ECE110 Homework 1

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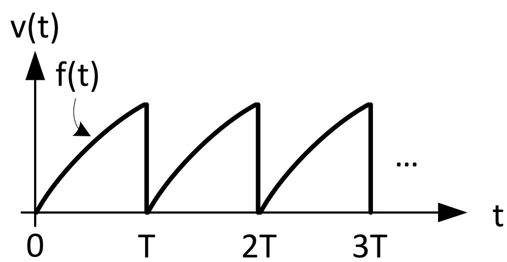
Sections covered: Review of ECE10

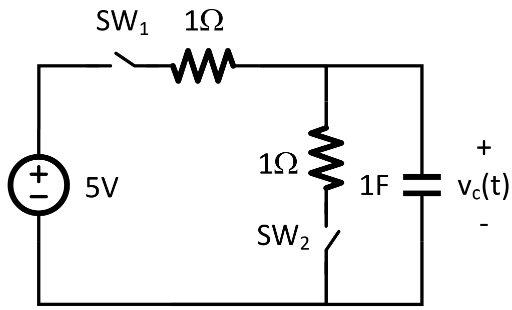
Total of 7 regular questions, 14 points each. The 5 optional problems have each 5 bonus points. Due: 6PM Friday of week 3.

1. The voltage waveform below ($v(t)$) is periodic, and is applied to a resistor R. $f(t) = 1 - e^{-t/T}$.
   a. Plot the energy of the resistor.
   b. Find the average power dissipated in the resistor in one cycle.

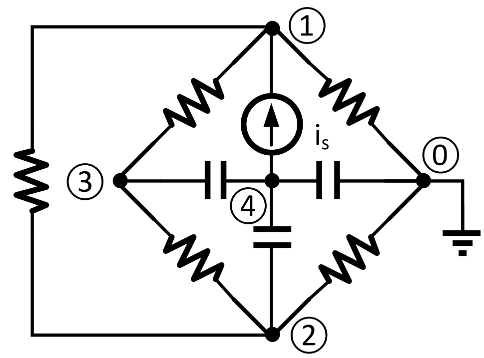


2. (Optional) In the circuit below, switch SW_1 is on between 0 and $\frac{T}{2}$, while SW_2 is off, and is off between $\frac{T}{2}$ and T, while SW_2 is on. The pattern repeats. Assume $T = 2S$.
   a. Plot the capacitor voltage $v_C(t)$ quantitatively.
   b. Find the steady state values.
   c. Find the average power delivered to the network by the 5V voltage source when the circuit reaches the steady state.

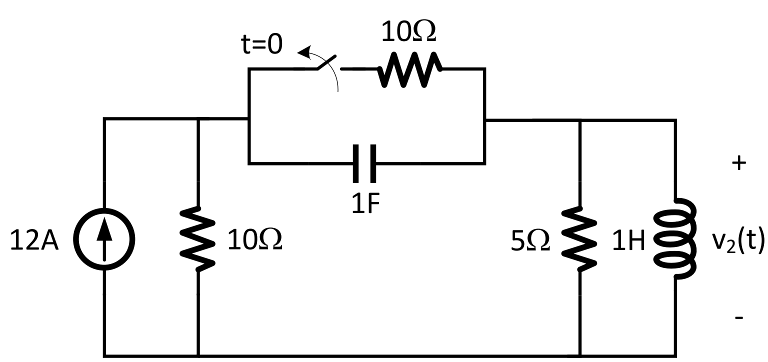
      Hint: Try to replace the source, switches, and resistors with a square-wave voltage source and one series resistance.



3. (Optional) In the network below, the datum node is indicated on the right. Assume all the resistors are 1Ω, and all the capacitors are 1F. Using node voltage analysis, write the corresponding differential equations (node voltages being the variables, i_s being the input) describing the network. You do not need to solve the equations.

