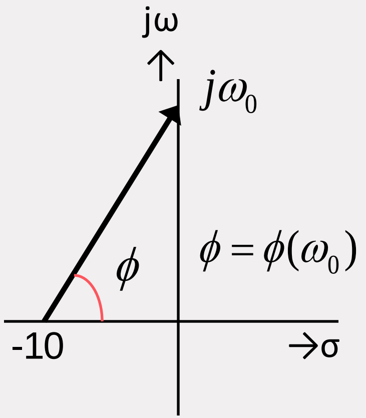
$$A = (s+10)Y(s)\Big|_{s=-10} = \frac{10\omega_0}{100+\omega_0^2}$$

$$B = (s+j\omega_0)Y(s)\Big|_{s=-j\omega_0} = \frac{10\omega_0}{10-j\omega_0}.\frac{1}{-2j\omega_0} = \frac{10}{100+\omega_0^2}.\frac{10+j\omega_0}{-2j} = \frac{10}{100+\omega_0^2}.\frac{\sqrt{100+\omega_0^2}}{-2j}.e^{j\phi}$$

$$C = (s-j\omega_0)Y(s)\Big|_{s=j\omega_0} = \frac{10\omega_0}{10+j\omega_0}.\frac{1}{2j\omega_0} = \frac{10}{100+\omega_0^2}.\frac{10-j\omega_0}{2j} = \frac{10}{100+\omega_0^2}.\frac{\sqrt{100+\omega_0^2}}{2j}.e^{-j\phi}$$

Bode diagrams

from: $Y(s)=\dfrac{A}{s+10}+\dfrac{B}{s+j\omega_0}+\dfrac{C}{s-j\omega_0}$



$$y(t)=\frac{10}{100+\omega_0^2}e^{-10t}-\frac{10}{\sqrt{100+\omega_0^2}}\frac{e^{j\phi}}{2j}e^{-j\omega_0 t}+\frac{10}{\sqrt{100+\omega_0^2}}\frac{e^{-j\phi}}{2j}e^{j\omega_0 t}$$

we are interested in the stationary state response, in this case the sinusoidal term:

$$y_{ss}(t)=-\frac{10}{\sqrt{100+\omega_0^2}}\frac{e^{j\phi}}{2j}e^{-j\omega_0 t}+\frac{10}{\sqrt{100+\omega_0^2}}\frac{e^{-j\phi}}{2j}e^{j\omega_0 t}=\frac{10}{\sqrt{100+\omega_0^2}}\cdot\frac{e^{j(\omega_0 t-\phi))}-e^{-j(\omega_0 t-\phi))}}{2j}=$$

$$=\frac{10}{\sqrt{100+\omega_0^2}}\sin\left(\omega_0 t-\phi\right)=\left|H(j\omega_0)\right|\sin\left(\omega_0 t-\phi\right)$$

Bode diagrams

plot 1: amplitude against frequency

hor. axis: *logarithmic scale*

$\log_{10} \omega$ *rad/sec*

vert. axis: *linear scale*

$20\log_{10}|H(j\omega)|$ *dB*

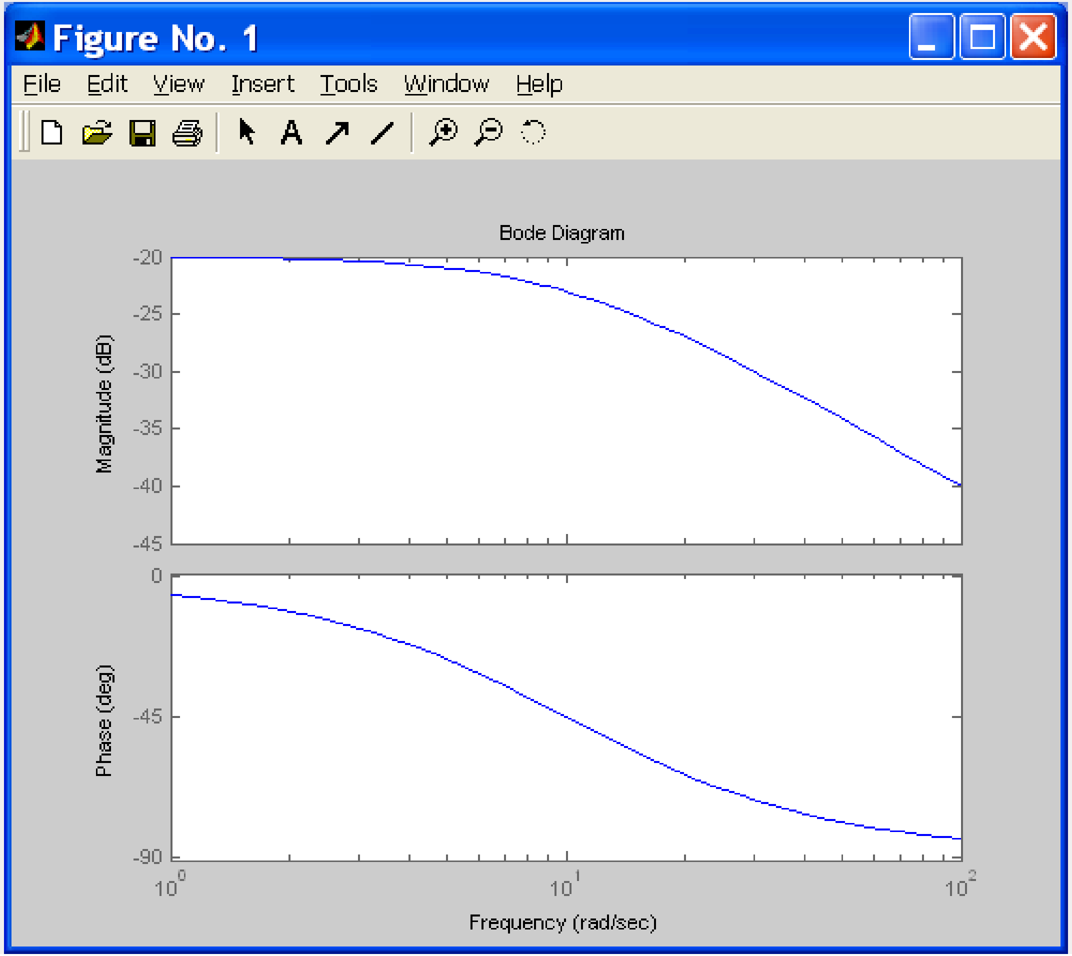
plot 2: phase against frequency

hor. axis: *logarithmic scale*

$\log_{10} \omega$ *rad/sec*

vert. axis: *linear scale*

$\theta(\omega)$ *radians or degrees*

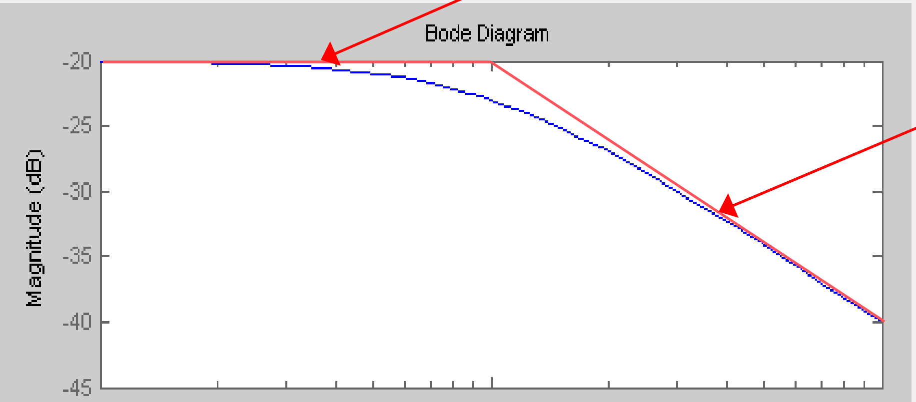


Bode diagrams

Example: $H_4(j\omega) = \dfrac{1}{j\omega + 10} \Rightarrow \left|H_4(j\omega)\right|_{dB} = 20\log_{10}\dfrac{1}{\sqrt{\omega^2+10^2}} = -20\log_{10}\sqrt{\omega^2+10^2}$

$$\left|H_4(j\omega)\right|_{dB} \approx -20\log_{10}\sqrt{10^2} = -20$$ *slope = 0 dB/dec*

$$\left|H_4(j\omega)\right|_{dB} = 20\log_{10}\frac{1}{\sqrt{\omega^2+10^2}} \approx -20\log_{10}\sqrt{\omega^2} = -20\log_{10}\omega$$ *slope = -20 dB/dec*



