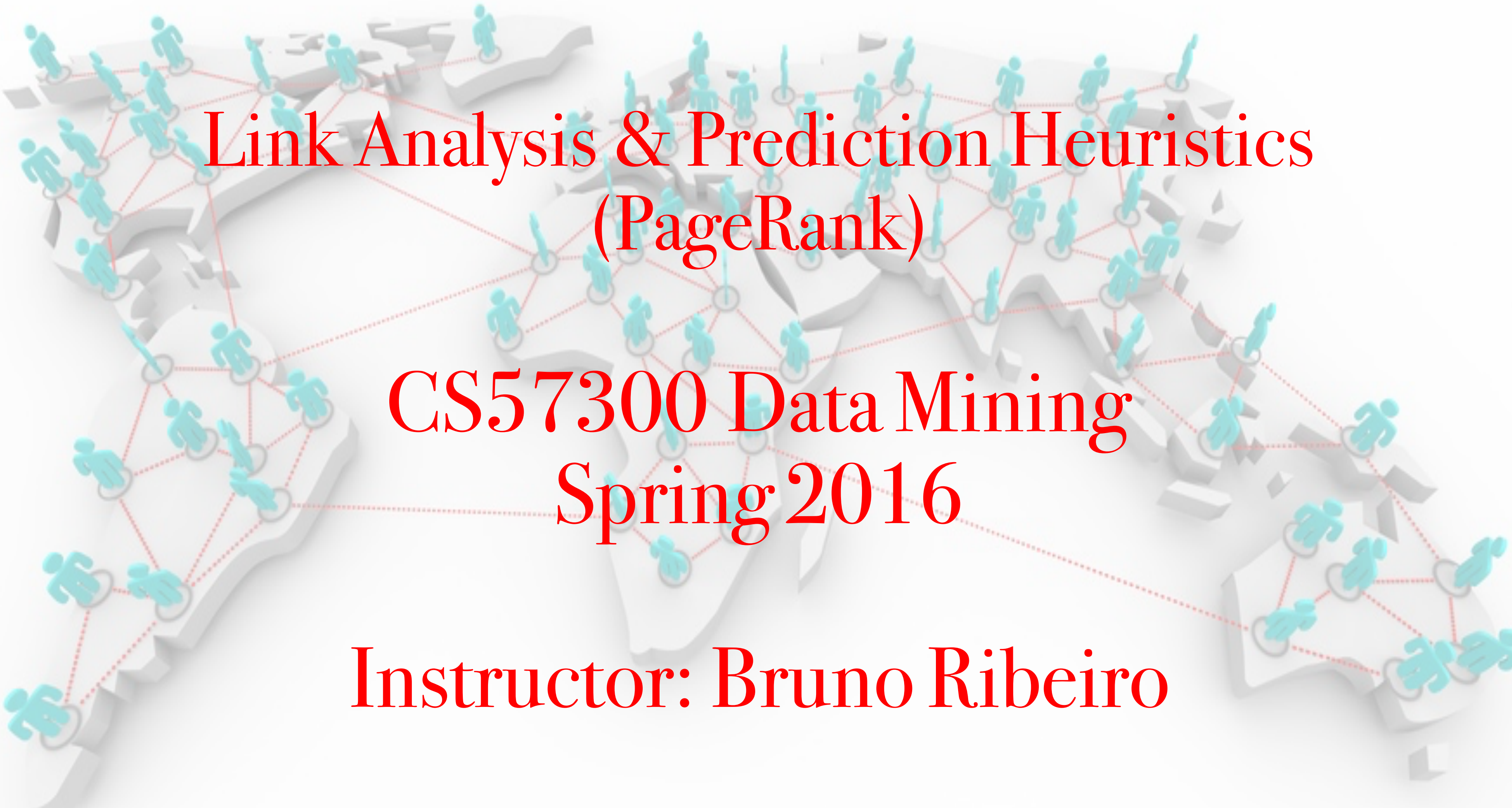
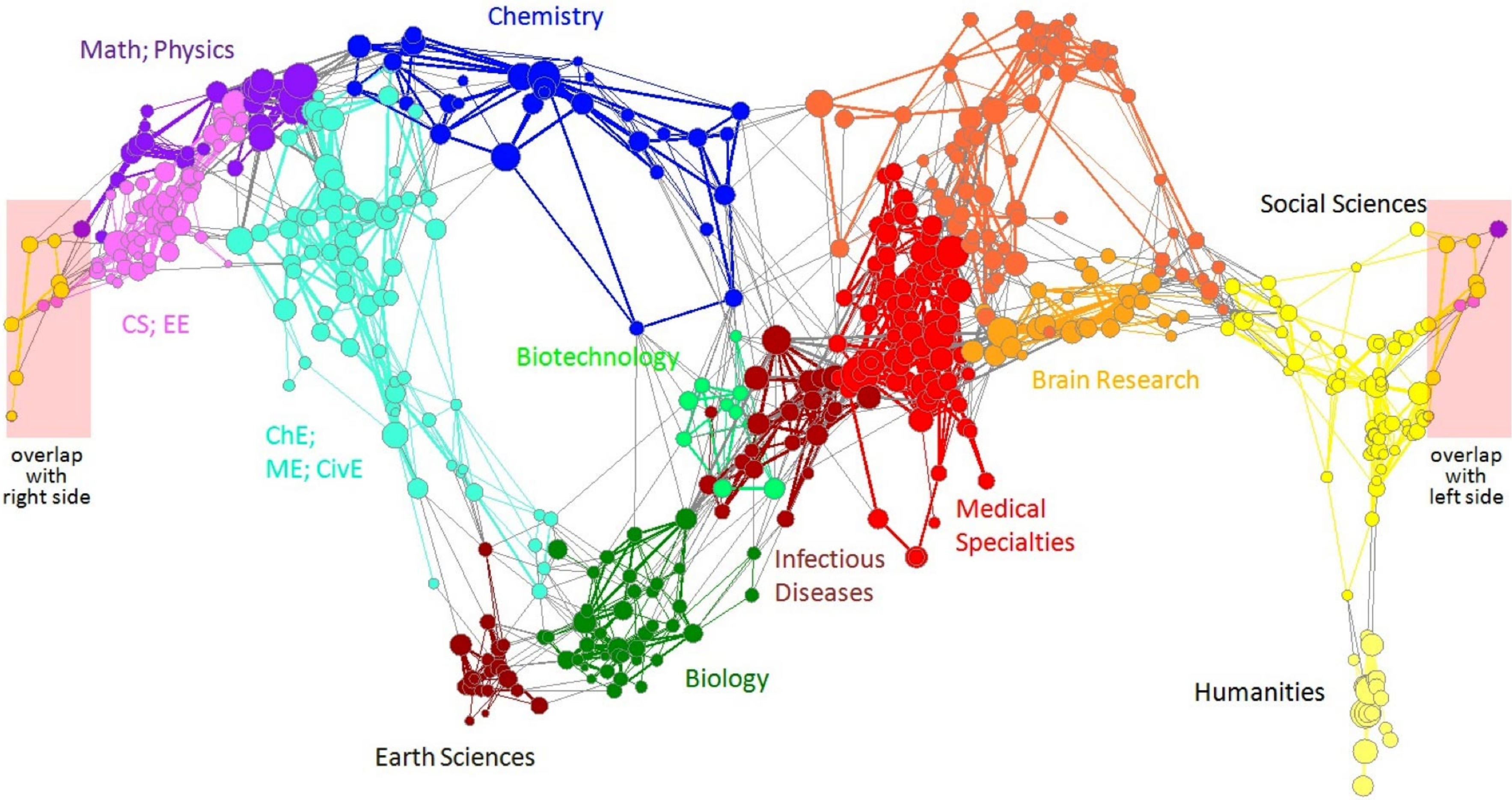


