Overview of lecture topics

- Topic 1: Course introduction
- Topic 2: Introduction to frequency domain and Laplace transformation
- Topic 3: Dynamic Behavior of Linear systems
- Topic 4: Frequency Response Analysis and Bode plots
- Topic 5: Mathematical Description of Chemical Systems
- Topic 6: Nonlinear ODE's, Linearity, Linearization Feedback, Stability, Root Locus
- Topic 7: Feedback Controller Design and Bode stability
- Topic 8: Advanced (Enhanced) Process Control

Process Dynamics and Process Control

Topic 3: Dynamic Behaviour of Linear Systems

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Course 6E8X0

Outline

Transfer functions

Linear systems: Some properties

Dynamic Behaviour of Low Order Systems

- First Order Systems
- Integrating Systems (Pure Capacity)
- Second Order Systems
- Lead/Lag Systems

Dynamic Behavior of High Order Systems

Dynamic Behavior of Some Typical Systems

- System With Time Delay
- Inverse Response Systems

Transfer Function Representation of Some Typical Dynamic Systems

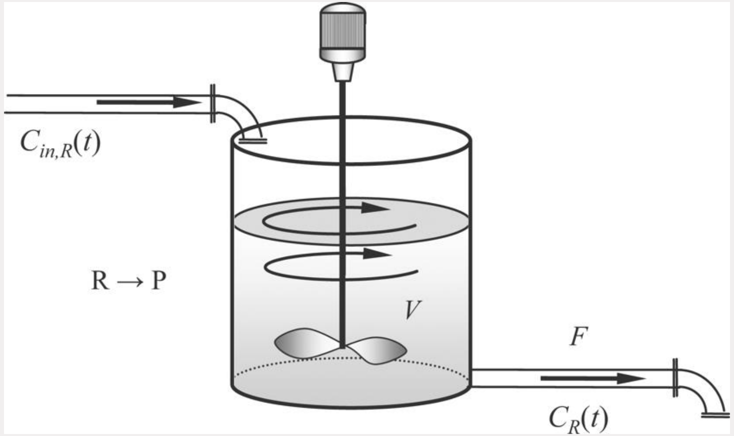
First Order Systems:

Example: CSTR,

$$\frac{d}{dt}(VC_R) = FC_{in,R} - FC_R - Vk_RC_R$$

$$\Big\downarrow \mathcal{L} \qquad C_R(0) = 0$$

$$sVC_R(s) - C_R(0) = FC_{in,R}(s) - FC_R(s) - Vk_RC_R(s)$$



An isothermal CSTR

$$\left(\frac{V}{Vk_r + F}s + 1\right)C_R(s) = \frac{F}{F + Vk_r}C_{R,in}(s) \Rightarrow \frac{C_R(s)}{C_{R,in}(s)} = \frac{\frac{F}{F+Vk_r}}{\left(\frac{V}{Vk_r+F}s + 1\right)}$$

Transfer Function Representation of Some Typical Dynamic Systems

First Order Systems, General Representation:

$$\tau \frac{dy(t)}{dt} = -y(t) + Ku(t)$$

$$(\tau s + 1)\, Y(s) = KU(s)$$

Transfer Function :

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1}$$

$K \longrightarrow$ Gain

$\tau \longrightarrow$ Time constant

$$= \frac{\text{Transformed forced function (output)}}{\text{Transformed forcing function (input)}}$$

At zero initial conditions

Transfer Function Representation of Linear Dynamic Systems

Differential Equation:

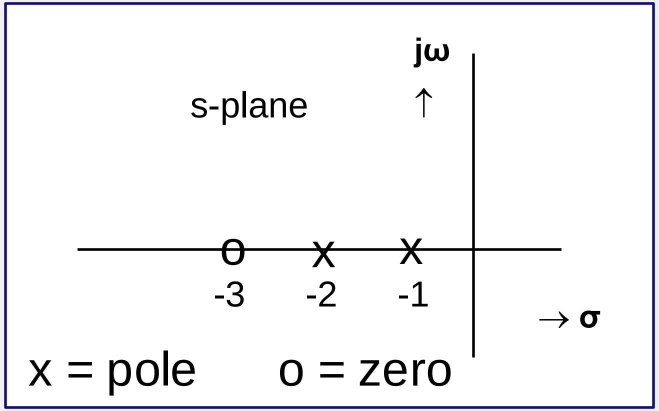
$$\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = \dot{x}(t) + 3x(t) \qquad \text{with} \quad \dot{y}(0) = y(0) = x(0) = 0$$

$$\mathcal{L}\left\{\ddot{y}(t) + 3\dot{y}(t) + 2y(t)\right\} = \mathcal{L}\left\{\dot{x}(t) + 3x(t)\right\}$$

$$Y(s)\left(s^2 + 3s + 2\right) = X(s)\,(s+3)$$

$$\frac{Y(s)}{X(s)} = \frac{s+3}{s^2+3s+2}$$

$$\boxed{H(s) = \frac{s+3}{(s+1)(s+2)}}$$



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Transfer Function Representation of Linear Dynamic Systems

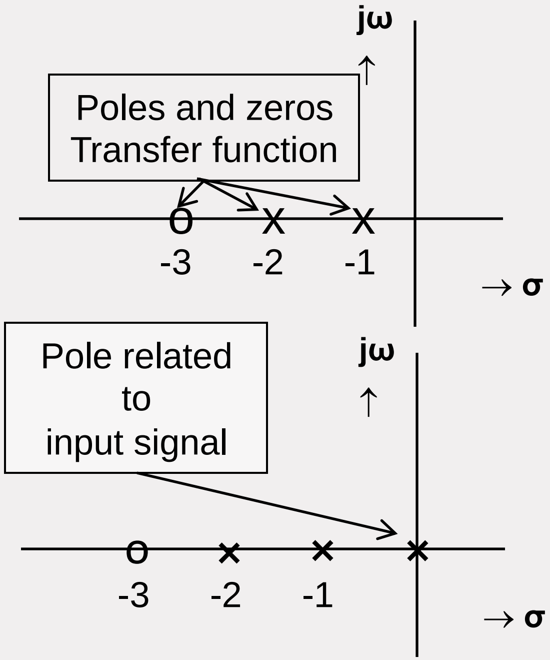
$$H(s) = \frac{s+3}{(s+1)(s+2)}$$

Transfer function

$$Y(s) = H(s)X(s) = \frac{s+3}{(s+1)(s+2)}\frac{1}{s}$$

$$\Big\downarrow \ \mathcal{L}^{-1}$$

$$y(t) = 1.5U(t) + 0.5e^{-2t} - 2e^{-t}$$



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Transfer Function Representation of Linear Dynamic Systems

General Linear Differential Equation:

$$\sum_{i=0}^{N} a_i \frac{d^i y(t)}{dt^i} = \sum_{j=0}^{M} b_j \frac{d^j x(t)}{dt^j} \qquad i = 0, \cdots, N \quad j = 0, \cdots, M$$

$$a_N \frac{d^N y}{dt^N} + a_{N-1} \frac{d^{N-1} y}{dt^{N-1}} + \cdots + a_1 \frac{dy}{dt} + a_0 y = b_M \frac{d^M x}{dT^M} + b_{M-1} \frac{d^{M-1} x}{dt^{M-1}} + \cdots + b_1 \frac{dx}{dt} + b_0 x$$

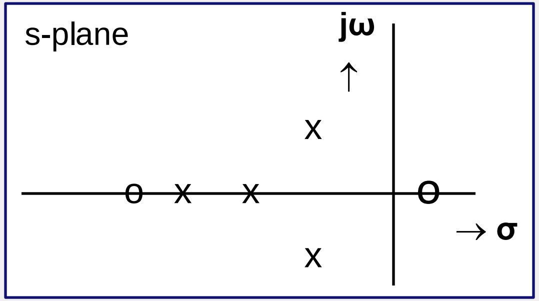
Laplace transform of DE (all initial conditions are assumed zero)

$$\overbrace{\left(a_N s^N + a_{N-1} s^{N-1} + \cdots + a_1 s + a_0\right)}^{D(s)} Y(s) = \overbrace{\left(b_M s^M + b_{M-1} s^{M-1} + \cdots + b_1 s + b_0\right)}^{N(s)} X(s)$$

$$\frac{Y(s)}{X(s)} = H(s) = \frac{N(s)}{D(s)} \qquad D(s) : \text{characteristic equation}$$

Transfer Function Representation of Linear Dynamic Systems

Given a transfer function: $$H(s) = \frac{N(s)}{D(s)}$$



$$\text{zeros of } N(s) \Leftrightarrow \textbf{zeros} \text{ of } H(s)$$

$$\text{zeros of } D(s) \Leftrightarrow \textbf{poles} \text{ of } H(s)$$

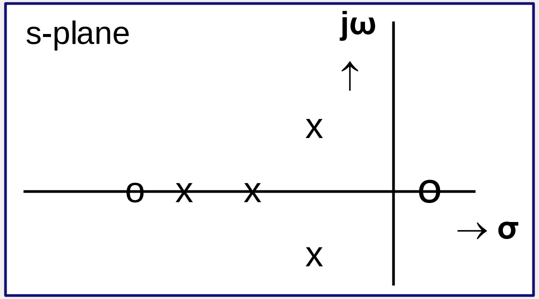
$$n^{\text{th}}\text{order DE} \Leftrightarrow n \text{ energy containers} \Leftrightarrow n \text{ integrators in simulation} \Leftrightarrow n^{\text{th}}\text{order system}$$
$$\Leftrightarrow n \text{ poles} \Leftrightarrow n\text{roots of characteristic equation}$$

DE has real coefficients (real world!) therefore:

$$\text{complex poles and zeros appear in conjugate pairs} \rightarrow \text{s-plane is symmetrical around real axis}$$

