Overview of lecture topics

- Topic 1: Course introduction
- Topic 2: Introduction to frequency domain and Laplace transformation
- Topic 3: Dynamic Behavior of Linear systems
- Topic 4: Frequency Response Analysis and Bode plots
- Topic 5: Mathematical Description of Chemical Systems
- Topic 6: Nonlinear ODE's, Linearity, Linearization, Feedback, Stability, Root Locus
- Topic 7: Feedback Controller Design and Bode stability
- Topic 8: Advanced (Enhanced) Process Control

Process Dynamics and Process Control

Lecture 7: Feedback Controller Design and Bode stability

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Course 6E8X0

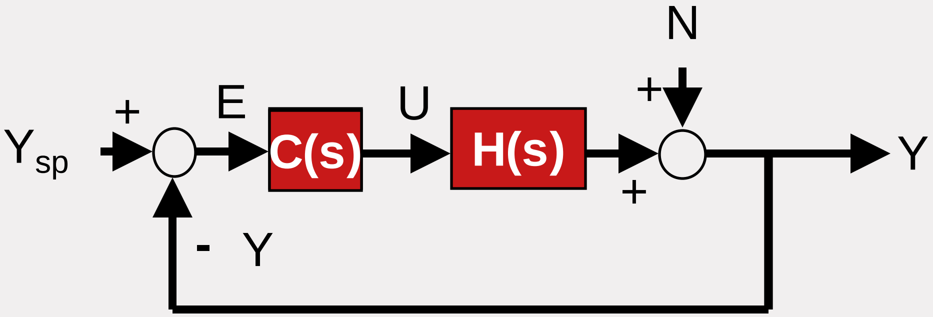
Outline

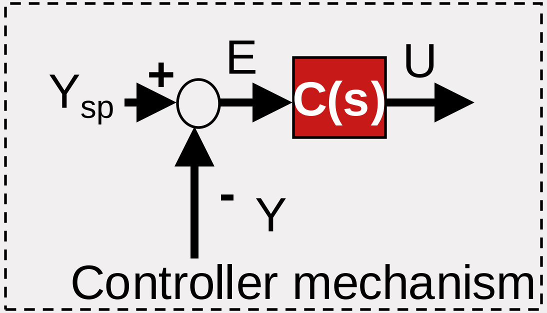
Conventional Feedback Controllers

- Type of Controllers
- Dynamic Behavior of Feedback Controlled Processes
- Controller Tuning Methods
  - Ziegler-Nichols Method
  - Reaction Curve Method
  - IMC Tuning

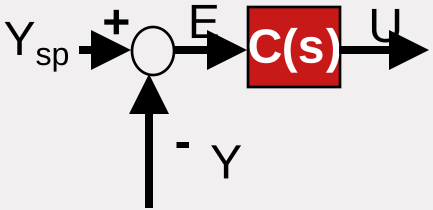
Bode Stability and Frequency Domain Based Controller Design

Conventional Feedback Controllers





Conventional Feedback Controllers



Basic Controller Types

Proportional controller (P only)

$$u'(t) = \bar{u} + Ke(t) \qquad \bar{u} : \text{bias}$$

$$u(t) = Ke(t) \overset{\mathcal{L}}{\Rightarrow} U(s) = KE(s) \Rightarrow C(s) = K$$

Integral Controller (I)

$$u(t) = \frac{K}{\tau_I}\int_0^t e(t')dt' \overset{\mathcal{L}}{\Rightarrow} U(s) = \frac{K}{\tau_I s}E(s) \Rightarrow C(s) = \frac{K}{\tau_I s}$$

Derivative Controller (D)

$$u(t) = k\tau_D\frac{de(t)}{dt} \overset{\mathcal{L}}{\Rightarrow} U(s) = K\tau_D sE(s) \Rightarrow C(s) = \frac{U(s)}{E(s)} = K\tau_D s$$

Conventional Feedback Controllers

PI(D) (Three/Two-mode) Controller

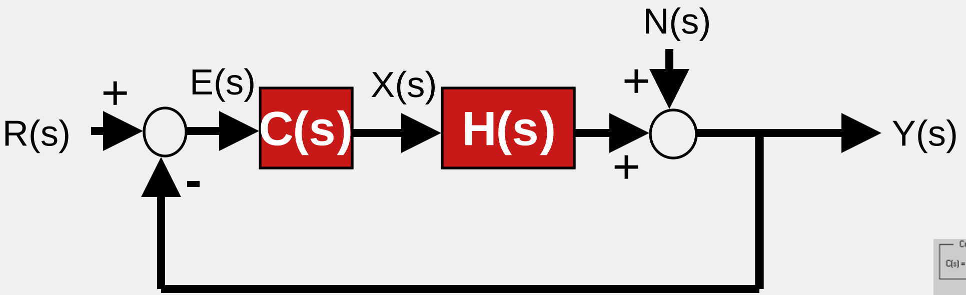
- Workhorses of the process industry
  - 90% of the loops are PI
  - The PID controller is described by:

$$u(t) = \bar{u} + K\left(e(t) + \frac{1}{\tau_I}\int_0^t e(t)dt + \tau_D\frac{de(t)}{dt}\right)$$

$$U(s) = K\left(1 + \frac{1}{\tau_I s} + s\tau_D\right)E(s)$$

Dynamic Behaviour of Feedback Controlled Systems

P-action:



