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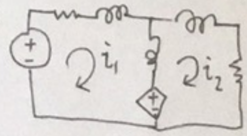
1. (a) there are three energying storing elements and an inductive cutset, so the order of the circuit and the number of natural frequencies is 2.

(b)



since $r_m = 5$, in time domain: $\begin{bmatrix} 2D+6 & -D \\ -D-5 & 2D+1 \end{bmatrix} \begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. in laplace domain:

$$\begin{bmatrix} 2s+6 & -s \\ -s-5 & 2s+1 \end{bmatrix} \begin{bmatrix} i_1(s) \\ i_2(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

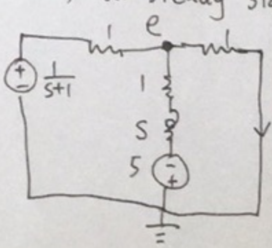
(c) $\det = 3s^2+9s+6 = 0$ $s_1 = -1$, $s_2 = -2$ so natural frequencies are -1 and -2.

(d) $I_{M_2}(s) = \dfrac{\det \begin{bmatrix} 2s+6 & V_1(s) \\ -s-5 & 0 \end{bmatrix}}{3s^2+9s+6} = \dfrac{(s+5)V_1(s)}{3s^2+9s+6}$ $\dfrac{V_2(s)}{V_1(s)} = \dfrac{I_{M_2}(s)}{V_1(s)} = \dfrac{s+5}{3s^2+9s+6}$

(e) zeros: $s_1 = -5$, $s_2 = \infty$ poles: $s_1 = -1$, $s_2 = -2$

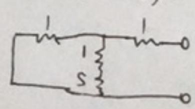
(f) $V_2(s) = \dfrac{s+5}{3s^2+9s+6} = \dfrac{4}{3(s+1)} - \dfrac{1}{s+2}$ $V_2(t) = \mathcal{L}^{-1}\{V_2(s)\} = \left(\frac{4}{3}e^{-t} - e^{-2t}\right)u(t)$

2. (a) at steady state, $i_L(0^-) = 5A$, $V_c(0^-) = 5V$



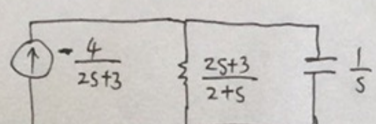
node analysis: $\dfrac{1}{s+1} - e(s) = \dfrac{e(s)+5}{1+s} + e(s)$ $e(s) = -\dfrac{4}{2s+3}$

$I_{sc}(s) = e(s) = -\dfrac{4}{2s+3}$



$R_{nor} = 1 + \dfrac{s+1}{s+2} = \dfrac{2s+3}{s+2}$

overall equivalent:



(b) $I_c(s) = -\dfrac{4}{2s+3} \times \dfrac{\frac{2s+3}{s+2}}{\frac{2s+3}{s+2} + \frac{1}{s}} = -\dfrac{4}{3s+5}$ $V_c(s) = \frac{1}{s} \cdot I_c(s) = \frac{4}{5} \cdot \dfrac{1}{s+\frac{5}{3}} - \frac{4}{5} \cdot \frac{1}{s}$

$V_c(t) = \mathcal{L}^{-1}\{V_c(s)\} = \left(\frac{4}{5}e^{-\frac{5}{3}t} - \frac{4}{5}\right)u(t)$

3. (a) $\dfrac{V_2(s)}{V(s)} = \dfrac{-I_2(s)}{I_1(s) + V_1(s)} = \dfrac{-I_2(s)}{\frac{2s+3}{s+1}I_1(s) + \frac{1}{s+1}I_2(s)}$ since $V_2(s) = \dfrac{1}{s+1}I_1(s) + \dfrac{s+2}{s+1}I_2(s) = -I_2(s)$,

$I_1(s) = -(2s+3)I_2(s)$. so $\dfrac{V_2(s)}{V(s)} = \dfrac{-I_2(s)}{-\frac{(2s+3)^2}{s+1}I_2(s) + \frac{1}{s+1}I_2(s)} = \dfrac{s+1}{4s^2+12s+8}$

(b) $V_2(s) = \dfrac{s+1}{4s^2+12s+8} \times \dfrac{1}{s} = \frac{1}{8} \cdot \frac{1}{s} - \frac{1}{8} \cdot \frac{1}{s+2}$ $V_2(t) = \mathcal{L}^{-1}\{V_2(s)\} = \left(\frac{1}{8} - \frac{1}{8}e^{-2t}\right)u(t)$