

# **Binomial Options Pricing Model: Reference Design**



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## Prerequisites

To run the reference design, you need to

- mount Alveo U200 card in your system and
- install the following mandatory components:

SDAccel Version	<b>2019.1</b>
Target Platform	<b>Alveo U200 card:</b> <ul style="list-style-type: none"> <li>• <b>201830_2</b> deployment shell: <b>xilinx_u200_xdma_201830_2</b></li> <li>• <b>201830_2</b> development shell: <b>xilinx_u200_xdma_dev_201830_2</b></li> </ul>
XRT Version	<b>2019.1</b>

Refer to the **UG1301** and **UG1238** user guides for more information regarding Alveo and SDAccel installation.

The SDAccel development environment can be run using GUI and Makefile modes. This tutorial uses the GUI mode.

## Introduction

The goal of this reference design (tutorial) is by using a Binomial Options Pricing Model (further referenced as Binomial Model) to:

- Demonstrate performance advantage of the Xilinx Alveo-based solution vs. the Intel® Xeon® CPU one
- Introduce SDAccel flow, various analysis tools and optimization techniques allowing you to improve design performance significantly

The figure below shows the original description of the Binomial Model obtained from Wikipedia ([https://en.wikipedia.org/wiki/Binomial\\_options\\_pricing\\_model](https://en.wikipedia.org/wiki/Binomial_options_pricing_model))

The original Binomial Model algorithm is rewritten in a C/C++ programming language by using single precision floating point operations and keeping the C/C++ coding style as close as possible to the original version.

```
function americanPut(T, S, K, r, sigma, q, n) {
    ' T... expiration time
    ' S... stock price
    ' K... strike price
    ' q... dividend yield
    ' n... height of the binomial tree

    deltaT := T / n;
    up := exp(sigma * sqrt(deltaT));

    p0 := (up*exp(-q * deltaT) - exp(-r * deltaT)) / (up^2 - 1);
    p1 := exp(-r * deltaT) - p0;

    ' initial values at time T
    for i := 0 to n {
        p[i] := K - S * up^(2*i - n);
        if p[i] < 0 then p[i] := 0;
    }

    ' move to earlier times
    for j := n-1 down to 0 {
        for i := 0 to j {
            p[i] := p0 * p[i+1] + p1 * p[i];    ' binomial value
            exercise := K - S * up^(2*i - j);    ' exercise value
            if p[i] < exercise then p[i] := exercise;
        }
    }

    return americanPut := p[0];
}
```

To start, implement the original C/C++ description on Xilinx Alveo card and x86 CPU and compare their performance.

- Run the original algorithm on CPU with a single thread
- Implement the same algorithm on an Alveo card as a fully sequential process for data movement and kernel execution

Then by using various optimization techniques, increase the Alveo based performance by the factor of **60.8x**. Compare the Alveo results with the ones obtained on CPU running in a multithread mode.

## Test Environment and Important Notes

All results represented in this tutorial are generated using a **DELL 8510 Workstation** with the following characteristics:

- **CPU:** Intel® Xeon® E5-1650 v3 @ 3.50 GHz
  - Number of threads per core: 6
  - Number of cores per socket: 2
- **RAM:** 64 GB
- **OS:** CentOS 7.4

**IMPORTANT:** the results you will obtain on your machine and the ones presented in this document may differ.

## General Flow for this reference design

- **Step 1:** Projects Setup.
- **Step 2:** Important design characteristics
- **Step 3:** Running SW Model on X86 CPU
- **Step 4:** Alveo: Original Design
- **Step 5:** Alveo: Host Optimization - Data Transfer and Kernel Execution Overlap
- **Step 6:** Alveo: Single Kernel Optimization - Data Movement
- **Step 7:** Alveo: Single Kernel Optimization - Running 4 Functions in Parallel
- **Step 8:** Alveo: Single Kernel Optimization - Additional Loops Unrolling
- **Step 9:** Alveo: Single Kernel Optimization - System Run Results
- **Step 10:** Alveo: Using Multiple Compute Units and Different DDRs

## Projects Setup

## Step 1

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The reference design consists of several projects, and all design source code is available on GitHub (<https://github.com/Xilinx/BinomialModel>). You need to import them to your local machine and then create projects using the instructions listed in the **Appendix A: Projects Setup: Detailed Steps**.

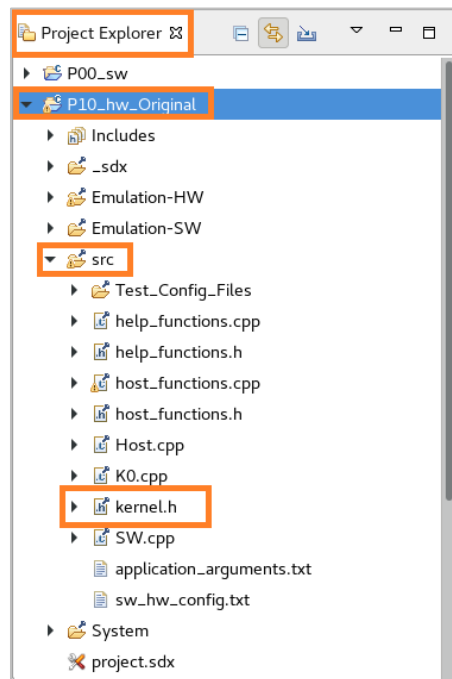
## Important Design Characteristics

## Step 2

You have successfully set up all projects, and they are ready to be run. However, before doing that, let's clarify several design aspects.

- In this tutorial, for demonstration purposes and for the sake of clarity, we were trying to keep the kernel coding style as simple as possible. For which, we assumed:
  - a Binomial tree height is limited to **1024** and
  - a Kernel can process maximum **1024** option positions
    - **IMPORTANT:** in this document, the **number of option positions** will be further referenced as a **number of test vectors**

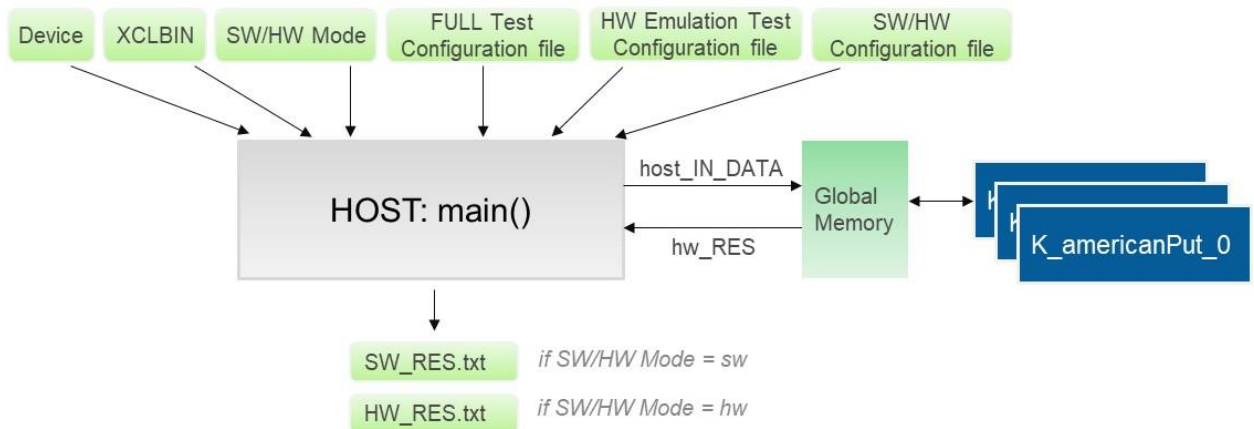
which is defined in a **kernel.h** file in each project. For example, open the **kernel.h** file from the **P10\_hw\_Original** project:



```
...
#define CONST_MAX_TREE_HEIGHT 1024
#define CONST_MAX_NB_OF_TESTS 1024
...
```

- As previously mentioned before, a **single precision floating point** data type and corresponding functions were used to implement the algorithm.
- The Host code is located in the **Host.cpp** file and kernels, depending on a project are located in **K0.cpp**, **K1.cpp** and **K2.cpp** files.

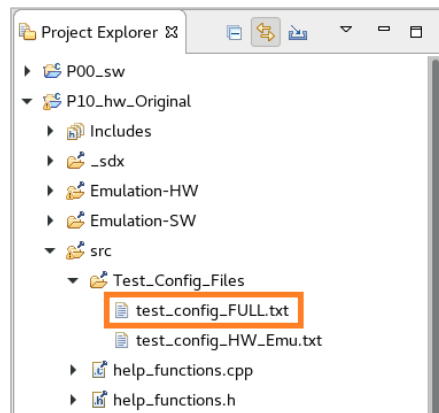
- The figure below represents the overall application structure:



The Host has 6 input arguments:

- **Device** – the target Alveo device.
  - For this design we use **xilinx\_u200\_xdma\_201830\_2**
- **XCLBIN** – the name of the Xilinx binary container where the kernel(s) are compiled
  - In our case, this is a **binary\_container\_1.xclbin** file
- **SW/HW Mode** - defines if the application runs as :
  - SW Model (**sw**) on an x86 CPU or
  - HW implementation (**hw**) on the Alveo card
- **FULL Test Configuration file** - defines Binomial application arguments to be used for
  - SW Model (SW/HW Mode = **sw**)
  - SW Emulation and System Run mode (SW/HW Mode = **hw**)

The name of this file is **test\_config\_FULL.txt**, and it is available in each project.



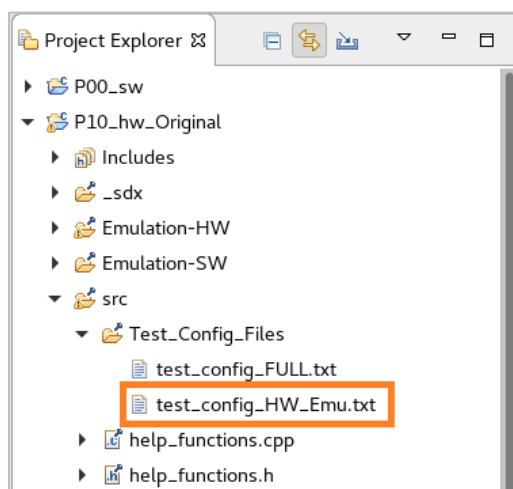
Reduced content of this file is represented in the following figure (please review this file for more information):

Binomial settings									Test Vector Configurations	
Company	T	S	K	r	sigma	q	n		K_Step	NB_OF_TESTS
Comp_1	1	110.0	100.0	0.025	0.2	0.1	1024		1.0	64
Comp_2	1	80.0	85.0	0.025	0.2	0.1	1024		1.0	64
Comp_3	1	32.0	33.0	0.025	0.2	0.1	1024		1.0	64
Comp_4	1	55.0	60.0	0.025	0.2	0.1	1024		1.0	64

- **HW Emulation Test Configuration file** - defines Binomial application arguments (reduced complexity) to be used in HW Emulation mode (SW/HW Mode = **hw**). It ensures fast execution runtimes in HW Emulation mode.

- This file is ignored when the SW Model is run on CPU (SW/HW Mode = **sw**).

The name of this file is **test\_config\_HW\_Emu.txt**, and it is available in each project:



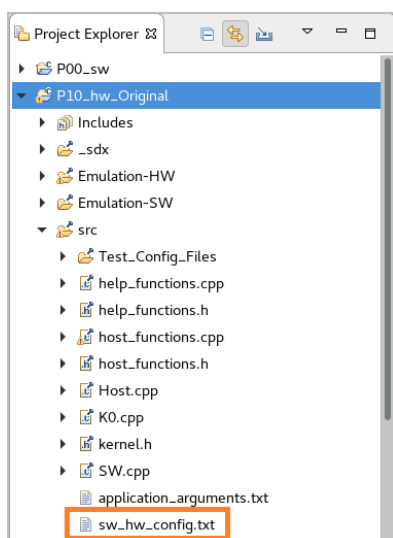
Reduced content of this file is represented on the following figure (please review this file for more information):

Binomial settings									Test Vector Configurations	
Company	T	S	K	r	sigma	q	n		K_Step	NB_OF_TESTS
Comp_1	1	110.0	100.0	0.025	0.2	0.1	<b>10</b>		1.0	<b>64</b>

- **SW / HW Configuration file** - defines
  - The number of threads to be used when running the SW Model on CPU (SW/HW Mode = **sw**)
  - Kernel configuration implemented on the Alveo card (SW/HW Mode = **hw**):
    - Number of Kernels
    - Number of Compute Units (CUs) per kernel
    - Number of parallel Binomial functions per CU

The name of the configuration file for **all** projects (except P00\_sw) is **hw\_sw\_config.txt**:





Reduced content of this file is represented on the following figure (please review this file for more information):

SW Resources		Available HW Resources	
NB_OF_THREADS	NB_KERNELS	NB_OF_CUs_PER_KERNEL	NB_OF_PARALLEL_FUNCTIONS_PER_CU
1	2	4	4

The **P00\_sw** project has two configuration files: **hw\_sw\_config\_1\_Thread.txt** and **hw\_sw\_config M\_Thread.txt**.