Transfer Function Representation of Linear Dynamic Systems

Given a transfer function: $$H(s) = \frac{N(s)}{D(s)}$$



$H(s)$ has n poles and m zeros; $n \geq m$

poles in LHP: stable behaviour

poles in RHP: unstable behaviour

Linear systems

differential equation (DE) ⇔ time domain

$$\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = \dot{x}(t) + 3x(t)$$

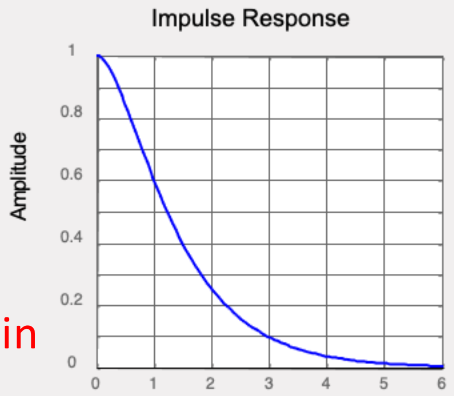
transfer function (TF) ⇔ complex frequency domain

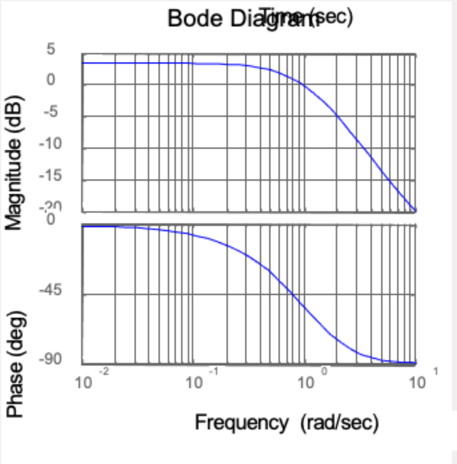
$$H(s) = \frac{s+3}{(s+1)(s+2)}$$

impulse response (IR) ⇔ time domain

Bode plots (BP)*) ⇔ frequency domain

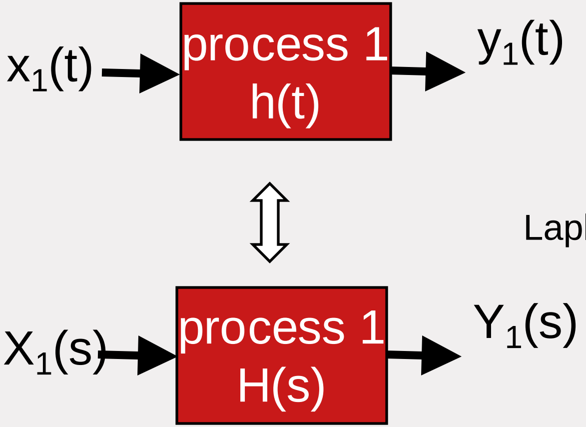
*)Bode Plots will be introduced and explained in Lecture 4





Linear systems

Block diagram representation: visualization of system

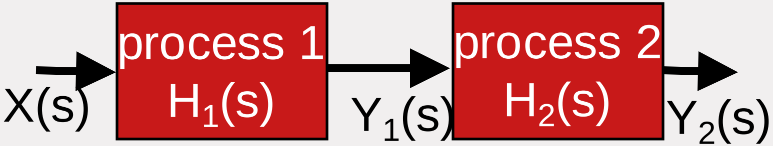


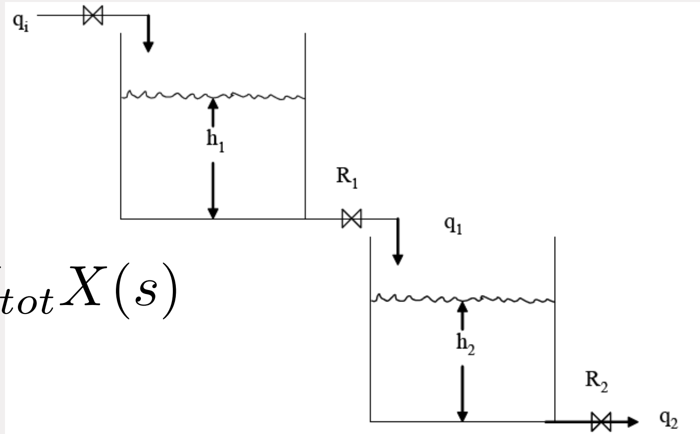
$$y_1(t) = h(t) \otimes x_1(t) = \int_0^{\infty} h(\tau) x_1(t-\tau) d\tau$$

Laplace transform $\Updownarrow$

$$Y_1(s) = H(s) \cdot X_1(s)$$

Linear systems





$$Y_1(s) = H_1(s)X(s)$$
$$Y_2(s) = H_2(s)Y_1(s) = H_2(s)H_1(s)X(s) = H_{tot}X(s)$$

$$Y_1(s) = X(s)H_1(s)$$
$$Y_2(s) = Y_1(s)H_2(s) = X(s)H_1(s)H_2(s) = H_1(s)H_2(s)X(s)$$

$$\boxed{H_{\mathrm{tot}}(s) = H_2(s)H_1(s) = H_1(s)H_2(s)}$$

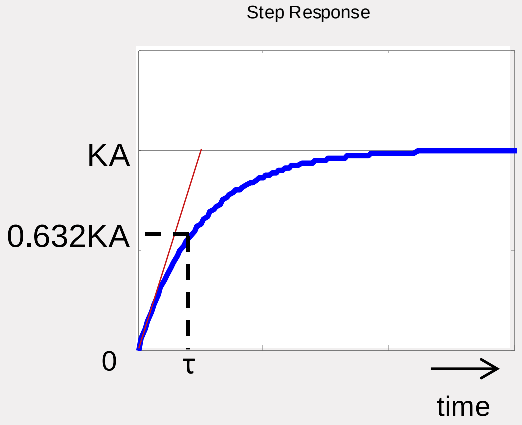
Transfer Function Representation of Some Typical Dynamic Systems

First Order Systems:

Step response :

$$U(s) = \frac{A}{s}$$

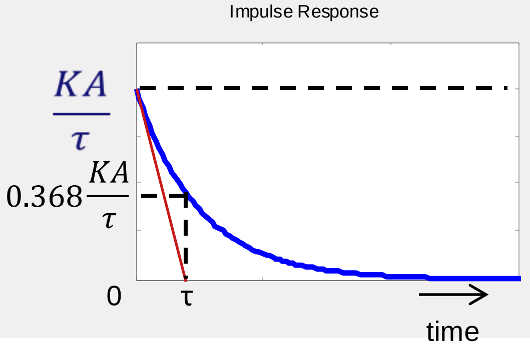
$$Y(s) = \frac{KA}{s(\tau s + 1)} \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\Longleftrightarrow}} y(t) = KA\left(1 - e^{-\frac{t}{\tau}}\right)$$



Impulse response :

$$U(s) = A$$

$$Y(s) = \frac{KA}{(\tau s + 1)} \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\Leftrightarrow}} y(t) = \frac{KA}{\tau} e^{-\frac{t}{\tau}}$$



Transfer Function Representation of Some Typical Dynamic Systems

First Order Systems:

Response to Ramp Input :

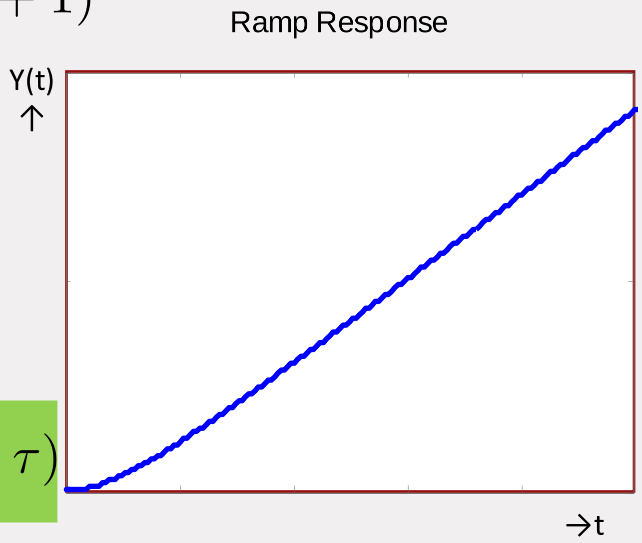
$$U(s) = \frac{A}{s^2} \Rightarrow Y(s) = \frac{K}{(\tau s + 1)} \frac{A}{s^2} = \frac{c_1}{s^2} + \frac{c_2}{s} + \frac{c_3}{(\tau s + 1)}$$

$$c_1 = s^2 \, Y(s)\big|_{s=0} = KA$$

$$c_2 = \left. \frac{d(s^2 Y(s))}{ds} \right|_{s=0} = -KA\tau$$

$$c_3 = (\tau s + 1) Y(s)\big|_{s=-\frac{1}{\tau}} = KA\tau^2$$

$$Y(s) = \frac{KA}{s^2\,(\tau s + 1)} \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\Longleftrightarrow}} y(t) = KA\tau e^{-\frac{t}{\tau}} + KA(t - \tau)$$



Transfer Function Representation of Some Typical Dynamic Systems

First Order Systems:

Response to sinusoidal input :

$$u(t) = A\sin(\omega t) \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\Longleftrightarrow}} U(s) = \frac{A\omega}{s^2+\omega^2}$$

$$Y(s) = \frac{K}{(\tau s+1)}\frac{A\omega}{(s^2+\omega^2)} = \frac{c_1}{\tau s+1} + \frac{c_2}{s+j\omega} + \frac{c_3}{s-j\omega}$$

$$c_1 = (\tau s+1)Y(s)\big|_{s=\frac{-1}{\tau}} = \frac{KA\omega\tau}{(1+\omega^2\tau^2)}$$

$$c_2 = (s+j\omega)Y(s)\big|_{s=-j\omega} = \frac{-KA\omega\tau}{2(\omega^2\tau^2+j\omega\tau)} \qquad c_3 = (s-j\omega)Y(s)\big|_{s=j\omega} = \frac{-KA\omega\tau}{2(\omega^2\tau^2-j\omega\tau)}$$