Bode diagrams

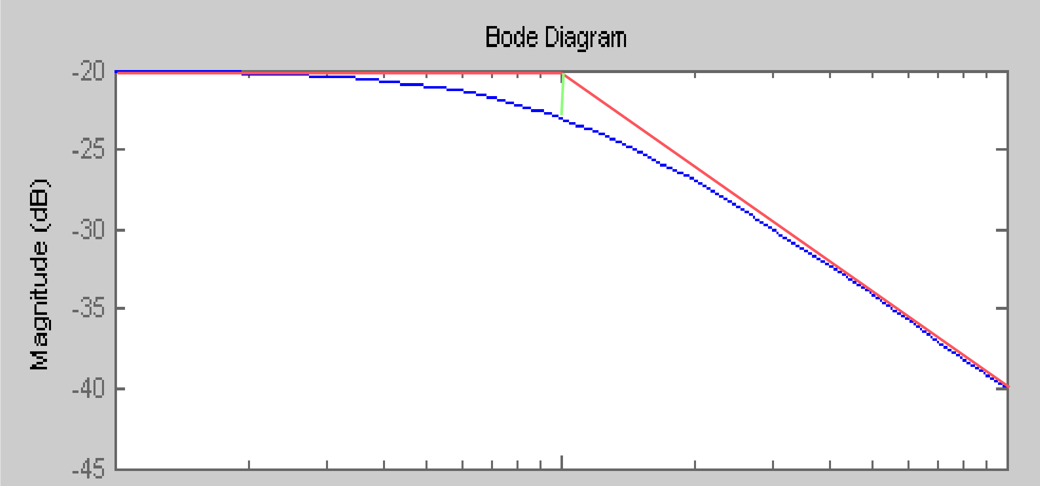
Example: $H_4(j\omega) = \dfrac{1}{j\omega + 10} \quad \Rightarrow \quad |H_4(j\omega)|_{dB} = 20\log_{10}\dfrac{1}{\sqrt{\omega^2 + 10^2}}$

for ω = 10:

$$|H_4(j10)|_{dB} = 20\log_{10}\frac{1}{\sqrt{10^2+10^2}} = -20\log_{10}\sqrt{200} =$$

$$= -20\log_{10}10 - 20\log_{10}\sqrt{2} = -20 - 3 = -23dB$$

difference between true (blue) value and approximated (red) value of |H(j10)| is 3 dB this is maximum deviation between true and approximated plots



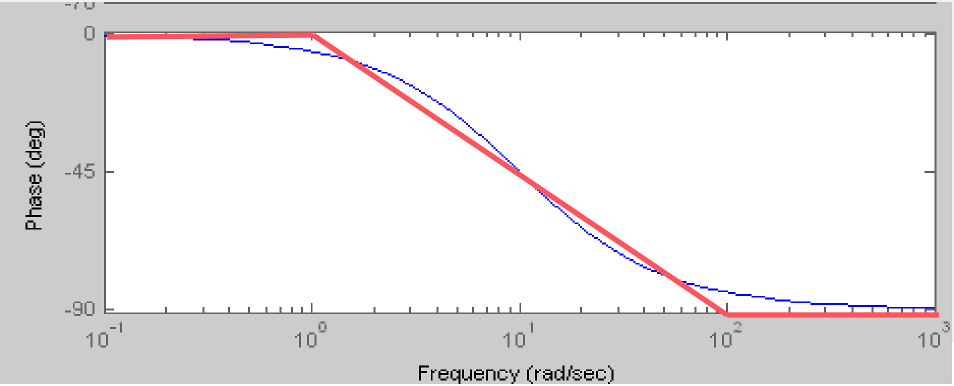
Bode diagrams

$$H_4(j\omega) = \frac{1}{j\omega + 10}$$

<u>*ω «10:*</u> $H_4(j\omega) \approx \frac{1}{10}$ $\arg\{H_4(j\omega)\} = 0^\circ$

<u>*ω »10:*</u> $H_4(j\omega) \approx \frac{1}{j\omega} = -\frac{j}{\omega}$ $\arg\{H_4(j\omega)\} = -90^\circ$

<u>*ω =10:*</u> $H_4(j10) = \frac{1}{j10 + 10} = \frac{1}{20}(1 - j)$ $\arg\{H_4(j\omega)\} = -45^\circ$



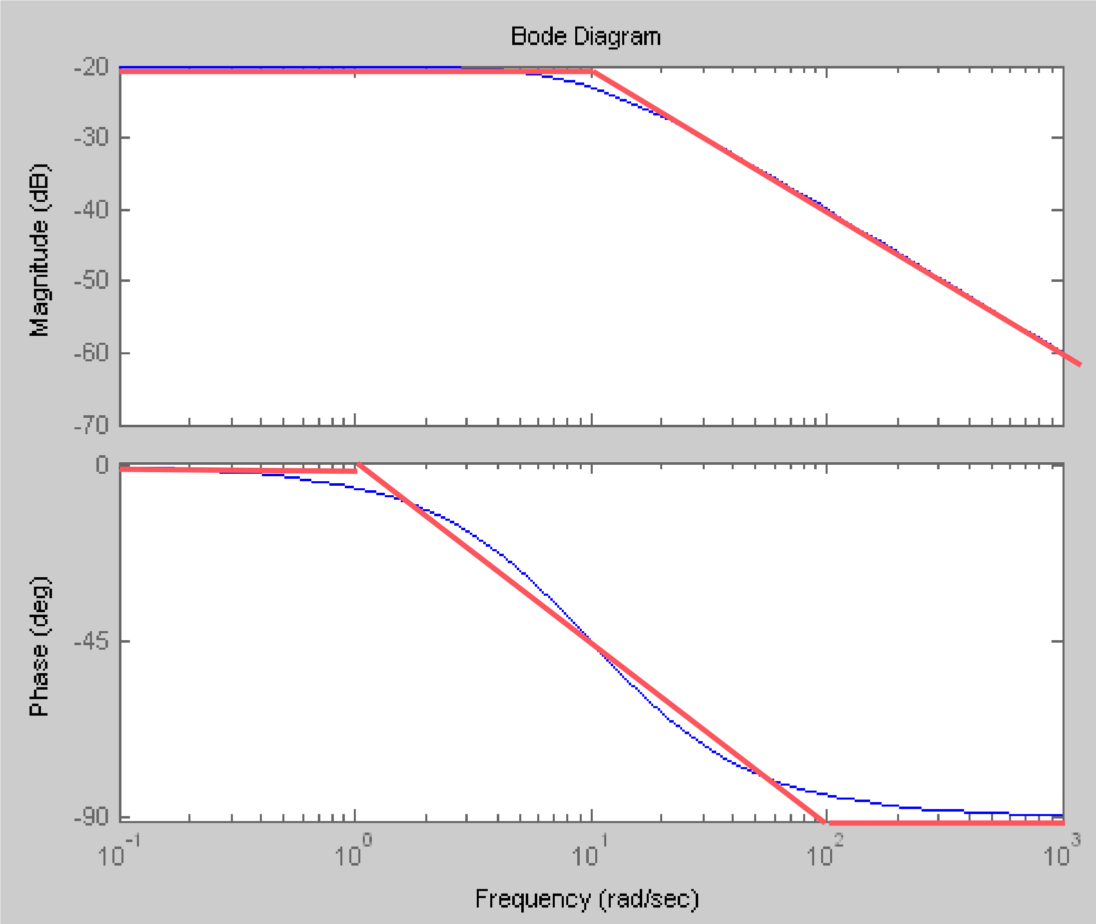
TU/e

Bode diagrams

$$H_4(j\omega) = \frac{1}{j\omega + 10}$$

Total diagram

- cross over frequency: ω=10
- amplitude diagram:
  - slope = -20 dB/dec
- phase diagram:
  - phase shift 0° → -90°



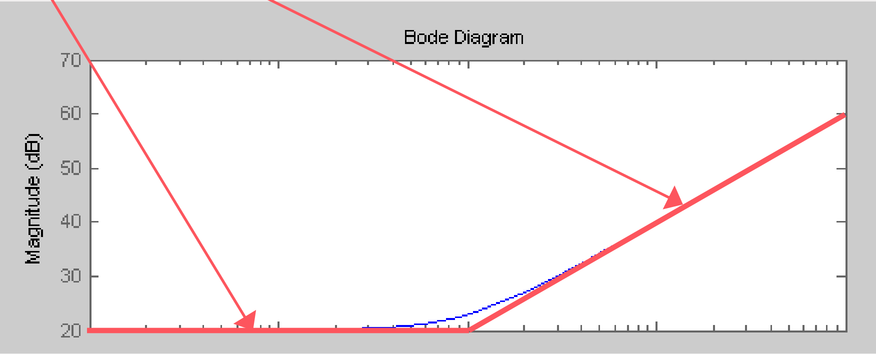
Bode diagrams

Example $H_5(j\omega) = j\omega + 10 \;\Rightarrow\; |H_5(j\omega)|_{dB} = 20\log_{10}\sqrt{\omega^2 + 10^2}$

ω «10: $|H_5(j\omega)|_{dB} \approx 20\log_{10}\sqrt{10^2} = 20$ slope = 0 dB/dec

ω »10: $|H_5(j\omega)|_{dB} \approx 20\log_{10}\sqrt{\omega^2} = 20\log_{10}\omega$ slope = 20 dB/dec

ω =10: $|H_5(j\omega)|_{dB} = 20\log_{10}\sqrt{10^2 + 10^2} = 20\log_{10}\sqrt{200} = 23dB$



