EE 110 Homework 2 Name: Xilai Zhang

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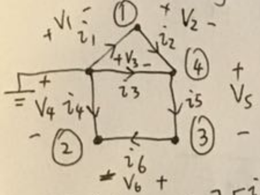
1. a. I labeled my nodes differently in the oriented graph.

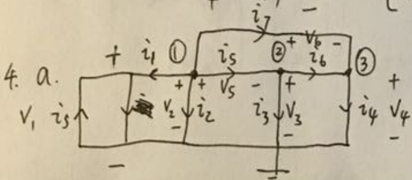


b. $$\begin{array}{c} ① \\ ② \\ ③ \\ ④ \end{array}\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & -1 & -1 & 0 & 1 & 0 \end{bmatrix}$$

c. $$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & -1 & -1 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = 0 \qquad \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix}$$

d. cutset: $i_4 + i_5 = 0$ KCL: $\begin{cases} -i_1 + i_2 = 0 \cdots ① \\ i_1 + i_3 + i_4 = 0 \cdots ② \\ -i_2 - i_3 + i_5 = 0 \cdots ③ \end{cases}$ $① + ② + ③ = i_4 + i_5 = 0 =$ cutset eq.

4. a.



b. $$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix} = 0 \qquad \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix}$$

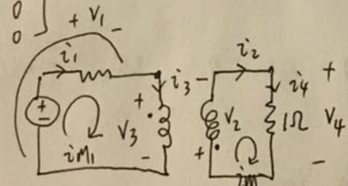
c. ①: $jC_1 w e_1 + G_1 e_1 - i_s + jC_3 w (e_1 - e_3) + \frac{1}{jL_1 w}(e_1 - e_2) = 0$

② $\frac{1}{jL_1 w}(e_2 - e_1) + \frac{1}{jL_2 w}(e_2 - e_3) + jC_2 w e_2 = 0$

③ $jC_3 w (e_3 - e_1) + \frac{1}{jL_2 w}(e_3 - e_2) + G_2 e_3 = 0$

$$\Rightarrow \begin{bmatrix} G_1 + jC_1 w + jC_3 w + \frac{1}{jL_1 w} & -\frac{1}{jL_1 w} & -jC_3 w \\ -\frac{1}{jL_1 w} & \frac{1}{jL_1 w} + \frac{1}{jL_2 w} + jC_2 w & -\frac{1}{jL_2 w} \\ -jC_3 w & -\frac{1}{jL_2 w} & jC_3 w + \frac{1}{jL_2 w} + G_2 \end{bmatrix} \cdot \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} I_s \\ 0 \\ 0 \end{bmatrix}$$

6. a.



$M = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ $Z_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2D & D & 0 \\ 0 & D & D & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $Z_m = M Z_b M^T =$

$\begin{bmatrix} D+1 & D \\ D & 2D+1 \end{bmatrix}$ $Z_m \cdot i_m = -M \cdot V_s$ $i_m = Z_m^{-1}(-M \cdot V_s) = \frac{1}{D^2+3D+1}\begin{bmatrix} 2D+1 & -D \\ -D & D+1 \end{bmatrix}\begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{2D+1}{D^2+3D+1} V_s \\ -\frac{D}{D^2+3D+1} V_s \end{bmatrix}$

$D = jw = 2j$, so $\begin{bmatrix} i_{M_1} \\ i_{M_2} \end{bmatrix} = \begin{bmatrix} \frac{1+4j}{6j-3} V_s \\ \frac{-2j}{6j-3} V_s \end{bmatrix}$

b. $\frac{V_s}{I_{M_1}} = \frac{6j-3}{4j+1} = \frac{18j+21}{17}$

7. a. $\mathcal{L}\{e^{-t}\cos t + te^{-t}\} = \frac{s+1}{(s+1)^2+1} + \frac{1}{(s+1)^2}$

b. $\mathcal{L}\{t^2-2t+1\} = \frac{2}{s^3} - \frac{2}{s^2} + \frac{1}{s}$

c. $\mathcal{L}\{e^{-\alpha t} f(t)\} = F(s+\alpha)$

8. a. $1 + \frac{2s+5}{s^2+4s+3} = 1 + \frac{1.5}{s+1} + \frac{0.5}{s+3}$ $\quad \mathcal{L}^{-1}\left(1 + \frac{1.5}{s+1} + \frac{0.5}{s+3}\right) = \delta(t) + 1.5e^{-t} + 0.5e^{-3t}$

b. $\frac{1}{s(s+1)^2} = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}$ $\quad \mathcal{L}^{-1} = 1 - e^{-t} - te^{-t}$