- **Result files** – the name of the result file is
  - **SW\_RES.txt** if SW/WH Mode = sw
  - **HW\_RES.txt** if SW/WH Mode = hw

Here is an example of a result file where the **BOPM\_Result** column contains Binomial Model results:

```
=====
Binomial Options Pricing Model: HW results
=====

-----
Company: Comp_1 (K_Step=1.000 #tests=64)
-----
```

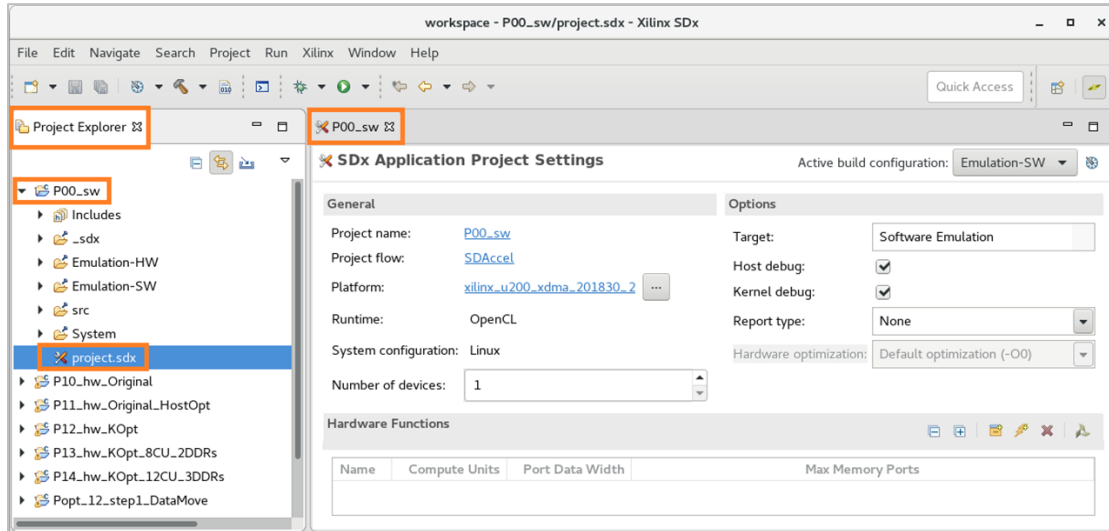
T	S	K	r	sigma	q	n	BOPM_Result
1	110.000	100.000	0.025	0.200	0.100	1024	7.14233
1	110.000	101.000	0.025	0.200	0.100	1024	7.64949
1	110.000	102.000	0.025	0.200	0.100	1024	8.17568
1	110.000	103.000	0.025	0.200	0.100	1024	8.72474
1	110.000	104.000	0.025	0.200	0.100	1024	9.29092
1	110.000	105.000	0.025	0.200	0.100	1024	9.87369
...							

## Running SW Model on X86 CPU

## Step 3

Projects: **P00\_sw**

Open **P00\_sw** project by double-clicking on the **project.sdx** under the **P00\_sw** in **Project Explorer**:



This project represents a SW Model (**SW.cpp** file) of the Binomial Model to be run on CPU.

In this file, the **sw\_calc\_p0** function represents a Binomial Model algorithm. If you compare this description with the one published on Wikipedia, then you will see that the coding styles are identical in both cases.

```
void K_americanPut_sw_model
(t_in_data* host_IN_DATA, float* sw_RES,
 int NB_OF_TESTS, int Nb_Of_Threads)
{
    int Nb_of_Test_Vectors_per_Task =
        NB_OF_TESTS/Nb_Of_Threads;

    for (int i=0; i<Nb_Of_Threads; i++)
        t[i] = thread(K_americanPut_sw_model_task, ...,
            Nb_of_Test_Vectors_per_Task, ...);
    ...
}

NB_Of_Threads ↓

void K_americanPut_sw_model_task (...)
{
    for (int i = 0; i<Nb_Of_Tests; i++) {
        int indx = Start_Index + i;
        sw_RES[indx] = sw_calc_p0 (...);
    }
}
```

BOPM – core function (SW)


```
float sw_calc_p0
(int T, float S, float K, float r, float sigma,
 float q, int n) {

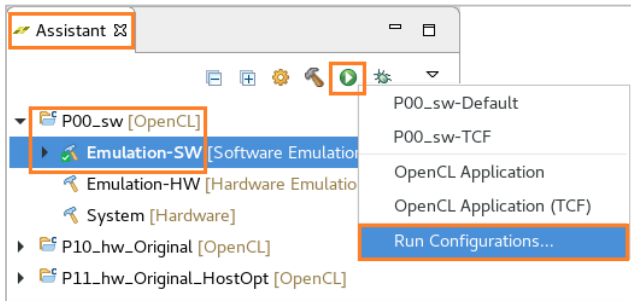
    float deltaT, up, p0, p1, exercise;
    float p[CONST_MAX_TREE_HEIGHT];
    ...
    for (int i = 0; i < n; i++) {
        p[i] = K - S * powf(up, (2*i - n)); // up^(2*i - n)
        if (p[i] < 0) p[i] = 0;
    }
    ...
    return (p[0]);
}
```

The **sw\_calc\_p0** is called multiple times by **K\_americanPut\_sw\_model\_task** and **K\_americanPut\_sw\_model** functions to execute several test vectors. The **K\_americanPut\_sw\_model** is called from a host (**Host.cpp**).

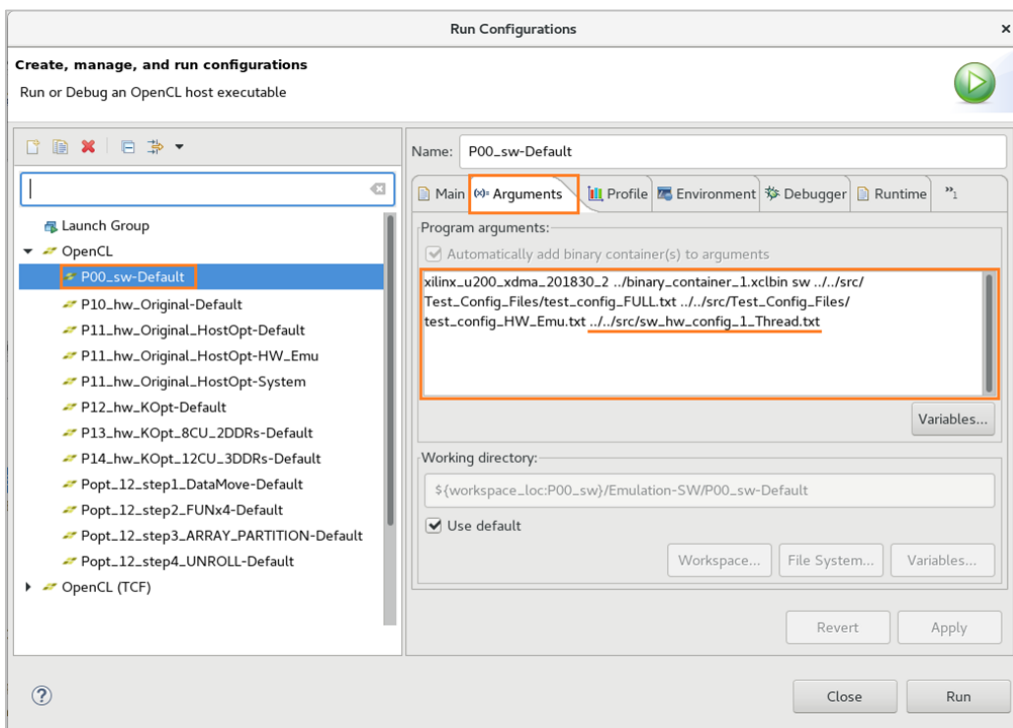
As you can also notice, the **K\_americanPut\_sw\_model** can be configured to run multiple threads. However, at this step of the tutorial, a single thread is used, which is defined in the **sw\_hw\_config\_1\_Thread.txt** configuration file


#	SW Resources		Available HW Resources	
#	NB_OF_THREADS	NB_KERNELS	NB_OF_CUs_PER_KERNEL	NB_OF_PARALLEL_FUNCTIONS_PER_CU
#	1	1	1	1

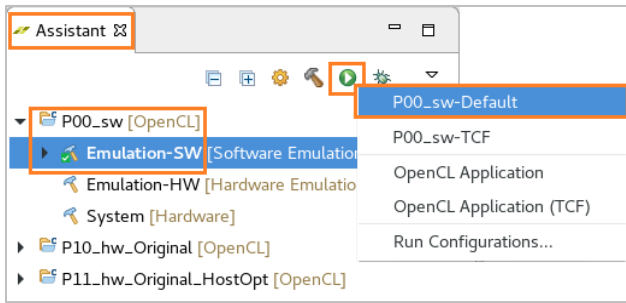
which is passed to the host program as a command line option. To access the command line options, in the **Assistant** tab under the **P00\_sw** project select **Emulation-SW**, then click on the  icon and select **Run Configurations** menu:



In the opened dialog box ensure that **P00\_sw-Default** run is selected in the left-side window. Select **Arguments** tab to access the command line arguments passed to the application:



To run the SW Model on CPU with a single thread, in the **Assistant** tab under the **P00\_sw** project select **Emulation-SW**, then click on the  icon and select **P00\_sw-Default**:



The application execution is successfully completed, and CPU execution time is reported in a Console window:


```
HOST-Info: =====
HOST-Info: Step: Run SW Model
HOST-Info: =====
HOST-Info: SW Model Execution
HOST-Info:   # Threads   = 1
HOST-Info:   Runtime (ms) = 9834.2
```

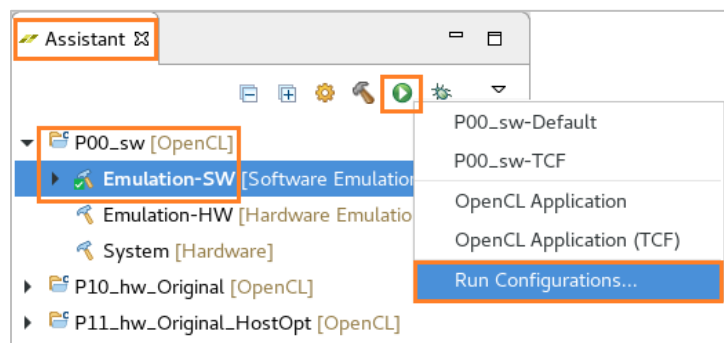
*Recall: the results you obtain on your machine and the ones represented in this document may differ.*


Later on (step 4), you will compare this result against the original code implementation on the Alveo card.

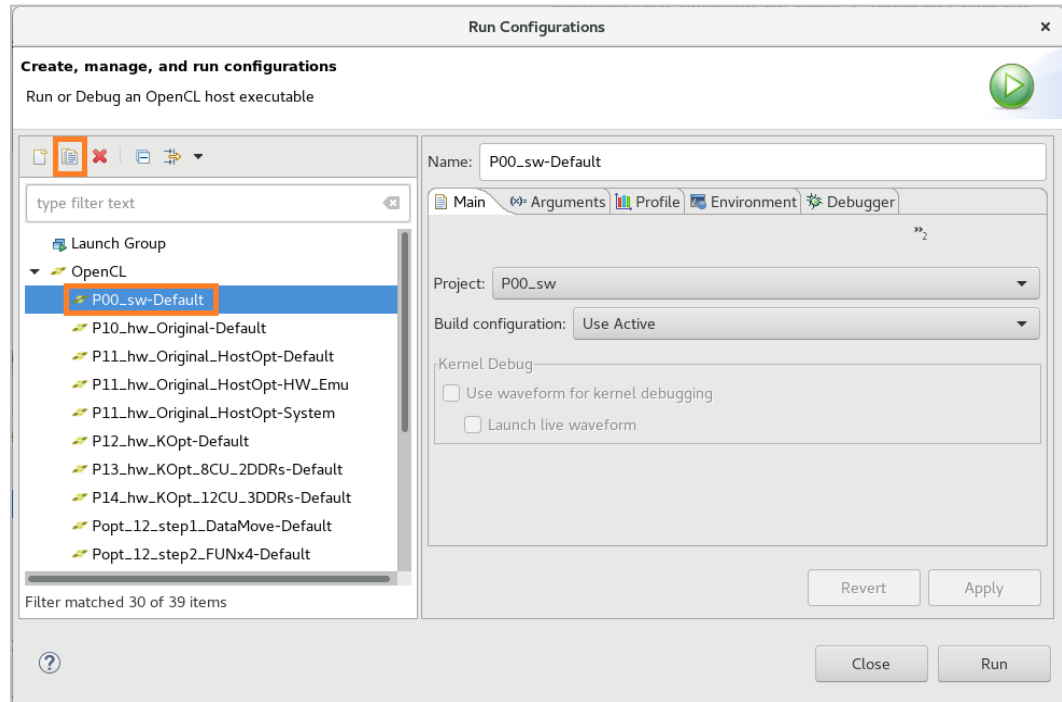
As previously mentioned, the optimized Alveo implementation will be compared against the CPU result executed in a multithreading mode. For the sake of time, let's generate the multithreading result now and use it in later steps.

To achieve this, you need to create an additional run, called **P00\_sw-M\_Thread** and specify the **sw\_hw\_config\_M\_Thread.txt** configuration file in the command line options.

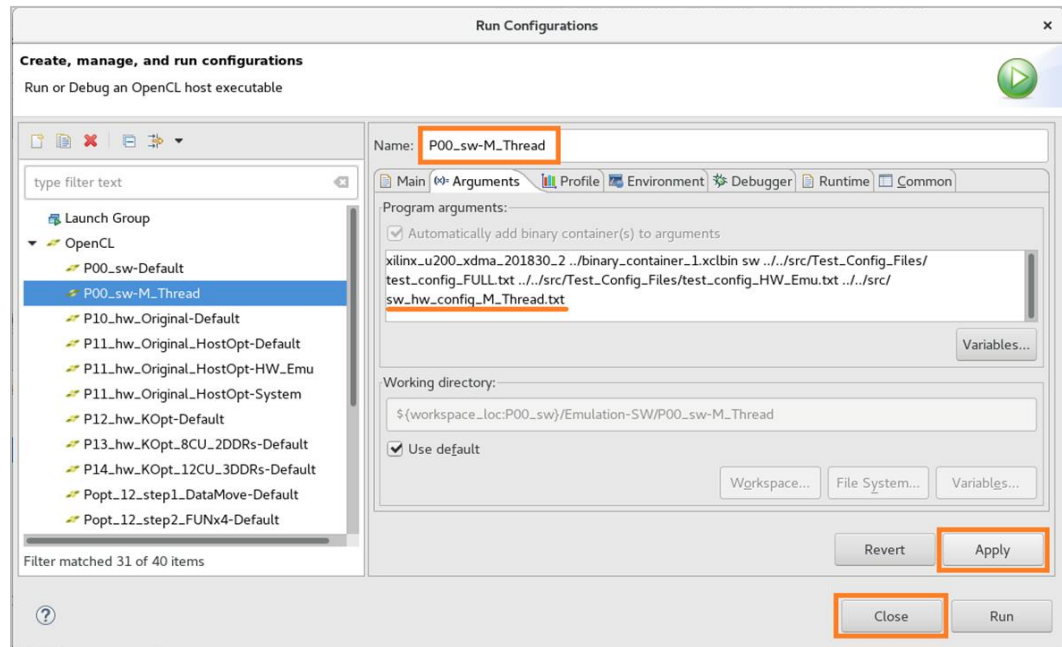
- In the **Assistant** tab under the **P00\_sw** project select **Emulation-SW**, click on the  icon and select **Run Configurations** menu



- In the opened dialog box, ensure that **P00\_sw\_Default** run is selected and then press on the  icon to duplicate the default run



- Then
  - Name the newly created run as **P00\_sw-M\_Thread**
  - In the **Arguments** tab, replace the **sw\_hw\_config\_1\_Thread.txt** file by the **sw\_hw\_config\_M\_Thread.txt** one:



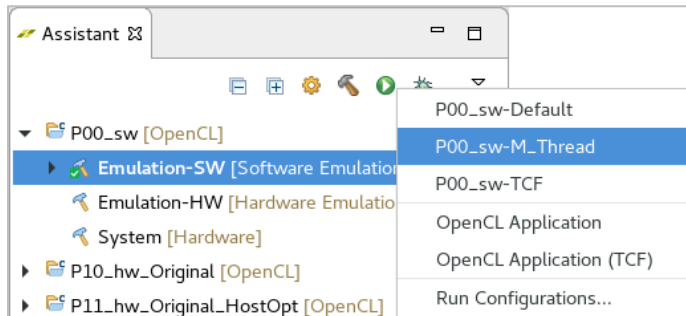
Then press **Apply** and **Close**.

