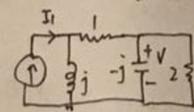
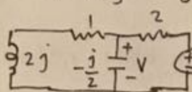
EE110 Homework 4 Name: Xilai Zhang UID: 804796478

1. Superposition: when only current source is present, 

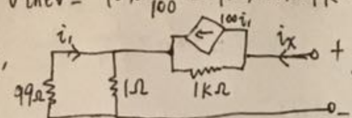
, $I_1 = e^{j\cdot 0} \cdot \frac{j}{j+\frac{2-3j}{2-j}} = \frac{2j+1}{3-j}$,

$V = I_1 \times \frac{-2j}{2-j} = \frac{30+10j}{50}$ thus $V(t) = 0.63\sin(t+\arctan(\frac{1}{3}))$. when only voltage source is present,

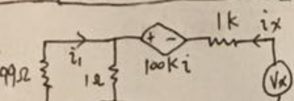


, $V = e^{j\cdot 0} \cdot \frac{\frac{1-0.5j}{1+0.5j}}{2+\frac{1-0.5j}{1+0.5j}} = \frac{1.75-4j}{15.25}$, thus $V(t) = 0.29\cos(2t+\arctan(-\frac{16}{7}))$. The

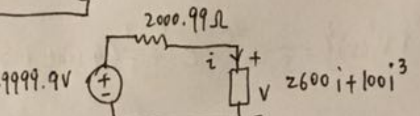
overall steady voltage across capacitor is the sum of the two, $V(t) = 0.63\sin(t+\arctan(\frac{1}{3})) + 0.29\cos(2t+\arctan(-\frac{16}{7}))$

2. a. $V_{thev} = 10\times\frac{1}{100} - 100\times 0.1\times 1k = -9999.9$ V. To find R_{ther}, we replace voltage source with a short, 

, and replace dependent current source with dependent

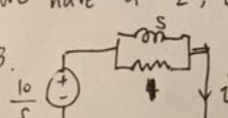
voltage source 

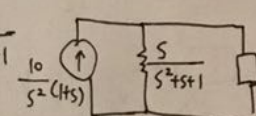
, $i_x = -100i_1$, $V_x = -99i_1 - 100ki_1 - 1ki_x$, $R_{TH} = \frac{V_x}{i_x} = 0.99+2k = 2000.99\ \Omega$

the thevin equivalent is 

b. $-9999.9 = 2000.99i + 2600i + 100i^3$ approximate the equation to $-10k = 2ki + 2600i + 100i^3$,

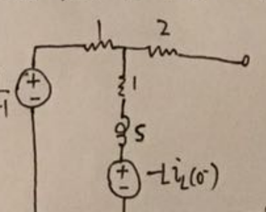
we have $i_1 = -2$, $i_{2,3} = 1\pm 7i$. Thus $V_1 = -6000$, $V_2 = -12000-14000i$, $V_3 = -12000+14000i$.

3. 

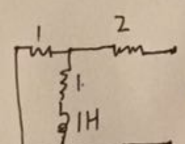
$i_{sc} = \frac{10}{s^2}(1+s)$ $R_{norton} = \frac{1}{s+1+\frac{1}{s}} = \frac{s}{s^2+s+1}$ 

a. $\frac{10}{s^2}(1+s) \times \frac{\frac{s}{s^2+s+1}}{\frac{s}{s^2+s+1}+1} = \frac{10}{s(s+1)}$ thus $V(t) = (10-10e^{-t})\,u(t)$

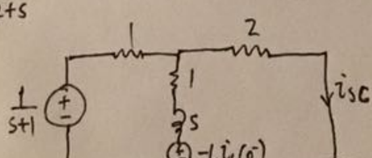
b. $\frac{10}{s^2}(1+s) \times \frac{\frac{1}{s^2+s+1}}{\frac{s}{s^2+s+1}+\frac{1}{s}} = \frac{10(1+s)}{s(2s^2+s+1)}$ thus $V(t) = 10-10e^{-\frac{t}{4}}\cos(\frac{\sqrt{7}}{4}t) + e^{-\frac{t}{4}}\frac{10\sqrt{7}}{7}\sin(\frac{\sqrt{7}}{4}t)$

4. at steady state, $V_c(\infty) = 5V$, $i_L(\infty) = 5A$. 

