MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.
   (a) A government issues bond to raise funds to support government spending.

   (b) The yield curve demonstrates the yields (interest rates) of government bonds at different maturities, and helps governments analyze economic activity.

   (c) The government can sell bonds to reduce money supply by removing cash from the economy in exchange for bonds.

2. To begin with, I will eliminate long-term bonds (i.e., at least 30 years), including CAN 10.50 Mar 15, CAN 9.75 Jun 1, CAN 9.25 Jun 1, CAN 9.00 Jun 1, CAN 8.00 Jun 1, CAN 5.75 Jun 1 and CAN 8.00 Jun 1. As noticed, these bonds have high coupon rates due to long time to maturity and the illiquidity. If they are included in the calculation, they will put more weights on earlier spot rates and thus the geometric mean of spot rate, YTM, will slightly variate. Apart from that, I will omit on-the-run bonds that are maturing in over 5 years. Since we only need those maturing in 0-5 years to draw the curve. Hence, CAN 1.25 Mar 1, CAN 1.00 Jun 1, CAN 2.00 Jun 1, 2.25 Jun 1, 1.5 Jun 1 and CAN 2.25 Jun 1 will be safely eliminated. After filtration, I will choose from the remaining 19 bonds, in a way that bonds are maturing in about every half year. Therefore, I will select CAN 1.5 Mar 1, CAN 0.75 Sep 1, CAN 0.75 Mar 1, CAN 0.75 Sep 1, CAN 0.5 Mar 1, CAN 2.75 Jun 1, CAN 1.75 Mar 1, CAN 1.5 Jun 1, CAN 2.25 Mar 1, CAN 1.5 Sep 1. Among these bonds, CAN 2.75 Jun 1 and CAN 1.5 Jun 1, maturing in 2.33 and 3.33 years, are included because we do not observe any maturing in 2.6 and 3.6 years.

3. Since the covariance matrix of a sequence of stochastic processes is symmetric, we can use spectral decomposition to expand such matrix in its eigenvalues and orthogonal eigenvectors. Each eigenvalue corresponds to each row linear combination of points/processes in the covariance matrix. Also, it is noticed that eigenvalues decrease exponentially from λ_1 to λ_n. In other words, if we see the covariances as time series, the future rates and yields tend to be eventually stable compared to current ones.

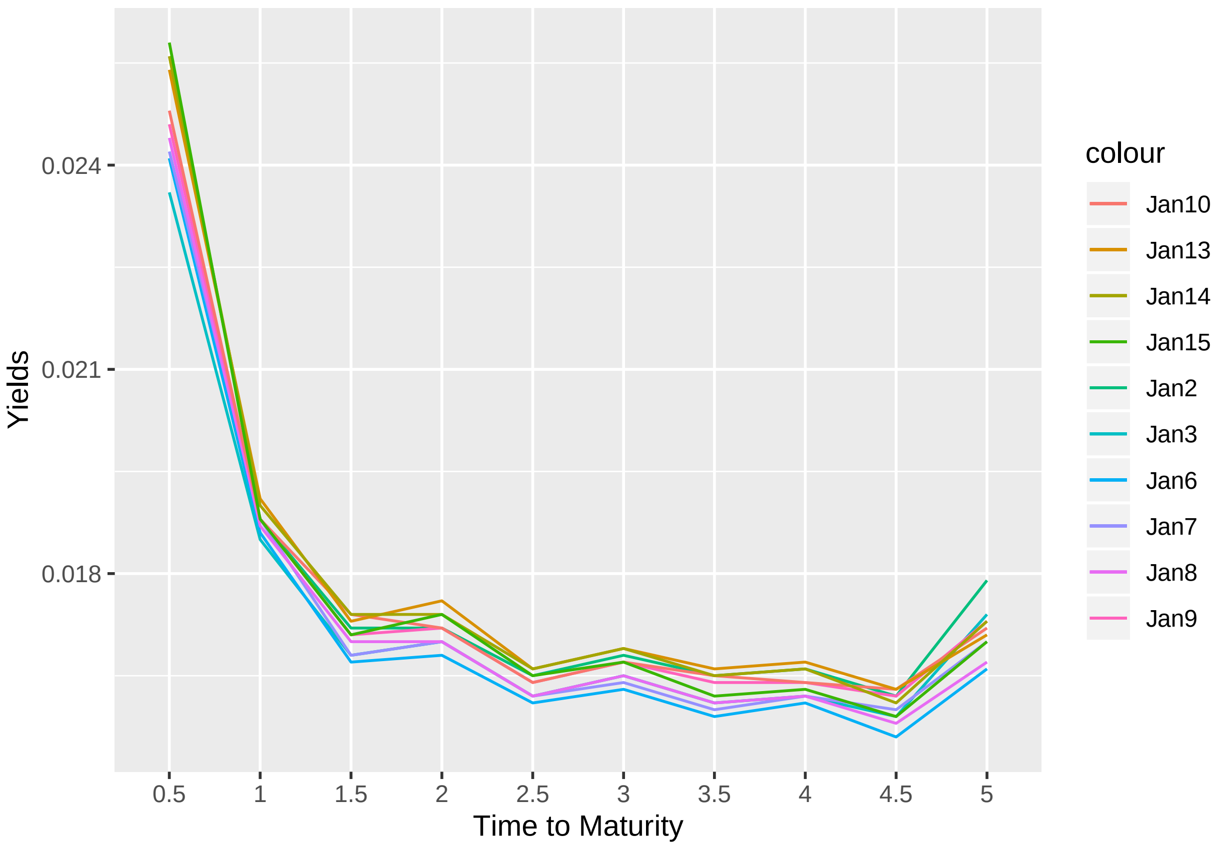
Empirical Questions - 75 points

4.
   (a)