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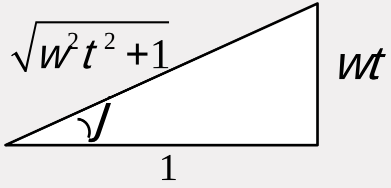
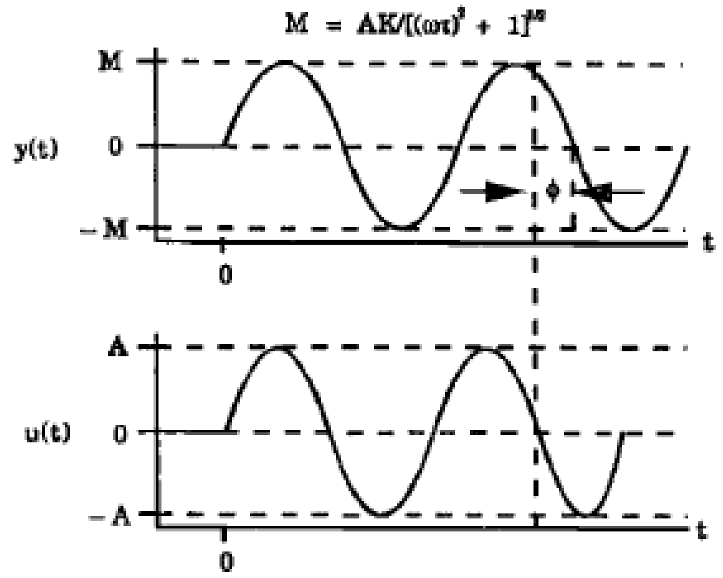
Transfer Function Representation of Some Typical Dynamic Systems

First Order Systems:

Response to sinusoidal input :

$$y(t) = KA\left(\frac{\omega\tau}{\omega^2\tau^2+1}\right)e^{\frac{-t}{\tau}} + \frac{KA}{\sqrt{\omega^2\tau^2+1}}\sin\left(\omega t - \varphi\right)$$





$$\sin\left(\omega t\right)\cos\left(\varphi\right) - \cos\left(\omega t\right)\sin\left(\varphi\right) = \sin\left(\omega t - \varphi\right)$$

Transfer Function Representation of Some Typical Dynamic Systems

$$Y(s) = \frac{KA\omega}{(\tau s + 1)(s^2 + \omega^2)} = \frac{c_1}{\tau s + 1} + \frac{c_2}{s + j\omega} + \frac{c_3}{s - j\omega}$$

$$c_1 = (\tau s + 1)Y\,(s)\Big|_{s=-1/\tau} = \frac{KA\omega\tau}{(1 + \omega^2\tau^2)}$$

$$c_2 = (s + j\omega)\,Y(s)\Big|_{s=-j\omega} = \frac{-KA\omega\tau}{2(\omega^2\tau^2 + j\omega\tau)}$$

$$c_3 = \frac{-KA\omega\tau}{2(\omega^2\tau^2 - j\omega\tau)}$$

$$Y(s) = \frac{KA\omega}{(\tau s + 1)(s^2 + \omega^2)} = \frac{\frac{KA\omega\tau}{(1 + \omega^2\tau^2)}}{\tau s + 1} + \frac{\frac{-KA\omega\tau}{2(\omega^2\tau^2 + j\omega\tau)}}{s + j\omega} + \frac{\frac{-KA\omega\tau}{2(\omega^2\tau^2 - j\omega\tau)}}{s - j\omega}$$

$$y(t) = KA\left(\frac{\omega\tau}{\omega^2\tau^2 + 1}\right)e^{-t/\tau} + \frac{-KA\omega\tau}{2(\omega^2\tau^2 + j\omega\tau)}\,e^{-j\omega t} + \frac{-KA\omega\tau}{2(\omega^2\tau^2 - j\omega\tau)}\,e^{j\omega t}$$

Transfer Function Representation of Some Typical Dynamic Systems

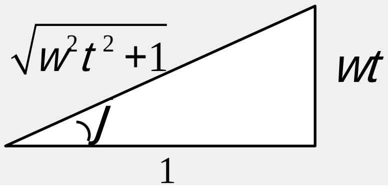
$$y(t) = KA\left(\frac{\omega\tau}{\omega^2\tau^2+1}\right)e^{-t/\tau} + \frac{-KA\omega\tau}{2(\omega^2\tau^2+j\omega\tau)}e^{-j\omega t} + \frac{-KA\omega\tau}{2(\omega^2\tau^2-j\omega\tau)}e^{j\omega t}$$

$$y(t) = KA\left(\frac{\omega\tau}{\omega^2\tau^2+1}\right)e^{-t/\tau} + \frac{KA}{(\sqrt{\omega^2\tau^2+1})}\left(\frac{-\omega\tau}{2(\sqrt{\omega^2\tau^2+1})}\left(e^{-j\omega t}+e^{j\omega t}\right)+\frac{1}{2j(\sqrt{\omega^2\tau^2+1})}\left(-e^{-j\omega t}+e^{j\omega t}\right)\right)$$

$$y(t) = KA\left(\frac{\omega\tau}{\omega^2\tau^2+1}\right)e^{-t/\tau} + \frac{KA}{(\sqrt{\omega^2\tau^2+1})}\left(\frac{-\omega\tau}{(\sqrt{\omega^2\tau^2+1})}\left(\frac{e^{j\omega t}+e^{-j\omega t}}{2}\right)+\frac{1}{(\sqrt{\omega^2\tau^2+1})}\left(\frac{e^{j\omega t}-e^{-j\omega t}}{2j}\right)\right)$$

$$y(t) = KA\left(\frac{\omega\tau}{\omega^2\tau^2+1}\right)e^{-t/\tau} + \frac{KA}{(\sqrt{\omega^2\tau^2+1})}\left(-\sin(\varphi)\cos(\omega t)+\cos(\varphi)\sin(\omega t)\right)$$

$$y(t) = KA\left(\frac{\omega\tau}{\omega^2\tau^2+1}\right)e^{-t/\tau} + \frac{KA}{(\sqrt{\omega^2\tau^2+1})}\sin(\omega t-\varphi)$$



Transfer Function Representation of Some Typical Dynamical Systems

Integrating Systems:

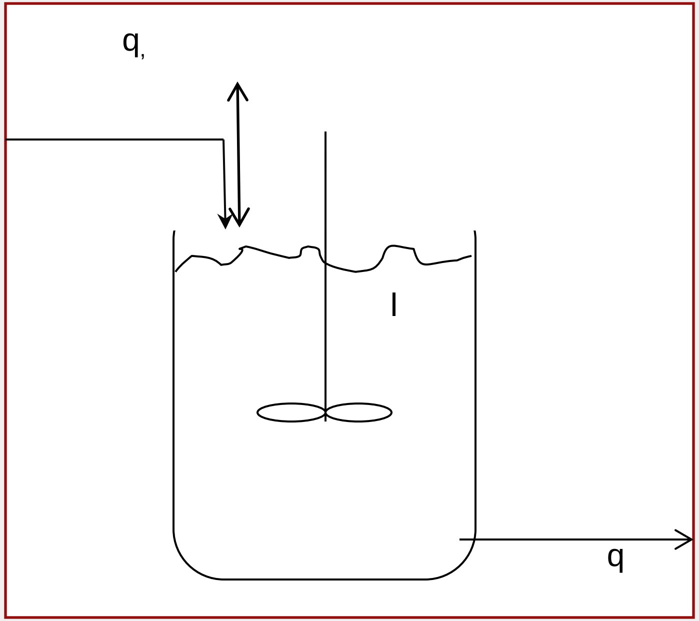
Example: Storage tank with constant outlet flow

$$A_r \frac{dl}{dt} = q_i - q$$ Mass balance

$$\mathcal{L}\left\{A_r \frac{dl}{dt}\right\} = \mathcal{L}\left\{q_i - q\right\}$$

$$A_r s L(s) = Q_i(s) - q$$

$$L(s) = \frac{\frac{1}{A_r}}{s} Q_i(s) - \frac{\frac{q}{A_r}}{s}$$



Transfer Function Representation of Some Typical Dynamical Systems

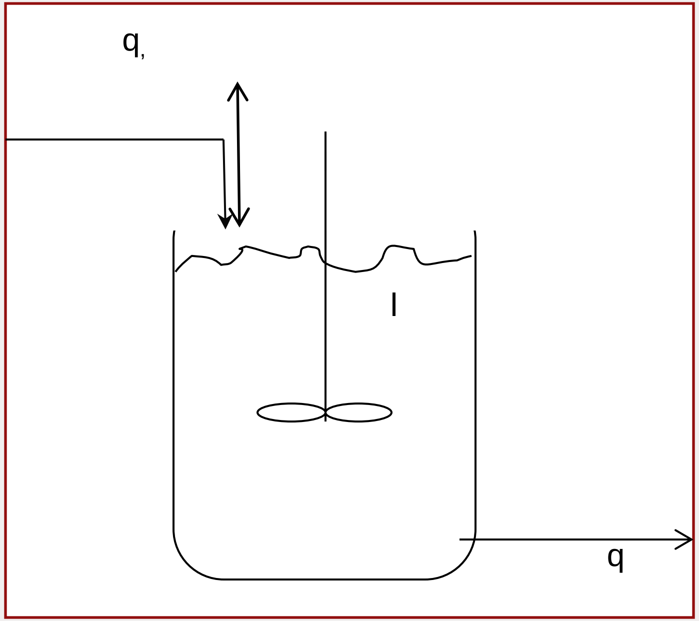
Integrating Systems:

General Representation:

$$\frac{dy(t)}{dt} = Ku(t)$$

Transfer Function:

$$\frac{Y(s)}{U(s)} = \frac{K}{s}$$



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Transfer Function Representation of Some Typical Dynamical Systems

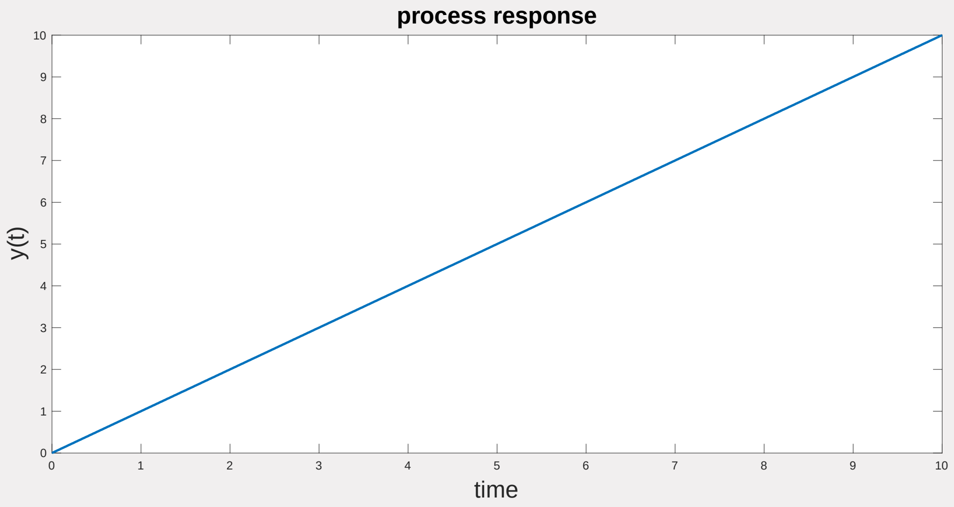
Integrating Systems:

Step response:

$$U(s) = \frac{A}{s}$$

$$Y(s) = \frac{K}{s}\frac{A}{s} = \frac{KA}{s^2}$$

$$y(t) = KAt$$



Transfer Function Representation of Some Typical Dynamical Systems

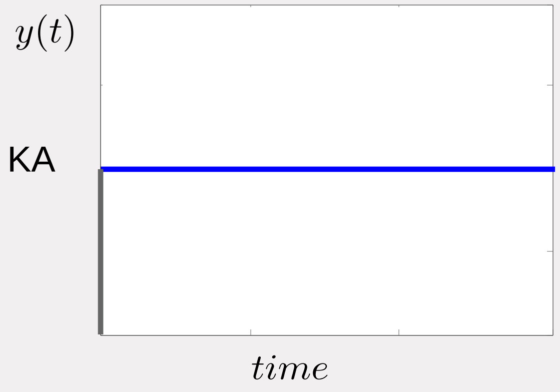
Integrating Systems:

Impulse response:

$$U(s) = A$$

$$Y(s) = \frac{K}{s} A$$

$$y(t) = KA$$



Transfer Function Representation of Some Typical Dynamical Systems

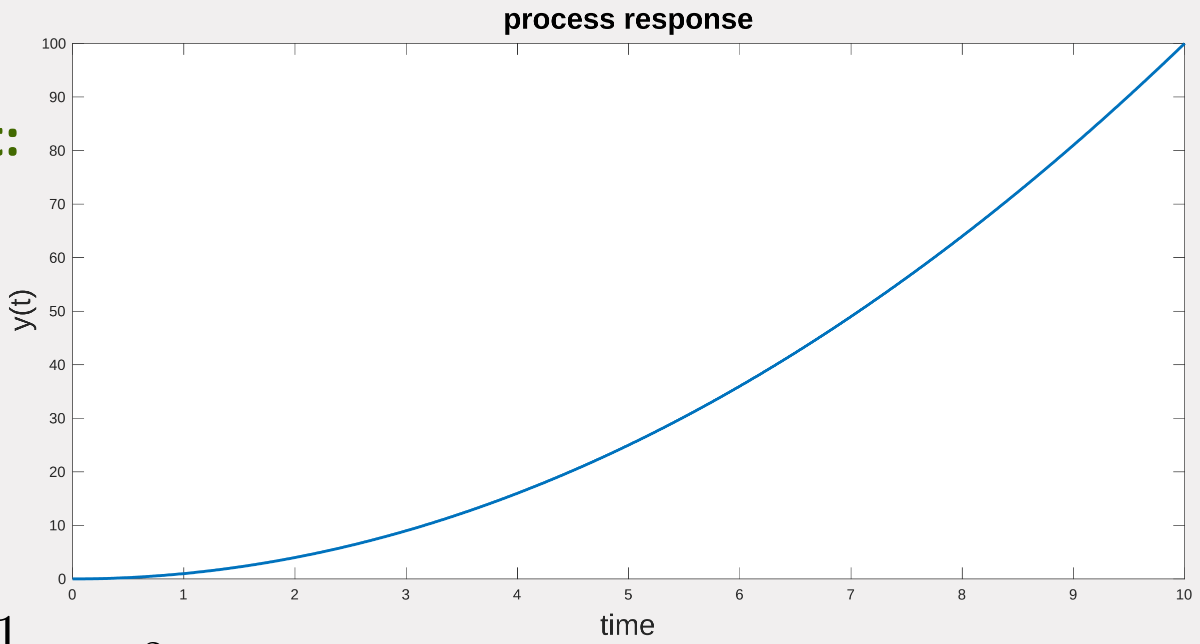
Integrating Systems:

Response to ramp input:

$$U(s) = \frac{A}{s^2}$$

$$Y(s) = \frac{K}{s}\frac{A}{s^2}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{KA}{s^3}\right\} = \frac{1}{2}KAt^2$$



Transfer Function Representation of Some Typical Dynamical Systems

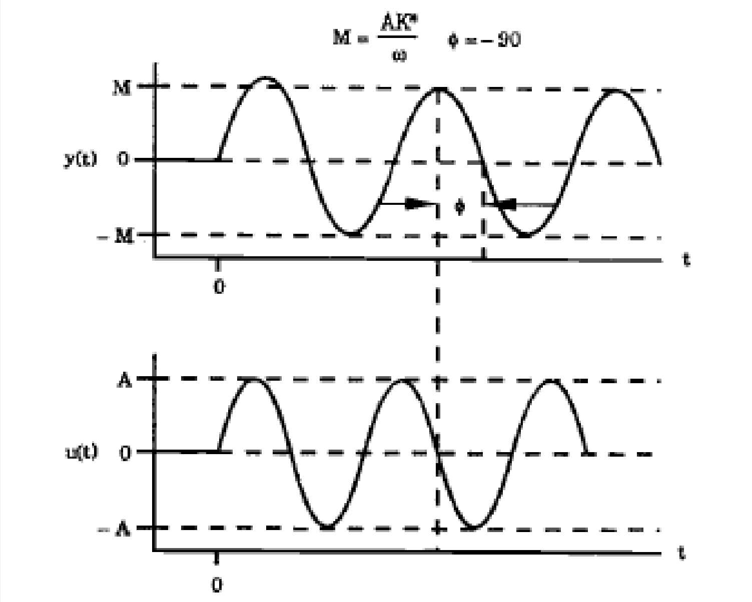
Integrating Systems:

Response to sinusoidal input:

$$U(s) = \frac{A\omega}{(s^2+\omega^2)}$$

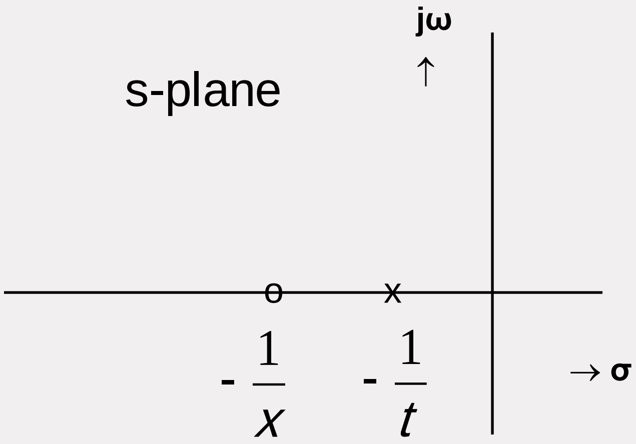
$$Y(s) = \frac{K}{s}\frac{A\omega}{(s^2+\omega^2)}$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t) = \frac{KA}{\omega}\left[1+\sin(\omega t - 90)\right]$$



Transfer Function Representation of Some Typical Dynamical Systems

Lead –Lag Systems:



$$G(s) = \frac{\xi s + 1}{\tau s + 1} = G_1(s).G_2(s)$$

$$G_1(s) = \xi s + 1 \qquad G_2 = \frac{1}{\tau s + 1}$$

$$Y_1(s) = \frac{\omega}{(s^2 + \omega^2)}(\xi s + 1) \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\Leftrightarrow}} y_1(t) = \omega\xi \cdot \cos \omega t + \sin \omega t$$

Phase lead

$$y_t = \sqrt{\omega^2\xi^2 + 1}\left(\frac{\omega\xi}{\sqrt{\omega^2\xi^2 + 1}}\cos(\omega t) + \frac{1}{\sqrt{\omega^2\xi^2 + 1}}\sin(\omega t)\right) = \sqrt{\omega^2\xi^2 + 1}\sin(\omega t + \varphi)$$

