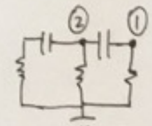
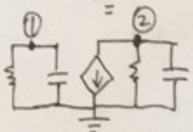
EE 110 Homework 3

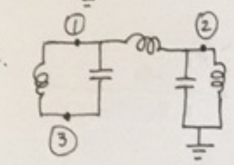
Name: Xilai Zhang UID: 804796478

1. 

node analysis, $Y_n = \begin{bmatrix} s+1 & -s \\ -s & 2s+2 \end{bmatrix}$ $\det[Y_n] = s^2+4s+2=0$ natural frequencies $s_{1,2} = -2 \pm \sqrt{2}$



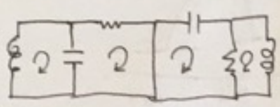
node analysis. $Y_n = \begin{bmatrix} 1+s & 0 \\ 1 & s+1 \end{bmatrix}$ $\det[Y_n] = s^2+2s+1$ natural frequencies $s_{1,2} = -1$

2. 

node analysis: $Y_n = \begin{bmatrix} 2s+\frac{2}{s} & -\frac{1}{s} & -2s-\frac{1}{s} \\ -\frac{1}{s} & 2s+\frac{2}{s} & 0 \\ -\frac{1}{s}-2s & 0 & \frac{1}{s}+2s \end{bmatrix}$ $\det[Y_n] = \frac{(2s^2+1)^2}{s^3} = 0$ natural frequencies

$s_{1,2} = \frac{\sqrt{2}}{2}j$

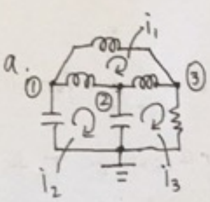
$s_{3,4} = -\frac{\sqrt{2}}{2}j$



mesh analysis: $Z_b = \begin{bmatrix} s+\frac{1}{s} & -\frac{1}{s} & 0 & 0 \\ -\frac{1}{s} & \frac{1}{s}+1 & 0 & 0 \\ 0 & 0 & \frac{1}{s}+2 & -2 \\ 0 & 0 & -2 & 2+s \end{bmatrix}$ $\det[Z_b] = 2s^2+\frac{2}{s^2}+3s+\frac{3}{s}+5=0$

natural frequencies $s_1 = -\sqrt[3]{-1}$

$s_2 = (-1)^{\frac{2}{3}}$ $s_{3,4} = -\frac{1}{4} \pm \frac{\sqrt{15}}{4}j$

3. a. 

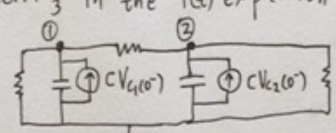
mesh analysis: $\begin{bmatrix} 3s & -s & -s \\ -s & s+\frac{2}{s} & -\frac{1}{s} \\ -s & -\frac{1}{s} & s+1+\frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} i_{L1}(0^-) - i_{L2}(0^-) - i_{L3}(0^-) \\ i_{L2}(0^-) + V_{C1}(0^-)/s - V_{C2}(0^-)/s \\ i_{L3}(0^-) + V_{C2}(0^-)/s \end{bmatrix}$

node analysis: $\begin{bmatrix} \frac{2}{s}+s & -\frac{1}{s} & -\frac{1}{s} \\ -\frac{1}{s} & s+\frac{2}{s} & -\frac{1}{s} \\ -\frac{1}{s} & -\frac{1}{s} & 1+\frac{2}{s} \end{bmatrix} \begin{bmatrix} e_1(s) \\ e_2(s) \\ e_3(s) \end{bmatrix} = \begin{bmatrix} -\frac{i_{L1}(0^-)}{s} - \frac{i_{L2}(0^-)}{s} + V_{C1}(0^-) \\ \frac{i_{L2}(0^-)}{s} - \frac{i_{L3}(0^-)}{s} + V_{C2}(0^-) \\ \frac{i_{L3}(0^-)}{s} + \frac{i_{L1}(0^-)}{s} \end{bmatrix}$

b. $\det[Z_n(s)] = \frac{(s+1)^2(s^2+3)}{s^2} = 0 \Rightarrow s_{1,2} = -1,\ s_{3,4} = \pm\sqrt{3}j$