$$C_p(s) = K_c$$

$$x(t) = K_c e(t)$$

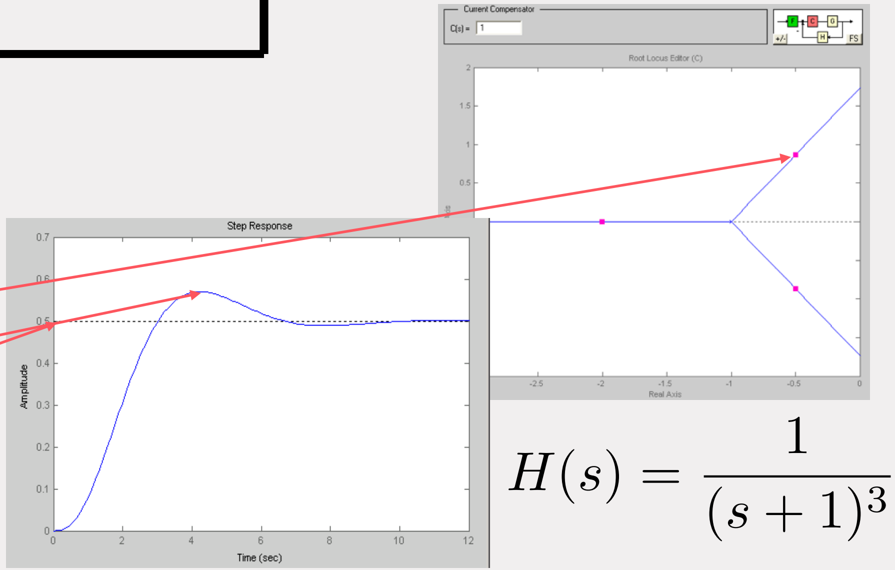
amplifies error e(t)

can improve speed +

often worse stability -

possible overshoot -

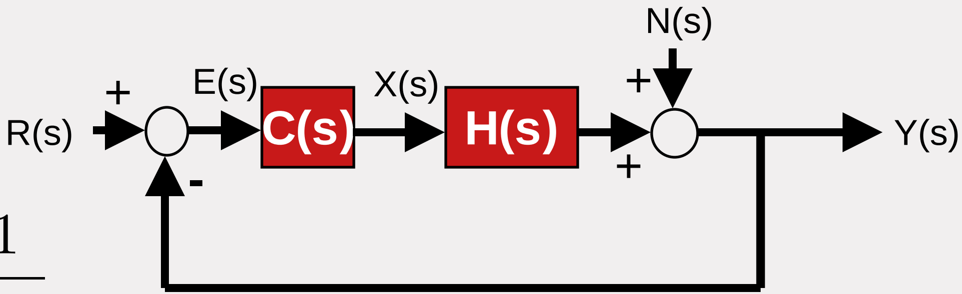
offset -



$$H(s) = \frac{1}{(s+1)^3}$$

Dynamic Behaviour of Feedback Controlled Systems

I-action:



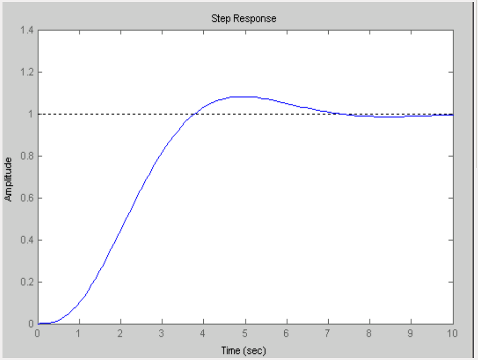
$$C_I(s) = \frac{1}{\tau_I s}$$

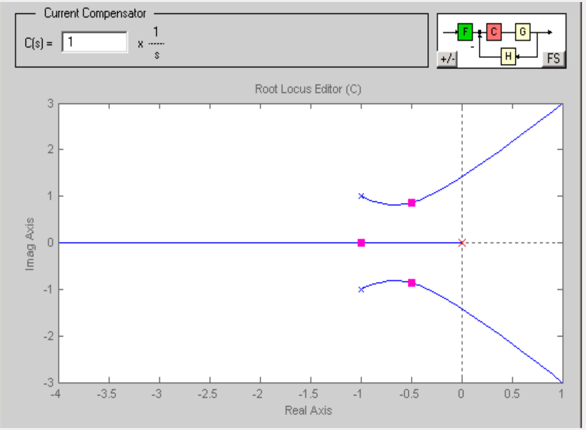
$$x(t) = \frac{K_c}{\tau_I} \int_0^t e(t)dt$$

integration of error e(t) adds pole in s = 0

offset = 0 +

worse stability -

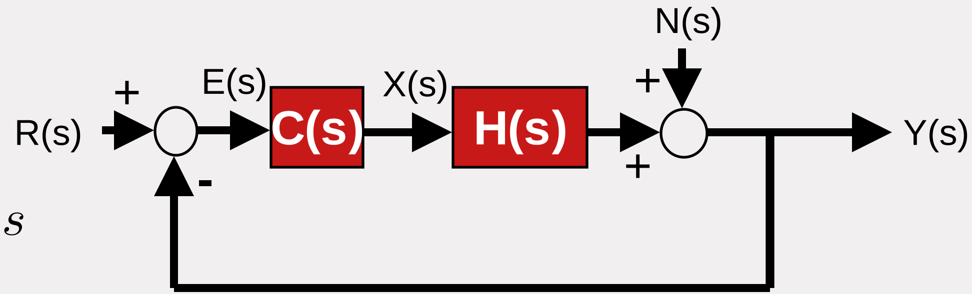




$$H(s) = \frac{1}{(s+1-j)(s+1+j)}$$

Dynamic Behaviour of Feedback Controlled Systems

D-action:



$$C_D(s) = \tau_D s$$

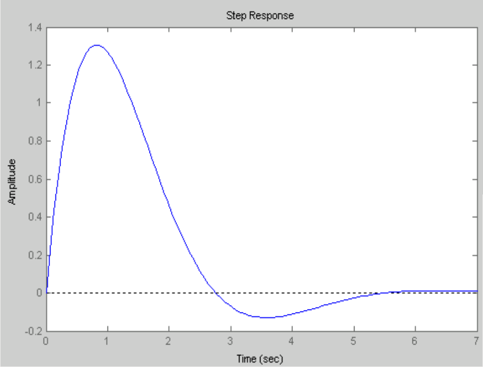
$$x(t) = K_c \tau_D \frac{e(t)}{dt}$$

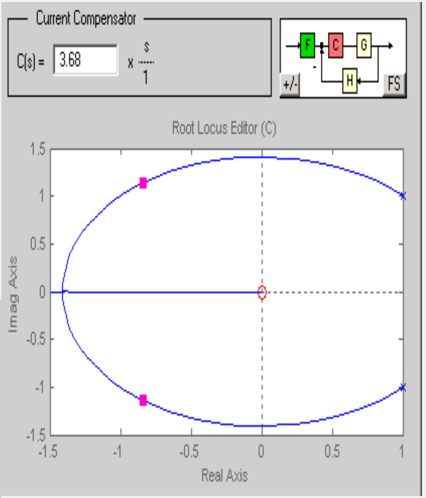
differentiation of error e(t) adds zero

controller zero in s=0 !!

zero steady state on step!!