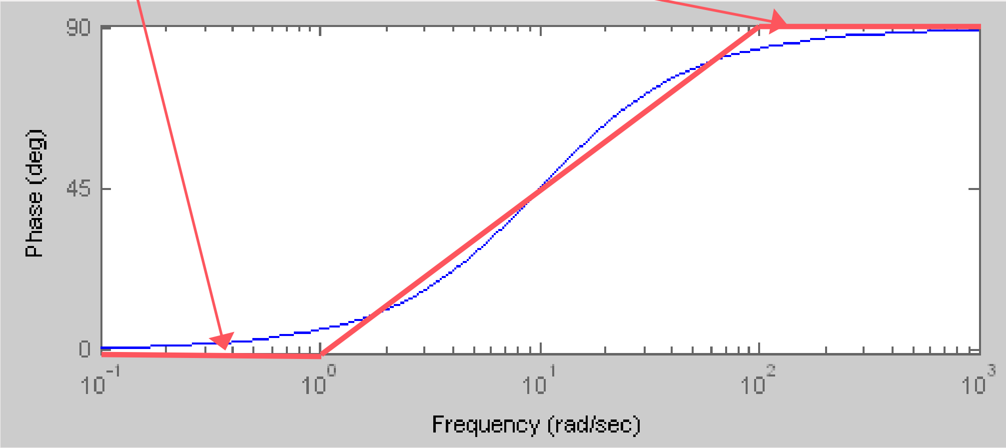
Bode diagrams

example $H_5(j\omega) = j\omega + 10$

ω «10: $H_5(j\omega) \approx 10$ $\arg\{H_5(j\omega)\} = 0^\circ$

ω »10: $H_5(j\omega) \approx j\omega$ $\arg\{H_5(j\omega)\} = 90^\circ$

ω =10: $H_5(j\omega) = j10 + 10$ $\arg\{H_5(j\omega)\} = 45^\circ$



Bode diagrams

example

$$H_5(j\omega) = j\omega + 10$$

Total diagram

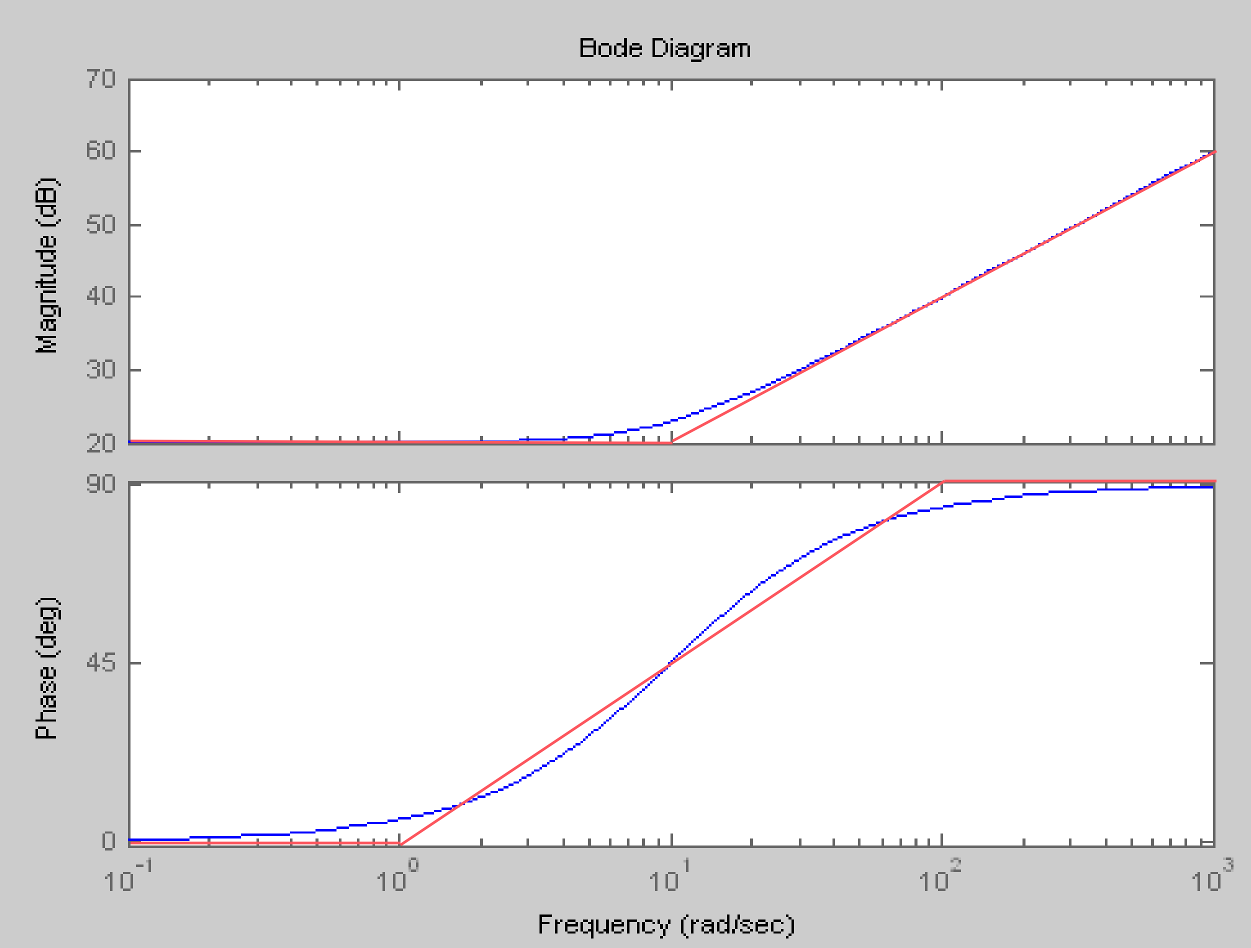
- cross over frequency: ω=10

- amplitude diagram:

slope = 20 dB/dec

-phase diagram:

phase shift $0^o \rightarrow 90^o$



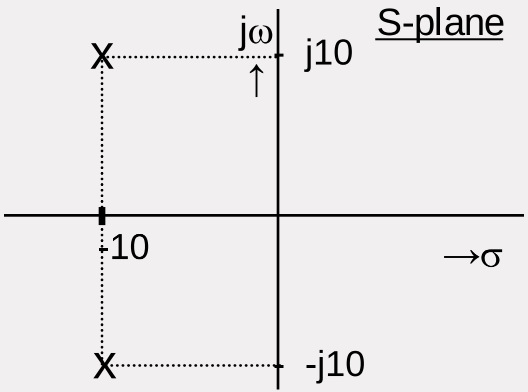
Bode diagrams

example

$$H_6(s) = \frac{200}{s^2 + 20s + 200} = \frac{200}{(s+10-j10)(s+10+j10)}$$

2 complex poles in $s_{1,2}$= -10 ± j10

$$H_6(j\omega) = \frac{200}{200 - \omega^2 + j20\omega}$$



Bode diagrams

example

$$H_6(j\omega) = \frac{200}{200 - \omega^2 + j20\omega}$$

ω small: $H_6(j\omega) \approx \frac{200}{200} = 1$ $\quad 20\log\left|H_6(j\omega)\right| \approx 0dB$ $\quad \arg\{H_6(j\omega)\} = 0^\circ$

ω large: $20\log\left|H_6(j\omega)\right| \approx 20\log 200 - 40\log\omega$ $\quad \arg\{H_6(j\omega)\} = -180^\circ$

for $\omega^2 = 200$ $\quad \omega = 10\sqrt{2}$ $\quad H_6(j\omega) = \frac{1}{j\sqrt{2}} = -\frac{j}{\sqrt{2}}$ $\quad \arg\{H_6(j\omega)\} = -90^\circ$

$$20\log|H_6(j\omega)|\Big|_{\omega^2=200} = 20log\frac{200}{20\sqrt{200}} = 20\log\frac{200}{200\sqrt{2}} = -20log\sqrt{2}$$

