P8106_hw2_xh2395

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Homework 2 Description

In this exercise, we build nonlinear models using the "College" data. The dataset contains statistics for 565 US Colleges from the 1995 issue of US News and World Report. The response variable is the out-of-state tuition (Outstate). In what follows, use the data excluding statistics for Columbia University (i.e., the 125th observation) to train the models.

Import and tidy the data

```r
#import data
college_c = read_csv("./data/College.csv")

#exclude Columbia University
college_df = college_c %>%
  filter(College != "Columbia University") %>%
  select(-College)
```

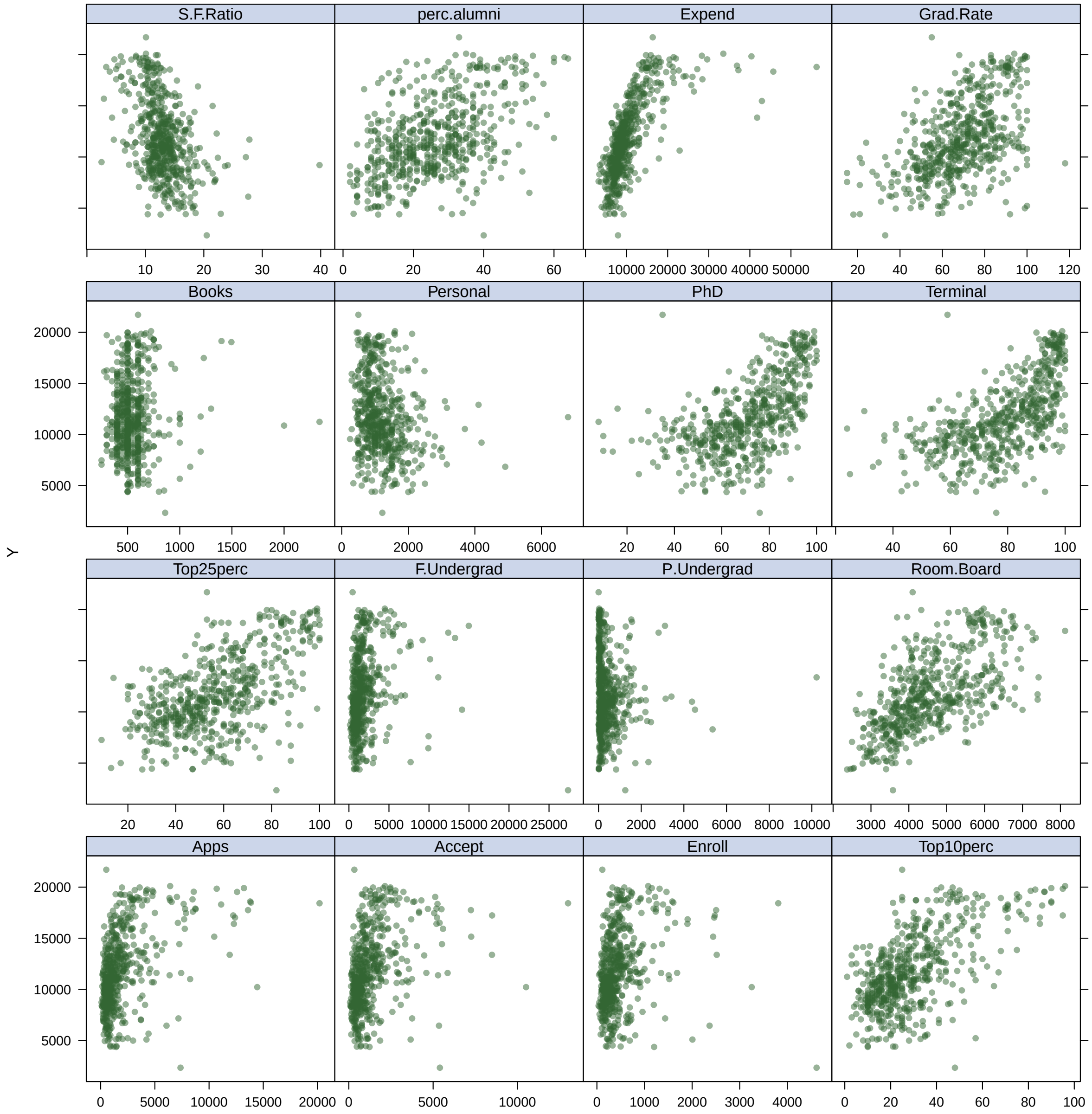
Set random seed

```r
set.seed(2020)
```

a) Create Scatter Plots

response vs predictors

```r
# matrix of predictors
x = model.matrix(Outstate ~ .,college_df)[,-1]
# vector of response
y = college_df$Outstate

featurePlot(x, y, plot = "scatter", labels = c("","Y"), type = c("p"), layout = c(4, 4))
```



b) Smoothing Spline Model

Fit a smoothing spline model using Terminal as the only predictor of Outstate for a range of degrees of freedom, as well as the degree of freedom obtained by generalized cross- validation, and plot the resulting fits. Describe the results obtained.