With: $\varphi = \tan^{-1}(\omega\xi)$

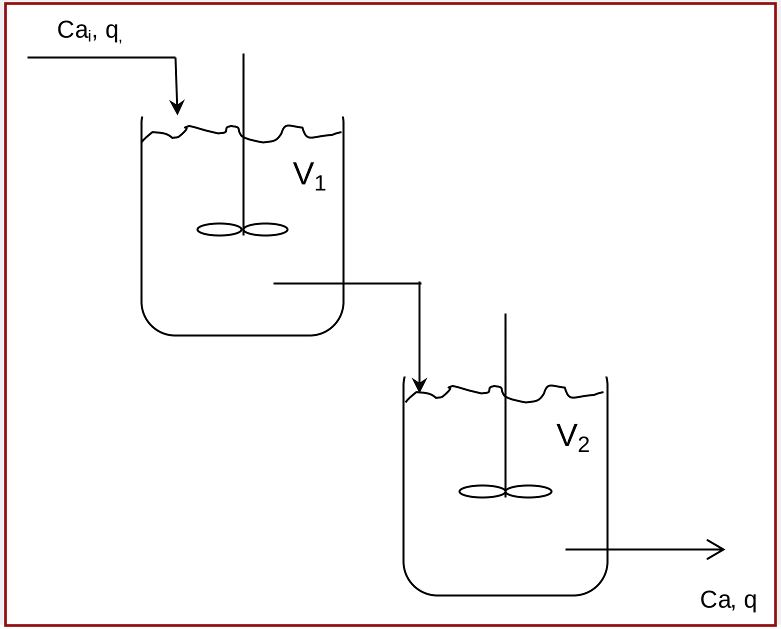
Transfer Function Representation of Some Typical Dynamical Systems

Second Order Systems:

$$V_1 \frac{dC_{a,1}}{dt} = qC_{a,i} - qC_{a,1}$$

$$V_2 \frac{dC_a}{dt} = qC_{a,1} - qC_a$$

$$\frac{C_a(s)}{C_{a,i}} = \frac{1}{\left(\frac{V_1}{q}s + 1\right)\left(\frac{V_2}{q}s + 1\right)}$$



Transfer Function Representation of Some Typical Dynamical Systems

Second Order Systems:

General Representation:

$$\tau^2 \frac{d^2 y(t)}{dt^2} + 2\xi\tau \frac{dy(t)}{dt} + y(t) = Ku(t)$$

- Naturally arises from two first order processes in series.
- Processes with a controller (feedback)

Transfer Function:

$$\boxed{\frac{Y(s)}{U(s)} = \frac{K}{\tau^2 s^2 + 2\xi\tau s + 1}}$$

Gain

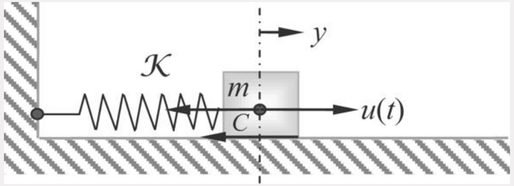
Damping coefficient

Time constant

Transfer Function Representation of Some Typical Dynamical Systems

Second Order Systems:

Mass Spring Systems:



$$m\frac{d^2y(t)}{dt^2} + C\frac{dy}{dt} + \mathcal{K}y(t) = u(t)$$

$$\Big\downarrow \mathcal{L}$$

$$\left(ms^2 + Cs + \mathcal{K}\right) Y(s) = U(s)$$

$\mathcal{K}$ Stiffness coefficient

C Friction coefficient

Transfer Function:

$$\frac{Y(s)}{U(s)} = \frac{1}{(ms^2 + Cs + \mathcal{K})}$$

$$\frac{Y(s)}{U(s)} = \frac{K}{(\tau^2 s^2 + 2\xi\tau s + 1)}$$

$$K = \frac{1}{\mathcal{K}} \qquad \xi = \frac{C}{2\sqrt{m\mathcal{K}}} \qquad \tau = \sqrt{\frac{m}{\mathcal{K}}}$$

Transfer Function Representation of Some Typical Dynamical Systems

Second Order Systems:

Roots (Poles of transfer function)

$$s_{1,2} = \frac{-2\xi\tau \pm \sqrt{4\xi^2\tau^2 - 4\tau^2}}{2\tau^2}$$

$$s_{1,2} = -\frac{\xi}{\tau} \pm \frac{1}{\tau}\sqrt{\xi^2 - 1}$$

Transient Response: Depends on the roots

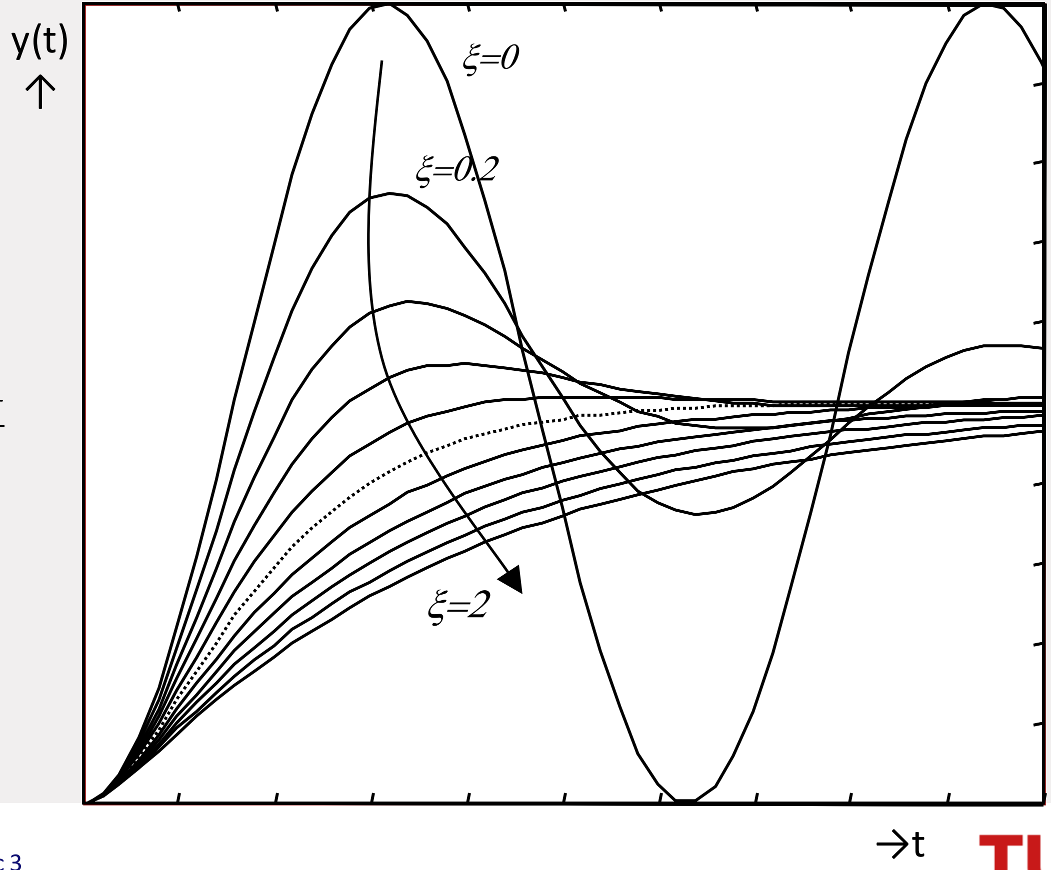
$\xi > 0$	Real parts of the roots negative (stable)
$\xi > 1$	Two distinct real roots (overdamped)
$\xi = 1$	repeated real roots (critically damped)
$0 < \xi < 1$	Complex roots (underdamped response)
$\xi = 0$	pure imaginary roots (undamped)

Transfer Function Representation of Some Typical Dynamical Systems

Second Order Systems:

Step Response:

$$Y(s) = \frac{K}{(\tau^2 s^2 + 2\xi\tau s + 1)}\frac{A}{s}$$



Transfer Function Representation of Some Typical Dynamical Systems

Second Order Systems:

Sinusoidal response:

$$Y(s) = \frac{K}{\tau^2 s^2 + 2\xi\tau s + 1} \frac{A\omega}{(s^2 + \omega^2)}$$

Ultimate response:

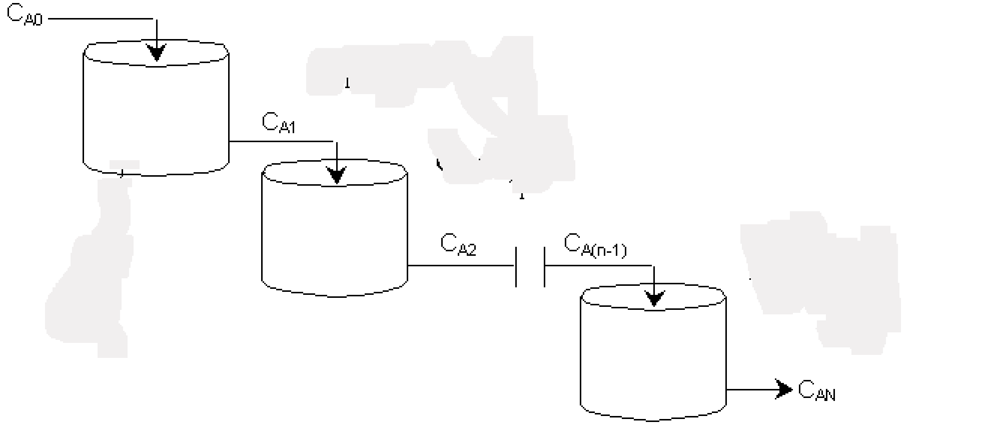
$$y(t)|_{t\to\infty} = \frac{KA}{\sqrt{\left[1-(\omega\tau)^2\right]^2}} \sin\left(\omega\tau + \varphi\right)$$

$$\varphi = -\tan^{-1}\left[\frac{2\xi\omega\tau}{1-(\omega\tau)^2}\right]$$

TU/e

Transfer Function Representation of Some Typical Dynamical Systems

High Order Systems:



$$V_1 \frac{dC_{A,1}}{dt} = qC_{A,0} - qC_{A,1}$$

$$V_2 \frac{dC_{A,2}}{dt} = qC_{A,1} - qC_{A,2}$$

$$\vdots$$

$$V_N \frac{dC_{A,N}}{dt} = qC_{A,N-1} - qC_{A,N}$$

$$\frac{C_{A,N}(s)}{C_{A,0}(s)} = \frac{1}{\left(\frac{V_1}{q}s+1\right)\left(\frac{V_2}{q}s+1\right)\cdots\left(\frac{V_N}{q}s+1\right)}$$

