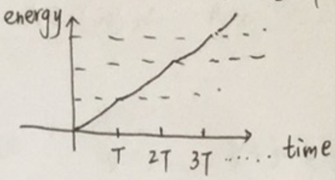
EE 110 Homework 1 Name: Xilai Zhang
UID: 804796478

1\. a. energy $= \frac{V^2}{R}t = \frac{1+e^{-2t/T}-2e^{-t/T}}{R}t$



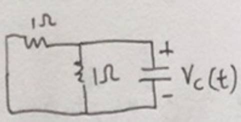
energy is accumulated in each period

b. $\frac{1}{T}\int_0^T \frac{(1-e^{-t/T})^2}{R}dt = \frac{1}{TR}\left[t-\frac{T}{2}e^{-\frac{2t}{T}}+2Te^{-t/T}\right]\Big|_0^T = \frac{1-\frac{1}{2}e^{-2}+2e^{-1}}{R}$

4\. $CV_c' + \frac{V_c}{10} + \frac{V_2+V_c}{10} = 12A = \frac{V_2}{5} + 6 + \frac{1}{L}\int_{0^-}^{0^+} V_2\,dt + \frac{V_2+60}{10} = 12\,A \quad V_2(0^+) = 0\,V$

$V_2(0^-)$ comes from previous state when switch is ~~open~~ closed, and $V_2(0^-) = 0V$ because the inductor acts like a short. the current in circuit, $i = \frac{V_2}{5} + \frac{1}{L}\int_{0^-}^{t} V_2\,dt + i_L(0^-) = \frac{120V - V_c(0^-) - V_c - V_2}{10\Omega}$, thus $\frac{di}{dt} = \frac{1}{5}V_2' + \frac{1}{L}V_2 = -\frac{1}{10}V_c' - \frac{1}{10}V_2 = -\frac{1}{10}\frac{i}{C} - \frac{1}{10}V_2 = -\frac{1}{10C}\left(\frac{V_2}{5} + i_L(0^-)\right) - \frac{1}{10}V_2$ at $t=0^+$, $\frac{1}{5}V_2'(0^+) = -\frac{1}{10}\cdot 6A = -\frac{3}{5}$, so $V_2'(0^+) = -3$

5\. a.



$V_c(t) = ke^{-2t}$ at $t=0$ $k = V_c(0)$, thus $V_c = V_c(0)e^{-2t}$

b. $V_c' + 2V_c = A\cos(wt+\varphi)$ let $V_c = k_1\sin(wt+\varphi) + k_2\cos(wt+\varphi)$ and plug V_c back into the differential equation, we get $\begin{cases} k_1 = \frac{Aw}{w^2+4} \\ k_2 = \frac{2A}{w^2+4}\end{cases}$, at $t=0$ $V_c = ce^{-2t} + k_1\sin\varphi + k_2\cos\varphi = 0$. thus $C=0$, thus $V_c = (-k_1\sin\varphi - k_2\cos\varphi)e^{-2t} + \frac{A}{\sqrt{w^2+4}}\cos(\varphi + \arctan(-\frac{w}{2}))$

c. in steady state e^{-2t} die out, $V_c = \frac{A}{\sqrt{w^2+4}}\cos(\varphi+\arctan(-\frac{w}{2}))$

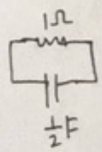
d. $V_c = Ae^{j\varphi}\frac{\frac{1}{jw+1}}{1+\frac{1}{jw+1}} = Ae^{j\varphi}\cdot\frac{1}{jw+2}$ $|V_c| = A\cdot\frac{1}{\sqrt{w^2+4}}$. $\varphi = \arctan(-\frac{w}{2})$

thus $V_c = \frac{A}{\sqrt{w^2+4}}\cos(\varphi+\arctan(-\frac{w}{2}))$

e. plug in the numbers, we get $V_c = \frac{3}{5}e^{-2t} + \frac{1}{\sqrt{5}}\cos(t+\arctan(-\frac{1}{2}))$

Xilai-Zhang_804796478

6. a. the circuit is second order because there are two energy storing elements.

b. 