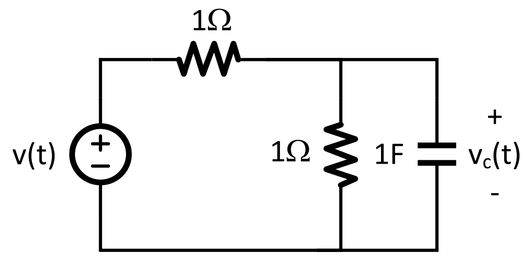


4. The circuit below has reached the steady state with the switch closed. At $t = 0$ the switch opens. Find $v_2(0^-)$, $v_2(0^+)$, and $\frac{dv_2}{dt}(0^+)$.

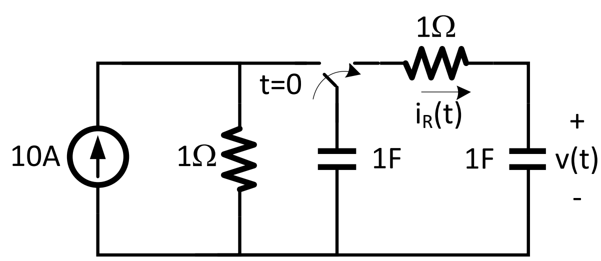


5. Consider the linear and time-invariant circuit below.
   a. Calculate the zero-input response of the circuit. Express your answer in terms of the capacitor initial voltage ($v_C(0)$).
   b. Find the zero-state response if $v(t) = Acos(\omega t + \varphi)$ by solving the differential equation.

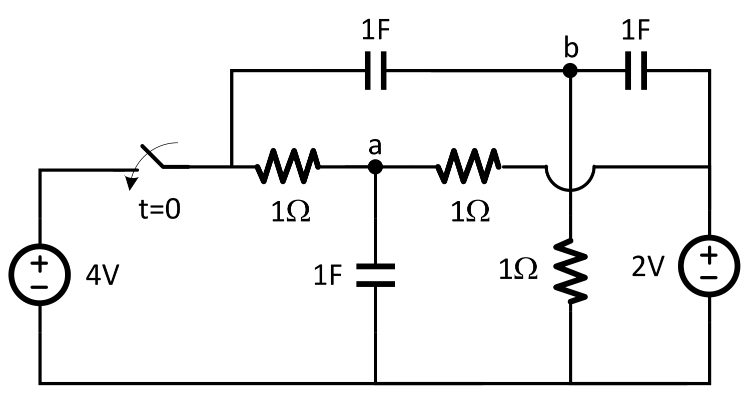
c. What is the steady-state capacitor voltage?
d. Redo part c using phasors.
e. Find the complete response, with $v_C(0) = 1V$, and $v(t) = cost$.



6. In the circuit below, the switch has been in the left position for a long time, and is flipped from the left to the right at $t = 0$. Assume $v(0^-) = 0$.
   a. Determine the order of the circuit (for $t > 0$).
   b. Find and plot the resistor current. **Hint**: Find $i_R(0^+)$, $i_R(\infty)$, and the time constant by inspection, and use the short-cut method to estimate the 1st-order circuits response. Is the circuit really 1st-order though?
   c. Find $v(t)$.
   d. What is the final voltage of the capacitor ($v(\infty)$)? Explain why $v(\infty)$ is not zero, despite the fact that the circuit is in zero-input mode (for $t > 0$).



7. (Optional) The circuit below has been idle for a long time. At $t = 0$, the switch is closed. Find the voltages $v_a(t)$, $v_b(t)$, and $v_{ab}(t)$. **Hint**: try to break the circuit into two independent (but 1st-order) pieces, and analyze each individually.