$V_{oc} = (\frac{1}{s+1}+5)(\frac{1+s}{2+s}) - 5 = \frac{-4}{s+2}$

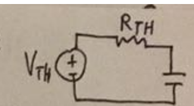
Rth: 

$R_{th} = 2+\frac{1+s}{2+s} = \frac{5+3s}{2+s}$

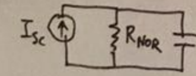


$i_{sc} = -\frac{4}{5+3s}$

a. $V_c = \frac{-4}{s+2} \times \frac{\frac{1}{s}}{\frac{s+3s}{2+s}+\frac{1}{s}} = \frac{-4}{3s^2+6s+2}$, $V_c(t) = \frac{2\sqrt{3}}{3}e^{(-1-\frac{\sqrt{3}}{3})t} - \frac{2\sqrt{3}}{3}e^{(\frac{\sqrt{3}}{3}-1)t}$



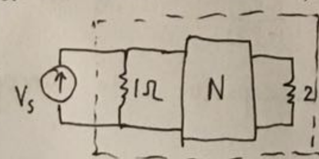
b. $V_c = -\frac{4}{s+3s} \times \frac{\frac{s+3s}{(2+s)s}}{\frac{s+3s}{2+s}+\frac{1}{s}} = \frac{-4}{3s^2+6s+2}$, $V_c(t) = \frac{2\sqrt{3}}{3}e^{(-1-\frac{\sqrt{3}}{3})t} - \frac{2\sqrt{3}}{3}e^{(\frac{\sqrt{3}}{3}-1)t}$



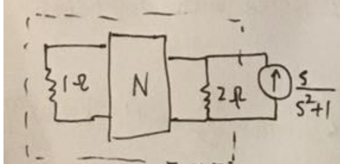
6. a. -1 and -2 are natural frequencies of V_1 and V_2.

b. the natural frequencies of the network include -1 and -2, and could have other natural frequencies.

c. transform network on the left into



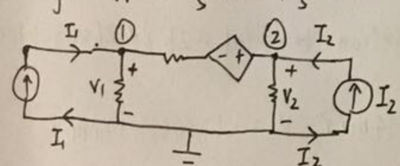
, then the network in the box is reciprocal. (view it as 2-port) we know $\begin{cases} V_1 = Z_{11}i_1 + Z_{12}i_2 = 0 \\ V_2 = Z_{21}i_1 + Z_{22}i_2 \end{cases}$, thus $\begin{cases} Z_{11} = 1+\frac{1}{s+1}+\frac{1}{s+2} \\ Z_{21} = Z_{12} = \frac{1}{s+1} - \frac{1}{s+2} \end{cases}$.



$\begin{cases} V_1' = Z_{12}i_2' = (\frac{2}{s+1} - \frac{1}{s+2})(\frac{s}{s^2+1}) = \frac{(s+3)s}{(s+1)(s+2)(s^2+1)} \\ V_2' = Z_{22}i_2' \end{cases}$ since Z_{22} is unknown,

V_2' is not known, $V_1'(t) = \mathcal{L}^{-1}\{V_1'(s)\} = \frac{2}{5}e^{-2t} - e^{-t} + \frac{1}{5}(4\sin t + 3\cos t)$ ~~whic~~ so the ~~so the~~ steady state voltage $V_1'(t) = \frac{4}{5}\sin t + \frac{3}{5}\cos t$.

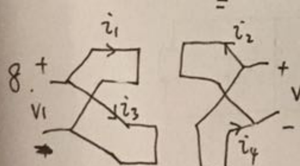
7.



node analysis: $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 4.5 & -1 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$ thus $y = \begin{bmatrix} 4.5 & -1 \\ -4 & 2 \end{bmatrix}$

and $Z = y^{-1} = \begin{bmatrix} 0.4 & 0.2 \\ 0.8 & 0.9 \end{bmatrix}$

8.



$\begin{cases} V_1 = \beta i_2 \\ V_2 = -\beta i_1 \\ V_1 = i_3 R_1 \\ V_2 = (i_4 - \alpha I_3)R_2 \end{cases} \Rightarrow \begin{cases} i_1 + i_3 = \frac{1}{R_1}V_1 - \frac{1}{\beta}V_2 \\ i_2 + i_4 = (\frac{\alpha}{R_1}+\frac{1}{\beta})V_1 + \frac{1}{R_2}V_2 \end{cases}$ thus $y = \begin{bmatrix} \frac{1}{R_1} & -\frac{1}{\beta} \\ \frac{\alpha}{R_1}+\frac{1}{\beta} & \frac{1}{R_2} \end{bmatrix}$