$i_R(0^+) = 10A \quad i_R(\infty) = 0A \quad i_R = 10e^{-2t}$

c. $10 - \int_{0^-}^{t} i\,dt = i + V$, $i = V'$, thus $10 - V = V' + V$, thus $V = Ce^{-2t} + 5$

at $t=0$, $V=0$, thus $C+5=0$, $C=-5$, thus $V(t) = -5e^{-2t} + 5$

d. $V(\infty) = 5V$. the final voltage is not 0 because when voltage on the two capacitors are the same, there will be no current in the circuit, and thus no more energy loss on the resistor, And at that state the circuit is steady.

8. a. $s^3 + 1 = 0 = (s+1)(s^2 - s + 1)$ $\quad s_1 = -1,\ s_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}j,\ s_3 = \frac{1}{2} - \frac{\sqrt{3}}{2}j$, thus the solution is of the form $k_1 e^{-t} + k_2 e^{\frac{t}{2}} \sin\frac{\sqrt{3}}{2}t + k_3 e^{\frac{t}{2}} \cos\frac{\sqrt{3}}{2}t + y_p(t)$. let $y_p(t) = C_1 \cos t + C_2 \sin t$, ~~plg~~ plug in $y_p(t)$ into equation we get $\begin{cases} C_1 = \frac{1}{2} \\ C_2 = -\frac{1}{2} \end{cases}$ thus $y_p(t) = \frac{1}{2}\cos t - \frac{1}{2}\sin t$, plug in general solution and use three initial conditions we get $\begin{cases} k_1 = \frac{1}{6} \\ k_2 = \frac{\sqrt{3}}{3} \\ k_3 = \frac{1}{3} \end{cases}$ thus $y(t) = \frac{1}{6}e^{-t} + \frac{\sqrt{3}}{3} e^{\frac{t}{2}} \sin\frac{\sqrt{3}}{2}t + \frac{1}{3} e^{\frac{t}{2}} \cos\frac{\sqrt{3}}{2}t + \frac{1}{2}\cos t - \frac{1}{2}\sin t$

b. $s^2 + 2s + 1 = 0 \quad (s+1)^2 = 0$ thus $y(t) = k_1 e^{-t} + k_2 t e^{-t}$ plug $y(t)$ back into the equation we get $\begin{cases} k_1 = 0 \\ k_2 = 1 \end{cases}$ thus $y(t) = te^{-t} u(t)$

c. by superposition, $\begin{cases} y' + 2y = 1 & 0 \le t < 1 \\ y' + 2y = -e^{-(t-1)} & 1 < t < \infty \end{cases}$ the $y' + 2y = 1$ gives us $y(t) = -\frac{1}{2}e^{-2t} + \frac{1}{2}$, the $y' + 2y = -e^{-(t-1)}$ has a solution of the form $k_1 e^{-(t-1)}$ and $k_1 = -1$, thus in the second part when $1 < t < \infty$, $y(t) = [C_1 e^{-2t} - e^{-(t-1)}]u(t-1)$, thus $y(t) =$

$$\begin{cases} -\frac{1}{2}e^{-2t} + \frac{1}{2} & 0 \le t < 1 \\ [C_1 e^{-2t} - e^{-(t-1)}]u(t-1) & 1 < t < \infty \end{cases}$$

Xilai_Zhang_804796478

10. a. $w_0 = 1 = \frac{1}{\sqrt{LC}}$ $Q = \frac{1}{2} = \frac{1}{R}\sqrt{\frac{L}{C}}$ thus $L = \frac{1}{C}$, $R = \frac{2}{C}$. $Li_L' + V_C(0^-) + \frac{1}{C}\int_0^t i_L dt + i_L R = 0$,

thus $Li_L'' + Ri_L' + \frac{i_L}{C} = 0$, thus $i_L'' + 2i_L' + i_L = 0$, let $i_L = k_1 e^{-t} + k_2 t e^{-t}$. $i_L(0) = 0$ thus

$k_1 = i_L(0)$. $Li_L'(0) + V_c(0) + Ri_L(0) = 0$ $i_L'(0) = \frac{-V_C(0) - Ri_L(0)}{L} = k_2 - k_1 = k_2 - i_L(0) = \frac{-V_C(0) - Ri_L(0)}{L}$

thus $k_2 = \frac{-V_c(0) + (L-R)i_L(0)}{L}$, thus $i_L(t) = i_L(0)e^{-t} + \frac{-V_c(0) + (L-R)i_L(0)}{L} te^{-t}$

b. $Li_L' + V_C(0) + \frac{1}{C}\int(i_L - \delta(t))\,dt + (i_L - \delta(t))R = 0$, thus $i_L'' + 2i_L' + 2i_L = \delta(t) + 2\delta'(t)$.

