Table I. IMC-Based PID Controller Parameters[a]

	model	$y/y_s = \tilde{g}_+ f$	controller	$k_c k$	τ_I	τ_D	τ_F	comments
A	$\frac{k}{\tau s + 1}$	$\frac{1}{\epsilon s + 1}$	$\frac{1}{k}\frac{\tau s + 1}{\epsilon s}$	$\frac{\tau}{\epsilon}$	τ	–	–	–
B	$\frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{1}{\epsilon s + 1}$	$\frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k\epsilon s}$	$\frac{\tau_1 + \tau_2}{\epsilon}$	$\tau_1 + \tau_2$	$\frac{\tau_1\tau_2}{\tau_1 + \tau_2}$	–	–
C	$\frac{k}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\frac{1}{\epsilon s + 1}$	$\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{k\epsilon s}$	$\frac{2\zeta\tau}{\epsilon}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$	–	–
D	$k\frac{-\beta s + 1}{\tau s + 1}$	$\frac{-\beta s + 1}{\epsilon s + 1}$	$\frac{\tau s + 1}{k(\beta + \epsilon)s}$	$\frac{\tau}{\beta + \epsilon}$	τ	–	–	(2, 3, 5)
E	$k\frac{-\beta s + 1}{\tau s + 1}$	$\frac{-\beta s + 1}{(\beta s + 1)(\epsilon s + 1)}$	$\frac{\tau s + 1}{ks(\beta\epsilon s + 2\beta + \epsilon)}$	$\frac{\tau}{2\beta + \epsilon}$	τ	–	$\frac{\beta\epsilon}{2\beta + \epsilon}$	(1, 4)
F	$k\frac{-\beta s + 1}{\tau^2 s^2 + 2\zeta\tau + 1}$	$\frac{-\beta s + 1}{\epsilon s + 1}$	$\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{k(\beta + \epsilon)s}$	$\frac{2\zeta\tau}{\beta + \epsilon}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$	–	(2, 3, 5)
G	$k\frac{-\beta s + 1}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\frac{-\beta s + 1}{(\beta s + 1)(\epsilon s + 1)}$	$\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{k(\beta\epsilon s + 2\beta + \epsilon)s}$	$\frac{2\zeta\tau}{2\beta + \epsilon}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$	$\frac{\beta\epsilon}{2\beta + \epsilon}$	(1, 4)
H	$\frac{k}{s}$	$\frac{1}{\epsilon s + 1}$	$\frac{1}{k\epsilon}$	$\frac{1}{\epsilon}$	–	–	–	–
I	$\frac{k}{s}$	$\frac{2\epsilon + 1}{(\epsilon s + 1)^2}$	$\frac{2\epsilon s + 1}{k\epsilon^2 s}$	$\frac{2}{\epsilon}$	2ϵ	–	–	(6)
J	$\frac{k}{s(\tau s + 1)}$	$\frac{1}{\epsilon s + 1}$	$\frac{\tau s + 1}{k\epsilon}$	$\frac{1}{\epsilon}$	–	τ	–	–
K	$\frac{k}{s(\tau s + 1)}$	$\frac{2\epsilon s + 1}{(\epsilon s + 1)^2}$	$\frac{(\tau s + 1)(2\epsilon s + 1)}{k\epsilon^2 s}$	$\frac{2\epsilon + \tau}{\epsilon^2}$	$2\epsilon + \tau$	$\frac{2\epsilon\tau}{2\epsilon + \tau}$	–	(6)
L	$k\frac{-\beta s + 1}{s}$	$\frac{-\beta s + 1}{\epsilon s + 1}$	$\frac{1}{k(\beta + \epsilon)}$	$\frac{1}{\beta + \epsilon}$	–	–	–	(2, 3, 5)
M	$k\frac{-\beta s + 1}{s}$	$\frac{-\beta s + 1}{(\beta s + 1)(\epsilon s + 1)}$	$\frac{1}{k(\beta\epsilon s + 2\beta + \epsilon)}$	$\frac{1}{2\beta + \epsilon}$	–	–	$\frac{\beta\epsilon}{2\beta + \epsilon}$	(1, 4)
N	$k\frac{-\beta s + 1}{s}$	$\frac{(-\beta s + 1)((\beta + 2\epsilon)s + 1)}{(\epsilon s + 1)^2}$	$\frac{(\beta + 2\epsilon)s + 1}{ks(\beta + \epsilon)^2}$	$\frac{\beta + 2\epsilon}{(\beta + \epsilon)^2}$	$\beta + 2\epsilon$	–	–	(5, 6)