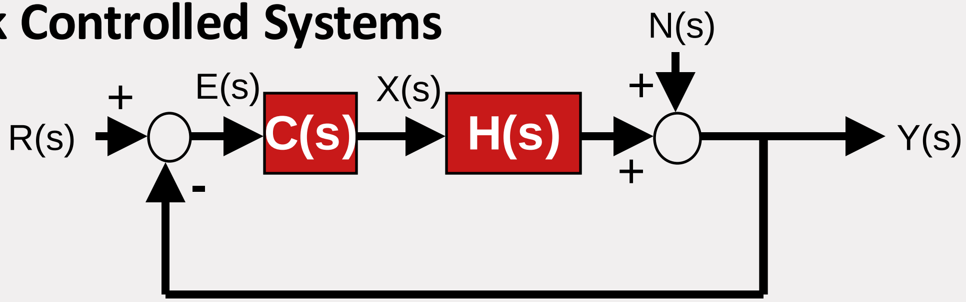
use PD instead of D controller!!





$$H(s) = \frac{1}{(s-1+j)(s-1-j)}$$

Dynamic Behaviour of Feedback Controlled Systems



combined PD action:

$$C_{PD}(s) = K_C(1 + \tau_D s)$$

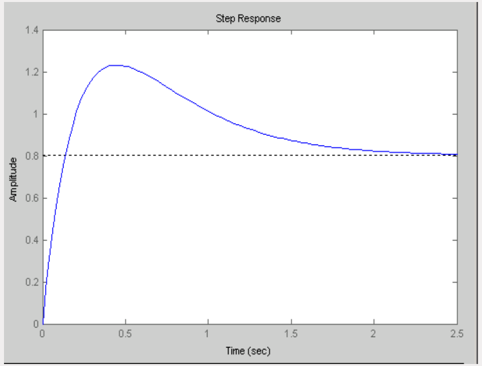
controller zero in

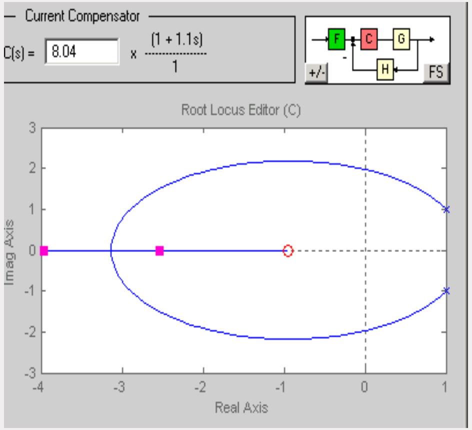
$$s = -\frac{1}{\tau_D}$$

improves stability +

offset -

possible overshoot -

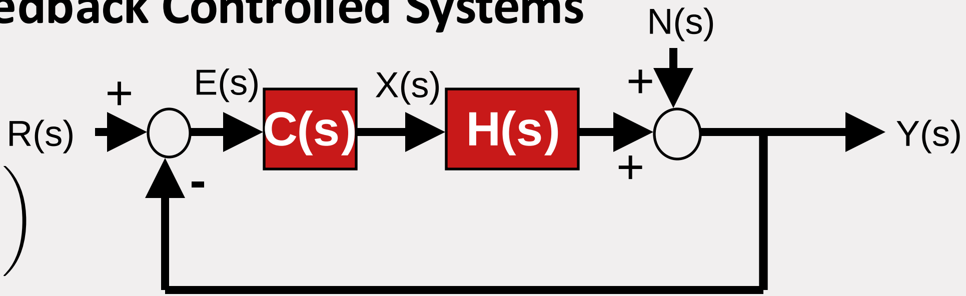




$$H(s) = \frac{1}{(s-1+j)(s-1-j)}$$

Dynamic Behaviour of Feedback Controlled Systems

combined PID action:



$$C_{PID}(s) = K_c \left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

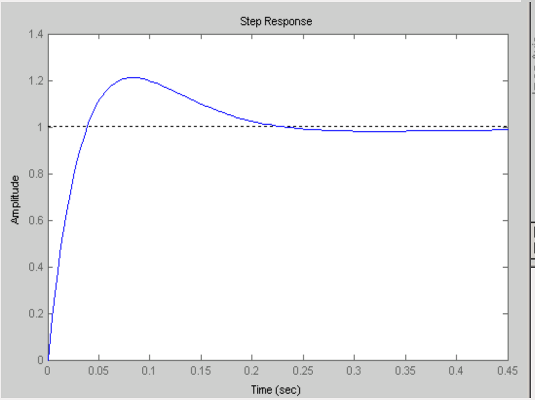
$$= K_c \left(\frac{\tau_D \tau_I s^2 + \tau_I s + 1}{\tau_I s}\right)$$

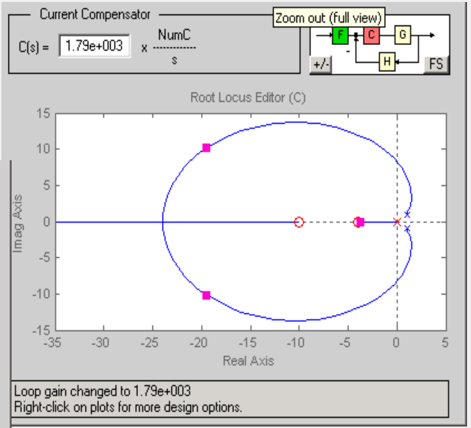
controller C(s): pole in s=0 and 2 zeros