The **sw\_hw\_config\_M\_Thread.txt** file is configured to run 12 Threads on CPU:

SW Resources		Available HW Resources		
NB_OF_THREADS	NB_KERNELS	NB_OF_CUs_PER_KERNEL	NB_OF_PARALLEL_FUNCTIONS_PER_CU	
12	1	1	1	

You need to adjust the number of threads delivering the best results for your machine configuration.

Then launch the **P00\_sw-M\_Thread** run:



The application execution is successfully completed, and a CPU execution time is reported in a Console window:

```

HOST-Info: =====
HOST-Info: Step: Run SW Model
HOST-Info: =====
HOST-Info: SW Model Execution
HOST-Info:   # Threads   = 12
HOST-Info:   Runtime (ms) = 1289.7

```

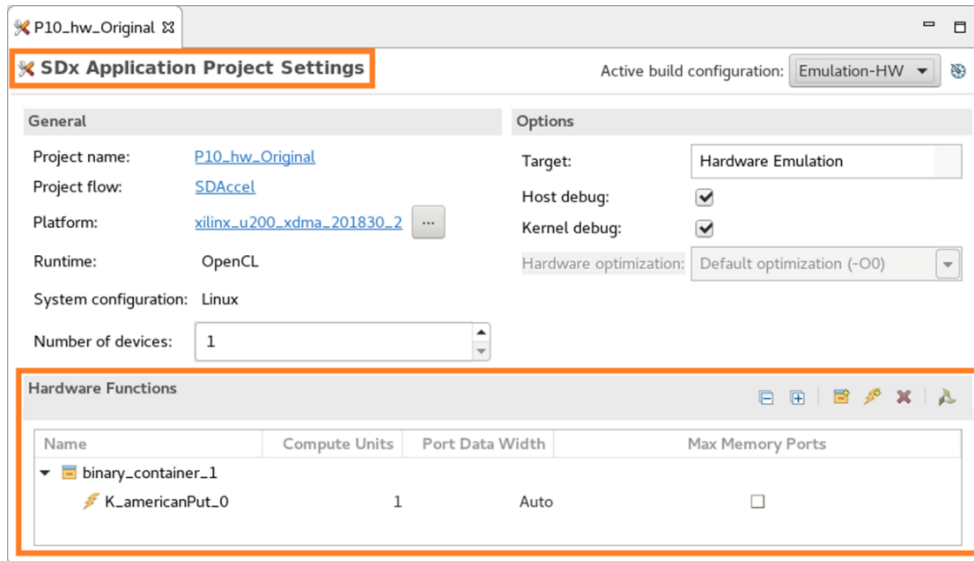
## Alveo: Original Design

## Step 4

### Projects: P10\_hw\_Original

Open the **P10\_hw\_Original** project. This project represents the original kernel implementation on the Alveo card.

The **Hardware Functions** in the **SDx Application Project Settings** contains the list of kernels which will be implemented on the Alveo card. In this project, we have one **K\_americanPut\_0** kernel, which will be implemented as a single compute unit (CU) in a **binary\_container\_1.xclbin** file.



Open the **K0.cpp** file, containing the kernel description.

- In this file, the **hw\_calc\_p0\_0** function represents the Binomial Model algorithm.
- The **hw\_calc\_p0\_0** function is called by **K\_americanPut\_0**, representing a HW kernel called from the host code (**Host.cpp**).

The original application on the Alveo card is organized as a simple sequential process where input data transfer from Host to Global Memory, kernel execution and results transfer from Global to Host memory is sequentially done for each test vector.

In Alveo implementation, the **T**, **S**, **K**, and other arguments are transferred from Host to the Global memory as elements of a **t\_in\_data** structure defined in the **kernel.h** file.

The structure elements are decomposed on separate variables in **hw\_calc\_p0\_0**, which represents the only difference between the **SW Model** (SW.cpp file in the P00\_sw project) and the code for Alveo implementation.

### Host iteratively calls K\_americanPut\_0

↓ NB\_OF\_TESTS

```
void K_americanPut_0 (t_in_data* IN_Data, float* Res) {
    #pragma HLS INTERFACE s_axilite port=IN_Data ...
    #pragma HLS INTERFACE m_axi port=IN_Data ...
    ...
    #pragma HLS DATA_PACK variable=IN_Data

    Res[0] = hw_calc_p0_0(IN_Data[0]);
}

float hw_calc_p0_0 (t_in_data in_d) {
    ...
    int T; float S; float K; ...
    float deltaT, up, p0, p1, exercise;
    float p[CONST_MAX_TREE_HEIGHT];
    ...
    for (int i = 0; i < n; i++) {
        #pragma HLS LOOP_TRIPCOUNT min=100 max=100 avg=100
        p[i] = K - S * powf(up, (2*i - n));
        if (p[i] < 0) p[i] = 0;
    }
    ...
    return(p[0]);
}
```


**BOPM – core function (HW)**

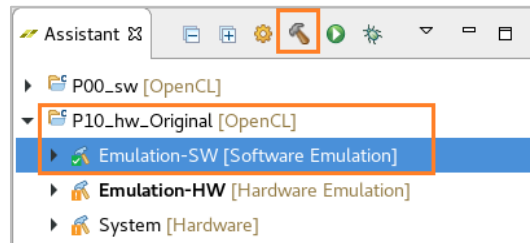
You can notice the presence of several pragmas in the **K0.cpp** file. Let's review them before continuing:

- The **INTERFACE** pragmas in **K\_americanPut\_0** instruct SDAccel compiler on how to connect the kernel to the Alveo shell.
- **DATA\_PACK** pragma ensures that the compiler considers all fields of the **t\_in\_data** structure as a single block of data without splitting them into individual elements.
- The **LOOP\_TRIPCOUNT** pragma in the **hw\_calc\_p0\_0** function has no impact on the design optimization and are exclusively used later on for design analysis.

#### 4-1. SW Emulation

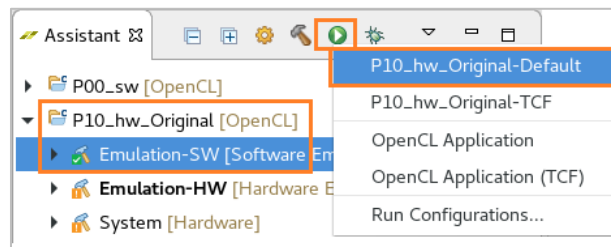
The main goal of SW Emulation is to validate the functional correctness of the design. In this mode, the runtime is very fast because both the host and the kernel code are compiled to be run on an x86 processor.

To compile the design for SW Emulation in the **Assistant tab**, under **P10\_hw\_Original** select **Emulation-SW**, and then press on the  icon:



The compilation process should complete successfully.

To run SW Emulation, in the **Assistant tab**, under **P10\_hw\_Original** select **Emulation-SW**, then click on the  icon and select **P10\_hw\_Original-Default**:



You should see the following messages in the Console window.

```
HOST-Info: =====
HOST-Info: Step: Run Application
HOST-Info: =====

HOST_Info: Waiting for application to be completed ...

HOST_Info: Test Passed
HOST-Info: Results stored in the HW_Res.txt file ...


HOST-Info: Application Completed
```

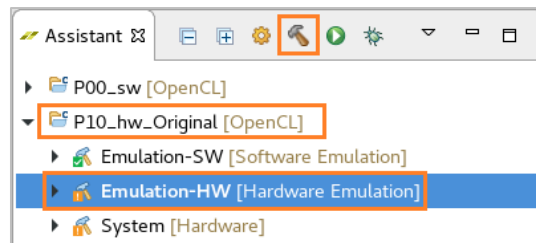


## 4-2. HW Emulation

While the SW Emulation flow is a good measure of functional correctness, it does not guarantee the correctness on the FPGA execution target. The Hardware Emulation flow enables checking of the correctness of the generated logic. This emulation flow invokes the hardware simulator in the SDAccel environment to test the logic functionality. As a consequence, the runtime in the Hardware Emulation flow is longer than in the SW Emulation flow.

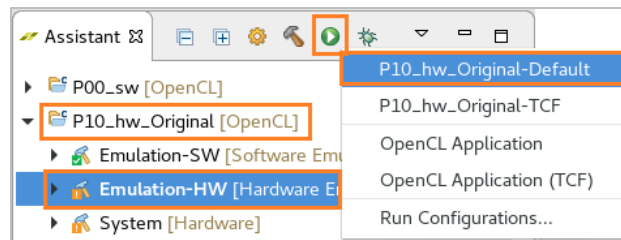
In addition, HW Emulation flow provides more comprehensive and precise profiling information allowing the user to analyze the application performance and identify the bottlenecks.

To compile the design for HW Emulation in the **Assistant tab**, under **P10\_hw\_Original** select **Emulation-HW**, and then press on the  icon:



The compilation process should complete successfully.

To run HW Emulation, in the **Assistant tab**, under **P10\_hw\_Original** select **Emulation-HW**, then click on the  icon and select **P10\_hw\_Original-Default**:



The application is successfully run, and you should see the following messages in the Console window.

```
HOST-Info: =====
HOST-Info: Step: Run Application
HOST-Info: =====

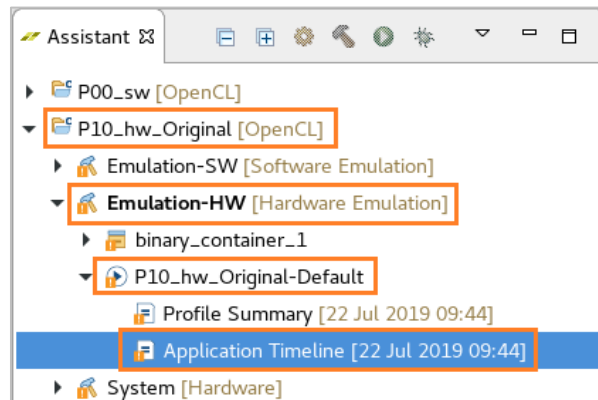
HOST-Info: Waiting for application to be completed ...

HOST-Info: Test Passed
HOST-Info: Results stored in the HW_Res.txt file ...

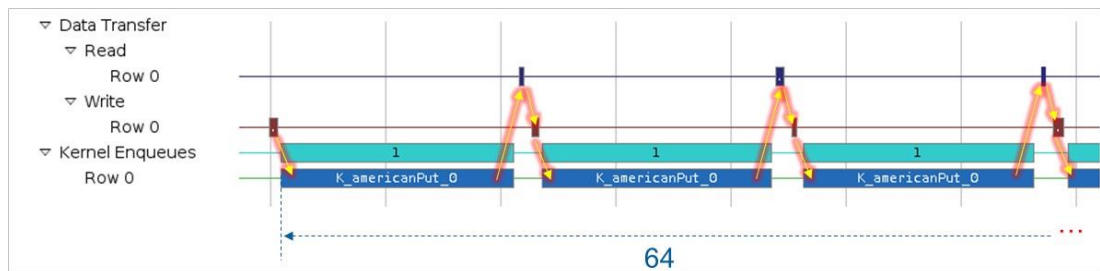
HOST-Info: Application Completed

INFO: [SDx-EM 22] [Wall clock time: 09:45, Emulation time: 0.483735 ms] Data transfer
between kernel(s) and global memory(s)
K_americanPut_0_1:m_axi_gmem_0-DDR[1]          RD = 2.000 KB          WR = 0.000 KB
K_americanPut_0_1:m_axi_gmem_1-DDR[1]          RD = 0.000 KB          WR = 0.250 KB
```

Then open Application Timeline report: in the **Assistant** tab, under **P10\_hw\_Original** / **Emulation-HW** / **P10\_hw\_Original-Default** double click on the **Application Timeline** to open it:




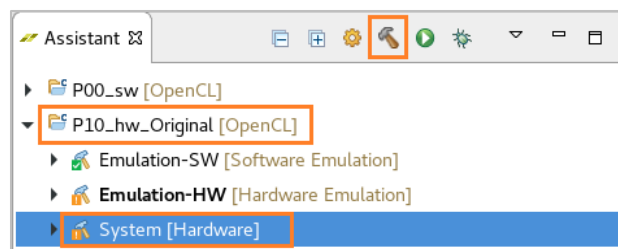
Zoom in and observe a sequential nature of the entire execution process on the Alveo card:




### 4-3. System Run

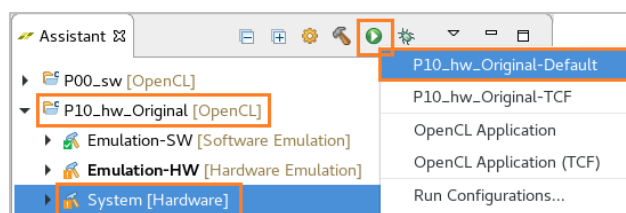
At this step of the flow, compile and run the design on the Alveo card. Please note that it may take ~1.5 hours to compile the application.

To compile the design for System Run in the **Assistant** tab, under **P10\_hw\_Original** select **System**, and then press on the  icon:



The compilation process should complete successfully.