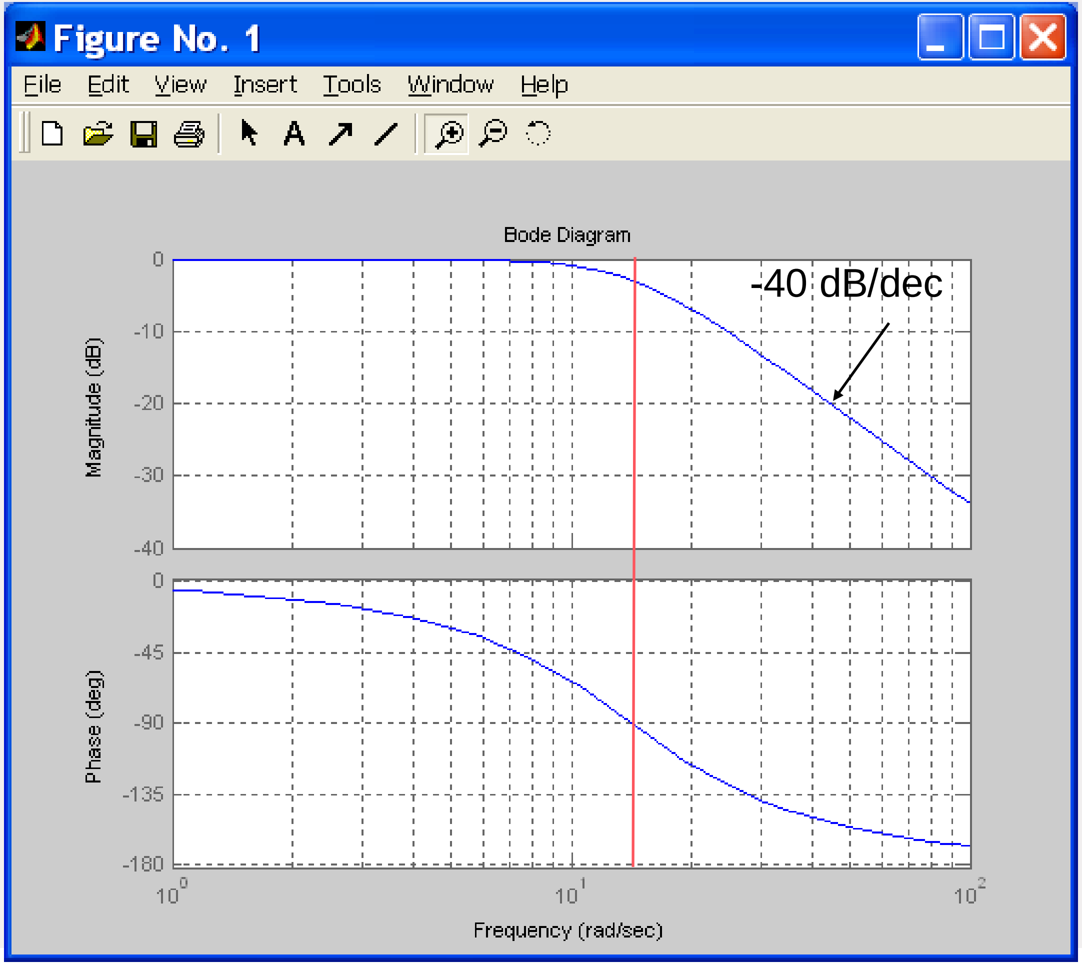
Bode diagrams

example 6

$$H_6(j\omega) = \frac{200}{200 - \omega^2 + j20\omega}$$

poles:

$S_{1,2}$=-10 ± j10



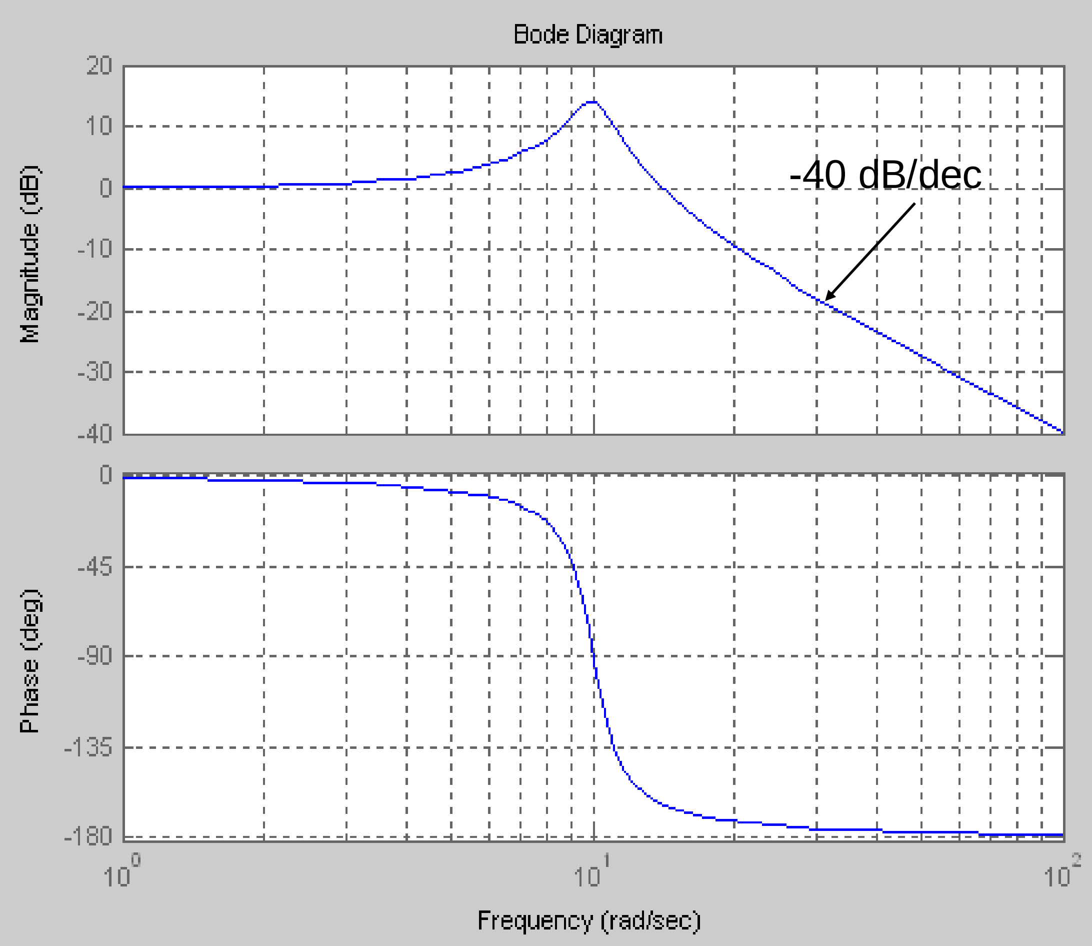
Bode diagrams

Example:

$$H_7(s) = \frac{101}{(s+1-j10)(s+1+j10)}$$

2 poles: $S_{1,2} = -1 \pm j10$

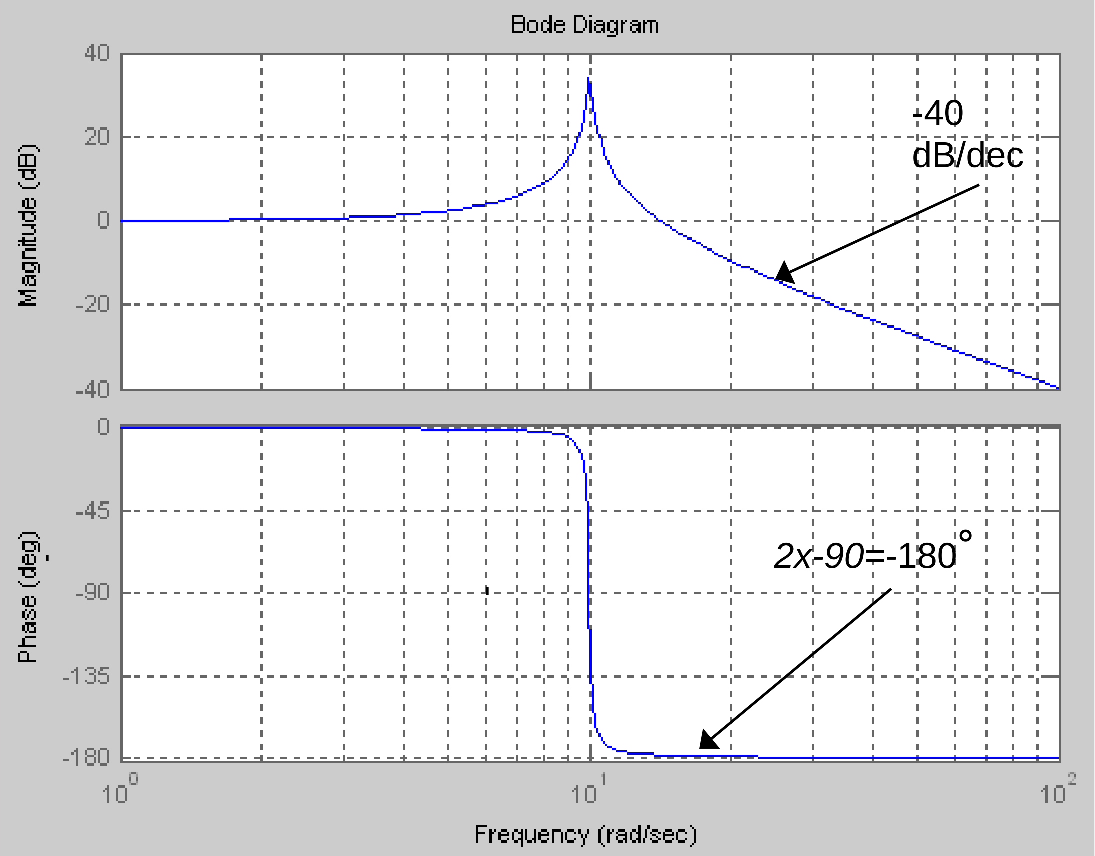
$$H_7(j\omega) = \frac{101}{101-\omega^2 + j2\omega}$$



Bode diagrams

Example:

$$H_8(s) = \frac{100}{(s+0.1-j10)(s+0.1+j10)}$$



Bode diagrams

Transfer function H(s) $H(s) = K \dfrac{(s+z_1)(s+z_2)\dots(s+z_M)}{(s+p_1)(s+p_2)\dots(s+p_N)}$

N poles in $-p_i$ and M zeros in $-z_i$ with $N \geq M$

rewrite H(s) as:

$$H(s) = \tilde{K} H_1(s) H_2(s) \dots . H_{N+M}(s)$$

with $H_i(s) = \dfrac{(s+z_i)}{z_i}$ and $H_i(s) = \dfrac{p_i}{s+p_i}$

then: $|H(s)| = \tilde{K} |H_1(s)| |H_2(s)| \dots . |H_{N+M}(s)|$

hence

$$|H(s)|_{dB} = 20\log_{10} \tilde{K} + 20\log_{10}|H_1(s)| + 20\log_{10}|H_2(s)| + \dots + 20\log_{10}|H_{N+M}(s)|$$