Link Analysis & Prediction Heuristics (PageRank)

CS57300 Data Mining
Spring 2016

Instructor: Bruno Ribeiro

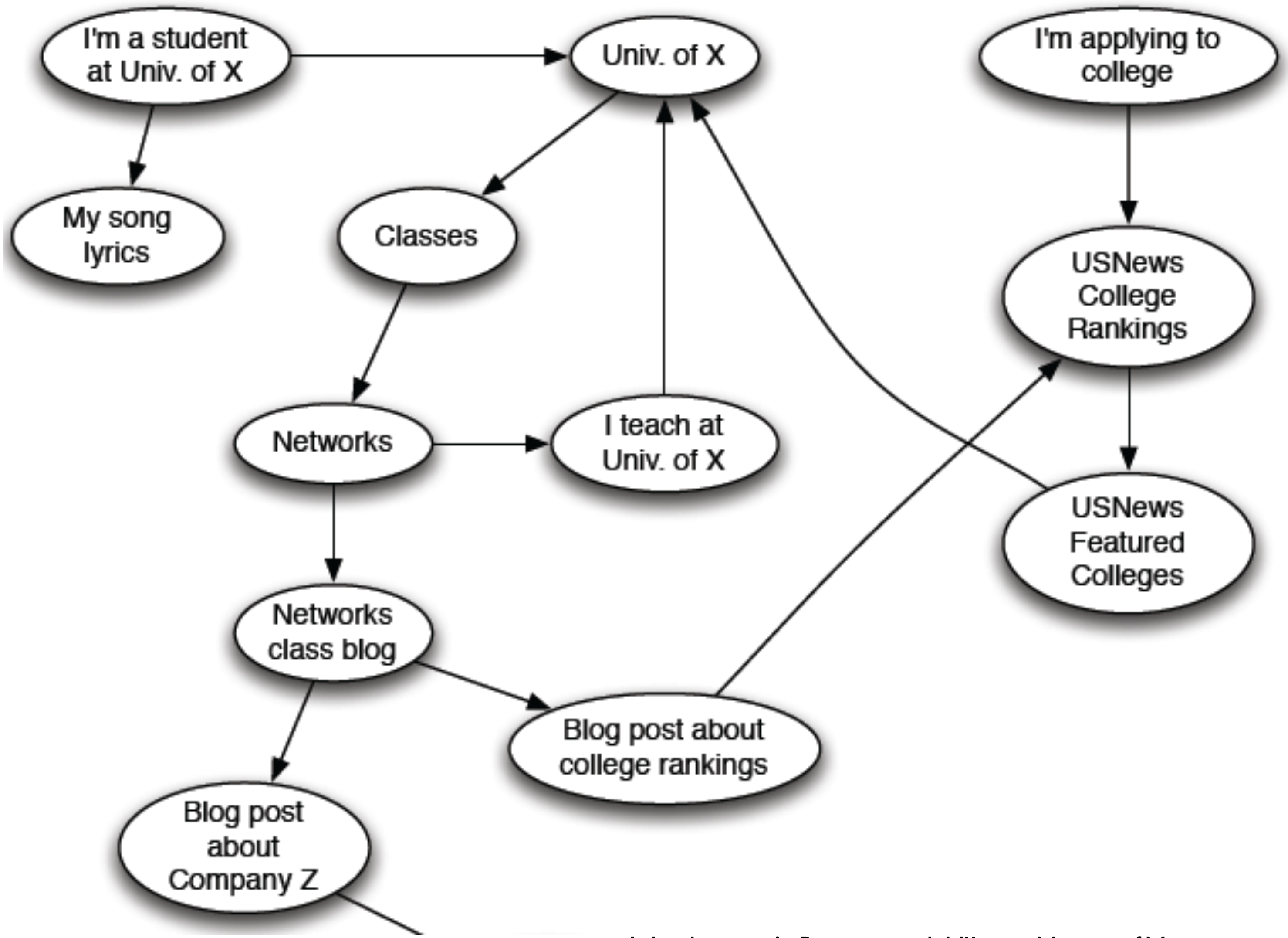
Graph Data: Information Nets



Citation networks and Maps of science

[Börner et al., 2012]

Web as a Directed Graph



Broad Question

- How to organize the Web?
- First try: Human curated Web directories
  - Yahoo, DMOZ, LookSmart
- Second try: Web Search
  - Information Retrieval investigates: Find relevant docs in a small and trusted set