improves stability +

no offset +

fast response +

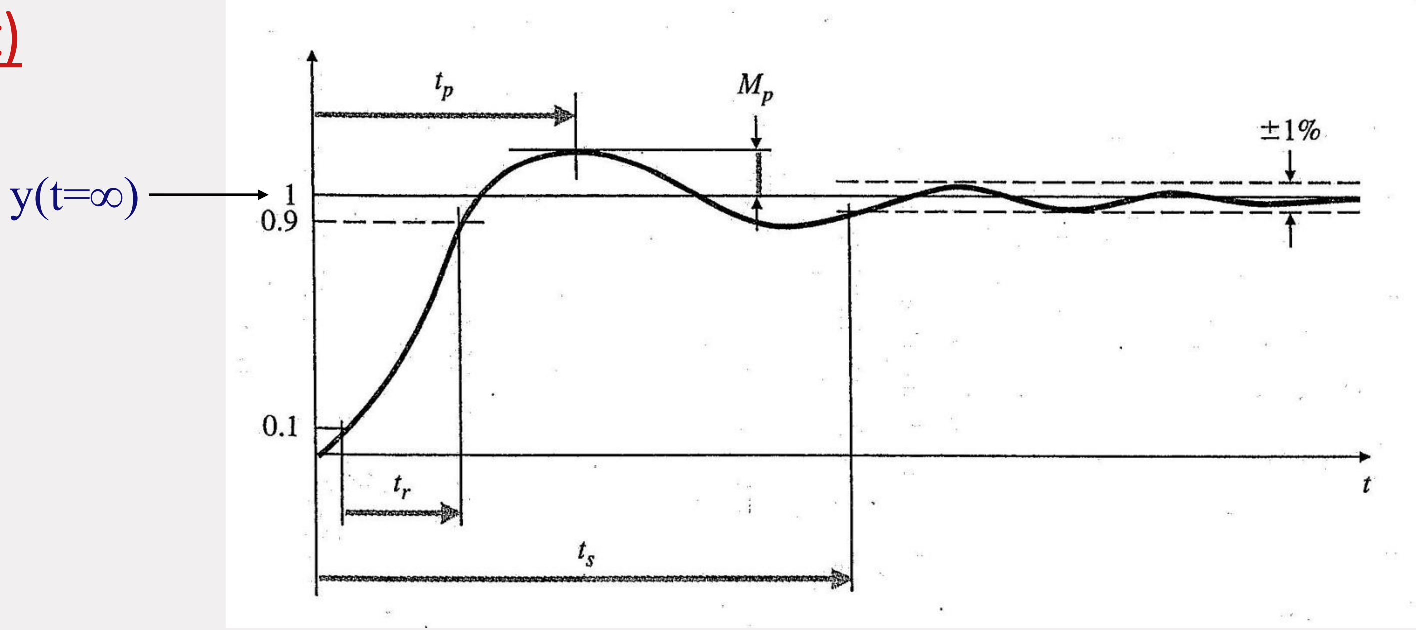




$$H(s) = \frac{1}{(s-1+j)(s-1-j)}$$

Design of Feedback Controllers

response y(t)



t_r: rise time

t_s: settling time

t_p: peak time

M_p: overshoot

offset: $r(t=\infty) - y(t=\infty)$

Design of Feedback Controllers

Performance Criteria

- Closed loop system must be stable
- Good disturbance rejection (effects of disturbances are minimized)
- Good set point tracking(rapid and smooth response to set point changes)
- Offset (steady state error) is minimum
- Control action is not excessive
- Control mechanism is robust (insensitive to inaccuracies in process model)

Controller tuning depends on criteria

Design of feedback controllers

In general the guidelines summarized in the table below apply for increase of any of the parameters K_c, τ_i, τ_d:

Controller parameter	Rise time	Overshoot	Settling time	Steady state error (Offset)
K_c	decrease	increase	Limited effect	decrease
τ_i	increase	decrease	decrease	zero
τ_d	Limited effect	decrease	decrease	No effect

The controller configuration assumed is given by:

$$C_{PID}(s) = K_c \left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Controller Tuning

Ziegler–Nichols Method (1942): Closed Loop Method for tuning a controller

- Controller: $$C_{PID}(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$
- For a certain small value of controller Gain $K_c \simeq 0$, Integration time $\tau_i = \infty$ and Differentiation time $\tau_d = 0$ apply a set point change and observe the response
- Increase K_c further until you obtain sustained oscillations (damping=0) with oscillation period time P_u
  - $K_u = K$ ultimate gain
  - $P_u = 2\pi/\omega_u$ ultimate period (ω_u: critical angular frequency)

Ziegler-Nicols Controller settings	K_c	τ_i	τ_d
P	$\frac{K_u}{2}$	∞	0
PI	$\frac{K_u}{2.2}$	$\frac{P_u}{1.2}$	0
PID	$\frac{K_u}{1.7}$	$\frac{P_u}{2}$	$\frac{P_u}{8}$

Controller Tuning

Ziegler–Nichols Method (1942): Closed Loop Method for tuning a controller

- The tuning results in a fast settling step response with damped oscillatory behavior
- Typical decay ratio DR of the damped oscillations: DR = $\frac{b}{a} \simeq 0.25$ (see plot)
- The tuning typically is used as starting point for further fine tuning
- <u>Example</u>

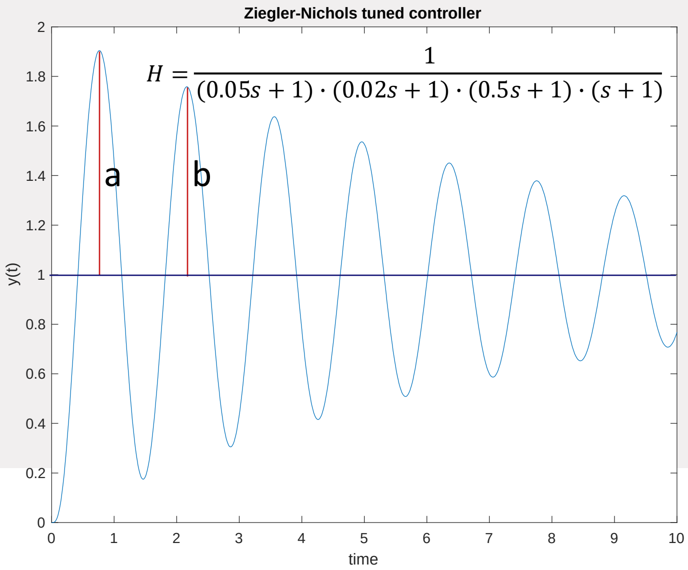
$$H(s) = \frac{1}{(0.05s+1)(0.02s+1)(0.5s+1)(s+1)}$$