To launch System Run, in the **Assistant** tab, under **P10\_hw\_Original** select **System**, then click on the  icon and select **P10\_hw\_Original-Default**:



The application execution is successfully completed, and the Alveo execution time is reported in a Console window:

```
...
HOST-Info: =====
HOST-Info: Step: Profiling
HOST-Info: =====
HOST-Info:  NUMBER_OF_KERNELS      :          1
HOST-Info:  NB_OF_TESTS                :          256
HOST-Info:  HW Execution Time (ms) :    1106.7
HOST-Info: -----
```

Comparing CPU (single thread) and the Alveo results, you can see that the Alveo solution outperforms CPU by the factor of **8.9x**:

Design	CPU (ms)	Alveo (ms)	Alveo Gain vs. CPU
Original	9834.2 1 Thread	1106.7	<b>8.9 x</b>

## Alveo: Host Optimization - Data Transfer and Kernel Execution Overlap

### Step 5

Projects: **P11\_hw\_Original\_HostOpt**

Using the same (original) kernel implementation from the **P10\_hw\_Original** project, we can improve design performance by overlapping data transfers with Kernel execution. This is achieved by modifying the host code (Host.cpp).

Such an approach is beneficial for processing a large amount of data.

- *Note: in our design, we deal with a small amount of data - 256 test vectors only.*
  - *Therefore, we do not expect significant performance gain by using this method.*
  - *However, we still want to demonstrate this approach when targeting Alveo cards.*

In this approach:

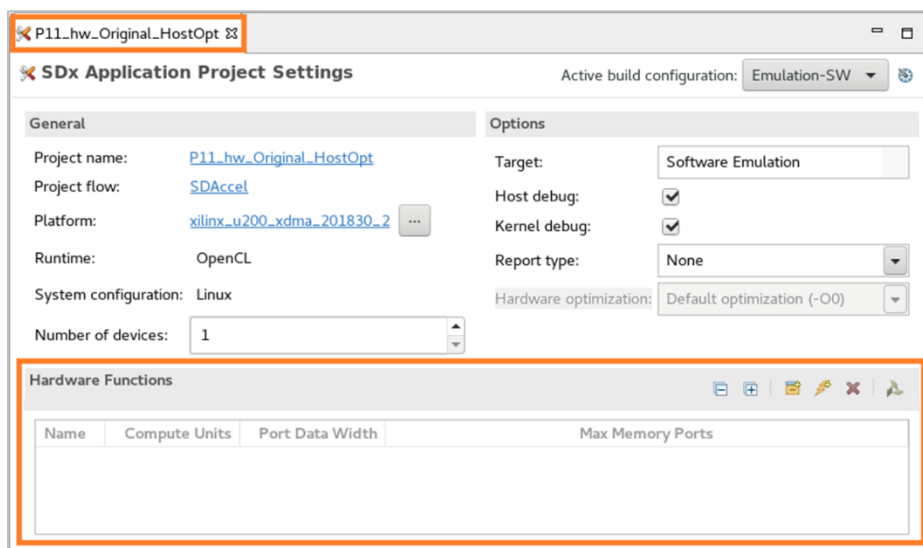
- Data is prefetched in advance for coming computations
- Ping-Pong buffers are used for Input test vectors and Results
- A single compute unit (CU) is used

This method is illustrated in the following figure:



Open the **P11\_hw\_Original\_HostOpt** project.

Please notice that there are no kernels listed in the Hardware Functions:



This is done to save time by avoiding kernel compilation in SW Emulation, HW Emulation and System Run modes. This is achieved by preconfiguring the **P11\_hw\_Original\_HostOpt-Default** run, where the **binary\_container\_1.xclbin** file is referenced from the **P10\_hw\_Original** project.

Compile (only the host will be compiled) and run the design in a **SW Emulation** mode to confirm that the design functionality is correct. You should see the following LOG messages:

```
HOST-Info: =====
HOST-Info: Step: Run Application
HOST-Info: =====

HOST_Info: Waiting for application to be completed ...

HOST_Info: Test Passed
...
```

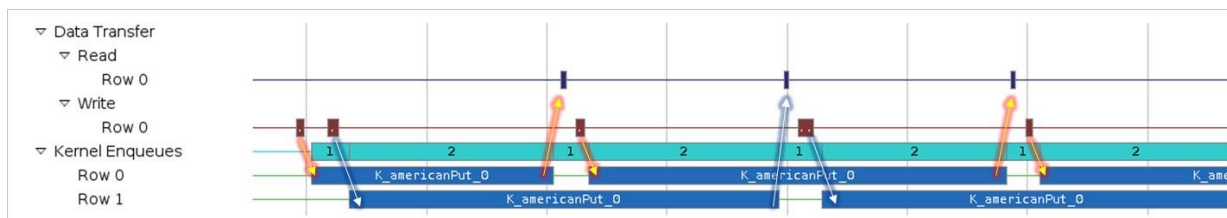
Compile and run the design in a **HW Emulation** mode. You should see the following LOG messages:

```
HOST-Info: =====
HOST-Info: Step: Run Application
HOST-Info: =====

HOST_Info: Waiting for application to be completed ...

HOST_Info: Test Passed
...
```

Then open **Application Timeline** trace report. Zoom in. You should observe an overlap between data transfer and kernel execution:



Compile and run the design in a **System Run** mode. The Alveo execution time is reported in a Console window:

```
HOST-Info: =====
HOST-Info: Step: Profiling
HOST-Info: =====
HOST-Info:   NUMBER_OF_KERNELS      :      1
HOST-Info:   NB_OF_TESTS              :    256
HOST-Info:   HW Execution Time (ms) :    958.1
HOST-Info: -----
```

Comparing CPU (multi-thread) and the Alveo results, you can see that the Alveo solution outperforms CPU by the factor of **1.3x**:

Design	CPU (ms)	Alveo (ms)	Alveo Gain vs. CPU
Original	9834.2 1 Thread	1106.7	8.9 x
Original (Host Optimization)	1289.7 12 Threads	958.1	1.3 x

## Alveo: Single Kernel Optimization - Data Movement

## Step 6

Projects: **P11\_hw\_Original\_HostOpt**, **Popt\_12\_step1\_DataMove**

To achieve significant performance acceleration, we need to replace the sequential execution of our application by a more efficient approach. This can be achieved by

- Changing a way data is transferred between
  - Host and Global Memory
  - Global Memory and Kernel
- Increasing the parallelization level of a computation process

### Important notes:

- To achieve faster design iterations and convergence, you are going to use SW Emulation and HW Emulation modes only during the single kernel optimization process. The final acceleration gain will be of course measured on the Alveo card via System Run.
- To improve execution time at the HW Emulation step, the complexity of the test vectors was reduced. The test vectors for the HW Emulation are stored in the **test\_config\_HW\_Emu.txt** file:

Binomial settings									Test Vector Configurations	
Company	T	S	K	r	sigma	q	n		K_Step	NB_OF_TESTS
Comp_1	1	110.0	100.0	0.025	0.2	0.1	10		1.0	64

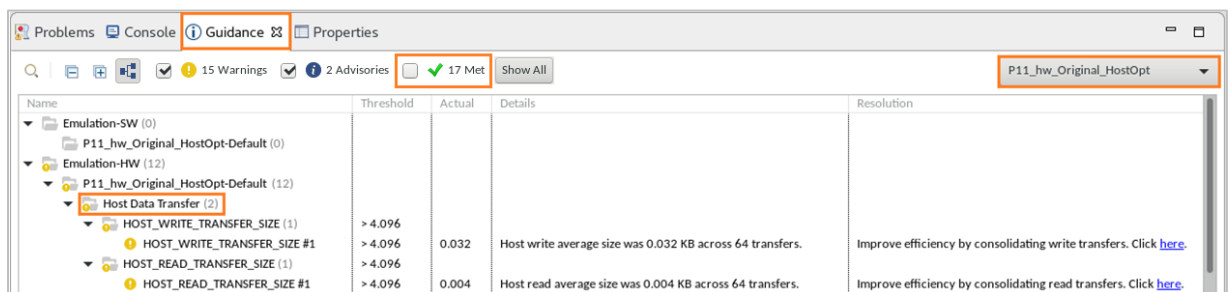
For HW Emulation:

- The number of test vectors was reduced to **64** (from of 256 used for SW Emulation and System Run)
- The height of the Binomial tree was reduced to **10** (from of 1024 used for SW Emulation and System Run)

Before optimizing the design, let's have a look at the suggestions reported by SDAccel flow at the HW Emulation step in the **P11\_hw\_Original\_HostOpt** project. These suggestions could be found on a **Guidance** tab.

### 6-1. Host ↔ Global Memory Data transfer

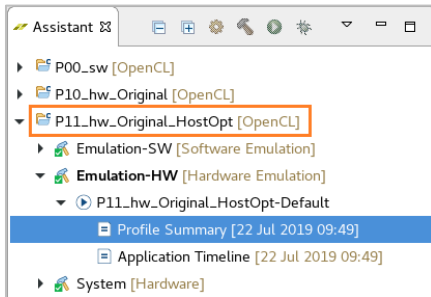
Make the **Guidance** tab active, then select the **P11\_hw\_Original\_HostOpt** project in the drop-down menu. Uncheck the rules which met the requirements, to focus on the suggestions only:



Navigate to the **Host Data Transfer** section. You will see that SDAccel reports that data transfer between Host and Global memory is done in small chunks of data and suggest to improve efficiency by consolidating read and write transfers.

Please note that there are other Host Data Transfer suggestions, which will be reviewed later.

Information about data transfer can be found in the **profile summary** report (generated during application run). To open this report, in the Assistant, under the **P11\_hw\_Original\_HostOpt / Emulation-HW / ...** double click on the **Profile Summary**:



In the opened report, navigate to the **Data Transfer** tab. In the **Host and Global Memory** section you will find **Number of Transfers** and **Average Size** for Read and Write operations:

P11\_hw\_Original\_HostOpt

Profile Summary

Profile Summary (sdaccel\_profile\_summary) for 'P11\_hw\_Original\_HostOpt-Default' launch configurati

Top Operations

Kernels & Compute Units

Data Transfers

OpenCL APIs

Data Transfer: Host and Global Memory

Context: Number of Devices	Transfer Type	Number Of Transfers	Average Size (KB)	Transfer Rate (MB/s)	Average Bandwidth Utilization (%)	Total Time (ms)	Average Time (ms)
context0:1	READ	64	0.004		N/A	N/A	N/A
context0:1	WRITE	64	0.032	N/A	N/A	N/A	N/A

Data Transfer: Kernels and Global Memory

Device	Compute Unit/ Port Name	Kernel Arguments	Memory Resources	Transfer Type	Number Of Transfers	Average Size (KB)
xilinx_u200_xdma_201830_2-0	K_americanPut_0_1/m_axi_gmem_0	IN_Data	DDR[1]	READ	64	0.032
xilinx_u200_xdma_201830_2-0	K_americanPut_0_1/m_axi_gmem_1	Res	DDR[1]	WRITE	64	0.004

## 6-2. Global Memory ↔ Kernel Data transfer

If you navigate to the **Kernel Data Transfer** section in the **Guidance** report, then you will find suggestions representing data transfer between Kernel and Global memory. SDAccel suggests to improve efficiency by using Read/Write **Burst** transfers:

Name	Threshold	Actual	Details	Resolution
Emulation-HW (12)				
P11_hw_Original_HostOpt-Default (12)				
Host Data Transfer (2)				
Resource Usage (1)				
Kernel Data Transfer (9)				
KERNEL_READ_TRANSFER_SIZE (1)	> 0.512			
KERNEL_READ_TRANSFER_SIZE #1	> 0.512	0.032	Kernel average read size on port K_americanPut_0_1/m_axi_gmem	Use burst transfers and packing into full memory data width. Click <a href="#">here</a> .
KERNEL_READ_TRANSFER_UTIL (1)	> 5.000			
KERNEL_WRITE_TRANSFER_SIZE (1)	> 0.512			
KERNEL_WRITE_TRANSFER_SIZE #1	> 0.512	0.004	Kernel average write size on port K_americanPut_0_1/m_axi_gmem	Use burst transfers and packing into full memory data width. Click <a href="#">here</a> .

Information about data transfer between **Kernel and Global memory** can be found in the **profile summary** report as well:

P11\_hw\_Original\_HostOpt **Profile Summary**

**Profile Summary (sdaccel\_profile\_summary) for 'P11\_hw\_Original\_HostOpt-Default' launch configuration**

Top Operations / **Kernels & Compute Units** / **Data Transfers** / OpenCL APIs

▼ **Data Transfer: Host and Global Memory**

Context: Number of Devices	Transfer Type	Number Of Transfers	Average Size (KB)	Transfer Rate (MB/s)	Average Bandwidth Utilization (%)	Total Time (ms)	Average Time (ms)
context0:1	READ	64	0.004	N/A	N/A	N/A	N/A
context0:1	WRITE	64	0.032	N/A	N/A	N/A	N/A

▼ **Data Transfer: Kernels and Global Memory**

Device	Compute Unit/ Port Name	Kernel Arguments	Memory Resources	Transfer Type	Number Of Transfers	Average Size (KB)
xilinx_u200_xdma_201830_2-0	K_americanPut_0_1/m_axi_gmem_0	IN_Data	DDR[1]	READ	64	0.032
xilinx_u200_xdma_201830_2-0	K_americanPut_0_1/m_axi_gmem_1	Res	DDR[1]	WRITE	64	0.004

Then open **Kernel & Compute Units** tab in the profile summary report and record total estimated time to run a kernel:

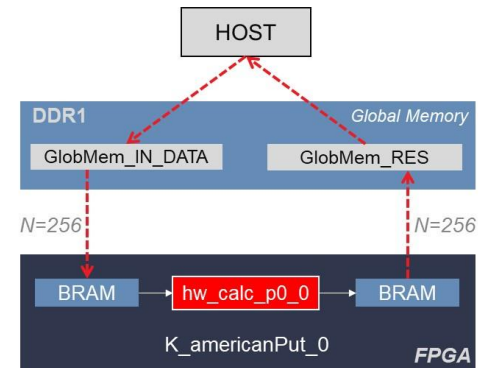
Top Operations / **Kernels & Compute Units** / Data Transfers / OpenCL APIs

▼ **Kernel Execution (includes estimated device times)**

Kernel	Number Of Enqueues	Total Time (ms)	Minimum Time (ms)	Average Time (ms)	Maximum Time (ms)
K_americanPut_0	64	0.594	0.006	0.009	0.013

In order to improve data transfer and therefore, design performance:

- The host should transfer ALL test vectors to Global Memory as a single block of data
- Then Kernel
  - Prefetches ALL test vectors to Local Memory (implemented usually using BRAM resources) in a burst mode
  - Processes ALL test vectors and stores ALL results in Local Memory
  - Transfers all results to Global Memory in Burst Mode
- Finally, Host transfers ALL results from to Host Memory



This can be represented using the following block diagram:

### 6-3. Open the Popt\_12\_step1\_DataMove project.

To implement this solution, we need to make changes in:

- **Host** code to transfer all data to/from Global Memory using a single API call per write and read operations (see **Host.cpp**):

```
...
// .....
// Copy ALL test vectors: host_IBuf -> GlobMem_IBuf
// .....
errCode = clEnqueueMigrateMemObjects(Command_Queue, 1, &(HW_Kernels[k_index].GlobMem_IBuf),
                                     0, 0, NULL, &Mem_wr_event[0]);
...
// .....
```



```

// Submit Kernel for execution
// .....
...
errCode = clEnqueueNDRangeKernel(Command_Queue, HW_Kernels[k_index].kernel, 1, NULL,
                                globalSize, localSize, 1,
                                &Mem_wr_event[0], &K_exe_event[0]);
...

// .....
// Copy ALL results: GlobMem_OBuf -> host_OBuf
// .....
errCode = clEnqueueMigrateMemObjects(Command_Queue, 1, &(HW_Kernels[k_index].GlobMem_OBuf),
                                CL_MIGRATE_MEM_OBJECT_HOST,
                                1, &K_exe_event[0], &Mem_rd_event[0]);
...

