Mathematical Notation

observation	time
1	165.3
2	158.8
3	173.9
4	190.0
5	211.6

Let i be an integer between 1 and 5, inclusive.

y_i is the time associated with obs i.

$y_1 = 165.3$, $y_2 = 158.8$ etc.

$$\sum_{i=1}^{5} y_i = y_1 + y_2 + y_3 + y_4 + y_5$$

same as $\sum_{j=1}^{5} y_j$

i is the indexing variable

practice:

$$\sum_{i=1}^{5} i = \qquad \sum_{i=1}^{5} i^2 =$$

$$\sum_{i=-1}^{4} i = \qquad \sum_{i=0}^{3} (-1)^i =$$

$$\sum_{i=1}^{6} (i-2) =$$

parentheses

$$\sum_{i=1}^{5} i = 1 + 2 + 3 + 4 + 5$$

$$\sum_{i=-1}^{4} i = -1 + 0 + 1 + 2 + 3 + 4$$

$$\sum_{i=1}^{6} (i-2) = -1 + 0 + 1 + 2 + 3 + 4$$

$$\sum_{i=1}^{5} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

$$= 1 + 4 + 9 + 16 + 25$$

$$\sum_{i=0}^{3} (-1)^i = (-1)^0 + (-1)^1 + (-1)^2 + (-1)^3$$

$$= 1 + (-1) + (1) + (-1)$$

Aside $\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n-2 + n-1 + n$

if n is even:

$$= (1+n) + (2+n-1) + (3+n-2) + \cdots$$

$$= \underbrace{(n+1) + (n+1) + (n+1) + \cdots}_{n/2 \text{ times}}$$

$$= (n+1)\frac{n}{2}$$

if n is odd:

$$= (1+n-1) + (2+n-2) + \cdots + n$$

$$= \underbrace{(n) + (n) + \cdots + (n)}_{\frac{n-1}{2} \text{ times}} + n$$

$$= n\frac{(n-1)}{2} + n = n\left(1 + \frac{n-1}{2}\right) = \frac{n(n+1)}{2}$$

<u>Practice writing these in summation notation</u>

$1 + 1/2 + 1/3 + 1/4 + 1/5 =$

$1/2 + 1/4 + 1/8 + 1/16 =$

$5 + 4 + 3 + 2 + 1 =$

$2 + 4 + 6 + 8 + 10 + 12 =$

$y_2 + y_4 + y_6 + y_8 =$

$$1 + 1/2 + 1/3 + 1/4 + 1/5 = \sum_{i=1}^{5} \frac{1}{i}$$

$$1/2 + 1/4 + 1/8 + 1/16 = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} = \sum_{i=1}^{4} \frac{1}{2^i}$$

$$5 + 4 + 3 + 2 + 1 = \sum_{i=1}^{5} i$$

$$2 + 4 + 6 + 8 + 10 + 12 = \sum_{i=1}^{6} (2i)$$

$$y_2 + y_4 + y_6 + y_8 = \sum_{i=1}^{4} y_{2i}$$

obs	runner	race	time
1	1	1	162.4
2	1	2	163.1
3	1	3	160.0
4	2	1	175.6
5	2	2	176.7
6	2	3	172.2

Notation for times:

$y_1, y_2, y_3, y_4, y_5, y_6$ time of observation i

$y_1, y_2, y_3, y_1, y_2, y_3$ Not ok because y_1 means two different things

$y_{11}, y_{12}, y_{13}, y_{21}, y_{22}, y_{23}$ y_{ij} = time of runner i in race j.

Careful with double notation.

Always define what the subscripts mean

Indexing functions.

Define $j(i)$ = runner who produced obs i

$j(1) = 1$, $j(2) = 1$, $j(3) = 1$, $j(4) = 2$, $j(5) = 2$, $j(6) = 2$

likewise $k(i)$ = race for obs i

$j(1) =$ $j(2) =$ $j(4) =$ $j(6) =$

$k(1) =$ $k(2) =$ $k(4) =$ $k(6) =$

suppose b_1 is expected time for runner 1

b_2 is expected time for runner 2

b_3 is expected time for runner 3

Define: $e_i = y_i - b_{j(i)}$ = residual for observation i.

$e_1 = y_1 - b_1$, $e_2 = y_2 - b_1$, $e_3 = y_3 - b_1$, $e_4 = y_4 - b_2$, ...

e_{ij} = residual for runner i in race j.

Write these in terms of y_{ij} and b_i

$e_{11} =$

$e_{12} =$

$e_{13} =$

$e_{21} =$

$e_{22} =$

$e_{23} =$

$e_{ij} =$

In this class, we'll be writing things like

$$Y_i = a_0 + a_{j(i)} + b_{k(i)} + (ab)_{j(i),k(i)} + c_0 X_i + \varepsilon_i$$

We'll build up to it slowly, step by step.