$$C_{PID}(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

$$K_u = 24.2661 \quad P_u = 0.95$$

Ziegler-Nichols PI-controller tuning:

$$K_c = \frac{K_u}{2.2} = 11.03 \quad \tau_i = \frac{P_u}{1.2} = 0.7917$$

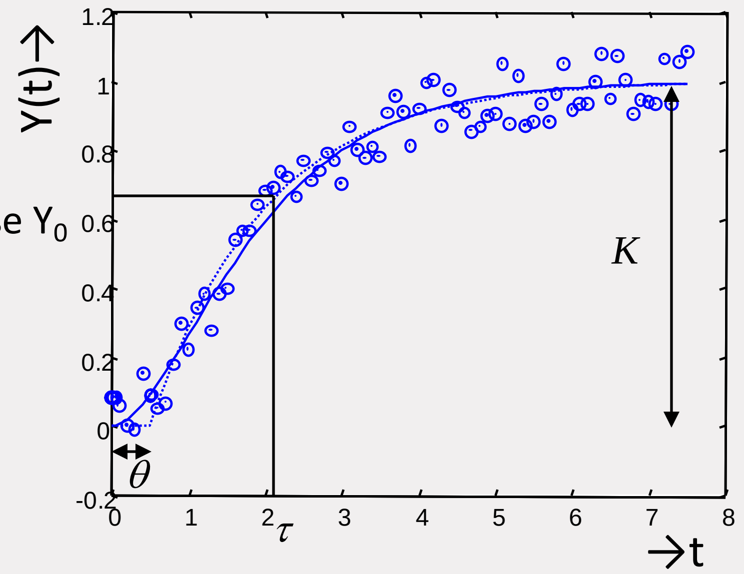


Controller Tuning

Reaction Curve Method:

- Step U_0 is introduced in u(t)
- Y(t) is recorded
- Determine:
  - deadtime θ, time constant τ and steady state reponse Y_0
    - Steady state response amplitude Y_0
    - response time constant τ
  - Calculate:

$$K_0 = \frac{Y_0}{U_0}\frac{\tau}{\theta}$$



- Fit a first order process
  - with deadtime model:

$$Y_m(s) = \frac{\frac{Y_0}{U_0}}{(\tau s+1)} e^{-\theta s} U(S)$$

Controller Tuning

Process Reaction Method based controller tuning:

- Do the open loop test and determine the step response curve
- From the response obtained determine:
  - Dead time θ
  - Time constant τ
  - Steady state response amplitude Y_0
- Calculate:

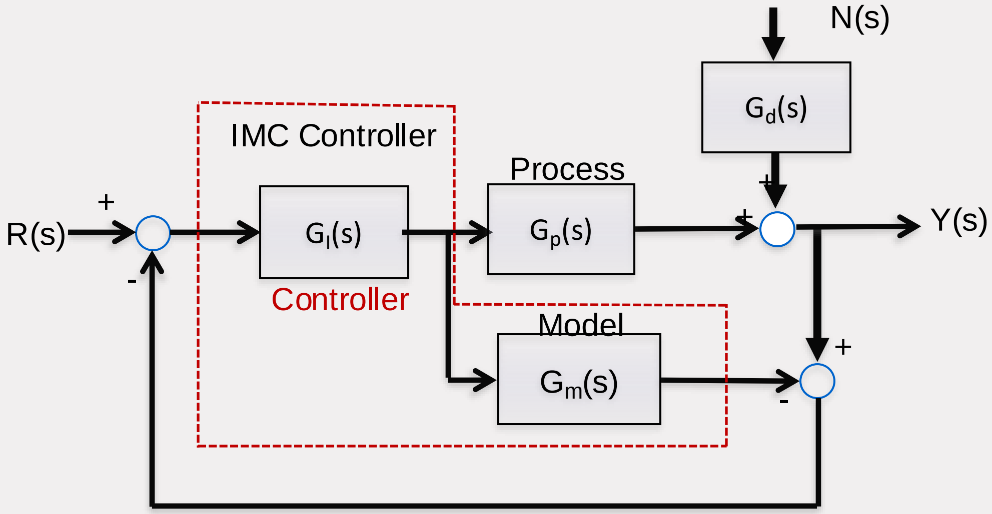
$$K_0 = \frac{U_0}{Y_0}\frac{\tau}{\theta}$$

- Determine controller parameters:

Reaction method tuning	K_c	τ_i	τ_d
P	K_0		
PI	$0.9K_0$	3.3θ	
PID	$1.2K_0$	2θ	0.5θ