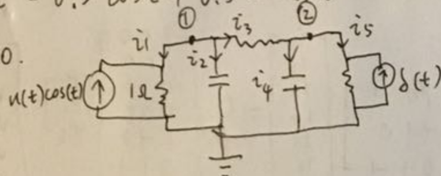
C. $\frac{1}{(s+1)(s^2+s+1)} = \frac{1}{s+1} + \frac{-s}{s^2+s+1} = \frac{1}{s+1} - \frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}} + \frac{2\sqrt{3}}{3} \cdot \frac{\frac{\sqrt{3}}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}}$

$\mathcal{L}^{-1} = e^{-t} - e^{-\frac{1}{2}t} \cos\frac{\sqrt{3}}{2}t + \frac{2\sqrt{3}}{3} e^{-\frac{1}{2}t} \sin\frac{\sqrt{3}}{2}t$

d. $\frac{0.5s}{s^2+1} + \frac{0.5s}{s^2+3} = \cancel{\frac{\frac{1}{2}(s+\frac{1}{2}) - \frac{1}{4}\frac{2}{\sqrt{3}}\frac{\sqrt{3}}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}} + \frac{}{(s+\frac{1}{2})}} = \frac{0.5s}{s^2+1} + \frac{0.5s}{s^2+(\sqrt{3})^2} =$

$\mathcal{L}^{-1} = 0.5\cos t + 0.5\cos\sqrt{3}t$

10.



$$\begin{bmatrix} 1+jc\omega+1 & -G_2 \\ -G_2 & 1+jc\omega+1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} u(t)\cos(t) \\ \delta(t) \end{bmatrix}$$

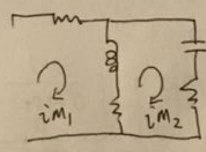
$$\begin{cases} (2+s)e_1(s) - e_2(s) = \frac{s}{s^2+1} \\ -e_1(s) + (s+2)e_2(s) = 1 \end{cases} \Rightarrow \begin{cases} e_1(s) = \frac{2s^2+2s+1}{(s^2+1)(s+1)(s+3)} \\ e_2(s) = \frac{s^3+2s^2+2s+2}{(s^2+1)(s+1)(s+3)} \end{cases} \Rightarrow \begin{cases} e_1(t) = \left[\frac{1}{10}(4\cos t + 3\sin t) + \frac{1}{4}e^{-t} - \frac{13}{20}e^{-3t}\right]u(t) \\ e_2(t) = \left[\frac{1}{10}(\cos t + 2\sin t) + \frac{1}{4}e^{-t} + \frac{13}{20}e^{-3t}\right]u(t) \end{cases}$$

$$\begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix} = \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix}$$

12. a. $|I| = \frac{V_s}{2} = \frac{2 \cdot e^{j \cdot 0}}{2} = 1$ $\quad \angle I = 0$ $\quad I_{(t)} = \cos(t)$

b. at sinusoidal steady state, $V_c = \frac{\sqrt{2}}{2}\cos(t-45^\circ)$, $V_c(0^-) = 0.5$. $\quad i_{M_2} = \frac{\sqrt{2}}{2}\cos(t+45^\circ)$

$i_{M_2}(0^-) = 0.5$



$$\begin{cases} i_{M_1} + L(i_{M_1} - i_{M_2})' + i_{M_1} - i_{M_2} = 4 \\ (i_{M_2} - i_{M_1}) + L(i_{M_2} - i_{M_1})' + \frac{1}{C}\int i_{M_2} + V_c(0^-) + i_{M_2} = 0 \end{cases}$$

C. $$\begin{cases} (s+2)I_{M_1}(s) - (s+1)I_{M_2}(s) = \frac{4}{s} + 0.5 \\ (-1-s)I_{M_1}(s) + (s+2+\frac{1}{s})I_{M_2}(s) = -\frac{0.5}{s} - 0.5 \end{cases} \Rightarrow \begin{cases} i_{M_1}(s) = \frac{2}{s} \\ i_{M_2}(s) = \frac{1.5}{s+1} \end{cases} \Rightarrow \begin{cases} i_{M_1}(t) = 2u(t) \\ i_{M_2}(t) = 1.5e^{-t}u(t) \end{cases}$$

d. $I(0^+) = 2A$. $\quad I(0^-) = 1A$.

2. $$\begin{cases} V_1(-\hat{i}_1) + V_2\hat{i}_2 + \sum V_k \cdot \hat{i}_k = 0 \\ \hat{V}_1 \cdot (-i_1) + \hat{V}_2 \cdot i_2 + \sum \hat{V}_k \cdot i_k = 0 \end{cases} \Rightarrow \begin{cases} 4 \times (-2) + 1 \times \frac{\hat{V}_2}{2} + \sum V_k \cdot \hat{i}_k = 0 \\ 5.5 \times (-1) + \hat{V}_2 \cdot 1 + \sum \hat{V}_k \cdot i_k = 0 \end{cases} \Rightarrow \hat{V}_2 = -5V$$