

Estimation of cosmological parameters using Genetic Algorithms.



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Introduction

A **Genetic Algorithm (GA)** consists of an optimization and search technique inspired on the principles of the evolution via natural selection. It simulates the development of a population of possible solutions over several generations. The most appropriate solutions are more likely to survive and pass on their characteristics to subsequent generations.

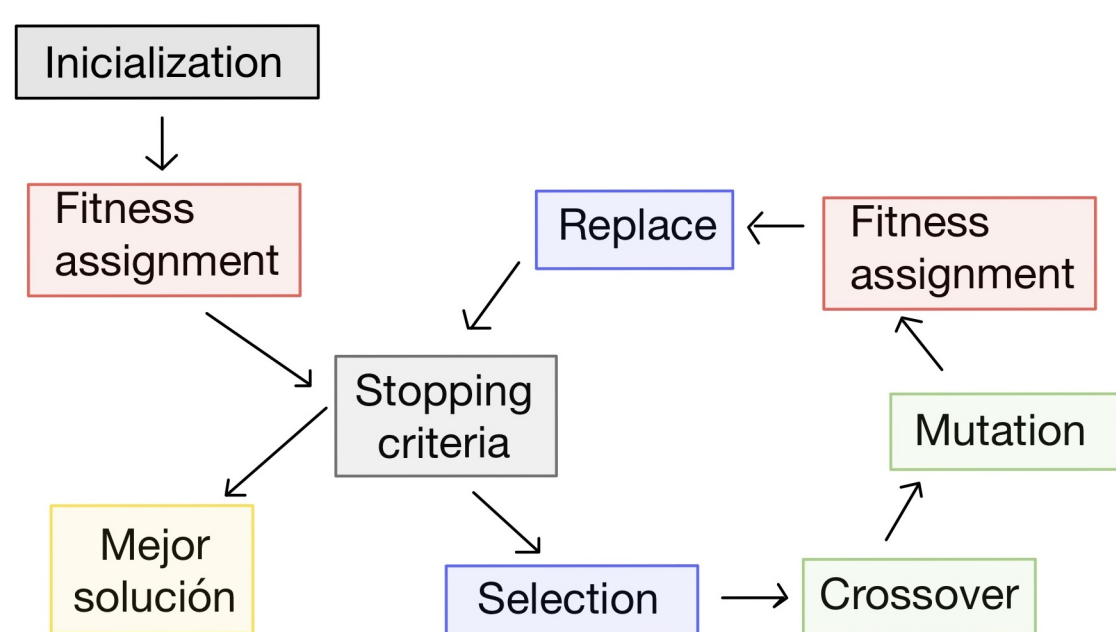


Figura 1: Flowchart of a genetic algorithm operation.

- **Initialization:** The initial population is randomly generated. It consists on a set of chromosomes, in this case functions, which represent the possible solutions to the problem.

- **Fitness assignment:**

A fitness function is applied to each chromosome to evaluate how accurately it describes the data. In this case, the function used is χ^2 , amount to be minimized

$$\chi^2(f) = \sum_{i=1}^N \left(\frac{y_i - f(x_i)}{\sigma_i} \right)^2.$$

- **Stopping criteria:**

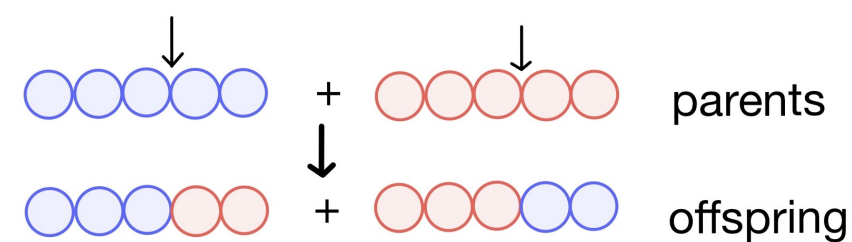
The algorithm works until a pre-determined maximum number of generations.

- **Selection:**

The most suitable chromosomes

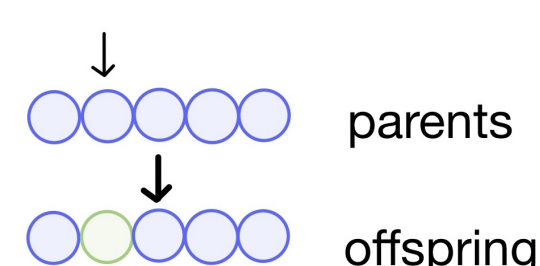
are selected and will be crossed in the next generation. In this case, a selection rate of 20% is used.