O	$k\frac{-\beta s+1}{s}$	$\frac{(-\beta s+1)(2(\beta+\epsilon)s+1)}{(\beta s+1)(\epsilon s+1)^2}$	$\frac{2(\beta+\epsilon)s+1}{ks(\beta\epsilon^2 s+\epsilon^2+4\beta\epsilon+2\beta^2)}$	$\frac{2(\beta+\epsilon)}{2\beta^2+4\beta\epsilon+\epsilon^2}$	$2(\beta+\epsilon)$	–	$\frac{\beta\epsilon^2}{2\beta^2+4\beta\epsilon+\epsilon^2}$	(4, 6)
P	$k\frac{-\beta s+1}{s(\tau s+1)}$	$\frac{-\beta s+1}{\epsilon s+1}$	$\frac{\tau s+1}{k(\beta+\epsilon)}$	$\frac{1}{\beta+\epsilon}$	–	τ	–	(2, 3, 5)
Q	$k\frac{-\beta s+1}{s(\tau s+1)}$	$\frac{-\beta s+1}{(\beta s+1)(\epsilon s+1)}$	$\frac{\tau s+1}{k(\beta\epsilon s+2\beta+\epsilon)}$	$\frac{1}{2\beta+\epsilon}$	–	τ	$\frac{\beta\epsilon}{2\beta+\epsilon}$	(1, 4)
R	$k\frac{-\beta s+1}{s(\tau s+1)}$	$\frac{(-\beta s+1)((\beta+2\epsilon)s+1)}{(\epsilon s+1)^2}$	$\frac{(\tau s+1)((\beta+2\epsilon)s+1)}{ks(\beta+\epsilon)^2}$	$\frac{\beta+2\epsilon+\tau}{(\beta+\epsilon)^2}$	$\beta+2\epsilon+\tau$	$\frac{\tau(\beta+2\epsilon)}{\beta+2\epsilon+\tau}$	–	(5, 6)
S	$k\frac{-\beta s+1}{s(\tau s+1)}$	$\frac{(-\beta s+1)(2(\beta+\epsilon)s+1)}{(\beta s+1)(\epsilon s+1)^2}$	$\frac{(\tau s+1)(2(\beta+\epsilon)s+1)}{ks(\beta\epsilon^2 s+\epsilon^2+4\beta\epsilon+2\beta^2)}$	$\frac{2(\beta+\epsilon)+\tau}{2\beta^2+4\beta\epsilon+\epsilon^2}$	$2(\beta+\epsilon)+\tau$	$\frac{2\tau(\beta+\epsilon)}{2(\beta+\epsilon)+\tau}$	$\frac{\beta\epsilon^2}{2\beta^2+4\beta\epsilon+\epsilon^2}$	(4, 6)

[a] Controller form: $c = [k_c/(\tau_F s + 1)](1 + [1/(\tau_I s)] + \tau_D s)$. ϵ is the only adjustable parameter; for most cases ϵ is equivalent to the closed-loop time constant and $1/\epsilon$ is approximately the closed-loop bandwidth. In all cases, there exists no offset for step set-point/disturbance changes. Comments: 1. ISE optimal for step set-point changes when $\epsilon = 0$. 2. IAE optimal for step set-point changes when $\epsilon = 0$. 3. ISE optimal for step set-point changes when $\epsilon = \beta$. 4. Filter/factorization option 1 (64). Practical recommendation $\epsilon > \beta/2$. 5. Filter/factorization option 2 (66). Practical recommendation $\epsilon > \beta$. 6. No offset for ramp set-point/disturbance changes.