    - Newspaper articles, Patents, etc.
  - <u>But</u>: Web is huge, full of untrusted documents, random things, web spam, etc.

Web Search: 2 Challenges

2 challenges of web search:

(1) Web contains many sources of information
Who to "trust"?

- Trick: Trustworthy pages may point to each other!

(2) What is the "best" answer to query "newspaper"?

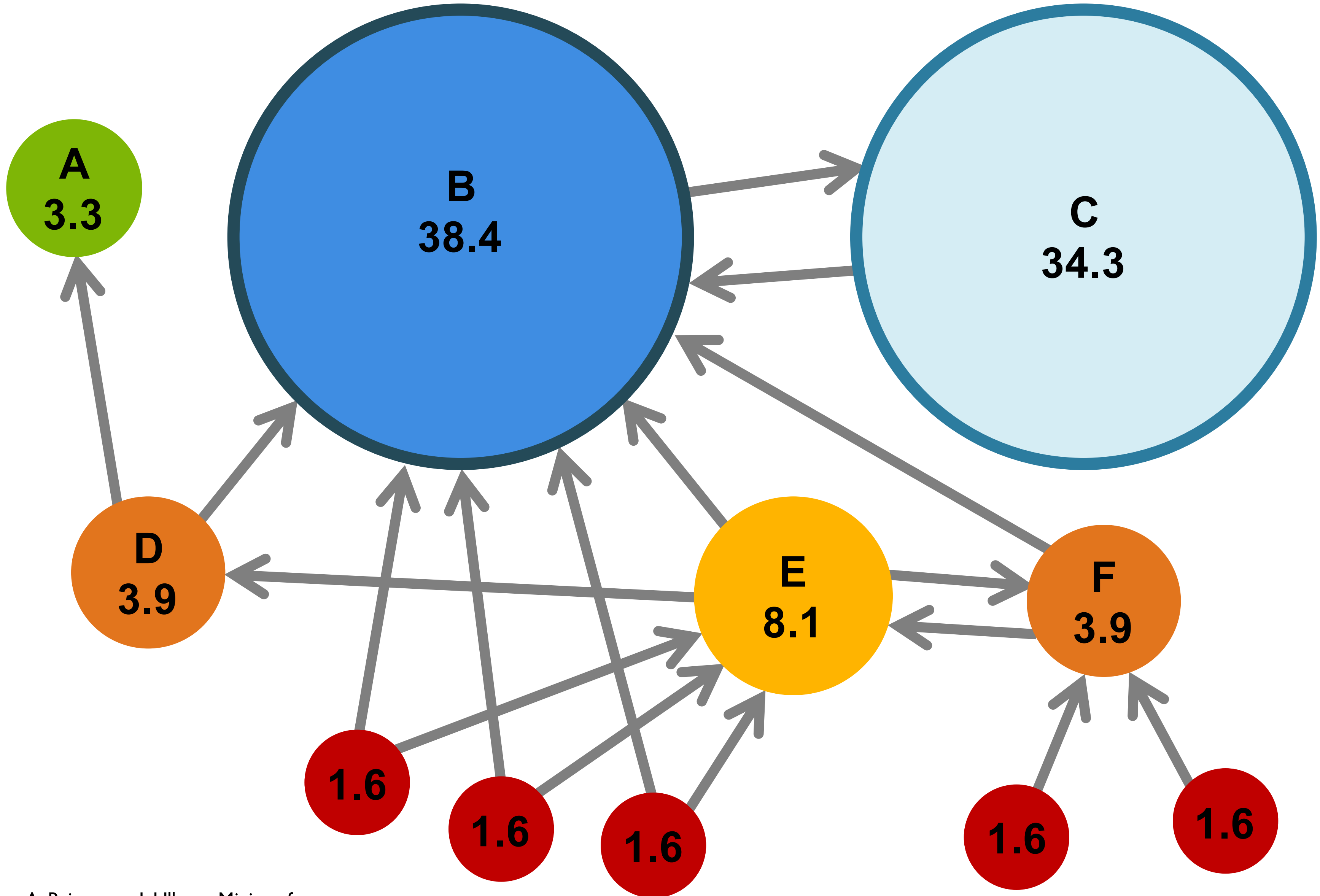
- No single right answer
- Trick: Pages that actually know about newspapers might all be pointing to many newspapers