Transfer Function Representation of Some Typical Dynamical Systems

High Order Systems:

General Representation:

$$\frac{Y_N(s)}{U(s)} = \left(K \prod_{i=1}^{N} \frac{1}{\tau_i s + 1} \right)$$

N first order systems in series

General Nth Representation:

N^{th} order system with zeros

$$\frac{Y_N(s)}{U(s)} = \frac{(\xi_1 s + 1)(\xi_2 s + 1)(\xi_3 s + 1)\cdots(\xi_M s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)\cdots(\tau_N s + 1)} = \frac{K \prod_i^M (\xi_i s + 1)}{\prod_{j=1}^N (\tau_j s + 1)}$$

Transfer Function Representation of Some Typical Dynamical Systems

High Order Systems:

Step response of Nth order system in series

Partial fractions

$$Y_N(s) = \left(K \prod_{i=1}^{N} \frac{1}{\tau_i s + 1} \right) \cdot \frac{1}{s}$$

$$Y_N(s) = K \cdot \left[\frac{A_0}{s} + \sum_{i=1}^{N} \frac{A_i}{\tau_i s + 1} \right]$$

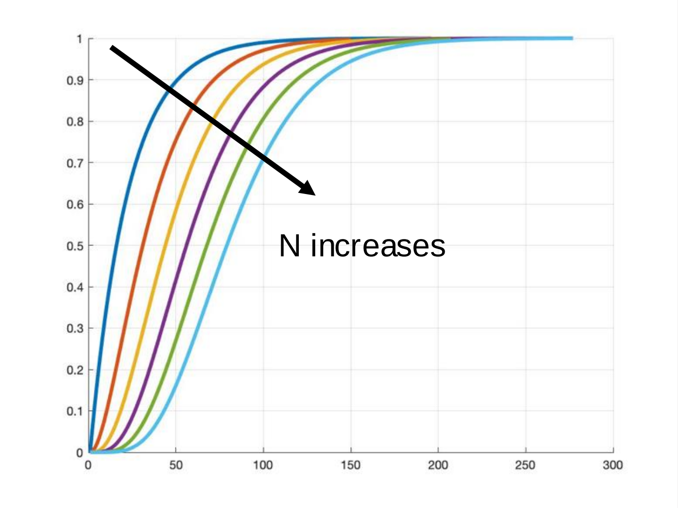
$$A_o = 1; \quad A_i = \lim_{s \to -\frac{1}{\tau_i}} \left[\frac{(\tau_i s + 1) \cdot Y_N(s)}{K} \right]$$

$$y_N(t) = K \cdot \left[1 + \sum_{i=1}^{N} \frac{A_i}{\tau_i} \cdot e^{-\frac{t}{\tau_i}} \right]$$

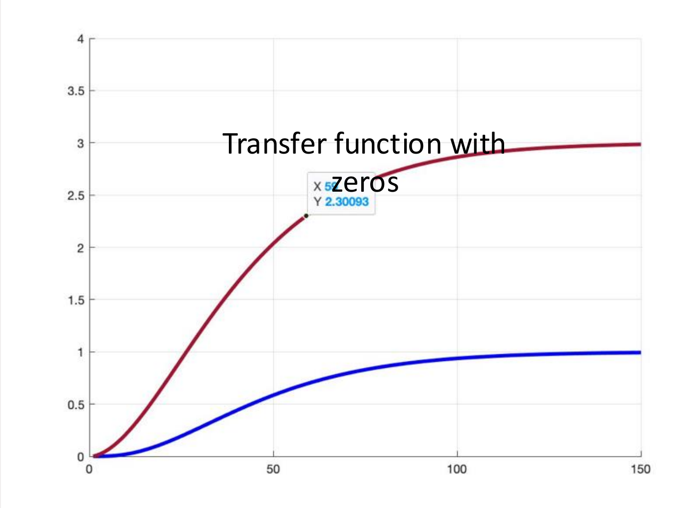
TU/e

Transfer Function Representation of Some Typical Dynamical Systems

High Order Systems:



Step Response



Step Response of high order systems with zeros

Transfer Function Representation of Some Typical Dynamical Systems

Systems with Time Delay (Deadtime):

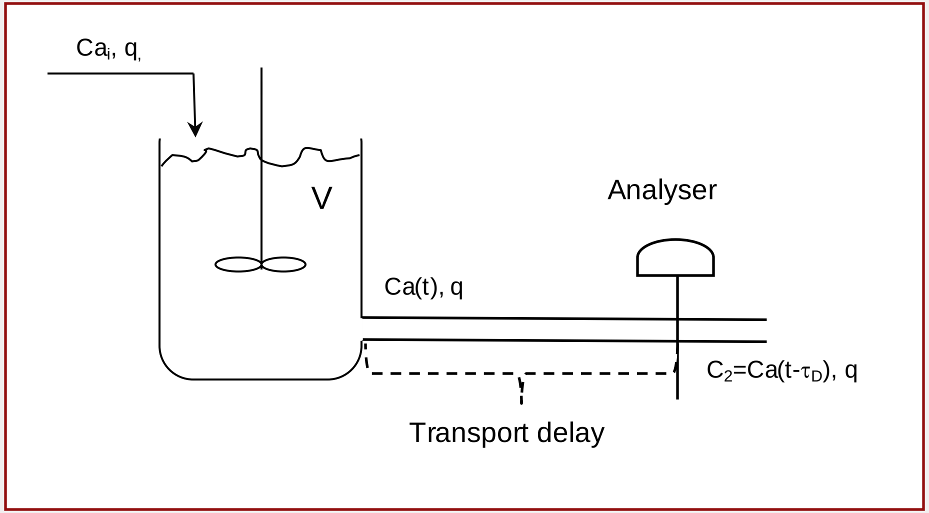
Example: A tank with transport delay at the outlet

$$V\frac{dC_a}{dt} = qC_{a,i} - qC_a$$

$$\frac{C_a(s)}{C_{a,i}(s)} = \frac{q}{Vs+q} = \frac{1}{\frac{V}{q}s+1}$$

$$C_2(t) = C_a(t-\tau_D)$$

$$C_2(s) = e^{-\tau_D s}C_a(s)$$

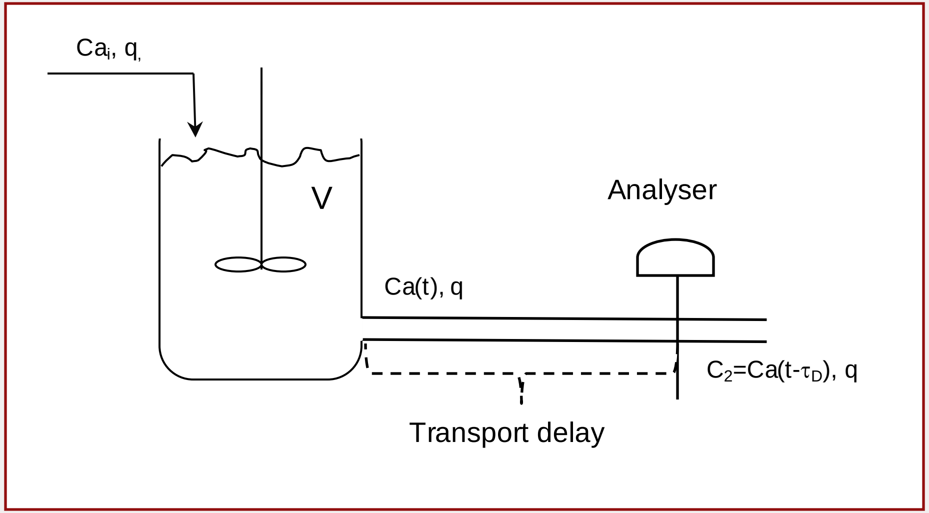


Transfer Function Representation of Some Typical Dynamical Systems

Systems with Time Delay (Deadtime):

Transfer function between $C_{a,i}$ and C_2

$$\frac{C_2(s)}{C_{a,i}(s)} = e^{-\tau_D s} \frac{1}{\frac{V}{q}s + 1}$$



Transfer Function Representation of Some Typical Dynamical Systems

Laplace Transform of Systems with Time Delay (Deadtime):

$$h(t) = \begin{cases} 0 & t < \tau_D \\ f(t-\tau_D) & t >\geq \tau_D \end{cases}$$

$$\mathcal{L}\{h(t)\} = \int_0^{\tau_D} e^{-st} \cdot 0 \; dt + \int_{\tau_D}^{\infty} f(t-\tau_D) \cdot e^{-st} dt$$

$$\mathcal{L}\{h(t)\} = 0 + \int_0^{\infty} f(l) \cdot e^{-s(l+\tau_D)} dl$$

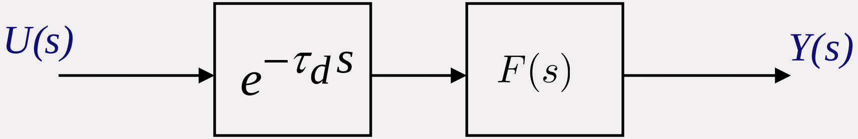
$$= e^{-s\tau_D} \underbrace{\int_0^{\infty} f(l) \cdot e^{-sl} dl}_{F(s)}$$

