CS 57300 Purdue University
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1. Matrix multiplication is *not commutative*
2. $(AB)^\intercal = B^\intercal A^\intercal$
   (a) symmetric matrix: $A = A^\intercal$
   (b) $(B^\intercal)^\intercal = B$
   (c) $(BB^\intercal)^\intercal = BB^\intercal$
3. inner product $\langle x, y\rangle = x^\intercal y = \sum_{\forall i} x_i y_i$
   (a) $\langle x, x\rangle \geq 0$
   (b) $\|x\|^2 \equiv \langle x, x\rangle$
   (c) $\langle x, y\rangle = 0$ iff $x \perp y$
   (d) θ- angle btw x and y
   $\cos\theta = \frac{\langle x,y\rangle}{\|x\|\,\|y\|}$
   (e) Cauchy–Schwarz inequality
   $|\langle x, y\rangle| \leq \|x\| \cdot \|y\|$
   (f) triangle inequality
   $\|x + y\| \leq \|x\| + \|y\|$
   (g) if $v_1, \ldots, v_n$ form an orthogormal basis of $\mathbb{R}^m$, $m \leq n$, then
   $x = \sum_{i=1}^{n} \langle x, v_i\rangle v_i, \quad \forall x \in \mathbb{R}^m.$
4. Trace
   (a) $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$
5. Matrix inverse
   (a) $(AB)^{-1} = B^{-1}A^{-1}$
6. Eigenvalues, $A_{n\times n} \geq 0$
   (a) $Ax = \lambda x$
      i. if $Ax = \lambda x$ then $A\alpha x = \lambda\alpha x$
      ii. convention: $\|x\| = 1$
   (b) $AV = V\Lambda$, $\Lambda = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$,
   $V = \begin{bmatrix} x_1, & \cdots, & x_n \end{bmatrix}$
   (c) $\rho(A) \equiv \max_i(|\lambda_i|)$ is the spectral radius of A
   (d) Perron-Frobenius Theorem
      i. $\Lambda \in \mathbb{R}^n$
      ii. $Ax = \rho(A)x, \;\; x \geq 0$
      iii. If A is irreducible
         A. unique eigenvector $x > 0$
         B. x is associated with $\rho(A) > 0$
   (e) If A full rank then
      i. $A = V\Lambda V^{-1}$
      ii. $A^k = V\Lambda^k V^{-1}$
      iii. symmetric matrices
         A. $\Lambda \in \mathbb{R}^n$
         B. V is an orthonormal basis of $\mathbb{R}^n$: $VV^\intercal = I$
   (f) positive definite
      i. $x^\intercal Ax > 0$, unless $x = 0$
      ii. implies $\Lambda > 0$
   (g) positive semi-definite
      i. $x^\intercal Ax \geq 0$
      ii. $\Lambda \geq 0$