PageRank: Flow Formulation

Links as Votes

- Idea: Links as votes
  - Page is more important if it has more links
    - In-coming links? Out-going links?
- Think of in-links as votes:
  - www.purdue.edu has 1,910 in-links
  - www.fake-school-name.com has 1 in-link
- Are all in-links are equal?
  - Links from important pages count more
  - Recursive question!

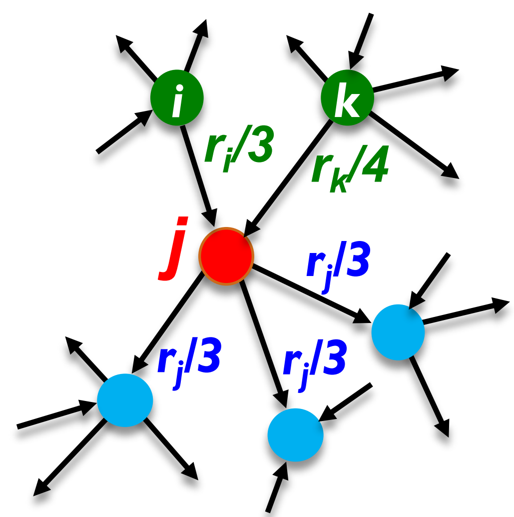
Example: PageRank Scores



Simple Recursive Formulation

- Each link's vote is proportional to the importance of its source page
- If page j with importance r_j has n out-links, each link gets r_j / n votes
- Page j's own importance is the sum of the votes on its in-links

$$r_j = r_i/3 + r_k/4$$



Solving the Flow Equations

- 3 equations, 3 unknowns, no constants
  - No unique solution
  - All solutions equivalent modulo the scale factor
- Additional constraint forces uniqueness:
  - $r_y + r_a + r_m = 1$
  - Solution: $r_y = \frac{2}{5},\ r_a = \frac{2}{5},\ r_m = \frac{1}{5}$
- Gaussian elimination method works for small examples, but we need a better method for large web-size graphs
- We need a new formulation!

Flow equations:

$$\mathbf{r_y = r_y/2 + r_a/2}$$
$$\mathbf{r_a = r_y/2 + r_m}$$
$$\mathbf{r_m = r_a/2}$$

PageRank: Matrix Formulation

- Stochastic adjacency matrix M
  - Let page i has d_i out-links
  - If $i \rightarrow j$, then $M_{ji} = \frac{1}{d_i}$ else $M_{ji} = 0$
    - M is a column stochastic matrix
      - Columns sum to 1
- Rank vector r: vector with an entry per page
  - r_i is the importance score of page i
  - $\sum_i r_i = 1$
- The flow equations can be written