TU/e

Transfer Function Representation of Some Typical Dynamical Systems

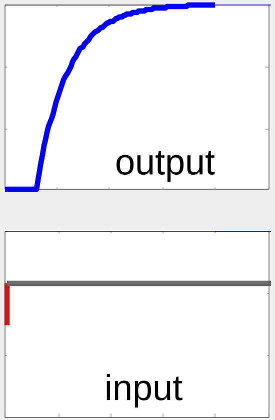
Laplace Transform of Systems with Time Delay (Deadtime):

Transfer Function representation



Deadtime is the amount of time that elapses between input change and process response

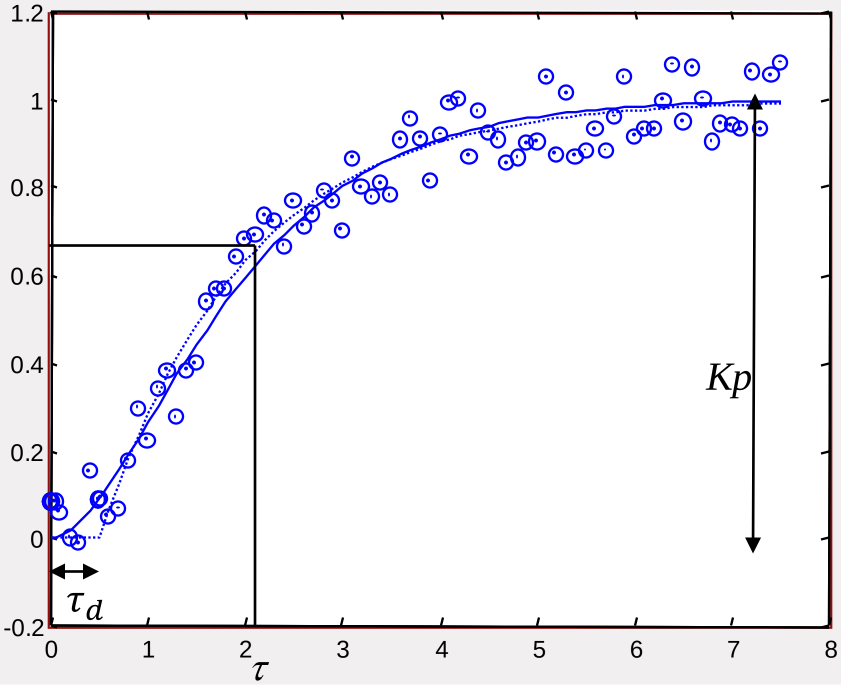
Step Response:



Transfer Function Representation of Some Typical Dynamical Systems

Complex processes can be approximated as first order process with deadtime

$$G(s) = \frac{K_p}{\tau s + 1} e^{-\tau_D s}$$

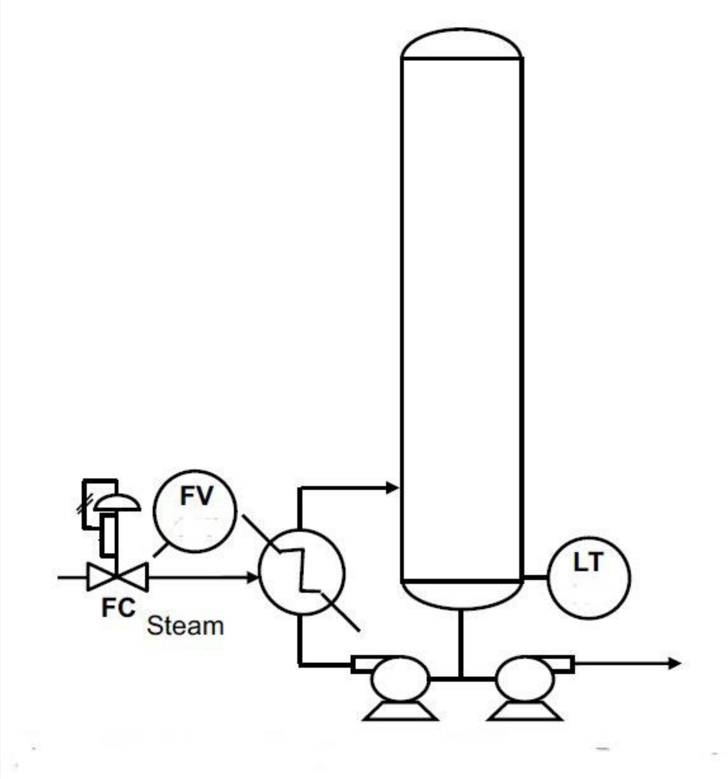


Transfer Function Representation of Some Typical Dynamical Systems

Inverse Response Systems

Example: the level in the bottom of the distillation

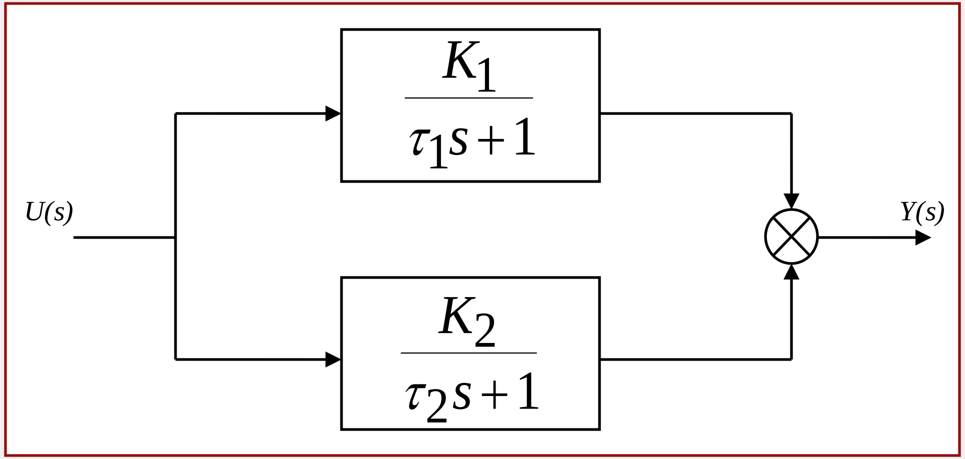
- The bottom level of the column illustrated may show an inverse response to a step change increase in heat input



Transfer Function Representation of Some Typical Dynamical Systems

Inverse Response Systems

- Can result from two processes in parallel



$$\frac{Y(s)}{U(s)} = H(s) = \frac{K_1}{\tau_1 s + 1} + \frac{K_2}{\tau_2 s + 1}$$

$$H(s) = \frac{K_1(\tau_2 s + 1) + K_2(\tau_1 s + 1)}{(\tau_1 s + 1) \cdot (\tau_2 s + 1)}$$

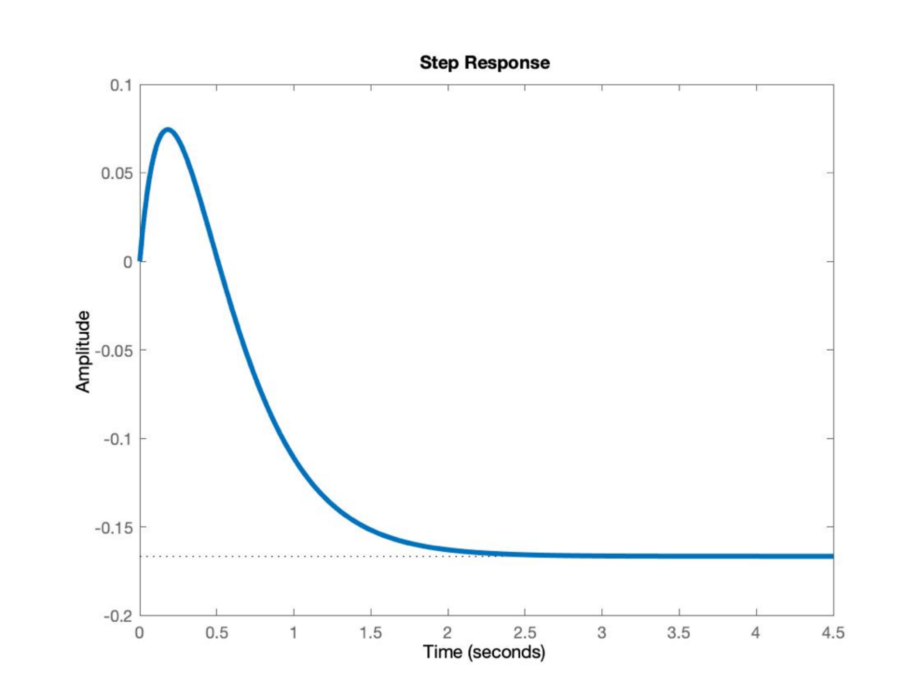
$$H(s) = \frac{(K_1\tau_2 + K_2\tau_1)s + (K_2 + K_1)}{(\tau_1 s + 1) \cdot (\tau_2 s + 1)}$$