Bode diagrams

Transfer function H(s)

$$H(s) = K\frac{(s+z_1)(s+z_2)...(s+z_M)}{(s+p_1)(s+p_2)...(s+p_N)}$$

$$H(s) = KH_1(s)H_2(s)....H_{N+M}(s)$$

with $H_i(s) = (s+z_i)$ or $H_i(s) = \dfrac{1}{s+p_i}$

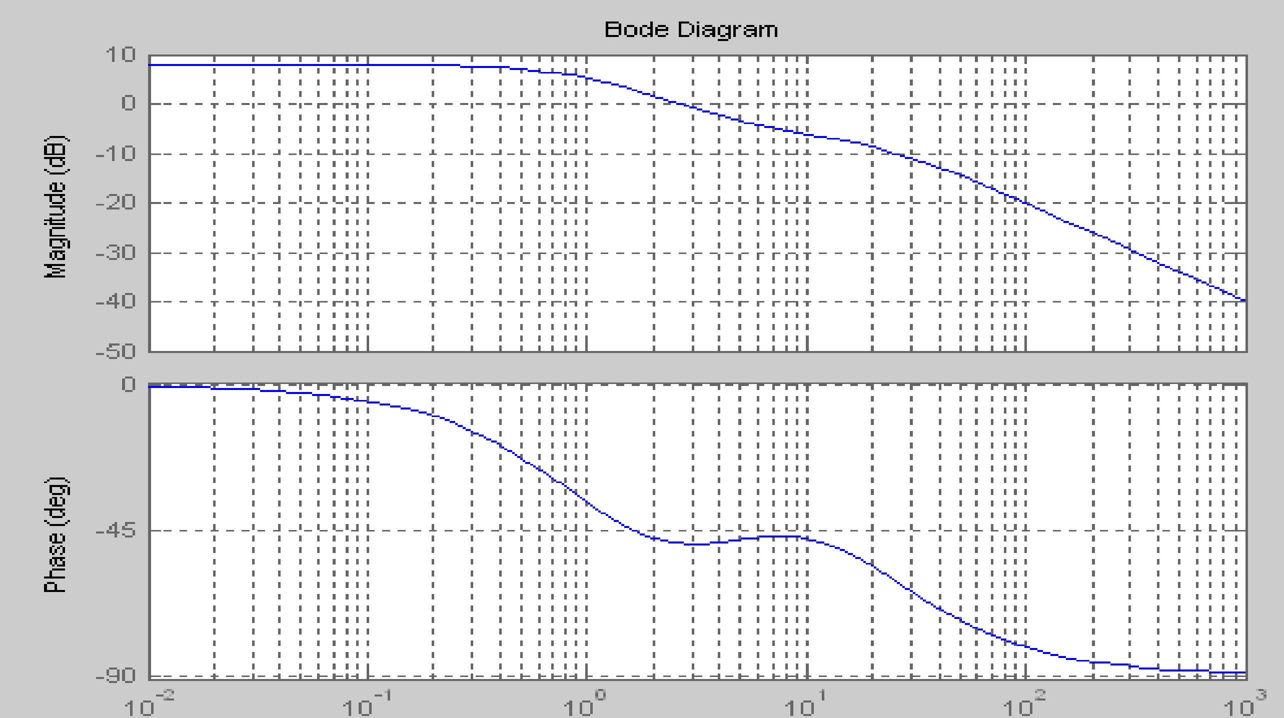
then: $\arg H(s) = \arg(K) + \arg H_1(s) + \arg H_2(s) + ... + \arg H_{N+M}(s)$

Conclusion:

- split H(s) into elementary transfers $H_i(s)$
- Bode amplitude diagram is sum of Bode diagrams of $H_i(s)$
- Bode phase diagram is sum of phase diagrams of $H_i(s)$

Bode diagrams

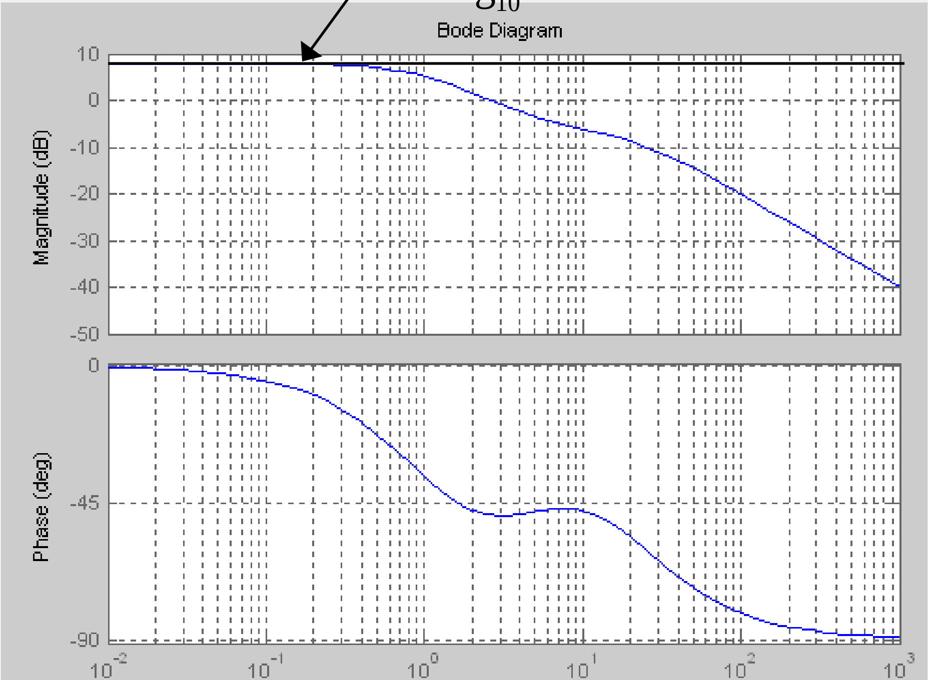
Example: $H(s) = 10\dfrac{s+5}{(s+1)(s+20)} = \dfrac{10}{4}\cdot\dfrac{1}{s+1}\cdot\dfrac{(s+5)}{5}\cdot\dfrac{20}{s+20}$



Bode diagrams

$$H(s)=10\frac{s+5}{(s+1)(s+20)}=\frac{10}{4}\cdot\frac{1}{s+1}\cdot\frac{(s+5)}{5}\cdot\frac{20}{s+20}$$