$$r = M \cdot r$$

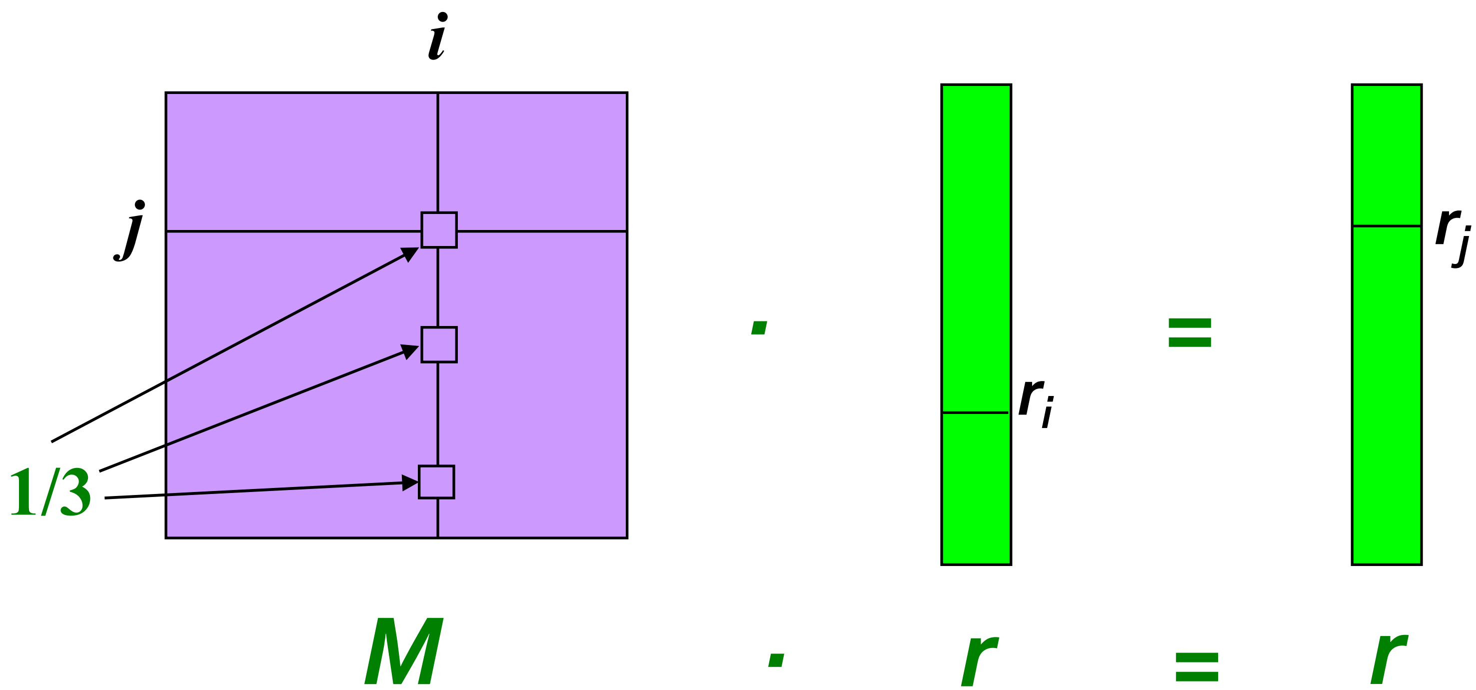
$$r_j = \sum_{i \rightarrow j} \frac{r_i}{\mathrm{d_i}}$$

Example

- **Remember the flow equation:** $r_j = \sum_{i \to j} \frac{r_i}{\mathrm{d_i}}$
- **Flow equation in the matrix form**

$$\boldsymbol{M} \cdot \boldsymbol{r} = \boldsymbol{r}$$

  - **Suppose page *i* links to 3 pages, including *j***



Eigenvector Formulation

- The flow equations can be written

$$r = M \cdot r$$

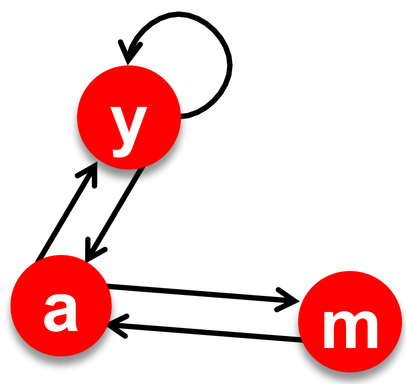
- So the rank vector r is an eigenvector of the stochastic web matrix M
  - In fact, its first or principal eigenvector, with corresponding eigenvalue *1*
    - Largest eigenvalue of M is 1 since M is column stochastic (with non-negative entries)
      - *We know r is unit length and each column of M sums to one, so* $\boldsymbol{Mr} \leq 1$

NOTE: $\boldsymbol{x}$ is an eigenvector with the corresponding eigenvalue **λ** if:

$$\boldsymbol{Ax} = \boldsymbol{\lambda x}$$

- We can now efficiently solve for *r via Power iteration*

Example: Flow Equations & M



	y	a	m
y	½	½	0
a	½	0	1
m	0	½	0

$$r = M \cdot r$$

$$r_y = r_y/2 + r_a/2$$
$$r_a = r_y/2 + r_m$$
$$r_m = r_a/2$$

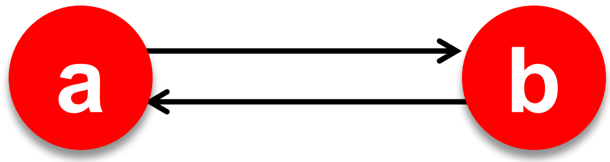
$$\begin{bmatrix} y \\ a \\ m \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 1 \\ 0 & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} y \\ a \\ m \end{bmatrix}$$

PageRank: Three Questions

$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{\mathrm{d}_\mathrm{i}} \quad \text{or equivalently} \quad r = Mr$$

- Does this converge?
- Does it converge to what we want?
- Are results reasonable?