- It is observed when the zero lies in the right half of the s-plane

Transfer Function Representation of Some Typical Dynamical Systems

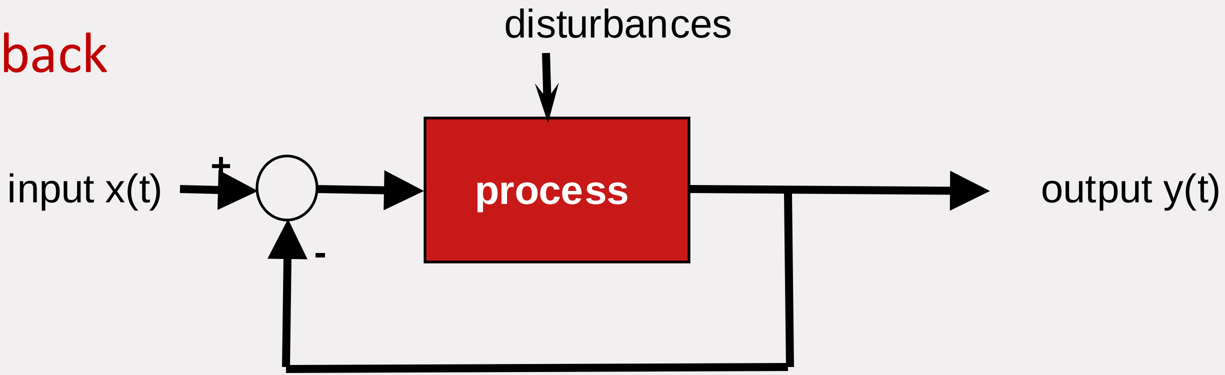
Inverse Response Systems

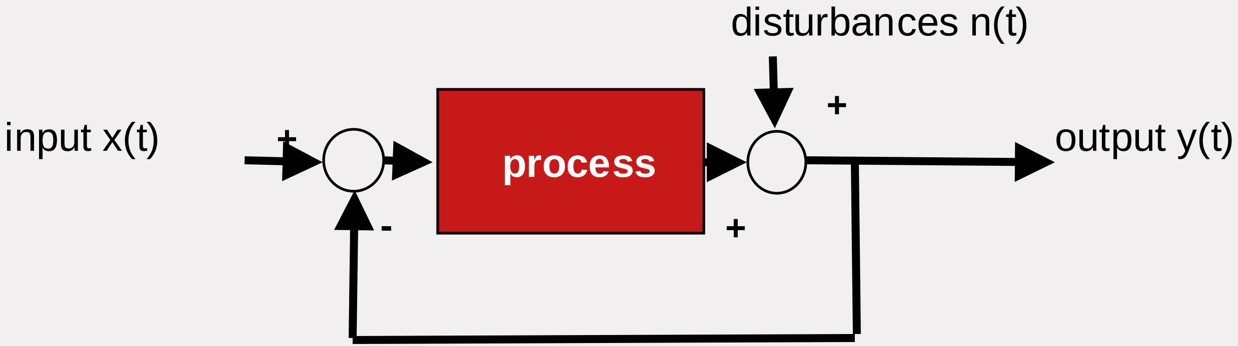
Step Response:



Linear systems

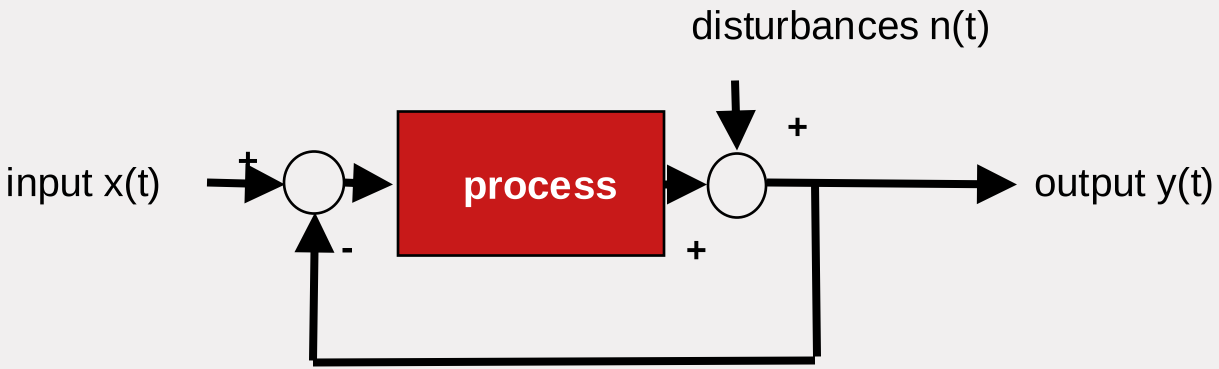
Feedback





Linear systems

Feedback

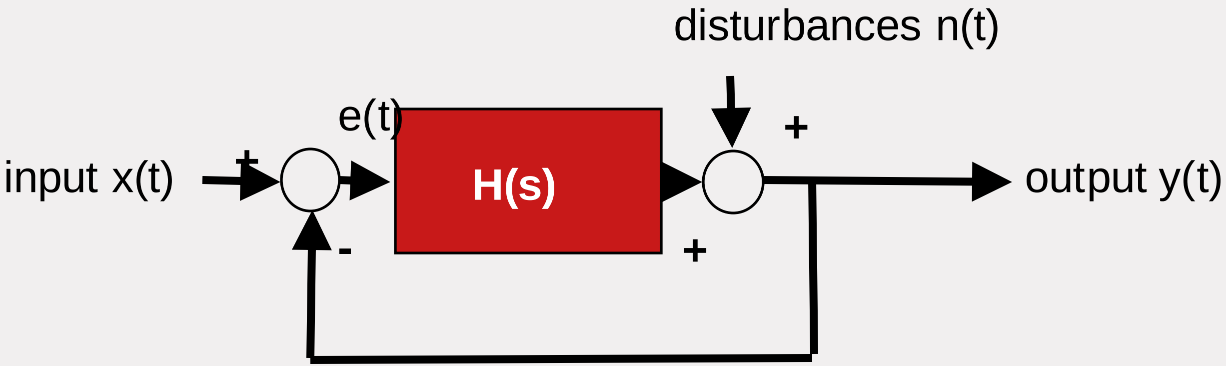


why feedback?

- stability
- change of dynamic behaviour
- reduction of influence of disturbance on output tracking

Linear systems

Feedback



$$Y(s) = H(s)E(s) + N(s) = H(s)\left[X(s) - Y(s)\right] + N(s)$$

$$Y(s) = \frac{H(s)}{1+H(s)}X(s) + \frac{1}{1+H(s)}N(s)$$

Linear systems

Closed loop system:

Open loop transfer system: $$H(s) = \frac{Num(s)}{Den(s)}$$

Closed loop transfer system: $$H_{cl}(s) = \frac{H(s)}{1+H(s)} = \frac{\frac{Num(s)}{Den(s)}}{1+\frac{Num(s)}{Den(s)}} = \frac{Num(s)}{Den(s)+Num(s)}$$

$$\text{zeros of } \; H_{cl}(s): \quad \text{zeros of } Num(s) = 0$$

$$\text{poles of } \; H_{cl}(s): \quad \text{poles of } Den(s) + Num(s) = 0$$

Poles are different than the poles of H(s)

The dynamic behaviour of transfer function H(s) is changed by feedback!

Linear systems

Example:

Open loop Process: $Y(s) = H(s)X(s) + N(s)$

Assume $H(s) = \dfrac{1}{s+1}, \quad X(s) = \dfrac{1}{s}, \quad N(s) = \dfrac{1}{s}$

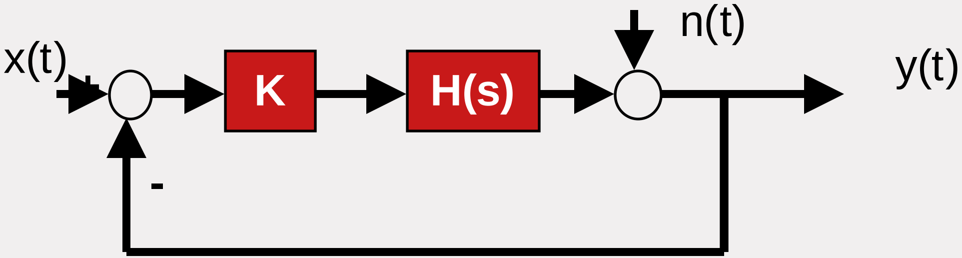
Output $Y(s) = \dfrac{1}{s+1} \cdot \dfrac{1}{s} + \dfrac{1}{s}$

Final Value of $y(t)$

$$y(\infty) = \lim_{s\to 0} (sY(s)) = \lim_{s\to 0} \left(s\frac{1}{s+1}\frac{1}{s} + s\frac{1}{s} \right) = 1 + 1 = 2$$

Linear systems

Feedback



$$Y(s) = H_{xy,cl}(s)X(s) + H_{ny,cl}N(s) = \frac{KH(s)}{1+KH(s)}X(s) + \frac{1}{1+KH(s)}N(s)$$

Linear systems

Closed loop Transfer Function: $H(s) = \dfrac{1}{s+1}$

from input $x(t)$ to output $y(t)$

$$H_{xy,cl} = \frac{KH(s)}{1+KH(s)} = \frac{\frac{K}{s+1}}{1+\frac{K}{s+1}} = \frac{K}{s+1+K}$$

no zero; pole: $-(1+K)$, **Faster response for** $K > 0$

response to input step, final value

$$y_{xy}(\infty) = \lim_{s\to 0}\left(sY_{xy}(s)\right) = \lim_{s\to 0}\left(s\frac{K}{s+1+K}\cdot\frac{1}{s}\right) = \frac{K}{1+K}$$

Linear systems

Closed loop Transfer Function: $$H(s) = \frac{1}{s+1}$$

from disturbance $n(t)$ to output $y(t)$

$$Y_{ny} = H_{ny,cl}N(s) = \frac{1}{1+KH(s)}N(s) = \frac{1}{1+\frac{K}{s+1}} \cdot \frac{1}{s}$$

response to disturbance step, final value

$$y_{ny}(\infty) = \lim_{s\to 0}\left(sY_{ny(s)}\right) = \lim_{s\to 0}\left(s\frac{s+1}{s+1+K}\cdot\frac{1}{s}\right) = \frac{1}{1+K}$$