- **Crossover:** It generates a new potential solution given two previously selected parents.



- **Mutation:**

It randomly alters a portion of the chromosome of an individual, enabling the exploration of different solutions. A mutation rate of 5% is utilized.



Based on the articles [1] and [2], it was possible to comprehend the functioning of the algorithm and identify the pertinent parameters to utilize.

Motivation

When a galaxy recedes from an observer, its emitted spectral lines shift toward the red of the electromagnetic spectrum, phenomenon known as redshift (z).

Based on data from type IA supernovae we can determine the expansion rate of the universe described by the Hubble parameter

$$H(z) = H_0 \left[\sum_i \Omega_{0,i} (1+z)^{3(1+\omega_i)} \right]^{\frac{1}{2}}, \quad (1)$$

that typically takes into account the following variables:

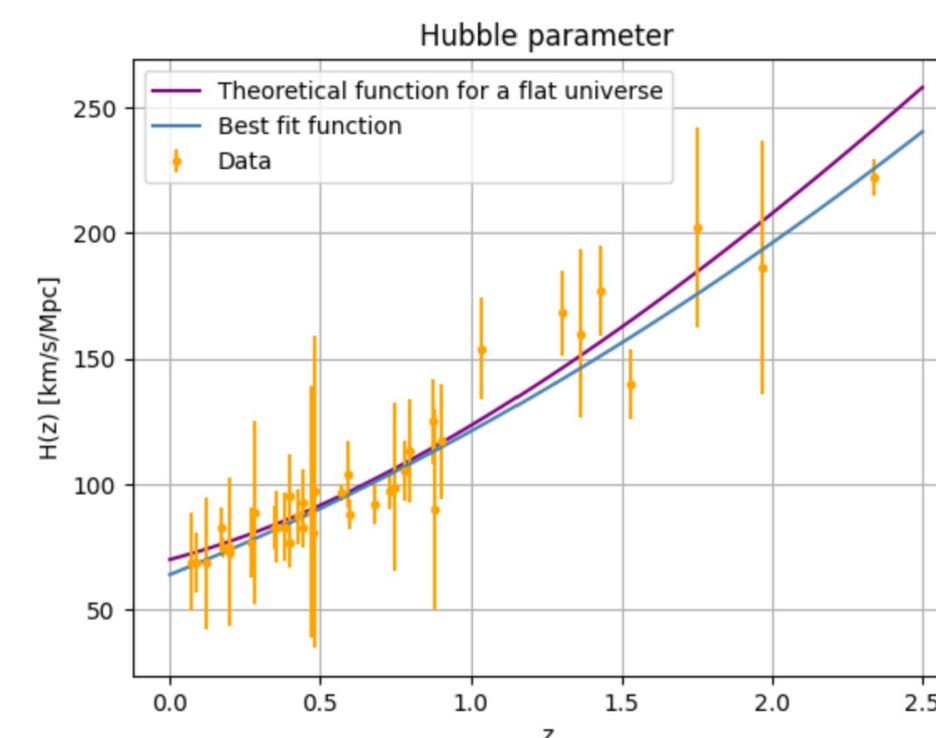
$$H(z) = H_0 \sqrt{\Omega_{0,r}(1+z)^4 + \Omega_{0,m}(1+z)^3 + \Omega_{0,\lambda}(1+z)^{3(1+\omega_{DE})} + \Omega_{0,k}(1+z)^2}. \quad (2)$$