Does this converge?



$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{\mathrm{d}_i}$$

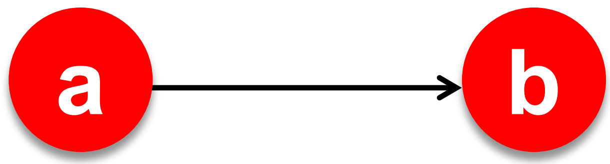
- **Example:**

r_a	1	0	1	0
r_b	0	1	0	1

=

Iteration 0, 1, 2, …

Does it converge to what we want?



$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{\mathrm{d}_i}$$

- **Example:**

r_a	1	0	0	0
r_b	0	1	0	0

=

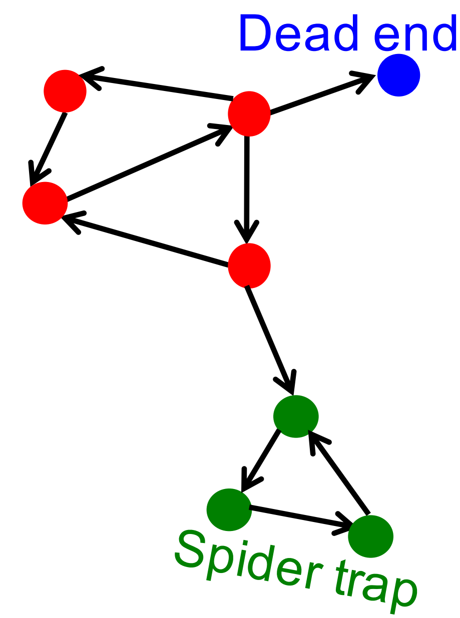
Iteration 0, 1, 2, …

PageRank: Problems

2 problems:

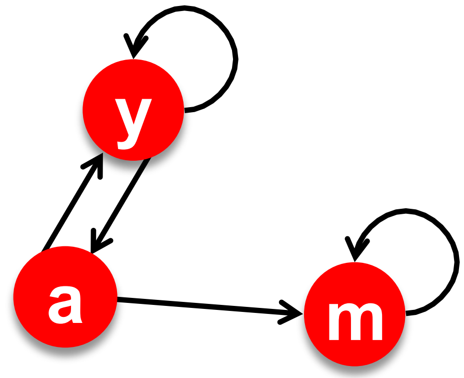
- (1) Some pages are dead ends (have no out-links)
  - Random walk has "nowhere" to go to
  - Such pages cause importance to "leak out"

- (2) Spider traps: (all out-links are within the group)
  - Random walked gets "stuck" in a trap
  - And eventually spider traps absorb all importance



Problem: Spider Traps

- Power Iteration:
  - Set $r_j = 1$
  - $r_j = \sum_{i \to j} \frac{r_i}{d_i}$
    - And iterate



m is a spider trap

	y	a	m
y	½	½	0
a	½	0	0
m	0	½	1

$$\mathbf{r_y = r_y/2 + r_a/2}$$
$$\mathbf{r_a = r_y/2}$$
$$\mathbf{r_m = r_a/2 + r_m}$$

- **Example:**

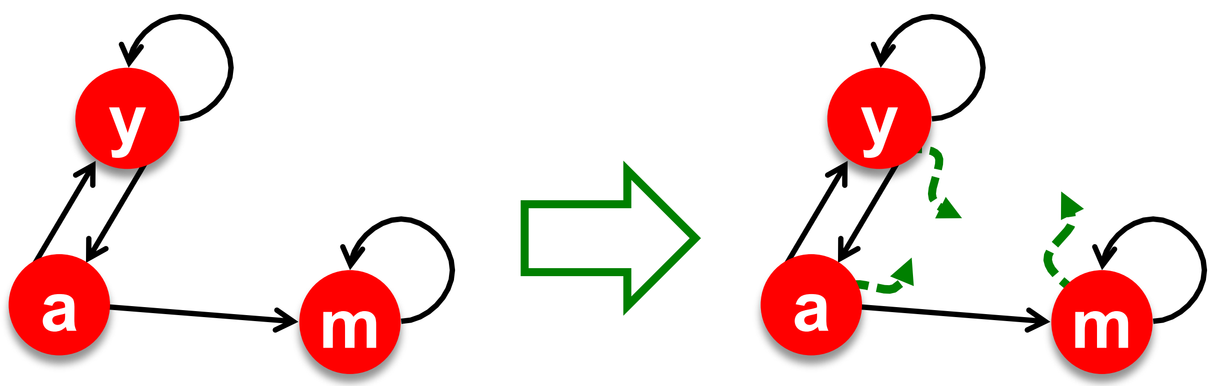
$$\begin{bmatrix} r_y \\ r_a \\ r_m \end{bmatrix} = \begin{array}{cccccc} 1/3 & 2/6 & 3/12 & 5/24 & & 0 \\ 1/3 & 1/6 & 2/12 & 3/24 & \ldots & 0 \\ 1/3 & 3/6 & 7/12 & 16/24 & & 1 \end{array}$$