```

- In **K\_americanPut\_0** kernel function (no changes are required in **hw\_calc\_p0\_0**) (see **K0.cpp**)

```

...
void K_americanPut_0(t_in_data* IN_Data, float* Res,
                    int Nb_of_Tests, int Start_Index ) {

// ----- //
    #pragma HLS INTERFACE s_axilite port=IN_Data      bundle=control
    #pragma HLS INTERFACE s_axilite port=Res          bundle=control
    ...
// ----- //

    t_in_data tmp_IN_Data[CONST_MAX_NB_OF_TESTS];
    #pragma HLS DATA_PACK variable=tmp_IN_Data

    float      tmp_Res[CONST_MAX_NB_OF_TESTS];

    // -----
    // Transfer data: Global Memory -> BRAM
    // -----
    read_in_data_loop: for (int i = 0; i < Nb_of_Tests; i++)
        #pragma HLS LOOP_TRIPCOUNT min=100 max=100 avg=100
        tmp_IN_Data[i] = IN_Data[Start_Index+i];

    // -----
    // Calculate
    // -----
    calculalte: for (int i = 0; i < Nb_of_Tests; i++) {
        #pragma HLS LOOP_TRIPCOUNT min=100 max=100 avg=100
        tmp_Res[i] = hw_calc_p0_0(tmp_IN_Data[i]);
    }

    // -----
    // Transfer data: BRAM -> Global Memory
    // -----
    write_out_data_loop: for (int i = 0; i < Nb_of_Tests; i++)
        #pragma HLS LOOP_TRIPCOUNT min=100 max=100 avg=100
        Res[Start_Index+i] = tmp_Res[i];
}

```

In the above code:

- We added two input arguments: **Nb\_of\_Test** and **Start\_Index**. The first one defines the number of test vectors kernel need to process. The last one will be used at later steps of the flow when multiple CUs will be implemented on the Alveo card.
- We introduced local memory: **tmp\_IN\_Data**, **tmp\_Res**.
- In the **read\_in\_data\_loop** loop, we transfer input data from Global to Local memory in a burst mode.
- The **calculate** loop processes all test vectors and stores them in local memory.
- And finally, the **write\_out\_data\_loop** loop transfers all results to the Global memory in a burst mode.

Compile and run the project in **SW Emulation** mode to check its functionality.

Then compile and run the project in **HW Emulation** mode and then open Profile Summary report.

You will see that the number of transfers was significantly reduced:

Top Operations

Kernels & Compute Units

Data Transfers

OpenCL APIs

▼ Data Transfer: Host and Global Memory

Context: Number of Devices	Transfer Type	Number Of Transfers	Average Size (KB)	Transfer Rate (MB/s)	Average Bandwidth Utilization (%)	Total Time (ms)	Average Time (ms)
context0:1	READ	1	0.256	N/A	N/A	N/A	N/A
context0:1	WRITE	1	2.048	N/A	N/A	N/A	N/A

▼ Data Transfer: Kernels and Global Memory

Device	Compute Unit/Port Name	Kernel Arguments	Memory Resources	Transfer Type	Number Of Transfers	Average Size (KB)
xilinx_u200_xdma_201830_2-0	K_americanPut_0_1/m_axi_gmem_0	IN_Data	DDR[1]	READ	4	0.512
xilinx_u200_xdma_201830_2-0	K_americanPut_0_1/m_axi_gmem_1	Res	DDR[1]	WRITE	4	0.064

Then check Kernel execution time (estimated):

Top Operations		Kernels & Compute Units		Data Transfers		OpenCL APIs	
▼ Kernel Execution (includes estimated device times)							
Kernel	Number Of Enqueues	Total Time (ms)	Minimum Time (ms)	Average Time (ms)	Maximum Time (ms)		
K_americanPut_0	1	0.194	0.194	0.194	0.194		

You can see that it was reduced by the factor of **3x** compared to the **0.594 ms** obtained in the **P11\_hw\_Original\_HostOpt** project.

Now let's open **Guidance** report and review a couple of additional suggestions from the SDAccel flow. Please ensure that the **Popt\_12\_step1\_DataMove** project is selected:

11 Warnings 4 Advisories 12 Met Show All					Popt_12_step1_DataMove	
Name	Threshold	Actual	Details	Resolution		
Emulation-SW (0)						
Emulation-HW (11)						
Popt_12_step1_DataMove-Default (9)						
Host Data Transfer (2)						
Kernel Data Transfer (7)						
KERNEL_READ_TRANSFER_UTIL (1)	> 5.000					
KERNEL_WRITE_TRANSFER_SIZE (1)	> 0.512					
KERNEL_WRITE_TRANSFER_UTIL (1)	> 5.000					
MEMORY_WRITE_TRANSFER_UTIL (1)	> 5.000					
KERNEL_PORT_DATA_WIDTH (2)	= 512					
KERNEL_PORT_DATA_WIDTH #1	= 512	256	Port K_americanPut_0_1/m_axi_gmem_0 has a data width of 256.	Utilize the entire memory data width. Click <a href="#">here</a> .		
KERNEL_PORT_DATA_WIDTH #2	= 512	32	Port K_americanPut_0_1/m_axi_gmem_1 has a data width of 32.	Utilize the entire memory data width. Click <a href="#">here</a> .		

DDR data bus enables the transfer of **512** bits of data simultaneously. The guidance points out that for

- READ operation we use **256** bits (one test vector transferred)
- WRITE operation we use **32** bits (one result transferred)

and suggests using **512** bits to reduce the number of transfers further.

This can be achieved by using vectorization approach (ex: to transfer 16 floats (results) simultaneously). This technique is not considered in this example. Please refer to **UG1207** Optimization Guide for more information.

In addition, we have the following suggestion:

The screenshot shows the 'Guidance' window in the Xilinx IDE. It displays a table with columns: Name, Threshold, Actual, Details, and Resolution. The table lists various performance metrics for the 'Popt\_12\_step1\_DataMove' design. A warning icon is present next to the 'PLRAM\_USAGE #1' entry, which indicates that PLRAMs were used by 0 port(s). The resolution text suggests utilizing PLRAMs to maximize performance.

Name	Threshold	Actual	Details	Resolution
Emulation-SW (0)				
Popt_12_step1_DataMove-Default (0)				
Emulation-HW (15)				
Popt_12_step1_DataMove-Default (10)				
Host Data Transfer (2)				
Kernel Data Transfer (8)				
KERNEL_READ_TRANSFER_UTIL (1)	> 5.000			
KERNEL_WRITE_TRANSFER_SIZE (1)	> 0.512			
PLRAM_USAGE (1)	> 0			
PLRAM_USAGE #1	> 0	0	PLRAMs were used by 0 port(s).	Utilize PLRAMs to maximize performance. Click <a href="#">here</a> .

Global Memory can be implemented using **BRAM** resources instead of DDR (called **PLRAM**), providing faster data access, which is suggested here. This technique is not considered in this example. Please refer to UG1207 Optimization Guide for more information.

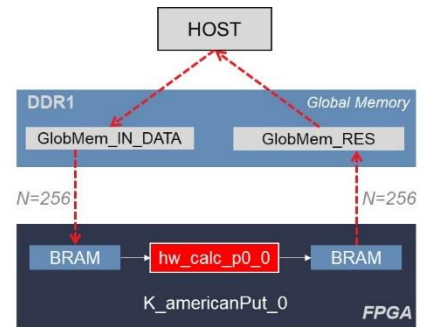
## Alveo: Single Kernel Optimization - Running 4 Functions in Parallel

### Step 7

Projects: **Popt\_12\_step2\_FUNx4**, **Popt\_12\_step3\_ARRAY\_PARTITION**

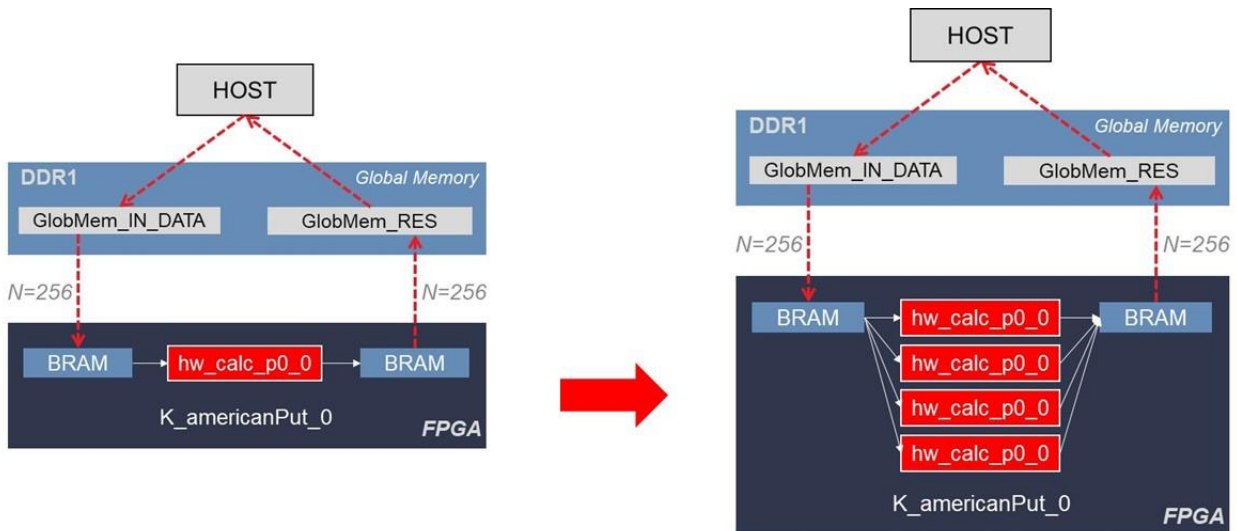
In the previous **Popt\_12\_step2\_DataMove** project we processed a single test vector per iteration:

```
void K_americanPut_0(t_in_data* IN_Data, float* Res,
                    int Nb_of_Tests, int Start_Index ) {
...
    calcualte: for (int i = 0; i < Nb_of_Tests; i++) {
        #pragma HLS LOOP_TRIPCOUNT min=100 max=100 avg=100
        tmp_Res[i] = hw_calc_p0_0(tmp_IN_Data[i]);
    }
...
}
```



### 7-1. Open the Popt\_12\_step2\_FUNx4 project.

To further boost performance, we can process several test vectors simultaneously. In our design, we chose to process four of them, which should provide an additional **4x** acceleration factor. This transformation is presented in the following figure:



For that we need to slightly modify the Kernel code by introducing an additional loop (**calculate\_sub\_i**) with 4 iterations and **unroll** it:

```
void K_americanPut_0(t_in_data* IN_Data, float* Res,
                    int Nb_of_Tests, int Start_Index ) {
...
    calculate_i: for (int i = 0; i < Nb_of_Tests/4; i++) {
        #pragma HLS LOOP_TRIPCOUNT min=25 max=25 avg=25
        calculate_sub_i: for (int sub_i = 0; sub_i < 4; sub_i++) {
            #pragma HLS UNROLL
            tmp_Res[i*4+sub_i] = hw_calc_p0_0(tmp_IN_Data[i*4+sub_i]);
        }
    }
...
}
```

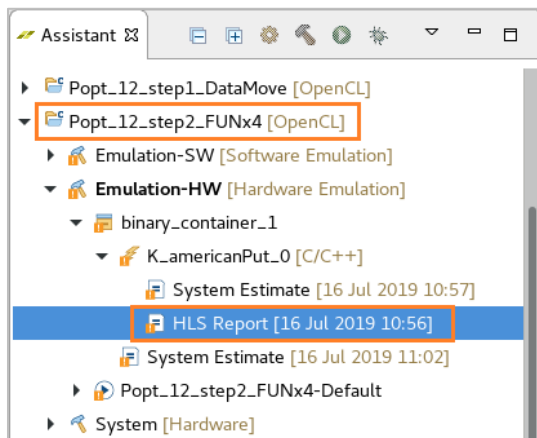
Compile and run the project in **SW Emulation** mode to check its functionality.

After they compile and run the project in **HW Emulation** mode and then open Profile Summary report.

Top Operations		Kernels & Compute Units	Data Transfers	OpenCL APIs	
▼ Kernel Execution (includes estimated device times)					
Kernel	Number Of Enqueues	Total Time (ms)	Minimum Time (ms)	Average Time (ms)	Maximum Time (ms)
K_americanPut_0	1	0.100	0.100	0.100	0.100

Comparing kernel execution time (**0.100 ms**) with the one obtained in the **Popt\_12\_step2\_DataMove** project (**0.194 ms**), we can observe that **2x** acceleration was only achieved instead of expected **4x**.