Hubble parameter

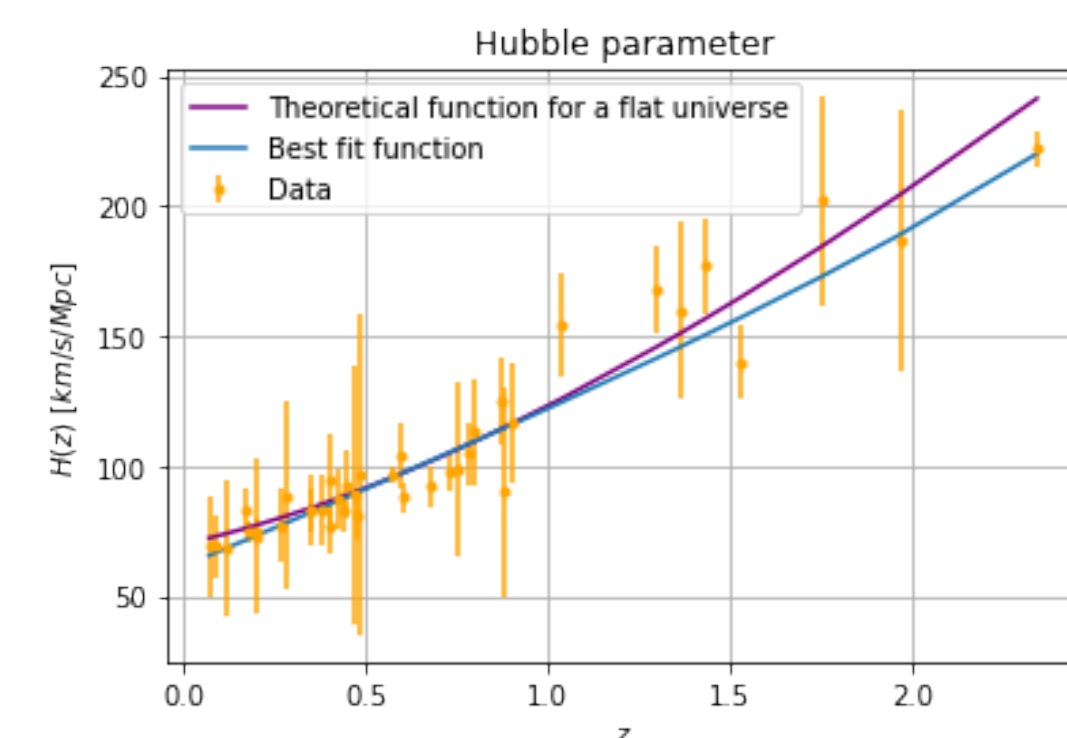
A genetic algorithm is utilized to estimate $H(z)$ based on the obtained data [3]. The objective is to find the solution that best fits this data.

Results

Each algorithm was implemented differently, however, both successfully yielded results by minimizing χ^2 and determining the best-fit function. The theoretical function for $H(z)$ was derived using $\Omega_{0,r} = 8.4 \times 10^{-5}$, $\Omega_{0,m} = 0.3$, $\Omega_{0,\lambda} = 0.69$, and $\omega_{DE} = -1$, characterizing a flat universe.



(a) Algorithm 1 (Paniagua)



(b) Algorithm 2 (Cisternas)

Figura 2: Hubble parameter $H(z)$ calculated from GA.

The estimation of the Hubble parameter was done using each algorithm:

$$H(z) = (z+1)^{(z+1)} + 37.14z + 74.44 \quad (3)$$

$$H(z) = z^3 + 59.559z + z^z + 60.635. \quad (4)$$

Algorithm 1, best-fit function.

Algorithm 2, best-fit function.

From the analysis of $H(z)$, two values for the present-day Hubble constant, denoted as $H_0 = H(0)$, were determined: 75.44 and 60.635 km/s/Mpc, respectively.

Conclusions

An equation for the Hubble parameter, $H(z)$, was derived from observational data using genetic algorithms. By employing 100 generations, the algorithm yielded minimum values of χ^2 37.77 and 21.22, respectively. In this process, algorithm 1 required 113.62 s to complete, while algorithm 2 required 220.96 s.

One of the main challenges in developing the code was obtaining a lower χ^2 after running the algorithm for more generations. For example, if 1000 generations are performed, the χ^2 value is 21.19, so there is no significant increase in its accuracy.

An additional issue arose with the utilization of SymPy symbolic functions, which, in contrast to NumPy functions, require more processing time. Consequently, the runtime of the code could fluctuate from minutes to hours.

Acknowledgements

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Supplementary material

Refer to the supplementary material to view the complete code.



(a) Repository algorithm 1, Paniagua



(b) Repository algorithm 2, Cisternas

Bibliography

- [1] S. Nesseris and J. García-Bellido, "A new perspective on dark energy modeling via genetic algorithms," *Journal of Cosmology and Astroparticle Physics*, vol. 2012, p. 033–033, Nov. 2012.
- [2] R. Medel-Esquivel, I. Gómez-Vargas, A. A. Morales Sánchez, R. García-Salcedo, and J. Alberto Vázquez, "Cosmological parameter estimation with genetic algorithms," *Universe*, vol. 10, p. 11, Dec. 2023.
- [3] M. Moresco, L. Pozzetti, A. Cimatti, R. Jimenez, C. Maraston, L. Verde, D. Thomas, A. Citro, R. Tojeiro, and D. Wilkinson, "A 6% measurement of the hubble parameter at $z \sim 0.45$: direct evidence of the epoch of cosmic re-acceleration," *Journal of Cosmology and Astroparticle Physics*, vol. 2016, p. 014–014, May 2016.