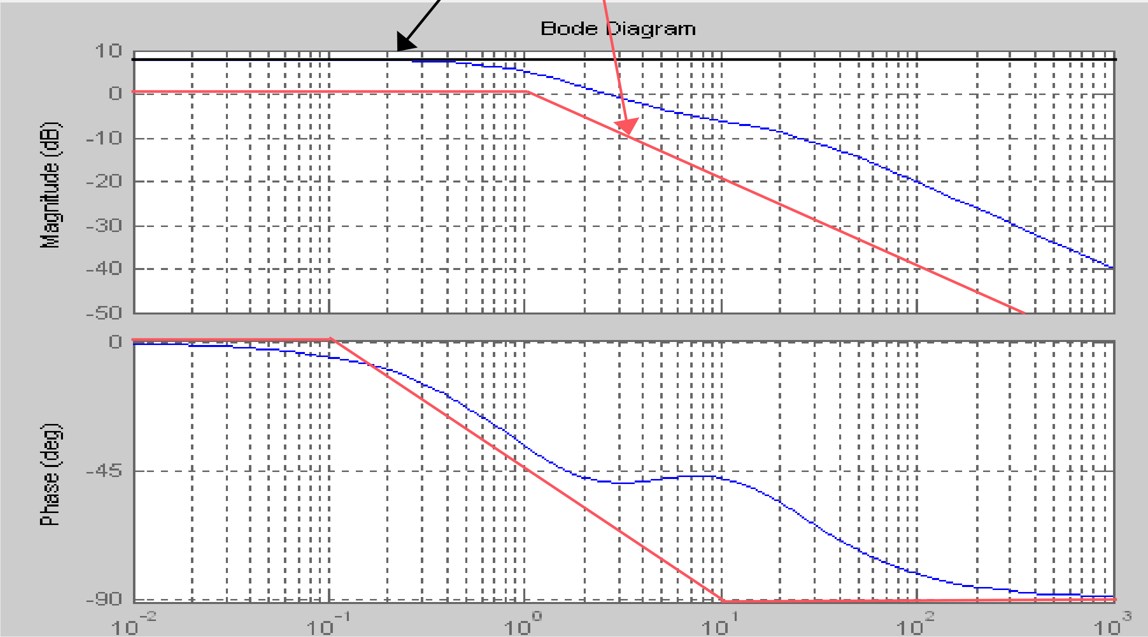
$$20\log_{10}2.5=7.96$$



Bode diagrams

Example

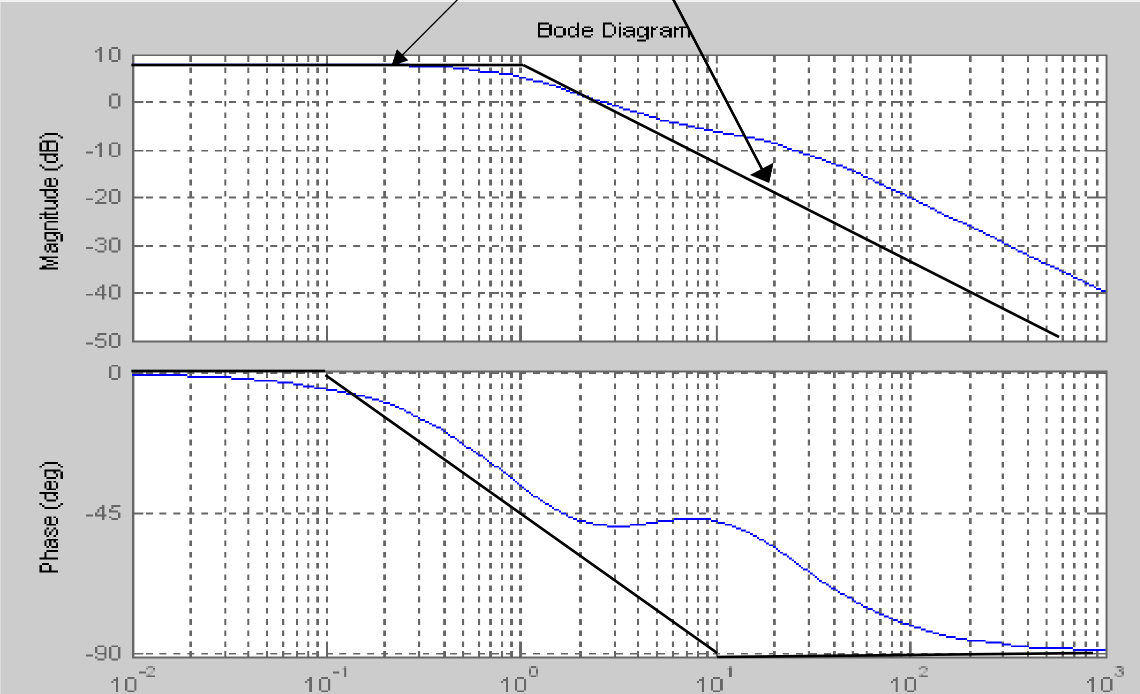
$$H(s) = 10\frac{s+5}{(s+1)(s+20)} = \frac{10}{\underline{4}} \cdot \frac{1}{s+1} \cdot \frac{(s+5)}{5} \cdot \frac{20}{s+20}$$



Bode diagrams

Example 10

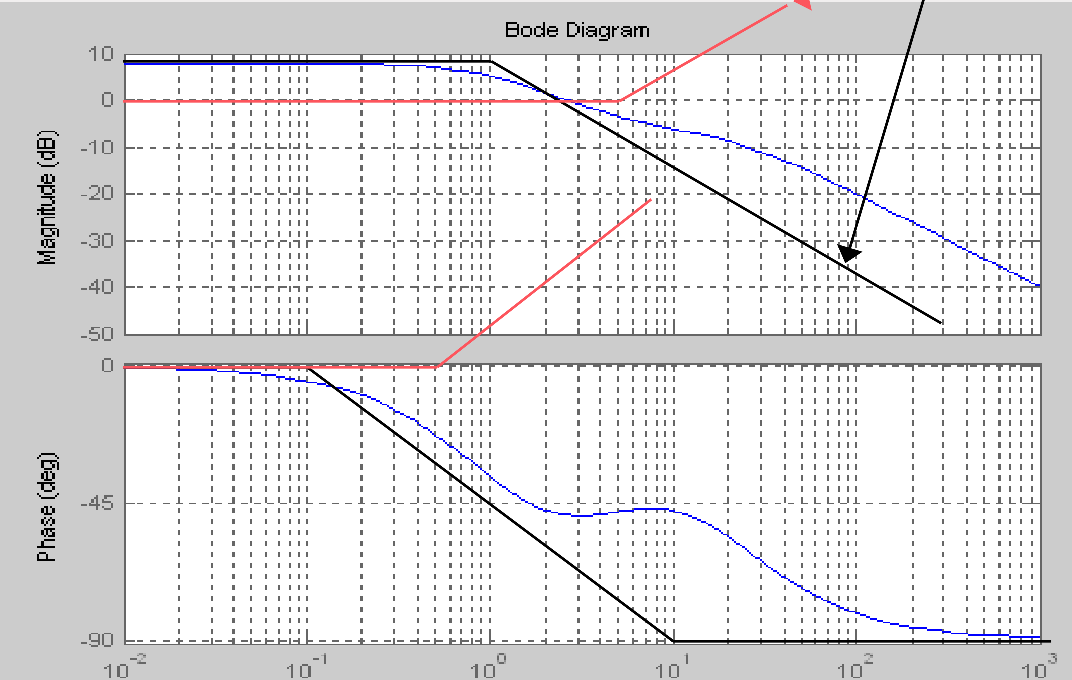
$$H(s) = 10\frac{s+5}{(s+1)(s+20)} = \frac{10}{4}\cdot\frac{1}{s+1}\cdot\frac{(s+5)}{5}\cdot\frac{20}{s+20}$$



Bode diagrams

Example 10

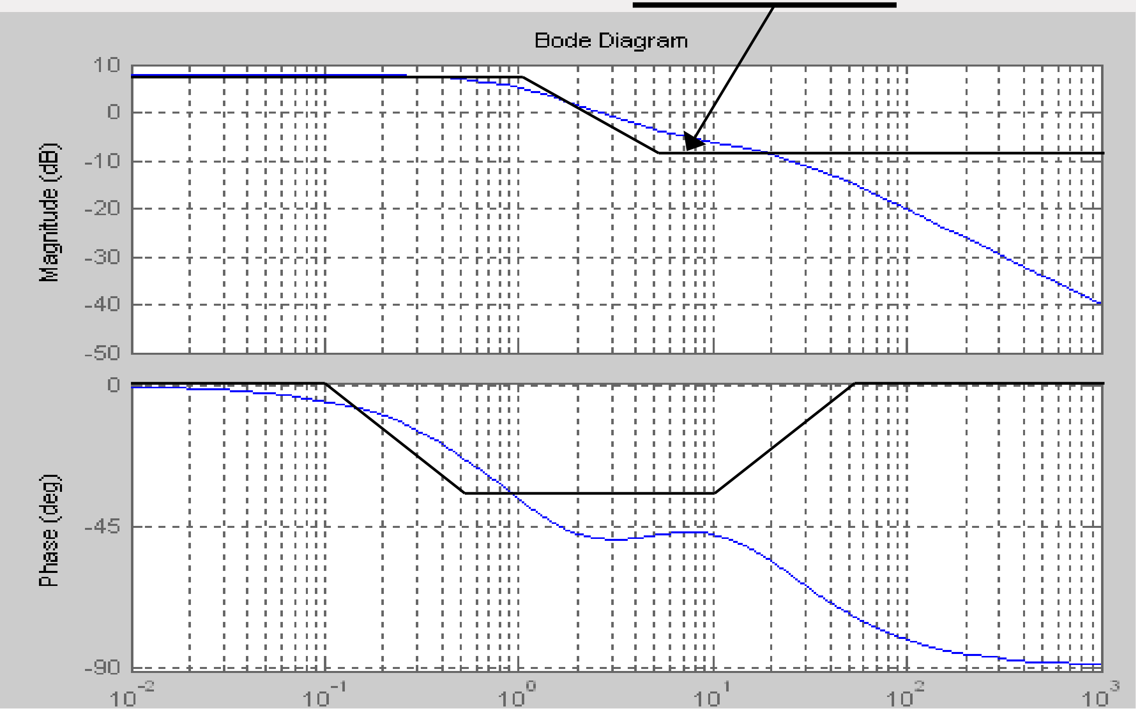
$$H(s) = 10\frac{s+5}{(s+1)(s+20)} = \underbrace{\frac{10}{4}\cdot\frac{1}{s+1}}\cdot\underbrace{\frac{(s+5)}{5}}\cdot\frac{20}{s+20}$$