let $i_L = k_1 e^{-t} u(t) + k_2 t e^{-t} u(t)$ and plug i_L back into the equation, we get $\begin{cases} k_1 = 2 \\ k_2 = -1 \end{cases}$

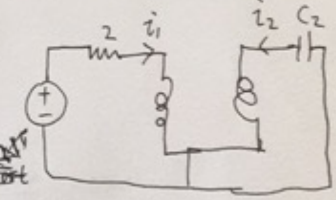
thus $i_L(t) = 2e^{-t}u(t) - te^{-t}u(t)$

c. $i_L(0^+) = 2$ $i_L'(0^+) = -2e^{-0} - (e^{-0} - 0) = -3$

12. a. in the V_{TH} topology, $V_{TH} = \frac{2jM}{2+2j} V_s$ in phasor domain.

in the short circuit current I_{TH} topology, we have

$\begin{cases} (2+2j)i_1 + 2Mj\,I_2 = V_s \\ (8j + \frac{1}{2jC})i_2 + 2Mj\,i_1 = 0 \end{cases}$ where i_1, i_2 is labeled as



~~replace with short~~

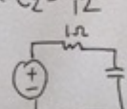
the two equations are transformed into matrices: $\begin{bmatrix} 2+2j & 2Mj \\ 2Mj & 8j + \frac{1}{2jC} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \end{bmatrix}$, and the

solution is $I_2 = \frac{-\frac{1}{2}Mj V_s}{4j + \frac{1}{4jC} - 4 + \frac{1}{4C} + M^2}$, thus $Z_{TH} = \frac{V_{TH}}{-i_2} = \frac{\frac{2jM}{2+2j}V_s}{\frac{\frac{1}{2}MjV_s}{4j + \frac{1}{4jC} - 4 + \frac{1}{4C} + M^2}} =$

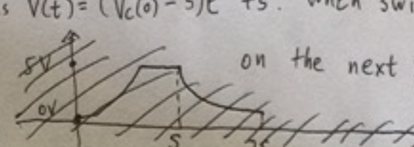
$M^2 + j(8 - M^2 - \frac{1}{2C_2})$

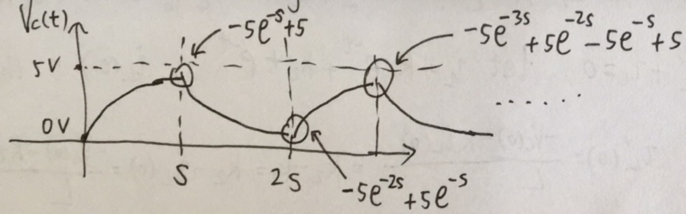
b. maximum power is achieved when $Z_{TH} = Z_L^*$, thus $M^2 + (8 - M^2 - \frac{1}{2C_2})j = (1-j)^* =$

$1+j$ solving this equation we get $\begin{cases} M = 1 \\ C_2 = \frac{1}{12} \end{cases}$

2. a. the circuit is transformed into  where v is a square wave of amplitude 5V

when switch is off, $V' + V = 5$, thus $V(t) = (V_c(0) - 5)e^{-t} + 5$. when switch is on, $V' - V = 0$, thus

$V(t) =$ $V_c(0)e^{-t}$ ← where $V_c(0)$ is the value at the beginning of period, thus the plot $V_c(t)$ is  on the next page, Xilai_Zhang_804796478



b. in steady state, $V_c(t)$ approaches $5V$ in the first part of the period, and becomes $0V$ in the second part of the period.

c. $\frac{1}{2S}\left[\int_0^S (V_c(0)-5)e^{-t}+5 + \int_0^S V_c(0)e^{-t}\right]$, where $V_c(0)$ is the voltage at the beginning of each S period. In steady state, $\frac{1}{2S}\left[\int_0^S(-5e^{-t}+5)+\int_0^S 5e^{-t}\right] = \frac{1}{2S}\cdot 5S = \frac{5}{2}$