Using Assistant, open **HLS synthesis** report which provides more detailed information regarding Kernel implementation:



In the HLS report, navigate to the **Performance Estimates / Instance** section:

Performance Estimates

Timing (ns)

Summary

Clock	Target	Estimated	Uncertainty
ap_clk	3.33	2.729	0.90

Latency (clock cycles)

Summary

Latency		Interval		
min	max	min	max	Type
1344491	1344494	1344491	1344494	none

Detail

Instance

		Latency		Interval		
Instance	Module	min	max	min	max	Type
grp_hw_calc_p0_0_fu_308	hw_calc_p0_0	26883	26883	26883	26883	none
grp_hw_calc_p0_0_fu_328	hw_calc_p0_0	26883	26883	26883	26883	none

Loop

In the kernel code, we asked to fully unroll a 4-iteration loop, expecting that **hw\_calc\_p0\_0** will be called four times simultaneously. However, as you can see from the HLS report, only two function calls were implemented.

The performance bottleneck is in accessing local memory: **tmp\_IN\_Data** and **tmp\_Res** arrays. These arrays are implemented as BRAMs, which can be seen from the **Utilization Estimates** section of the HLS report:

Memory									
Memory	Module	BRAM_18K	FF	LUT	Words	Bits	Banks	W*Bits*Banks	
tmp_IN_Data_UK_americanPut_0_tmp_IN_Data		15	0	0	1024	256	1	262144	
tmp_Res_U	K_americanPut_0_tmp_Res	2	0	0	1024	32	1	32768	
Total		2	17	0	2048	288	2	294912	

## 7-2. Open the Popt\_12\_step3\_ARRAY\_PARTITION project.

Architecturally, a BRAM has 2 ports and therefore enables to simultaneously read/write 2 pieces of information (maximum). Therefore, SDAccel compiler by default can implement only two **hw\_calc\_p0\_0** function calls instead of four. In order to implement four function calls, the algorithm should have the possibility to read/write 4 pieces of data from/to **tmp\_IN\_Data** and **tmp\_Res** simultaneously. This can be achieved via restructuring how data is stored in **tmp\_IN\_Data** and **tmp\_Res**, by using the **ARRAY\_PARTITION** pragma (please refer to the UG1207 and UG1253 users guide for more information).

According to the Binomial algorithm, the **tmp\_IN\_Data** and **tmp\_Res** arrays should be partitioned using a **cyclic** mode with the factor of 2 (K0.cpp):

```

void K_americanPut_0(t_in_data* IN_Data, float* Res,
                    int Nb_of_Tests, int Start_Index ) {
...
    t_in_data tmp_IN_Data[CONST_MAX_NB_OF_TESTS];
    #pragma HLS DATA_PACK variable=tmp_IN_Data
    #pragma HLS ARRAY_PARTITION variable=tmp_IN_Data cyclic factor=2 dim=1

    float      tmp_Res[CONST_MAX_NB_OF_TESTS];
    #pragma HLS ARRAY_PARTITION variable=tmp_Res      cyclic factor=2 dim=1
...
    calculate_i: for (int i = 0; i < Nb_of_Tests/4; i++) {
        #pragma HLS LOOP_TRIPCOUNT min=25 max=25 avg=25
        calculate_sub_i: for (int sub_i = 0; sub_i < 4; sub_i++) {
            #pragma HLS UNROLL
            tmp_Res[i*4+sub_i] = hw_calc_p0_0(tmp_IN_Data[i*4+sub_i]);
        }
    }
...
}

```

Compile and run the project in **SW Emulation** mode to check its functionality.

Then compile and run the project in **HW Emulation** mode.

Open **HLS report**:

**Performance Estimates**

☒ **Timing (ns)**

☒ **Summary**

Clock	Target	Estimated	Uncertainty
ap_clk	3.33	2.729	0.90

☒ **Latency (clock cycles)**

☒ **Summary**

Latency		Interval		
min	max	min	max	Type
672366	672369	672366	672369	none

☒ **Detail**

☒ **Instance**

Instance	Module	Latency		Interval		Type
		min	max	min	max	
grp_hw_calc_p0_0_fu_347	hw_calc_p0_0	26882	26882	26882	26882	none
grp_hw_calc_p0_0_fu_367	hw_calc_p0_0	26882	26882	26882	26882	none
grp_hw_calc_p0_0_fu_387	hw_calc_p0_0	26882	26882	26882	26882	none
grp_hw_calc_p0_0_fu_407	hw_calc_p0_0	26882	26882	26882	26882	none

☒ **Loop**

In this version, four **hw\_calc\_p0\_0** function calls were implemented, and as a consequence, kernel latency was reduced from 1344491 to 672366.

Open **Profile summary** report:

Top Operations		Kernels & Compute Units	Data Transfers	OpenCL APIs	
▼ Kernel Execution (includes estimated device times)					
Kernel	Number Of Enqueues	Total Time (ms)	Minimum Time (ms)	Average Time (ms)	Maximum Time (ms)
K_americanPut_0	1	0.051	0.051	0.051	0.051

Comparing kernel execution time (**0.051 ms**) with the one obtained in the **Popt\_12\_step2\_DataMove** project (**0.194 ms**), we can observe that we were able to achieve **4x** of acceleration.



## Alveo: Single Kernel Optimization - Additional Loops Unrolling

### Step 8

Projects: **Popt\_12\_step4\_UNROLL**

Open HLS report generated for the **Popt\_12\_step3\_ARRAY\_PARTITION** project, navigate to the **hw\_calc\_p0\_0** function and then expand the **Loop** subsection in **Performance Estimates**:

Module: K\_americanPut\_0 > hw\_calc\_p0\_0

Current Module : [K\\_americanPut\\_0](#) > hw\_calc\_p0\_0

Performance Estimates

Timing (ns)

Summary

Clock	Target	Estimated	Uncertainty
ap_clk	3.33	2.729	0.90

Latency (clock cycles)

Summary

Latency		Interval		
min	max	min	max	Type
26882	26882	26882	26882	none

Detail

Instance

Loop

Loop Name	Latency		Iteration Latency	Initiation Interval		Trip Count	Pipelined
	min	max		achieved	target		
- loop_init	162	162	64	1	1	100	yes
- loop_j	26600	26600	266	-	-	100	no
+ loop_i	263	263	66	2	1	100	yes

The **loop\_init** and **loop\_i** loops are described in the following code (K0.cpp in Popt\_12\_step3\_ARRAY\_PARTITION):

```
float hw_calc_p0_0 (t_in_data in_d) {
    float p[CONST_MAX_TREE_HEIGHT];
    ...
    loop_init: for (int i = 0; i < n; i++) {
        #pragma HLS LOOP_TRIPCOUNT min=100 max=100 avg=100
        p[i] = K - S * powf(up,(2*i - n));
        if (p[i] < 0) p[i] = 0;
    }

    loop_j: for (int j = n-1; j > 0; j--) {
        #pragma HLS LOOP_TRIPCOUNT min=100 max=100 avg=100

        loop_i: for (int i = 0; i < j; i++) {
            #pragma HLS LOOP_TRIPCOUNT min=100 max=100 avg=100
            p[i] = p0 * p[i+1] + p1 * p[i];
            exercise = K - S * powf(up,(2*i - j));
            if (p[i] < exercise) p[i] = exercise;
        }
    }
    ...
}
```

According to the HLS report, the **loop\_init** and **loop\_i** loops have a trip count of **100**. This means that they deal with a single set of **p** values per iteration, where **p** is a Local memory implemented as BRAM with two RW ports.

We can partially unroll these loops with a factor of **two** to take advantage of the two RW BRAM ports and as a consequence, increase performance (limiting the increase of FPGA resources).

Open the **Popt\_12\_step4\_UNROLL** project.

Then open the **K0.cpp** file, and observe how we applied the **UNROLL** pragmas :

```
float hw_calc_p0_0 (t_in_data in_d) {
    float p[CONST_MAX_TREE_HEIGHT];
    ...
    loop_init: for (int i = 0; i < n; i++) {
        #pragma HLS LOOP_TRIPCOUNT min=100 max=100 avg=100
        #pragma HLS UNROLL factor=2
        p[i] = K - S * powf(up,(2*i - n));
        if (p[i] < 0) p[i] = 0;
    }

    loop_j: for (int j = n-1; j > 0; j--) {
        #pragma HLS LOOP_TRIPCOUNT min=100 max=100 avg=100

        loop_i: for (int i = 0; i < j; i++) {
            #pragma HLS LOOP_TRIPCOUNT min=100 max=100 avg=100
            #pragma HLS UNROLL factor=2
            p[i] = p0 * p[i+1] + p1 * p[i];
            exercise = K - S * powf(up,(2*i - j));
            if (p[i] < exercise) p[i] = exercise;
        }
    }
    ...
}
```

Compile and run the project in **SW Emulation** mode to check its functionality.

Then compile and run the project in **HW Emulation** mode.

Open **HLS** report:

Module

- ▼ K\_americanPut\_0
  - ▶ hw\_calc\_p0\_0

Current Module : [K\\_americanPut\\_0](#) > hw\_calc\_p0\_0

Performance Estimates

□ Timing (ns)

□ Summary

Clock	Target	Estimated	Uncertainty
ap_clk	3.33	2.738	0.90

□ Latency (clock cycles)

□ Summary

Latency		Interval		Type
min	max	min	max	
22033	22033	22033	22033	none

□ Detail

□ Instance

□ Loop

Loop Name	Latency		Iteration Latency	Initiation Interval		Trip Count	Pipelined
	min	max		achieved	target		
- loop_init	113	113	64	1	1	50	yes
- loop_j	21800	21800	218	-	-	100	no
+ loop_i	215	215	68	3	1	50	yes

As a result of the applied optimization, the latency of the **hw\_calc\_p0\_0** function was reduced by **18%** (from 26882 to 22033).

## Alveo: Single Kernel Optimization - System Run Results

## Step 9

Projects: **P12\_hw\_KOpt**

Source files obtained in the **Popt\_12\_step4\_UNROLL** project were propagated to the **P12\_hw\_KOpt** one to compile the design for System run and get Alveo performance results.

Open the **P12\_hw\_KOpt** project. Compile and run it for **System Run**. Please note that it may take about 2 hours to complete the compilation process.

The Alveo execution time is reported in a Console window:

```
...
HOST-Info: =====
HOST-Info: Step: Profiling
HOST-Info: =====
HOST-Info:      NUMBER_OF_KERNELS      :          1
HOST-Info:      NB_OF_TESTS              :         256
HOST-Info:      HW Execution Time (ms) :      183.9
HOST-Info: -----
```

Comparing CPU (multi-thread) and the Alveo results, you can see that the optimized Alveo solution outperforms CPU by the factor of **7.0x**:

Design	CPU (ms)	Alveo (ms)	Alveo Gain vs CPU
Original	9834.2 <i>1 Thread</i>	1106.7	8.9 x
Original (Host Optimization)	1289.7	958.1	1.3 x
Kernel Optimization	<i>12 Threads</i>	183.9	7.0x

## Alveo: Using Multiple Compute Units and Different DDRs

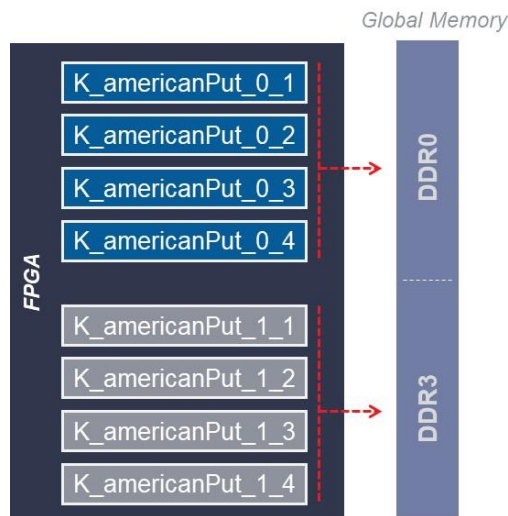
## Step 10

Projects: P13\_hw\_KOpt\_8CU\_2DDR, P14\_hw\_KOpt\_12CU\_3DDR

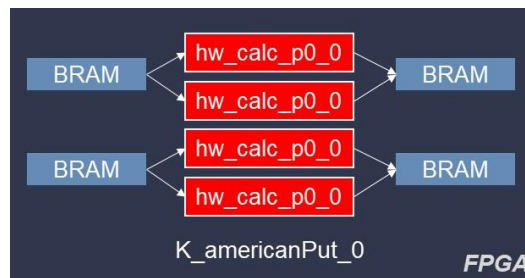
To further increase performance, we can use another technique and instantiate the optimized Kernel multiple times. In addition, we can use different DDR channels to accelerate further data movement, which is very beneficial for large data transfers.

### 10-1. Open the P13\_hw\_KOpt\_8CU\_2DDR

In this project, we will implement **8 CUs**. Four of the CUs will be connected to the **DDR0** channel and the other ones to **DDR3**, as shown in the following figure:



Taking into account that each CU is capable of processing 4 test vectors simultaneously:



the entire solution will be able to deal with 32 test vectors in parallel.