Controller Tuning

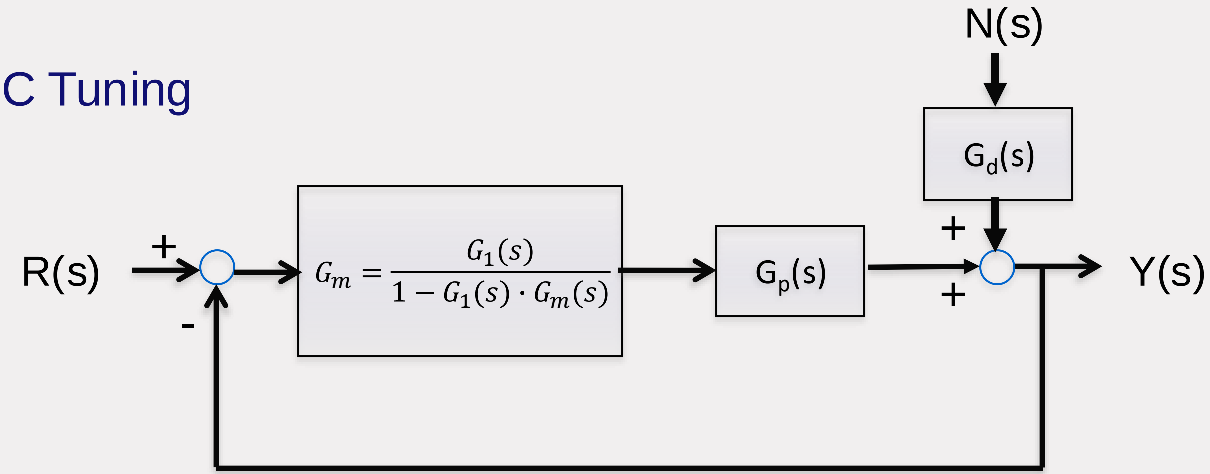
Internal Model Control Tuning, 1986



Block Diagram of IMC Controller

Controller Tuning

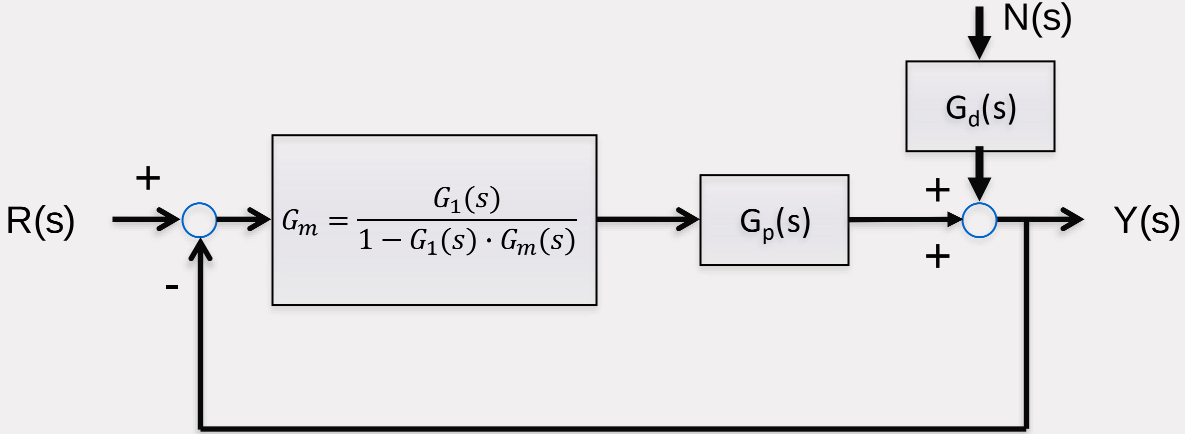
IMC Tuning



Corresponding conventional control structure

Controller Tuning

IMC Tuning



$$Y(s) = \frac{G_1(s)G_p(s)}{1 + G_1(s)\left(G_p(s) - G_m(s)\right)} R(s) + \frac{G_d(s)\left(1 - G_1(s)G_m(s)\right)}{1 + G_1(s)\left(G_p(s) - G_m(s)\right)} N(s)$$

Closed loop transfer functions

$$H_{YRcl}(s) = \frac{G_1(s)G_p(s)}{1 + G_1(s)\left(G_p(s) - G_m(s)\right)} \qquad H_{YNcl}(s) = \frac{G_d(s)\left(1 - G_1(s)G_m(s)\right)}{1 + G_1(s)\left(G_p(s) - G_m(s)\right)}$$

Controller Tuning

IMC Tuning

- Process Model is split into two parts

$$G_m(s) = G_m^+(s)G_m^-(s)$$

where $G_m^+(s)$ contains dead-time and RHP zeros and steady-state gain scaled to 1.

- Controller design

$$G_I = \frac{1}{G_m^-(s)} f(s)$$

IMC Filter $$f(s) = \frac{1}{(\tau_c s + 1)^r}$$

Controller Tuning

IMC Tuning

Morari &Zafirou, 1989

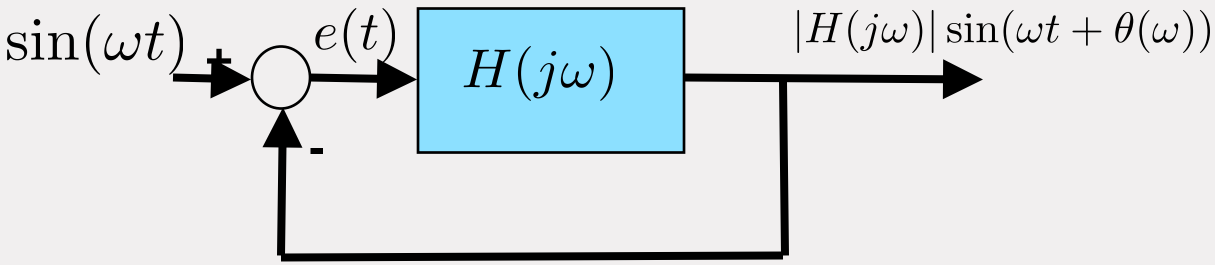
Table 6.1-1. IMC controllers for simple models interpreted as PID controllers with filter:

$$c = k_c\left(1 + \tau_D s + \frac{1}{\tau_I s}\right)\frac{1}{\tau_F s + 1}; \quad \gamma = \frac{2\beta\tau_2}{\beta + \tau_2}$$