$\det[Y_n(s)] = \frac{(s+1)^2(s^2+3)}{s^2} = 0 \Rightarrow s_{1,2} = -1,\ s_{3,4} = \pm\sqrt{3}j$ The natural frequencies are the same, also due to inductive loop, there is an extra natural frequency at 0.

c. $I_1(s) = \frac{I_0 \Delta_{11}}{Z_m(s)} = I_0 \frac{s^4+s^3+3s^2+2s+1}{s(s+1)^2(s^2+3)} \Rightarrow i(t) = \frac{I_0}{3} + k_1 e^{-t} + k_{12} t e^{-t} + k_3 \cos(\sqrt{3}t + \varphi_3)$ the constant term $\frac{I_0}{3}$ in the i(t) expression means that we have a natural frequency at $s=0$.

4. 

a. node analysis: $Y_n = \begin{bmatrix} s+2 & -1 \\ -1 & s+2 \end{bmatrix}$ $\det[Y_n] = 0 \Rightarrow s_1 = -1,\ s_2 = -3$

b. $e_2(s) = \frac{(s+2)V_{C2}(0^-) + V_{C1}(0^-)}{s^2+4s+3} = \frac{\frac{V_{C1}(0^-)+V_{C2}(0^-)}{2}}{s+1} + \frac{\frac{-V_{C1}(0^-)+V_{C2}(0^-)}{2}}{s+3}$ $e_1(s) = \frac{\frac{V_{C1}(0^-)+V_{C2}(0^-)}{2}}{s+1} + \frac{\frac{V_{C1}(0^-)-V_{C2}(0^-)}{2}}{s+3}$

to eliminate $s=-1$, let $V_{C1}(0^-) = -V_{C2}(0^-)$ would be enough.

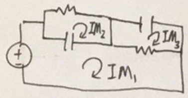
c. to eliminate $s=-3$, let $V_{C1}(0^-) = V_{C2}(0^-)$. $E = \frac{1}{2}C_1 V_{C1}(0^-)^2 + \frac{1}{2}C_2 V_{C2}(0^-)^2 = 1$ thus $V_{C1}(0^-) = V_{C2}(0^-) = 1V$

Name: Xilai Zhang
UID: 804796478

5. a. $H(s) = \mathcal{L}\{h(t)\} = 1 + \dfrac{(s+a)\cos\varphi - w\sin\varphi}{(s+a)^2 + w^2}$

b. $H(s) = \mathcal{L}\{h(t)\} = \dfrac{1}{s+1} - \dfrac{1}{(s+3)^2} + \dfrac{2}{s^2}$

c. $H(s) = \mathcal{L}\{h(t)\} = 1 + s$

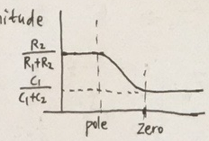
6. 

a. mesh analysis $\begin{bmatrix} \frac{1}{C_1 s} + R_2 & -\frac{1}{C_1 s} & -R_2 \\ -\frac{1}{C_1 s} & R_1 + \frac{1}{C_1 s} & 0 \\ -R_2 & 0 & \frac{1}{C_2 s} + R_2 \end{bmatrix} \begin{bmatrix} IM_1(s) \\ IM_2(s) \\ IM_3(s) \end{bmatrix} = \begin{bmatrix} V_1(s) \\ 0 \\ 0 \end{bmatrix}$

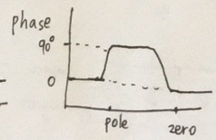
$$\frac{V_2(s)}{V_1(s)} = \frac{1}{C_2 s} IM_3(s) = \frac{R_1R_2C_1s + R_2}{R_1R_2C_1s + R_2 + R_1R_2C_2s + R_1} = \frac{R_1R_2C_1s + R_2}{(R_1R_2C_1 + R_1R_2C_2)s + R_1 + R_2}$$

zero: $-\dfrac{1}{R_1C_1}$ pole: $-\dfrac{R_1+R_2}{R_1R_2(C_1+C_2)}$

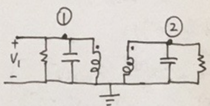
b. assume pole is smaller than zero, magnitude



phase



c. if $R_1C_1 = R_2C_2$, (let $R_1C_1 = R_2C_2 = x$, then) $G(s) = \dfrac{xR_2s + R_2}{xR_2s + R_2 + xR_1s + R_1} = \dfrac{R_2(xs+1)}{(R_1+R_2)(xs+1)} = \dfrac{R_2}{R_2+R_1} = \dfrac{\frac{C_1}{C_2}R_1}{\frac{C_1}{C_2}R_1 + R_1} = \dfrac{C_1}{C_1+C_2}$ for all s.