Table II. IMC-Based PID Parameters for $g(s) = ke^{-\theta s}/(\tau s + 1)$ and Practical Recommendations for ϵ/θ

controller	kk_c	τ_I	τ_D	recommended ϵ/θ (> $0.1\tau/\theta$ *always*)
PID	$(2\tau + \theta)/(2\epsilon + \theta)$	$\tau + (\theta/2)$	$\tau\theta/(2\tau + \theta)$	>0.8
PI	$\theta/\tau = 0.1$	1.54		>1.7
improved PI	$(2\tau + \theta)/2\epsilon$	$\tau + (\theta/2)$		>1.7

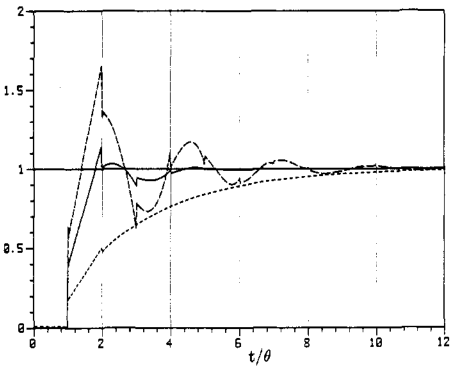

Figure 3. IMC-PID tuning rule. Effect of ϵ/θ on the closed-loop response to a unit step set-point change. $g(s) = ke^{-\theta s}/(\tau s + 1)$. (—) $\epsilon/\theta = 0.8$; (- - -) $\epsilon/\theta = 0.4$; (···) $\epsilon/\theta = 2.5$.

Option 2 (eq 66) was chosen for the filter for the first-order Padé approximation in order to get a PID controller without an additional lag term. These controllers are represented compactly in Table II. The closed-loop transfer functions for system (71) with these controllers indicate a number of advantages:

PID
$$y = \frac{e^{-\theta s}}{\dfrac{\left(\dfrac{\epsilon}{\theta} + \dfrac{1}{2}\right)\theta s}{1 + \dfrac{1}{2}\theta s} + e^{-\theta s}}(y_s - d) + d \qquad (76)$$

PI
$$y = \frac{e^{-\theta s}}{\left(\dfrac{\epsilon}{\theta}\right)\theta s + e^{-\theta s}}(y_s - d) + d \qquad (77)$$

The closed-loop response is independent of the system time constant τ. (The process lag $(1 + \tau s)$ is cancelled by the controller.) The time is scaled by θ. The shape of the response depends on ϵ/θ only.

In other words, specifying one value of ϵ/θ for any first-order lag with the dead-time model results in an identical response when the time is caled by θ, regardless of k, θ, and τ. For instance, if the dead time in system I is twice as long as the dead time in system II, then for a specific ϵ/θ, the response characteristics will be identical except that it will take the response of system I exactly twice as long to reach the same point as system II. The choice of the "best" ratio ϵ/θ must be based on performance and robustness considerations.

For the PID controller, Figure 3 demonstrates the dependence of the step response on ϵ/θ. $\epsilon/\theta = 0.4$ is fairly close to the value where instability occurs ($\epsilon/\theta = 0.145$), and the large overshoot and poorly damped oscillations are therefore not surprising. Note that $\epsilon/\theta = 0.5$ is the lower value recommended in Table I for models with a RHP zero factored according to (66). For $\epsilon/\theta = 0.8$, the response looks very good: the rise time is about 1.5θ and the settling time is 4.5θ; the overshoot is about 10%, and the decay