To implement such solution we have done the following changes with regards to a Kernel:

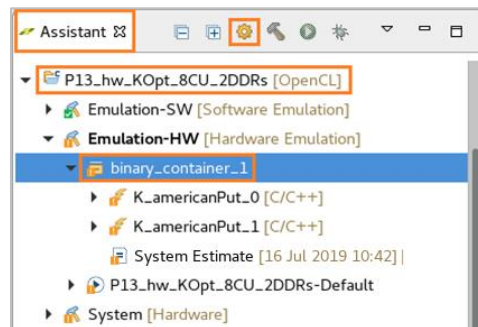
- We Created two identical kernels: **K\_americanPut\_0** (K0.cpp) and **K\_americanPut\_1** (K1.cpp) and added them to the Hardware Functions
- For each kernel we created 4 CUs:

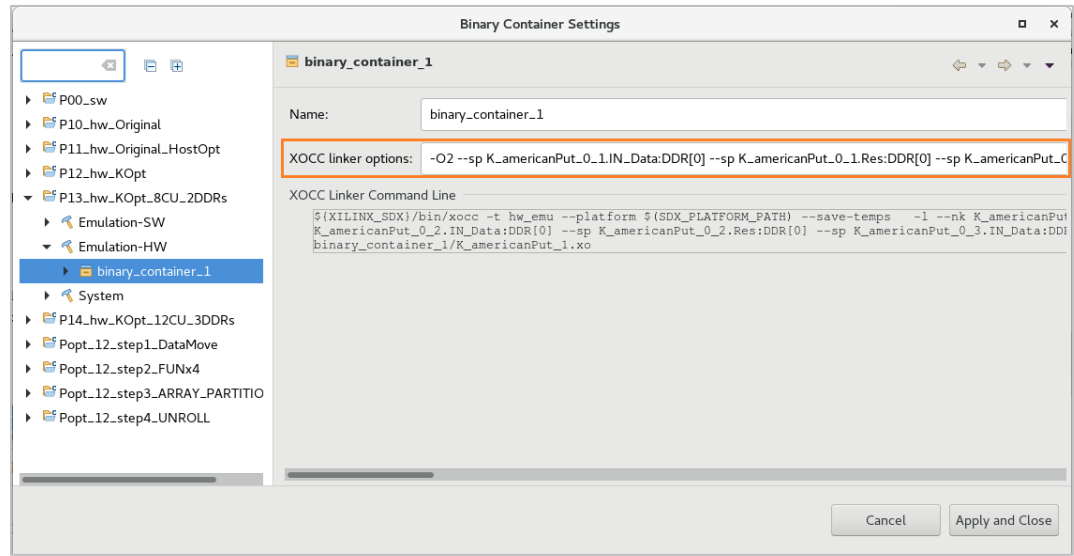
Hardware Functions			
Name	Compute Units	Port Data Width	Max Memory Ports
binary_container_1			
K_americanPut_0	4	Auto	<input type="checkbox"/>
K_americanPut_1	4	Auto	<input type="checkbox"/>

- Then we instructed SDAccel compiler how to connect the kernels to the DDR0 and DDR3 banks. This is done by specifying the following linker options:

```
--sp K_americanPut_0_1.IN_Data:DDR[0] -sp K_americanPut_0_1.Res:DDR[0]
--sp K_americanPut_0_2.IN_Data:DDR[0] -sp K_americanPut_0_2.Res:DDR[0]
--sp K_americanPut_0_3.IN_Data:DDR[0] -sp K_americanPut_0_3.Res:DDR[0]
--sp K_americanPut_0_4.IN_Data:DDR[0] -sp K_americanPut_0_4.Res:DDR[0]
--sp K_americanPut_1_1.IN_Data:DDR[3] -sp K_americanPut_1_1.Res:DDR[3]
--sp K_americanPut_1_2.IN_Data:DDR[3] -sp K_americanPut_1_2.Res:DDR[3]
--sp K_americanPut_1_3.IN_Data:DDR[3] -sp K_americanPut_1_3.Res:DDR[3]
--sp K_americanPut_1_4.IN_Data:DDR[3] -sp K_americanPut_1_4.Res:DDR[3]
```

To access these options, in the **Assistant** select **binary\_container\_1** under **Emulation-HW** or **System** and the press the  icon:





```

...
// .....
// Configure DDRs (using Xilinx Extension)
// Note: DDR Banks should be set for each implementation strategy
// .....
for (int i=0; i<(SW_HW_Config).NB_OF_KERNELS; i++) {
    HW_Kernels[i].GlobMem_IBuf_EXT.obj    = HW_Kernels[i].host_IBuf;
    HW_Kernels[i].GlobMem_IBuf_EXT.param = 0;
    HW_Kernels[i].GlobMem_OBuf_EXT.obj    = HW_Kernels[i].host_OBuf;
    HW_Kernels[i].GlobMem_OBuf_EXT.param = 0;
}

HW_Kernels[0].GlobMem_IBuf_EXT.flags = XCL_MEM_DDR_BANK0;
HW_Kernels[0].GlobMem_OBuf_EXT.flags = XCL_MEM_DDR_BANK0;
HW_Kernels[1].GlobMem_IBuf_EXT.flags = XCL_MEM_DDR_BANK3;
HW_Kernels[1].GlobMem_OBuf_EXT.flags = XCL_MEM_DDR_BANK3;
...

```

Run **SW Emulation** and **HW Emulation** to validate the design.

Then compile and run the design for **System Run** (please note that it may take about 4 hours to implement the design). The Alveo execution time is reported in a Console window:

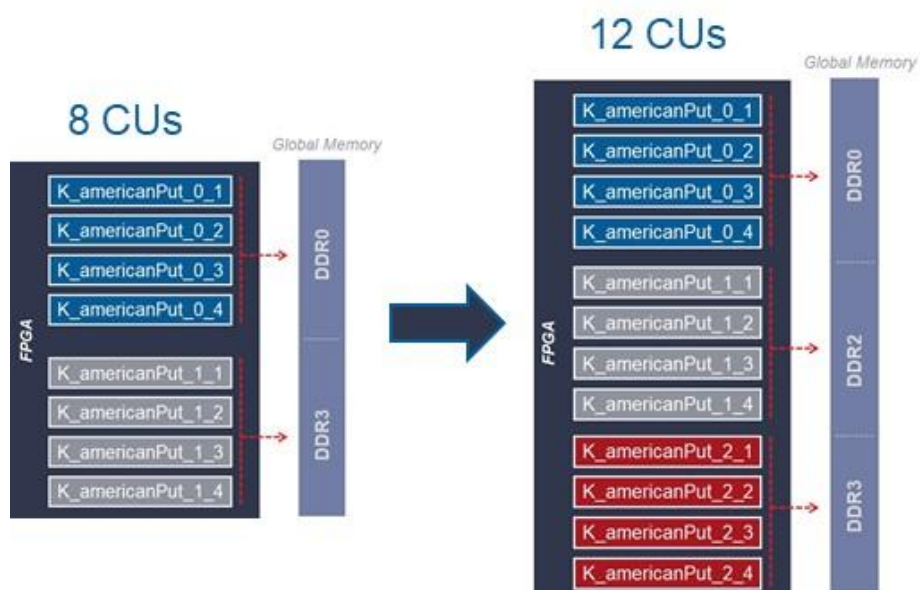
```
...
HOST-Info: =====
HOST-Info: Step: Profiling
HOST-Info: =====
HOST-Info:   NUMBER_OF_KERNELS      :           1
HOST-Info:   NB_OF_TESTS             :          256
HOST-Info:   HW Execution Time (ms) :       23.9
HOST-Info: -----
```

Comparing CPU (multi-thread) and the Alveo results, you can see that the optimized Alveo solution outperforms CPU by the factor of **54.0x**:

Design	CPU (ms)	Alveo (ms)	Alveo Gain vs CPU
Original	9834.2 1 Thread	1106.7	8.9 x
Original (Host Optimization)	1289.7 12 Threads	958.1	1.3 x
Kernel Optimization		183.9	7.0x
8 CUs and 2 DDRs		23.9	54.0x

## 10-2. Open the P14\_hw\_KOpt\_12CU\_3DDRr project

In this project we further increase the number of CUs up to 12 and connected them to them three DDRs as shown in the following figure:





Alveo-base implementationd shows additional performance improvements:

```

...
HOST-Info: =====
HOST-Info: Step: Profiling
HOST-Info: =====
HOST-Info:      NUMBER_OF_KERNELS      :          1
HOST-Info:      NB_OF_TESTS              :         256
HOST-Info:      HW Execution Time (ms) :      18.2
HOST-Info: -----

```

Comparing CPU (multi-thread) and the Alveo results, you can see that the optimized Alveo solution outperforms CPU by the factor of **70.9x**.

Comparing original Alveo performance (1106.7 ms) vs. the latest one (18.2 ms), we can see a **60.8x** improvement in the overall performance

Design	CPU (ms)	Alveo (ms)	Alveo Gain vs CPU
Original	9834.2 <i>1 Thread</i>	1106.7	<b>8.9 x</b>
Original (Host Optimization)	1289.7 <i>12 Threads</i>	958.1	<b>1.3 x</b>
Kernel Optimization		183.9	<b>7.0x</b>
8 CUs and 2 DDRs 12 CUs and 3 DDRs		23.9 18.2	<b>54.0x</b> <b>70.9x</b>

Please note that this design can be further optimized to achieve additional performance improvements.

## Conclusion:

In this tutorial by using a Binomial Options Pricing Model we:

- Introduced SDAccel flow and various analysis tools and optimization techniques allowing you to improve design performance significantly
- Demonstrated performance advantage of the Xilinx Alveo-based solution vs. the CPU one

## Projects Setup: Detailed Steps

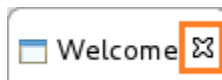
## Appendix A

The reference design consists of several projects, and all design source code is available on GitHub (<https://github.com/Xilinx/BinomialModel>). You need to import them to your local machine and then create projects using the following steps.

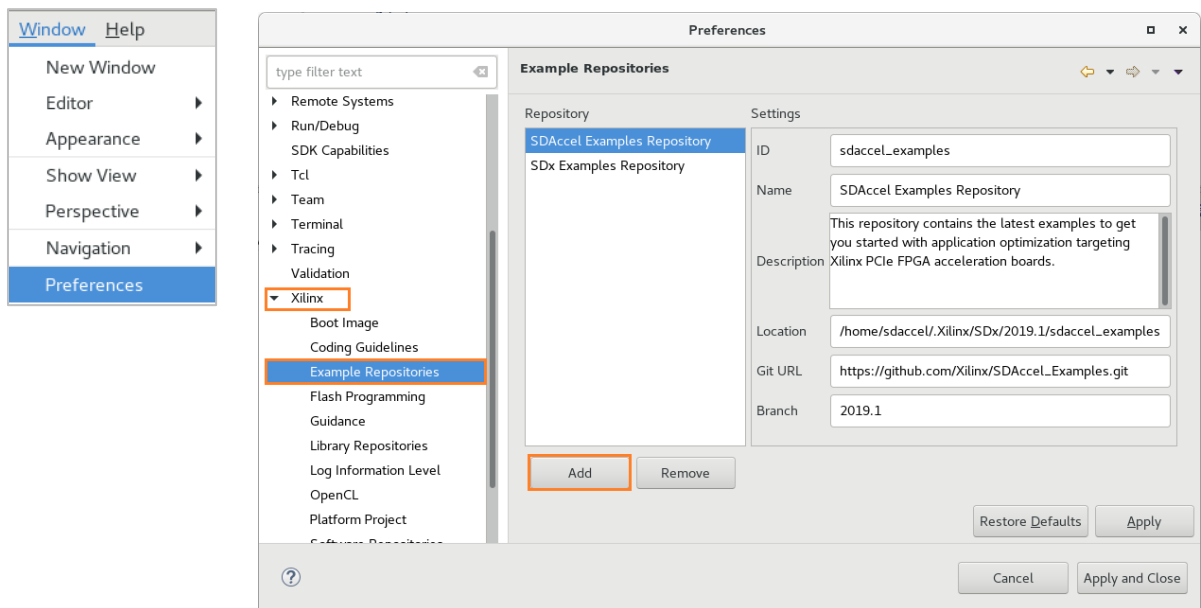
- Open **Terminal**
- Create a directory where you are going to place and run the reference design. Let's assume that this is a **/home/<user>/Binomial** folder.
- Go to the **/home/<user>/Binomial** folder
- Setup SDAccel environment by running the following command:  

```
<installation_path>/settings64.sh (bash shell)
<installation_path>/settings64.csh (c shell)
```
- Launch SDAccel by specifying **workspace** as a workspace:  

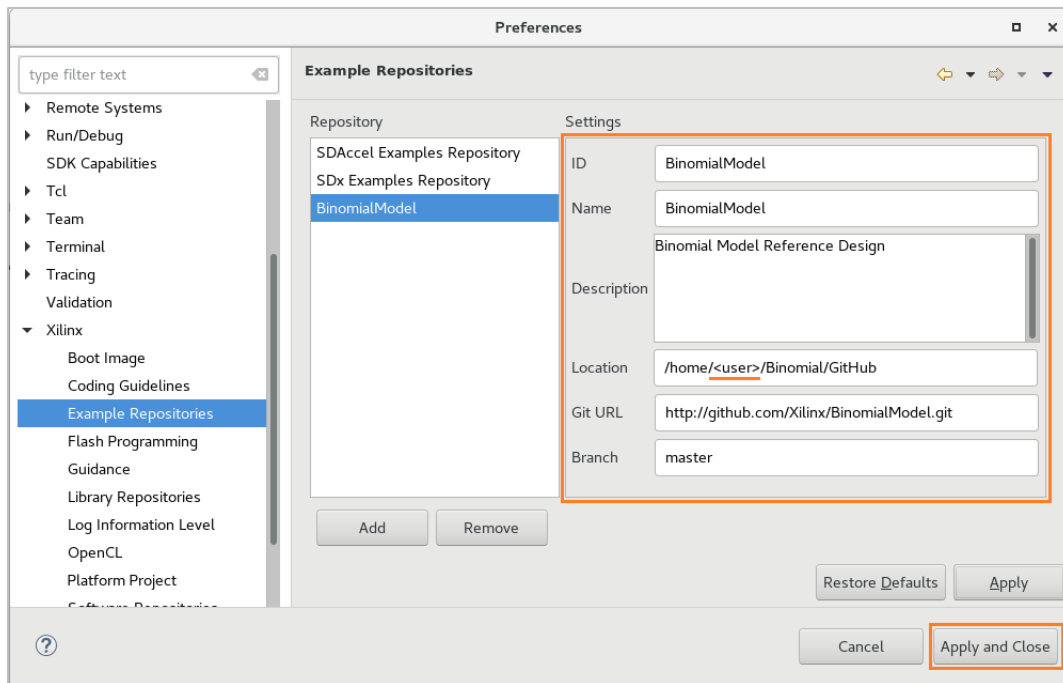
```
sdx -workspace workspace
```
- The Welcome window appears in the opened SDx environment. Press on the **x** button to Close this window before continue:



- In the opened SDAccel GUI, go to the **Window/Preferences** menu and then in the opened dialogue box navigate to the **Xilinx/Example Repositories**:



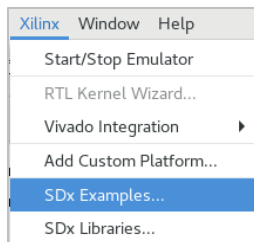
Press **Add** to add a new repository and then fill in a Settings part of the dialog box as shown below (note: replace `<user>` with your user name).