Iteration 0, 1, 2, …

All the PageRank score gets "trapped" in node m.

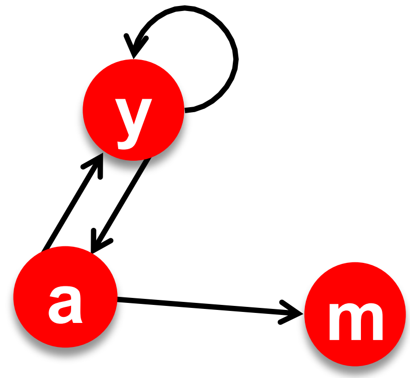
Solution: Teleports!

- The Google solution for spider traps: At each time step, the random surfer has two options
  - With prob. β, follow a link at random
  - With prob. $1-\beta$, jump to some random page
  - Common values for β are in the range 0.8 to 0.9
- Surfer will teleport out of spider trap within a few time steps



Problem: Dead Ends

- Power Iteration:
  - Set $r_j = 1$
  - $r_j = \sum_{i \to j} \frac{r_i}{d_i}$
    - And iterate



	y	a	m
y	½	½	0
a	½	0	0
m	0	½	0

$$\mathbf{r_y = r_y/2 + r_a/2}$$
$$\mathbf{r_a = r_y/2}$$
$$\mathbf{r_m = r_a/2}$$

- **Example:**

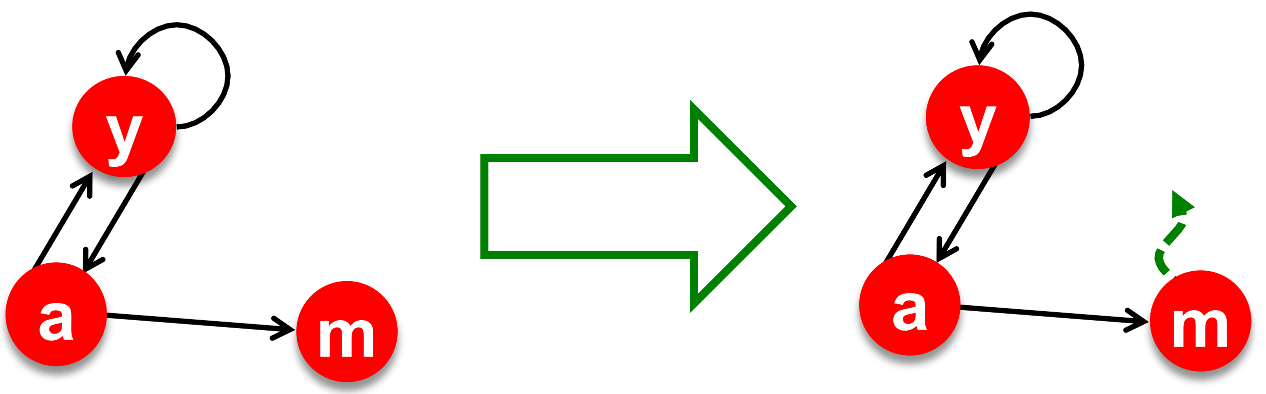
$$\begin{bmatrix} r_y \\ r_a \\ r_m \end{bmatrix} = \begin{array}{cccccc} 1/3 & 2/6 & 3/12 & 5/24 & & 0 \\ 1/3 & 1/6 & 2/12 & 3/24 & \ldots & 0 \\ 1/3 & 1/6 & 1/12 & 2/24 & & 0 \end{array}$$

Iteration 0, 1, 2, …

Here the PageRank "leaks" out since the matrix is not stochastic.

Solution: Always Teleport!

- Teleports: Follow random teleport links with probability 1.0 from dead-ends
  - Adjust matrix accordingly



	y	a	m
y	½	½	0
a	½	0	0
m	0	½	0

	y	a	m
y	½	½	⅓
a	½	0	⅓
m	0	½	⅓

Why Teleports Solve the Problem?

Why are dead-ends and spider traps a problem and why do teleports solve the problem?