Bode diagrams

Example 10

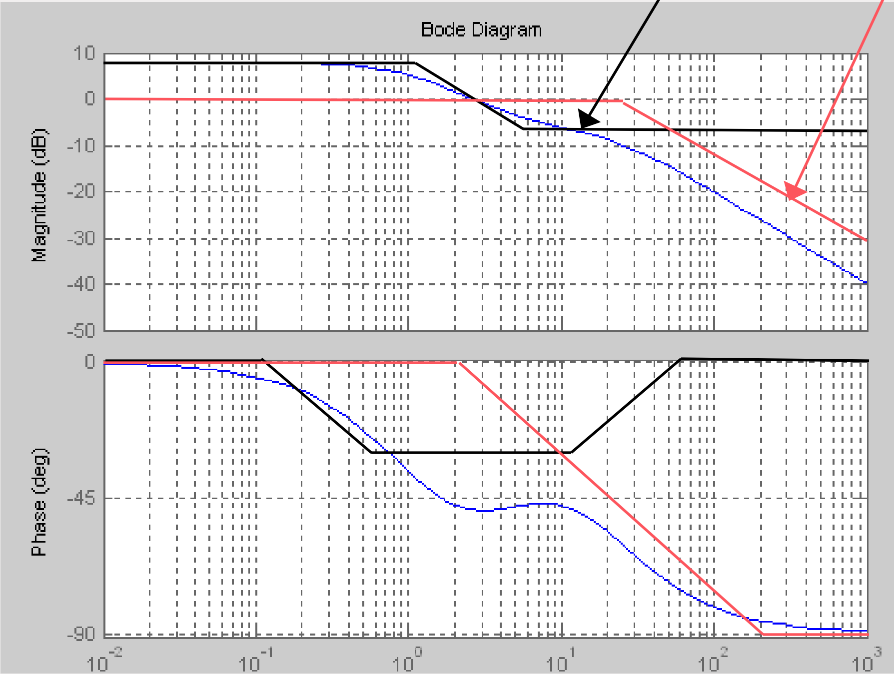
$$H(s) = 10\frac{s+5}{(s+1)(s+20)} = \frac{10}{4}\cdot\frac{1}{s+1}\cdot\underline{\frac{(s+5)}{5}}\cdot\frac{20}{s+20}$$



Bode diagrams

Example 10

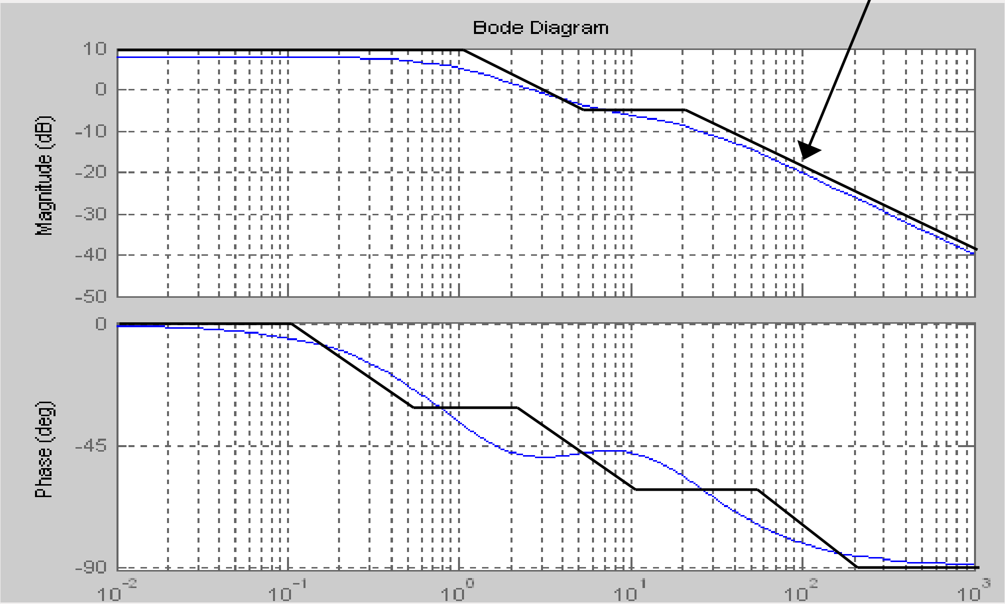
$$H(s) = 10\frac{s+5}{(s+1)(s+20)} = \frac{10}{4}\cdot\frac{1}{s+1}\cdot\frac{(s+5)}{5}\cdot\frac{20}{s+20}$$



Bode diagrams

Example

$$H(s) = 10\frac{s+5}{(s+1)(s+20)} = \frac{10}{4}\cdot\frac{1}{s+1}\cdot\frac{(s+5)}{5}\cdot\frac{20}{s+20}$$



Bode diagrams

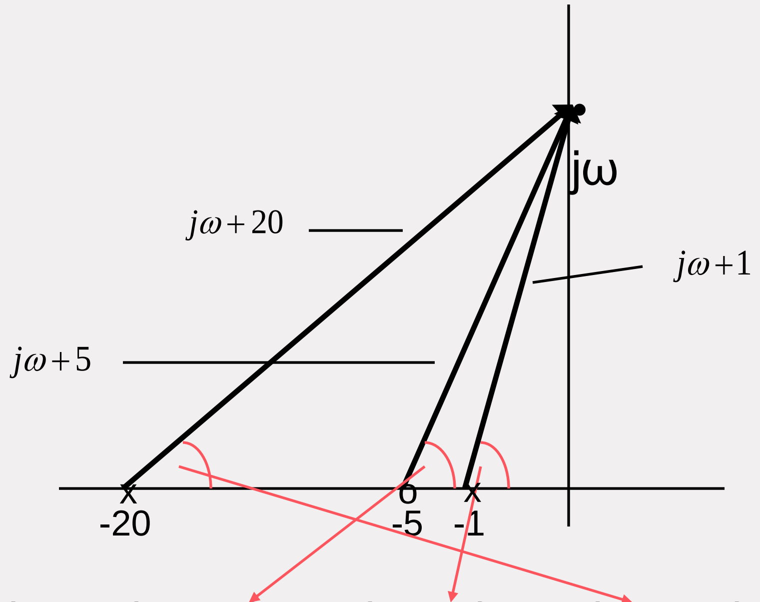
Summary:

- a pole gives a slope in the Bode diagram of – 20 dB/dec for high frequencies
- a zero gives a slope of + 20 dB/dec for high frequencies
- a process with N poles and M zeroes has consequently a slope of $(N - M) \cdot -20$ dB/dec for high frequencies
- if N > M then the resulting slope for high frequencies is negative; this means that high frequencies will be attenuated by the process
- if M > N then the resulting slope for high frequencies is positive; this means that very high frequencies will be extremely amplified by the process. This does not happen for physical processes

Bode diagrams

$$H(j\omega) = 10\frac{j\omega+5}{(j\omega+1)(j\omega+20)}$$

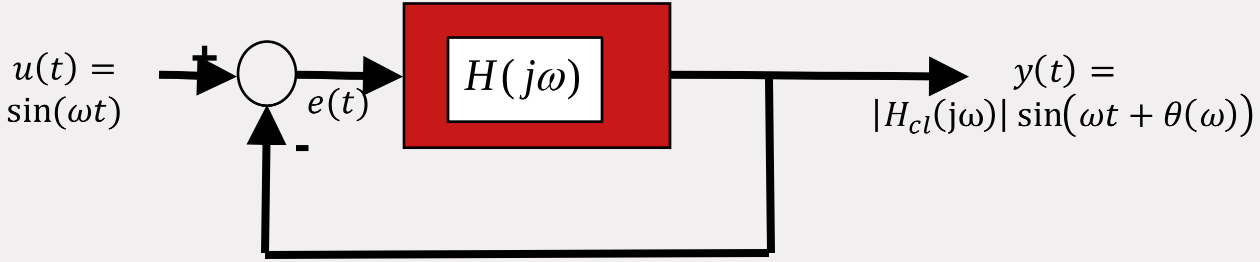
$$|H(j\omega)| = 10\frac{|j\omega+5|}{|j\omega+1||j\omega+20|}$$



$$\arg\{H(j\omega)\} = \arg\{j\omega+5\} - \arg\{j\omega+1\} - \arg\{j\omega+20\}$$

Bode diagrams

Stability



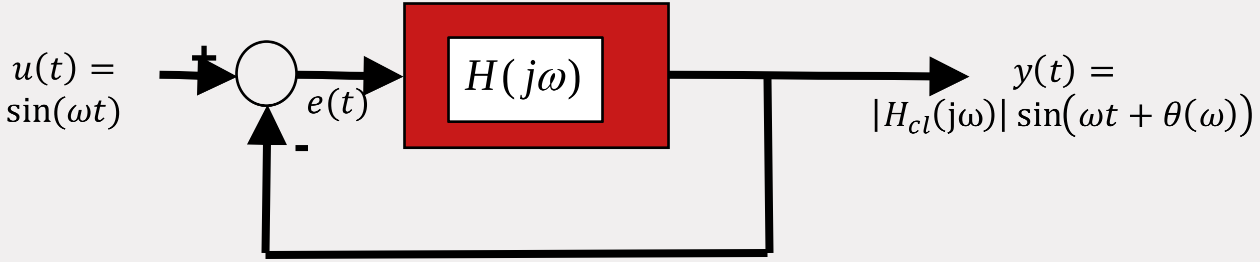
Assume for a certain frequency ω $\theta(\omega) = \pm\pi$ $\mathrm{AR} = |H(j\omega)| = 1$

$$Y(j\omega) = H(j\omega)E(j\omega) \quad \Rightarrow \quad y(t) = |H(j\omega)|\sin(\omega t \pm \pi) = -\sin(\omega t)$$

$$y(t) = |H_{cl}(j\omega)|\sin(\omega t + \vartheta(\omega)) = \sin(\omega t + \pi) = -\sin(\omega t)$$