	Model $\tilde{p}$	Input v_M	$\tilde{\eta} = \tilde{p}q = \tilde{p}\tilde{q}f$	$k_c k$	τ_I	τ_D	τ_F
A	$\frac{k}{\tau s+1}$	$\frac{1}{s}$	$\frac{1}{\lambda s+1}$	$\frac{\tau}{\lambda}$	τ	–	–
B	$\frac{k}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{1}{s}$	$\frac{1}{\lambda s+1}$	$\frac{\tau_1+\tau_2}{\lambda}$	$\tau_1 + \tau_2$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$	–
C	$\frac{k}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{1}{s}$	$\frac{1}{\lambda s+1}$	$\frac{2\zeta\tau}{\lambda}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$	–
D	$k\frac{-\beta s+1}{\tau s+1}$	$\frac{1}{s}$	$\frac{-\beta s+1}{(\beta s+1)(\lambda s+1)}$	$\frac{\tau}{2\beta+\lambda}$	τ	–	$\frac{\beta\lambda}{2\beta+\lambda}$
E	$k\frac{-\beta s+1}{\tau_1 s+1}$	$\frac{1}{\tau_2 s+1}\frac{1}{s}$	$\frac{(-\beta s+1)(\gamma s+1)}{(\beta s+1)(\lambda s+1)}$	$\frac{\gamma+\tau_1}{2\beta-\gamma+\lambda}$	$\gamma + \tau_1$	$\frac{\gamma\tau_1}{\gamma+\tau_1}$	$\frac{\beta(\gamma+\lambda)}{2\beta-\gamma+\lambda}$
F	$k\frac{-\beta s+1}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{1}{s}$	$\frac{-\beta s+1}{(\beta s+1)(\lambda s+1)}$	$\frac{2\zeta\tau}{2\beta+\lambda}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$	$\frac{\beta\lambda}{2\beta+\lambda}$
G	$\frac{k}{s}$	$\frac{1}{s}$	$\frac{1}{\lambda s+1}$	$\frac{1}{\lambda}$	–	–	–
H	$\frac{k}{s}$	$\frac{1}{s^2}$	$\frac{2\lambda s+1}{(\lambda s+1)^2}$	$\frac{2}{\lambda}$	2λ	–	–
I	$\frac{k}{s(\tau s+1)}$	$\frac{1}{s}$	$\frac{1}{\lambda s+1}$	$\frac{1}{\lambda}$	–	τ	–
J	$\frac{k}{s(\tau s+1)}$	$\frac{1}{s^2}$	$\frac{2\lambda s+1}{(\lambda s+1)^2}$	$\frac{2\lambda+\tau}{\lambda^2}$	$2\lambda + \tau$	$\frac{2\lambda\tau}{2\lambda+\tau}$	–
K	$k\frac{-\beta s+1}{s}$	$\frac{1}{s}$	$\frac{-\beta s+1}{(\beta s+1)(\lambda s+1)}$	$\frac{1}{2\beta+\lambda}$	–	–	$\frac{\beta\lambda}{2\beta+\lambda}$
L	$k\frac{-\beta s+1}{s}$	$\frac{1}{\tau_2 s+1}\frac{1}{s}$	$\frac{(-\beta s+1)(\gamma s+1)}{(\beta s+1)(\lambda s+1)}$	$\frac{1}{2\beta-\gamma+\lambda}$	–	γ	$\frac{\beta(\gamma+\lambda)}{2\beta-\gamma+\lambda}$
M	$k\frac{-\beta s+1}{s}$	$\frac{1}{s^2}$	$\frac{(-\beta s+1)(2\beta s+1)(2\lambda s+1)}{(\beta s+1)(\lambda s+1)^2}$	$\frac{2(\beta+\lambda)}{2\beta^2+\lambda^2}$	$2(\beta + \lambda)$	$\frac{2\beta\lambda}{\beta+\lambda}$	$\frac{\beta\lambda^2+4\beta^2\lambda}{2\beta^2+\lambda^2}$
N	$k\frac{-\beta s+1}{s(\tau s+1)}$	$\frac{1}{s}$	$\frac{-\beta s+1}{(\beta s+1)(\lambda s+1)}$	$\frac{1}{2\beta+\lambda}$	–	τ	$\frac{\beta\lambda}{2\beta+\lambda}$
O	$k\frac{-\beta s+1}{s(\tau s+1)}$	$\frac{1}{s^2}$	$\frac{(-\beta s+1)(2(\beta+\lambda)s+1)}{(\beta s+1)(\lambda s+1)^2}$	$\frac{2(\beta+\lambda)+\tau}{2\beta^2+4\beta\lambda+\lambda^2}$	$2(\beta + \lambda) + \tau$	$\frac{2\tau(\beta+\lambda)}{2(\beta+\lambda)+\tau}$	$\frac{\beta\lambda^2}{2\beta^2+4\beta\lambda+\lambda^2}$

Bode Stability

- stability



- if for a certain frequency ω: phase shift $\theta(\omega) = \pm\pi$ and gain $|H(j\omega)| = 1$
- then the output is: $y(t) = |H(j\omega)|\sin(\omega t + \theta(\omega)) = \sin(\omega t + \pi) = -\sin(\omega t)$
- and the error signal $e(t)$ will increase $\rightarrow$ instability

Bode stability

stability

If for certain frequency:

- the phase shift of a process H(jω) is ±180° and the gain is ≥1 (=0 dB) then the closed-loop process will become unstable

- we can use the Bode diagrams to check this: Use Matlab function margin(sys)

- phase margin: How much phase shift is left (to -180°) for that frequency ω_0 where the gain is 0 dB

- gain margin: How much gain is left (to 0 dB) for that frequency ω_0 where the phase shift is 180°

Bode stability

stability; gain margin and phase margin

example:

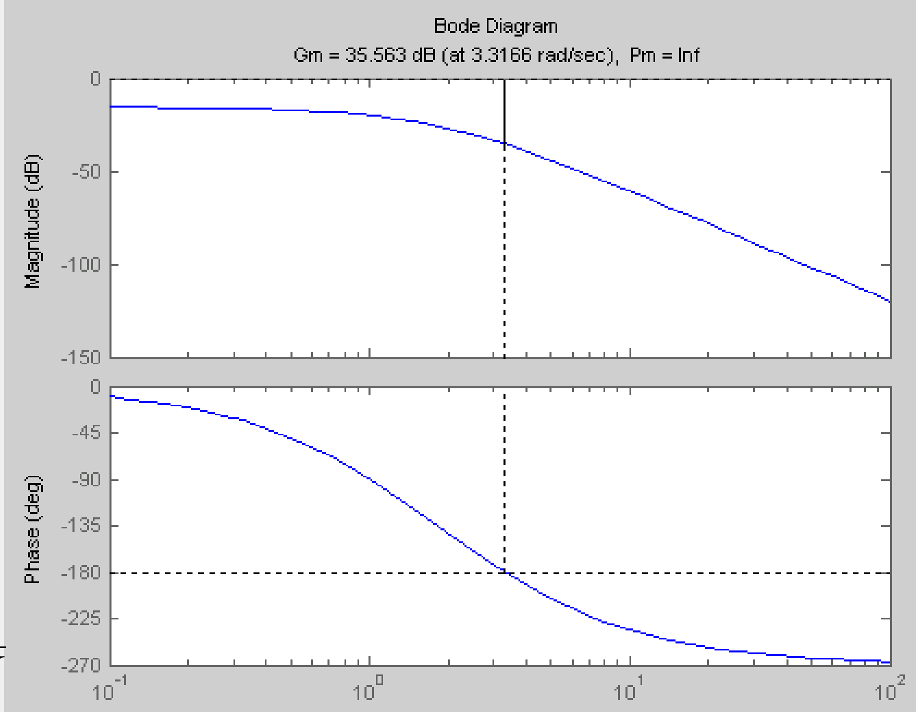
- open loop system H(s):

$$H(s) = \frac{1}{(s+1)(s+2)(s+3)}$$

- closed loop system $H_{cl}(s)$:

$$H_{cl}(s) = \frac{K \cdot H(s)}{1 + K \cdot H(s)} = \frac{K}{(s+1)(s+2)(s+3) + K}$$