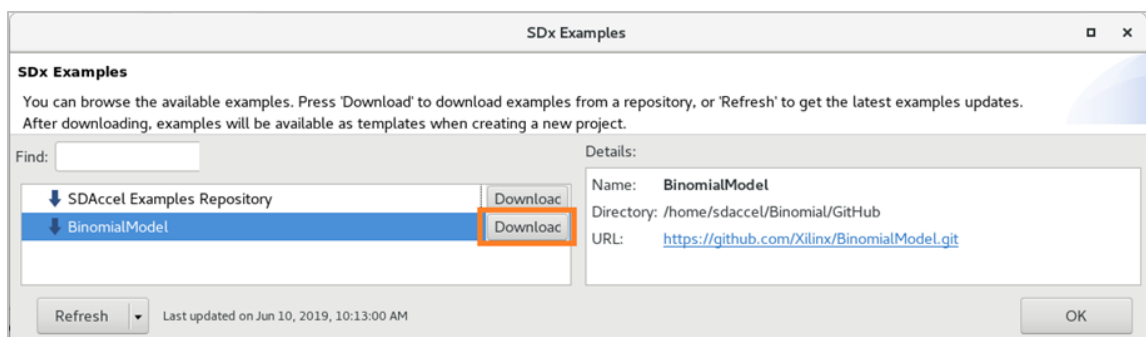
Imported project sources are placed in the **Binomial/GitHub** directory.

Then press **Apply and Close** to continue.

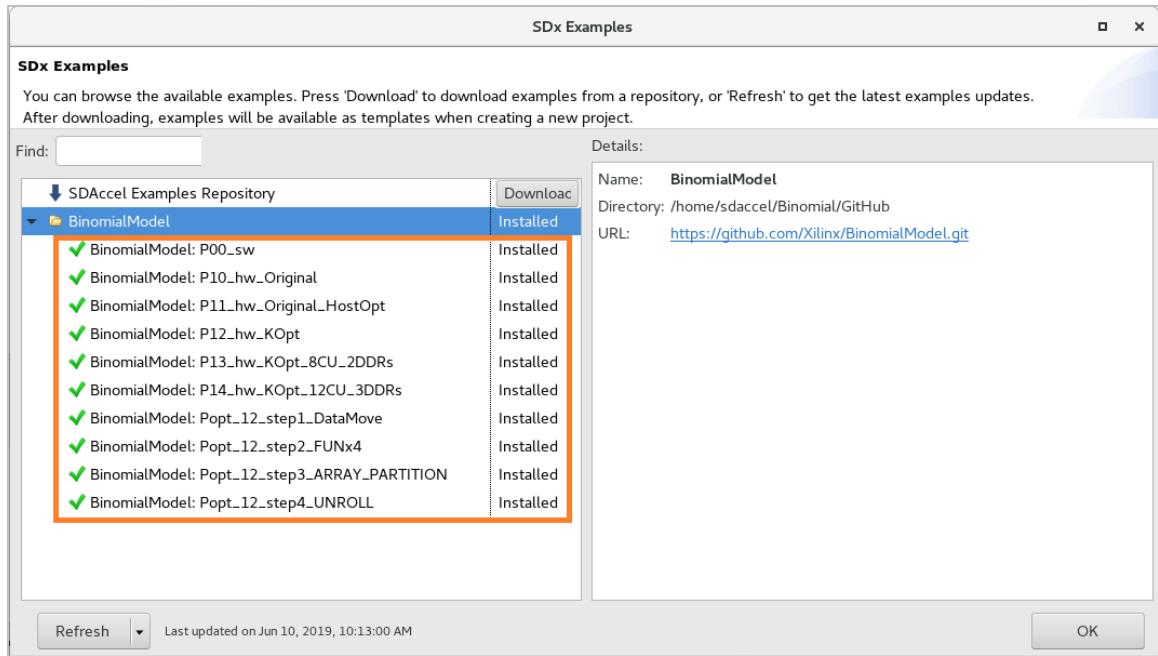
- To export the reference design, go to the **Xilinx/SDx Examples** menu



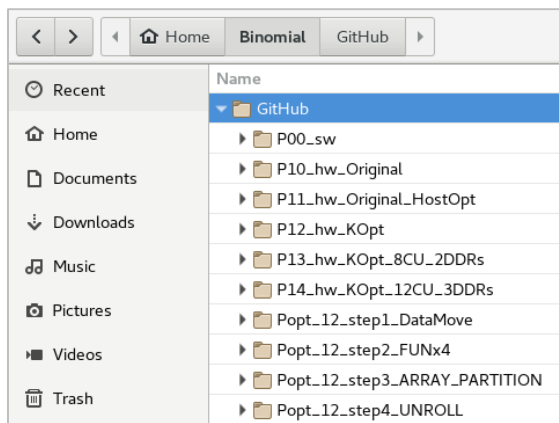
where you should see a **BinomialModel** repository. Press **Download** to import the project sources:



Once the import has completed, you should see the following list of project sources:



They are placed in the **/home/<user>/Binomial/GitHub** directory:

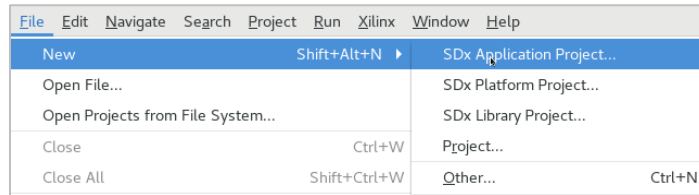


- Finally, you need to create projects using the following project names:

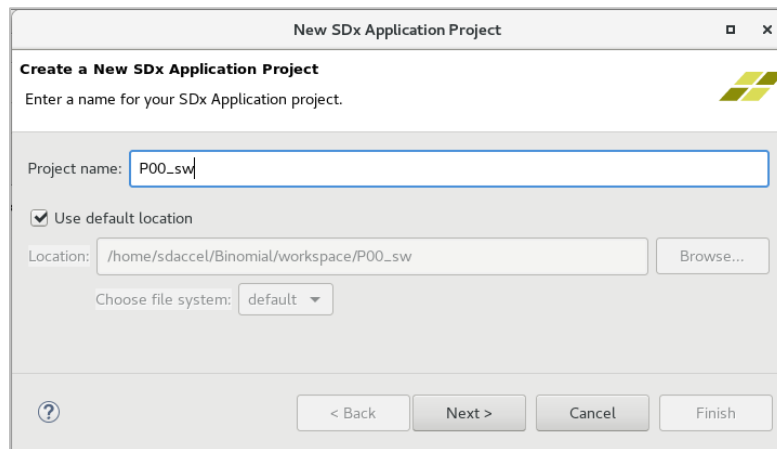
SDAccel Examples Repository	
BinomialModel	
✓ BinomialModel: P00_sw	→ P00_sw
✓ BinomialModel: P10_hw_Original	→ P10_hw_Original
✓ BinomialModel: P11_hw_Original_HostOpt	→ P11_hw_Original_HostOpt
✓ BinomialModel: P12_hw_KOpt	→ P12_hw_KOpt
✓ BinomialModel: P13_hw_KOpt_8CU_2DDRs	→ P13_hw_KOpt_8CU_2DDRs
✓ BinomialModel: P14_hw_KOpt_12CU_3DDRs	→ P14_hw_KOpt_12CU_3DDRs
✓ BinomialModel: Popt_12_step1_DataMove	→ Popt_12_step1_DataMove
✓ BinomialModel: Popt_12_step2_FUNx4	→ Popt_12_step2_FUNx4
✓ BinomialModel: Popt_12_step3_ARRAY_PARTITION	→ Popt_12_step3_ARRAY_PARTITION
✓ BinomialModel: Popt_12_step4_UNROLL	→ Popt_12_step4_UNROLL

using the following procedure (shown for a **P00\_sw** project):

- Select the **"SDx Application Project"** from the **File / New Menu**:

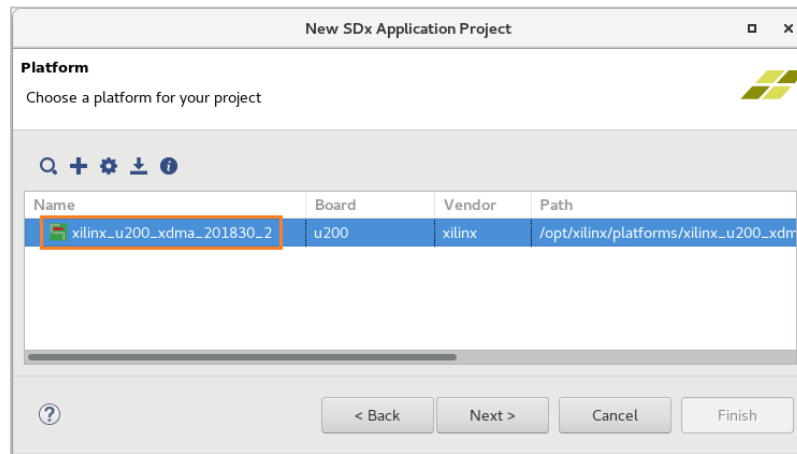


- In the **"Create a New SDx Project"** dialog box specify the following project name:  
**P00\_sw**



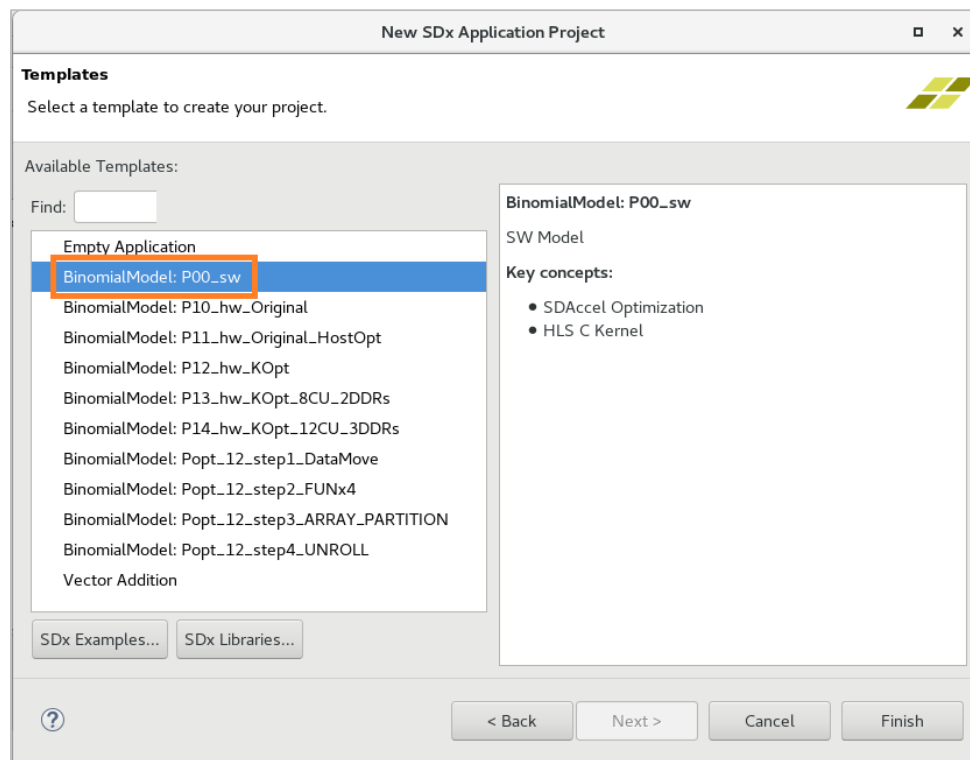
Then press **Next**

- In the "Choose Hardware Platform" dialog box select the "xilinx\_u200\_xdma\_201830\_2" platform:



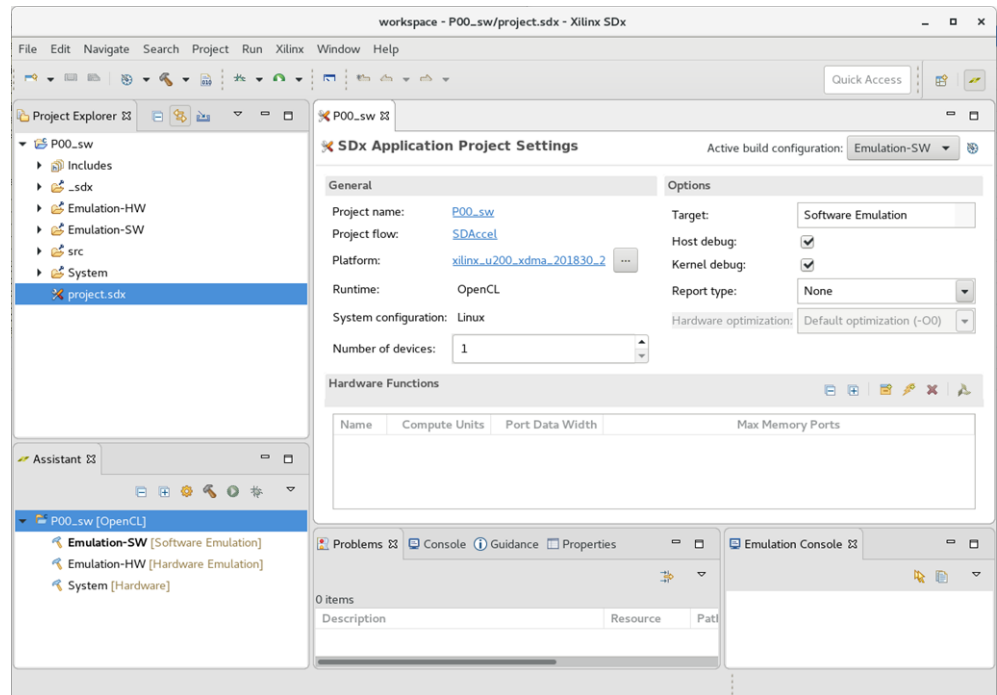
Then press **Next**

- In the "Templates" dialog box select **BinomialModel: P00\_sw**

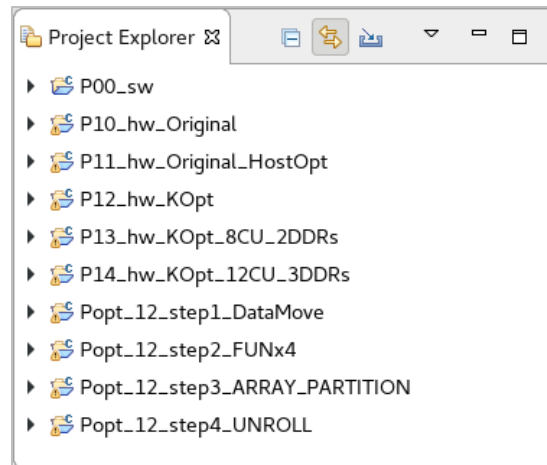


Then press **Finish** to finalize the project creation.

You will obtain the following environment:



- Repeat the same procedure for the remaining projects. After that you should see all the projects in the SDAccel GUI:



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