Bode diagrams

Stability



if for a certain frequency ω: phase shift θ(ω)=±π and gain |H(jω)|=1

then the output is:

$$Y(j\omega) = H(j\omega)E(j\omega) \quad \Rightarrow \quad y(t) = |H(j\omega)| \sin(\omega t \pm \pi) = -\sin(\omega t)$$

$$y(t) = |H_{cl}(j\omega)| \sin\big(\omega t + \vartheta(\omega)\big) = \sin(\omega t + \pi) = -\sin(\omega t)$$

and the error signal e(t) will increase → instability

Bode diagrams

Stability

If for a certain frequency:

the phase shift of a process $H(j\omega)$ is ±180° and the gain is ≥1 (=0 dB)

then the closed-loop process will become unstable

we can use the Bode diagrams to check this:

use: margin(sys)

phase margin: how much phase shift is left (to 180°) for that frequency where the gain is 0 dB

gain margin: how much gain is left (to 0 dB) for that frequency where the phase shift is 180°

Bode diagrams

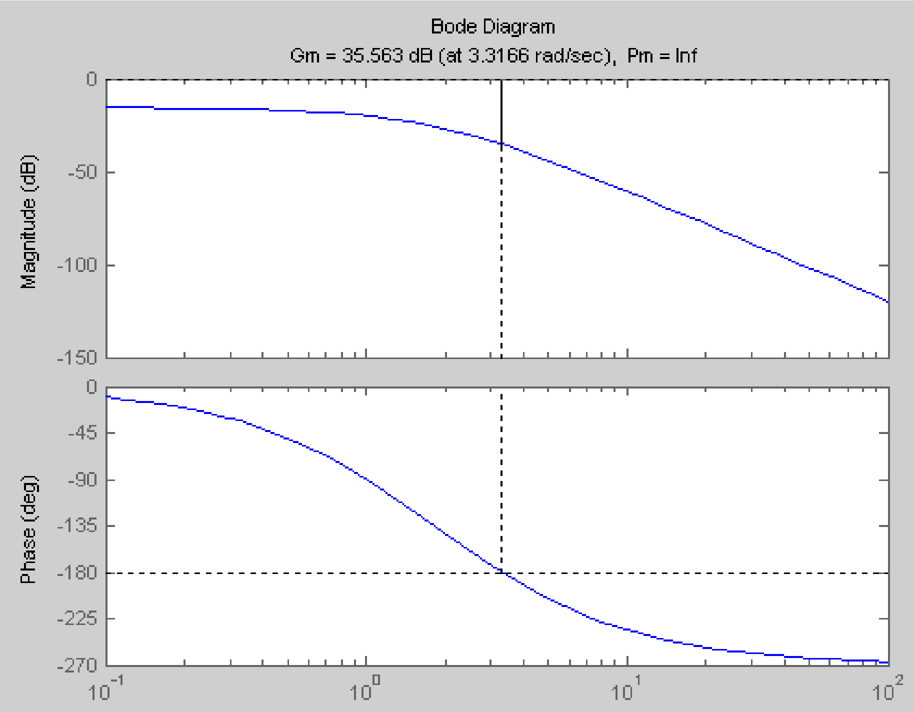
stability; gain margin and phase margin

Example:

open loop system:

$$H(s)=\frac{1}{(s+1)(s+2)(s+3)}$$

closed loop system will be stable



Bode diagrams

Stability; gain margin and phase margin

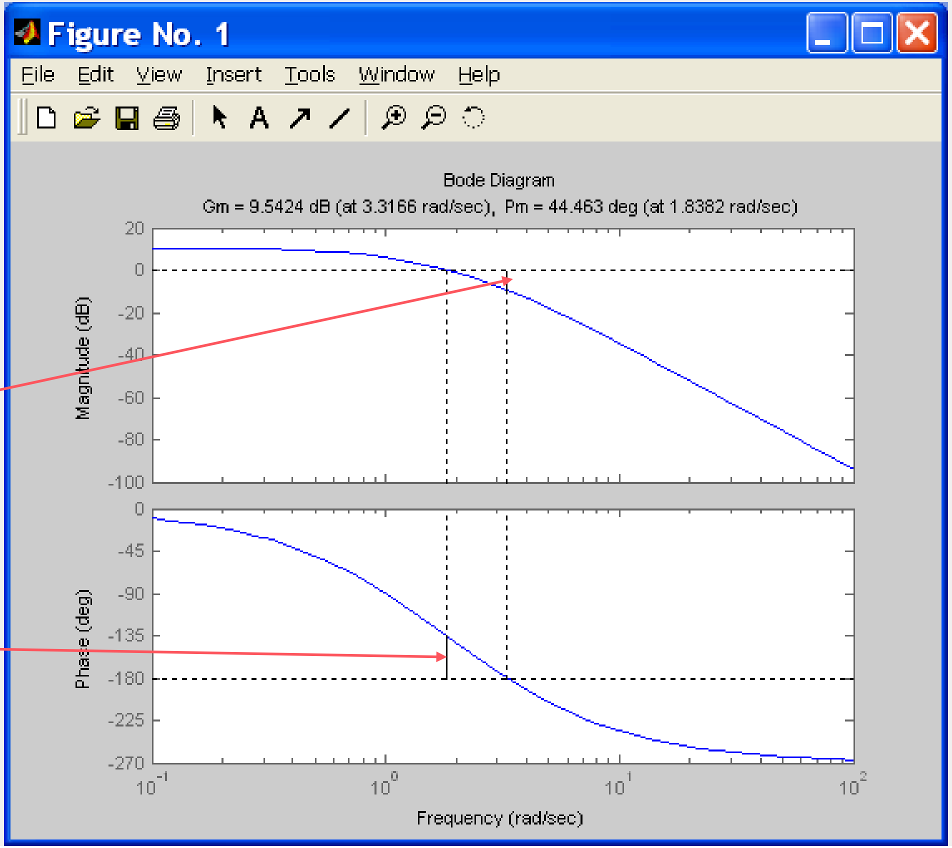
Example:

open loop system:

$$H(s) = \frac{20}{(s+1)(s+2)(s+3)}$$

Gain margin:

Phase margin:



Bode diagrams

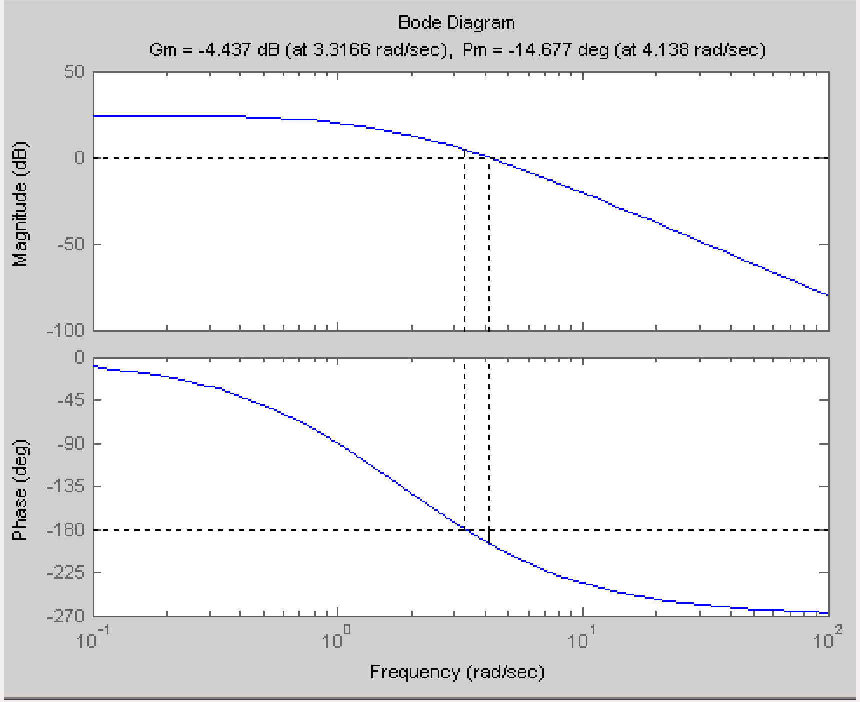
Stability; gain margin and phase margin

Example:

open loop system:

$$H(s) = \frac{100}{(s+1)(s+2)(s+3)}$$

closed loop system will be unstable



Summary

- Natural habitat of ordinary linear differential equations is frequency domain: A sine-wave excitation of a system in general is going to cause a sine-wave response with exactly the same frequency
- The corresponding transfer function can be decomposed in a series of (scaled) first order transfer functions
- If the system is excited with a pure sine-wave input the transfer function can be decomposed in an amplitude response $|H(j\omega)|$ and a phase response

$$|H(j\omega)| = \sqrt{\{Re(H(j\omega))\}^2 + \{Im(H(j\omega))\}^2}$$

and

$$\arg\big(H(j\omega)\big) = arctan\frac{Im(H(j\omega))}{Re(H(j\omega))}$$

- Using $20\log_{10}(|H(j\omega)|)$ for the amplitude response the amplitude response can be decomposed in a summation of scaled first order amplitude responses to compose the Bode amplitude diagram
- The phase response can be decomposed in a summation of first order phase responses to compose the Bode phase diagram

Summary

The Bode diagram enables immediate check of stability of the closed system by looking at the amplitude response of the open loop transfer function as function of $j\omega$ at the frequency point $\omega=\omega_1$ with phase

$$\Phi = -180^0 \pm n \cdot 360^0$$

- If the amplitude at ω_1 is <0dB the closed system is stable
- If the amplitude at ω_1 is ≥ 0dB the closed system is unstable