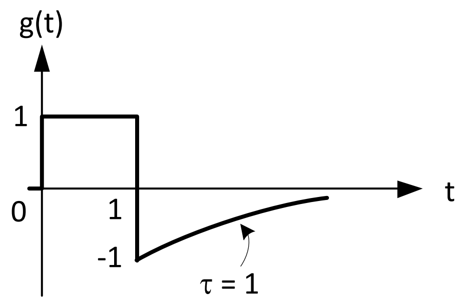


8. Find the solution of the following linear differential equations:

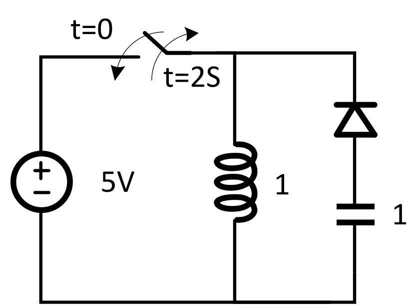
   a. $y^{(3)} + y = (cost)u(t)$, $\quad y(0^-) = 1$, $\quad y'(0^-) = 0$, $\quad y''(0^-) = 0$.

   b. $y'' + 2y' + y = \delta(t)$, $\quad y(0) = 0$, $\quad y'(0) = 0$.

   c. $y' + 2y = g(t)$, $\quad y(0^-) = 0$, where $g(t)$ is shown below:



9. (Optional) The circuit below is at zero state for $t < 0$. The switch closes at $t = 0$, and opens at $t = 2S$. Find and plot the inductor current and the capacitor voltage. The diode is ideal.

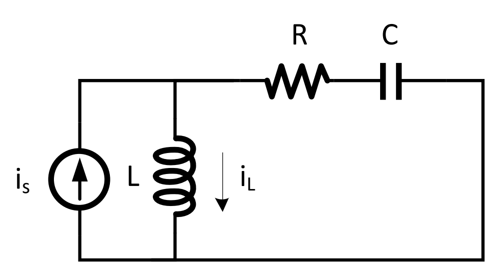


10. Given a series RLC circuit (below) with $\omega_0 = 1rad/s$, Q = 1/2,:

   a. Calculate the zero-input response of the circuit (i_L is the output). Express your answer in terms of $i_L(0)$ and $v_C(0)$.

   b. Find the impulse response by writing and solving the circuit differential equation.

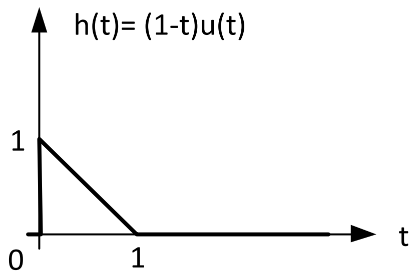
   c. Intuitively, what are the values of $i_L(0^+)$, and $i_L{}'(0^+)$?

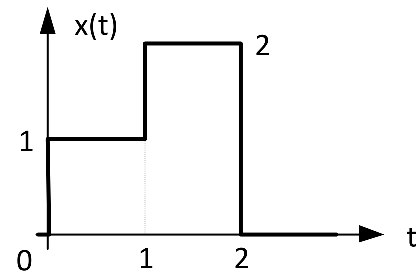


11. (Optional) The impulse response ($h(t)$) of an LTI circuit is shown on the left.

   a. Find the step response.

b. Find the response to the input ($x(t)$) shown on the right.





12. The following circuit is in sinusoidal steady state, and $v_s(t) = 2Cos2t$.

a. Find the Thevenin equivalent on the left side of nodes ab. **Hint**: Find the open circuit voltage by inspection, and the short circuit current by performing mesh analysis. Show that, $Z_{TH} = M^2 + j(8 - M^2 - \frac{1}{2C_2})$. Note that with $\begin{bmatrix} A & B \\ C & C \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \end{bmatrix}$, where I_1 and I_2 are the mesh currents, and A, B, C, D are complex numbers, $I_2 = \frac{-CV_s}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}} = \frac{-CV_s}{AD-BC}$.

b. Determine the optimum values for M, and C_2 to deliver the maximum power to the load Z_L.