- Spider-traps are not a problem, but with traps PageRank scores are not what we want
  - Solution: Never get stuck in a spider trap by teleporting out of it in a finite number of steps
- Dead-ends are a problem
  - The matrix is not column stochastic so our initial assumptions are not met
  - Solution: Make matrix column stochastic by always teleporting when there is nowhere else to go

Solution: Random Teleports

- Google's solution that does it all:
  At each step, random surfer has two options:
  - With probability β, follow a link at random
  - With probability $1\text{-}\beta$, jump to some random page

- PageRank equation [Brin-Page, 98]

$$r_j = \sum_{i \to j} \beta \frac{r_i}{d_i} + (1-\beta)\frac{1}{N}$$

d_i … out-degree of node i

This formulation assumes that $\boldsymbol{M}$ has no dead ends. We can either preprocess matrix $\boldsymbol{M}$ to remove all dead ends or explicitly follow random teleport links with probability 1.0 from dead-ends.

The Google Matrix

- PageRank equation [Brin-Page, '98]

$$r_j = \sum_{i \to j} \beta \frac{r_i}{d_i} + (1-\beta)\frac{1}{N}$$

- The Google Matrix *A*:

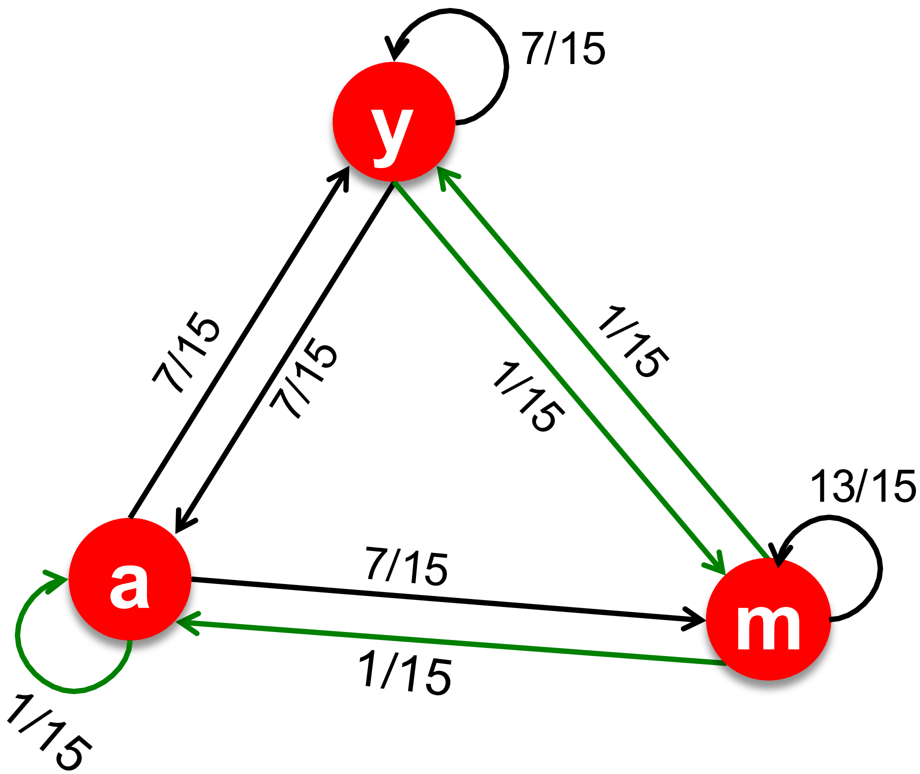
$$A = \beta\, M + (1-\beta)\left[\frac{1}{N}\right]_{N \times N}$$

$[1/N]_{NxN}$…N by N matrix where all entries are 1/N

- We have a recursive problem: $\boldsymbol{r = A \cdot r}$

- What is $\boldsymbol{\beta}$?
  - In practice $\beta = 0.8, 0.9$ (make 5 steps on avg., jump)

Random Teleports ($\beta = 0.8$)



$$0.8 \overset{\mathbf{M}}{\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}} + 0.2 \overset{\mathbf{[1/N]_{NxN}}}{\begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}}$$

$$\begin{array}{c} y \\ a \\ m \end{array} \underset{\mathbf{A}}{\begin{bmatrix} 7/15 & 7/15 & 1/15 \\ 7/15 & 1/15 & 1/15 \\ 1/15 & 7/15 & 13/15 \end{bmatrix}}$$

$$\begin{array}{c} y \\ a \\ m \end{array} = \begin{array}{c} 1/3 \\ 1/3 \\ 1/3 \end{array} \quad \begin{array}{c} 0.33 \\ 0.20 \\ 0.46 \end{array} \quad \begin{array}{c} 0.24 \\ 0.20 \\ 0.52 \end{array} \quad \begin{array}{c} 0.26 \\ 0.18 \\ 0.56 \end{array} \quad \ldots \quad \begin{array}{c} 7/33 \\ 5/33 \\ 21/33 \end{array}$$