Table 1: Yields to Maturity

	2nd	3rd	6th	7th	8th	9th	10th	13th	14th	15th
bond1	0.0241	0.0236	0.0241	0.0242	0.0244	0.0246	0.0248	0.0254	0.0256	0.0258
bond2	0.0188	0.0185	0.0186	0.0188	0.0187	0.0188	0.0188	0.0191	0.0190	0.0188
bond3	0.0172	0.0168	0.0167	0.0168	0.0170	0.0171	0.0174	0.0173	0.0174	0.0171
bond4	0.0172	0.0170	0.0168	0.0170	0.0170	0.0172	0.0172	0.0176	0.0174	0.0174
bond5	0.0165	0.0162	0.0161	0.0162	0.0162	0.0164	0.0164	0.0166	0.0166	0.0165
bond6	0.0168	0.0165	0.0163	0.0164	0.0165	0.0167	0.0167	0.0169	0.0169	0.0167
bond7	0.0165	0.0161	0.0159	0.0160	0.0161	0.0164	0.0165	0.0166	0.0165	0.0162

	2nd	3rd	6th	7th	8th	9th	10th	13th	14th	15th
bond8	0.0166	0.0162	0.0161	0.0162	0.0162	0.0164	0.0164	0.0167	0.0166	0.0163
bond9	0.0162	0.0159	0.0156	0.0160	0.0158	0.0162	0.0163	0.0163	0.0161	0.0159
bond10	0.0179	0.0174	0.0166	0.0170	0.0167	0.0173	0.0172	0.0171	0.0173	0.0170



(b)

```
spot <- vector()
for(i in 2:5){
  Calculate the spot rates for the first bond, which matures in 0.5 years,
  using the formula:
    spot rate = log((100+coupon/2)/price)/time_to_maturity
  Then get the lists of time_to_maturity and cashflows of the 2nd, 3rd, 4th,
  and 5th bonds
  for(j in 1:9)
  {
    Calculate the last discounted cashflow of each bond, saved as "denominator"
  }
  Then spot_rate = log(cashflow/"denominator")/time_to_maturity
}

for(i in 1:5)
{
  Again, calculate the last discounted cashflow of each bond from the
```

```
  6th to the 10th bond and the spot rates
}
```

5.

Table 2: Matrix of Log-Returns

bond1	bond2	bond3	bond4	bond5
0.0010960	0.0011051	0.0012672	0.0023271	0.0019322
0.0005973	0.0006023	0.0009737	0.0034302	0.0015228
-0.0002986	-0.0004015	-0.0016559	-0.0018145	-0.0010149
-0.0000996	0.0001004	0.0009744	0.0014116	0.0000000
-0.0008965	-0.0008034	-0.0014619	-0.0026231	-0.0018295
-0.0003987	-0.0001005	-0.0005854	0.0004040	-0.0005088
-0.0003988	-0.0009047	-0.0000976	0.0003029	-0.0001018
0.0003988	0.0005027	0.0007805	-0.0007069	0.0010174
0.0006976	0.0007033	0.0009748	0.0011106	0.0013211

Table 3: Covariance Matrix of Log-Returns

	bond1	bond2	bond3	bond4	bond5
bond1	4e-07	4e-07	6.0e-07	9.0e-07	8.0e-07
bond2	4e-07	5e-07	6.0e-07	9.0e-07	8.0e-07
bond3	6e-07	6e-07	1.3e-06	1.8e-06	1.3e-06
bond4	9e-07	9e-07	1.8e-06	3.7e-06	2.0e-06
bond5	8e-07	8e-07	1.3e-06	2.0e-06	1.6e-06

6.

Table 4: Eigenvalues for Covariance Matrix of Log-Return

x
6.6e-06
7.0e-07
2.0e-07
1.0e-07
0.0e+00

Table 5: Eigenvectors for Covariance Matrix of Log-Return

-0.2220606	0.3316013	-0.3304459	0.0122137	0.8552111
-0.2269795	0.3567976	-0.3522519	-0.7701682	-0.3223899
-0.4073879	0.2698750	0.8432841	-0.1900527	0.1181291
-0.7190236	-0.6781248	-0.1435987	-0.0455318	0.0214038
-0.4649834	0.4796356	-0.1870173	0.6070408	-0.3876418

The first eigenvalue weights about 86% of the sum of all eigenvalues from the covariance matrix, so the first pair of eigenvalue and eigenvector well explained the data.

References and GitHub Link to Code

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4. Bai, Z. (2010). The Generalized Eigenproblem. G.W. Stewart, 105–109. doi: 10.1007/978-0-8176-4968-5_9
5. Mossberg, M. (2008). Estimation of Continuous-Time Stochastic Signals From Sample Covariances. IEEE Transactions on Signal Processing, 56(2), 821–825. doi: 10.1109/tsp.2007.907829
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GitHub_link:https://github.com/Xin-Wei-Cynthia/APM466a1/tree/master