9. 

a. $L = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ $\Gamma = L^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ node analysis: $\begin{bmatrix} 1 + Cs + \Gamma_{11}\frac{1}{s} & \Gamma_{12}\frac{1}{s} \\ \Gamma_{21}\frac{1}{s} & \Gamma_{22}\frac{1}{s} + Cs + 1 \end{bmatrix} =$

$$\begin{bmatrix} 1 + s + \frac{1}{s} & -\frac{1}{s} \\ -\frac{1}{s} & \frac{2}{s} + s + 1 \end{bmatrix} \begin{bmatrix} e_1(s) \\ e_2(s) \end{bmatrix} = \begin{bmatrix} I_1(s) \\ 0 \end{bmatrix}$$

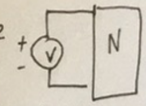
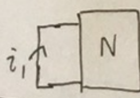
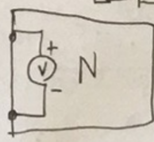
$e_1(s) = (\frac{2}{s} + s + 1) I_1(s) \times \dfrac{s^2}{s^4 + 2s^3 + 4s^2 + 3s + 1}$ $\dfrac{V_1(s)}{I_1(s)} = \dfrac{s^3 + s^2 + 2s}{s^4 + 2s^3 + 4s^2 + 3s + 1}$

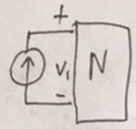
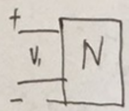
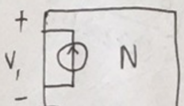
b. similarly, $\dfrac{V_2(s)}{I_1(s)} = \dfrac{e_2(s)}{I_1(s)} = \dfrac{s}{s^4 + 2s^3 + 4s^2 + 3s + 1}$

c. from a and b, $\dfrac{e_2(s)}{e_1(s)} = \dfrac{V_2(s)}{V_1(s)} = \dfrac{s}{s^3 + s^2 + 2s}$

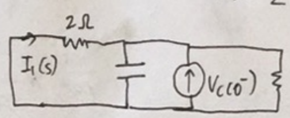
Xilai Zhang 804796478

10.

a. suppose we have a voltage source  , then when calculating network function we replace it with a short  , and the pole of the network function won't change, it is equivalent to  , where the voltage source becomes an initial state of the network.

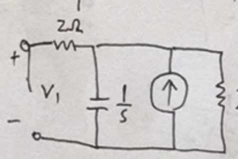
b. suppose we have a current source  , then when calculating network function we replace it with open  , and pole of network function won't change. We can move this current source into the nework  and make it an initial condition.

c. $Y(s) = \dfrac{\frac{1}{2}(s+\frac{1}{2})}{\frac{1}{2}+(s+\frac{1}{2})} = \dfrac{\frac{1}{2}(s+\frac{1}{2})}{s+1}$ $\quad Z(s) = 2+\dfrac{\frac{1}{s}\times 2}{\frac{1}{s}+2} = \dfrac{2(1+s)}{s+\frac{1}{2}}$ $\quad Y(s)$ has pole at -1, $Z(s)$ has pole at $s=-\frac{1}{2}$.



$$I_1(s) = -V_c(0^-)\times\frac{\frac{2\times\frac{1}{s}}{\frac{1}{s}+2}}{\frac{2\times\frac{1}{s}}{s+2}+2} = -\frac{V_c(0^-)}{2(1+s)}$$

Thus the initial state also creates a pole at $s=-1$ on the short circuit current, and $i(t)=ke^{-s_1t}$ is one of the terms.



, $V_1 = V_c(0^-)\times\dfrac{(\frac{1}{s}\times 2)}{\frac{1}{s}+2} = \dfrac{V_c(0^-)}{s+\frac{1}{2}}$, Thus the initial voltage on capacitor also creates a pole ~~on~~ at $s=-\frac{1}{2}$ on the open circuit voltage, and ke^{-s_2t} would be one of its terms.