- will be stable



Bode stability

stability; gain margin and phase margin

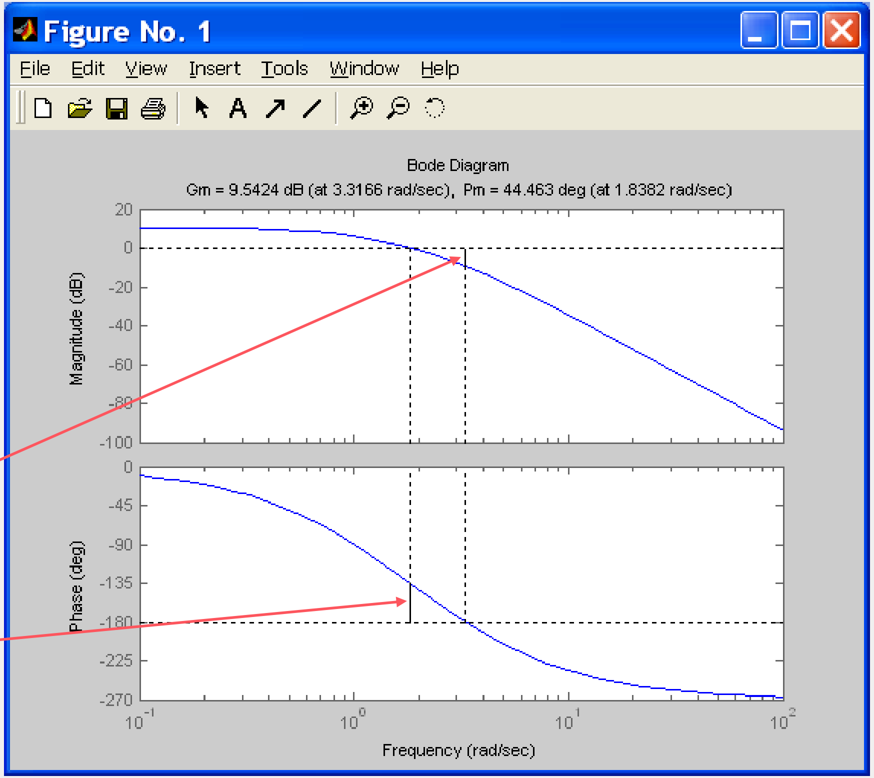
Example:

open loop system H(s):

$$H(s) = \frac{20}{(s+1)(s+2)(s+3)}$$

gainmargin: 9.54 dB at $\omega = 3.3166$ rad/s

phasemargin:44.463 deg at $\omega = 1.8362$ rad/s



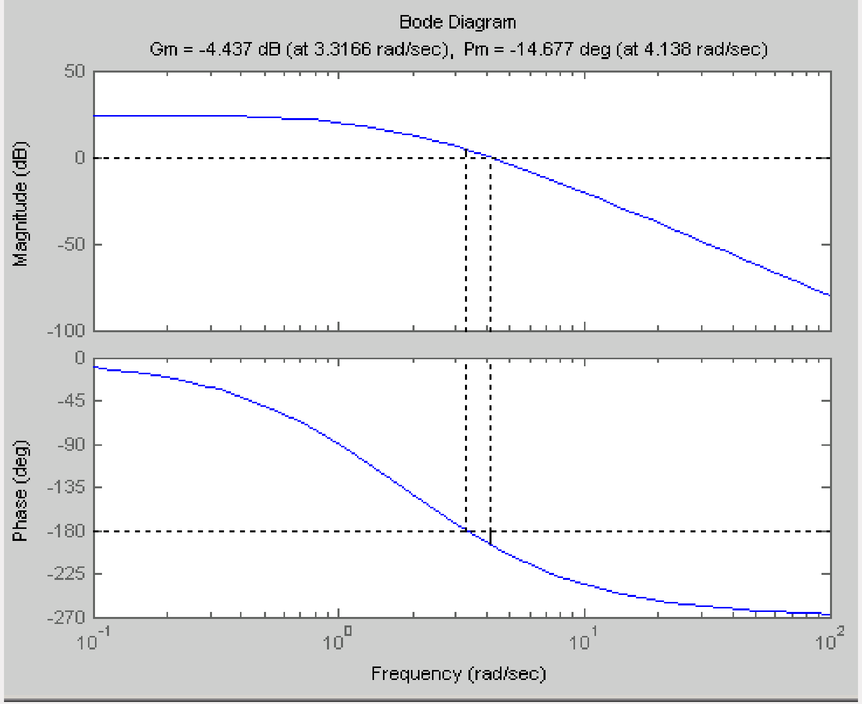
Bode stability

stability; gain margin and phase margin

Example:

- open loop system H(s):

$$H(s) = \frac{100}{(s+1)(s+2)(s+3)}$$



- closed loop system $H_{cl}(s)$ will be unstable:

$$H_{cl}(s) = \frac{H(s)}{1+H(s)} = \frac{100}{(s+1)(s+2)(s+3)+100}$$

- The instability is reflected by the negative gain and phase margins obtained

Summary Controller Tuning

Controller Tuning Methods

- Ziegler-Nichols closed loop Tuning Method
- Ziegler-Nichols open loop method (Process reaction curve)
- Internal Model Control Tuning method
- Frequency Response Techniques (Bode diagram using margins analysis)

- These methods give good starting values for the controller parameters.
- In practice, on-line tuning is done after the control system is installed.