The degree of freedom obtained by generalized cv

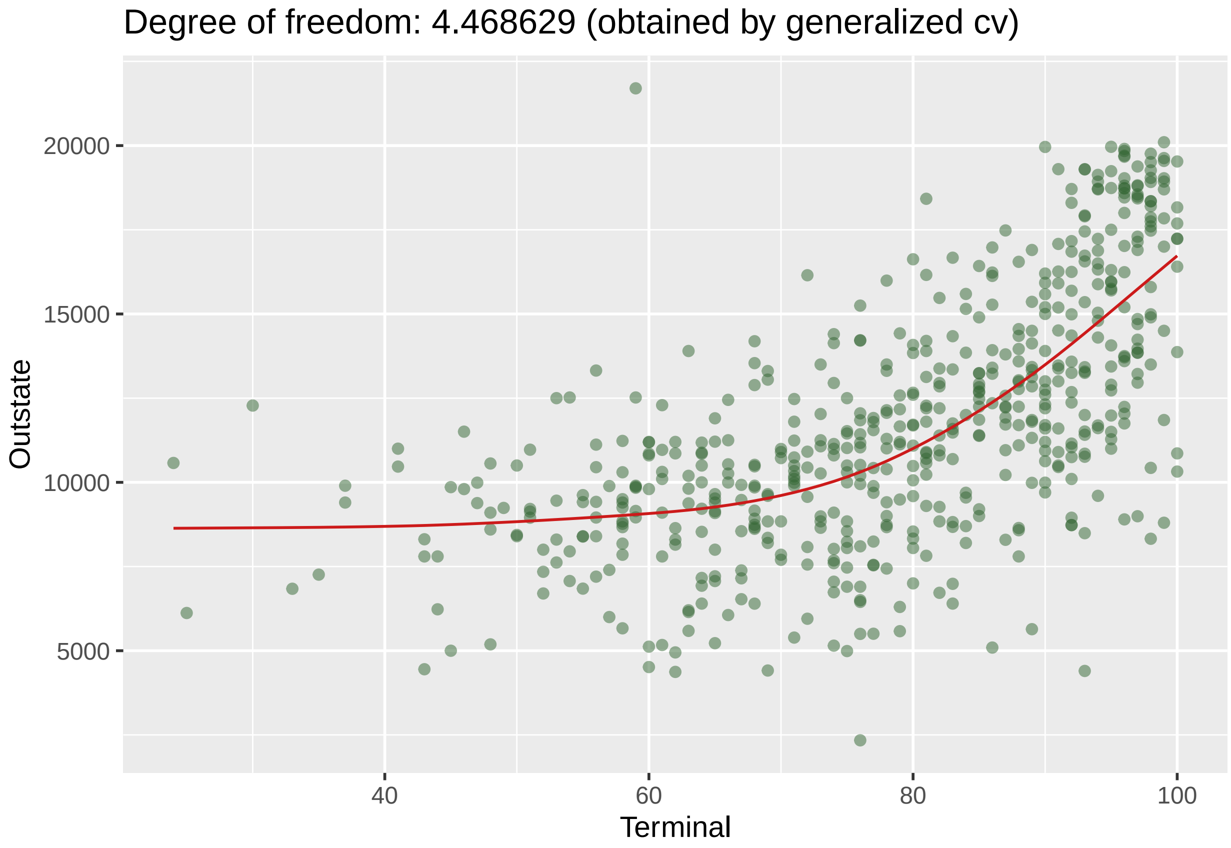
```
fit.ss = smooth.spline(college_df$Terminal, college_df$Outstate)
fit.ss$df
```

```
## [1] 4.468629
```

The degree of freedom obtained by generalized cv is 4.468629.

Plot the resulting fit

```
Terminallims = range(college_df$Terminal)
Terminal.grid = seq(from = Terminallims[1],to = Terminallims[2])

pred.ss = predict(fit.ss,
                  x = Terminal.grid)

pred.ss.df = data.frame(pred = pred.ss$y,
                        Terminal = Terminal.grid)

p_0 = ggplot(data = college_df, aes(x = Terminal, y = Outstate)) +
  geom_point(color = rgb(.2, .4, .2, .5))

p_ss = p_0 +
  geom_line(aes(x = Terminal, y = pred), data = pred.ss.df,
          color = rgb(.8, .1, .1, 1)) +
  labs(title = "Degree of freedom: 4.468629 (obtained by generalized cv)")

p_ss
```

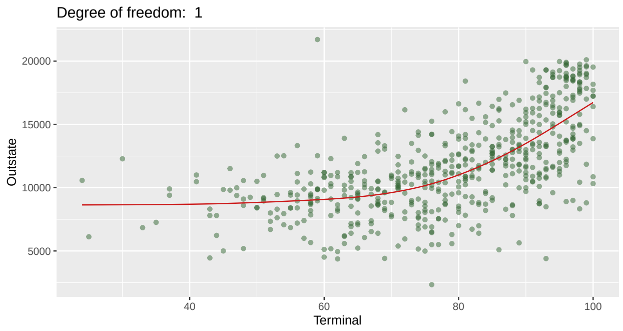


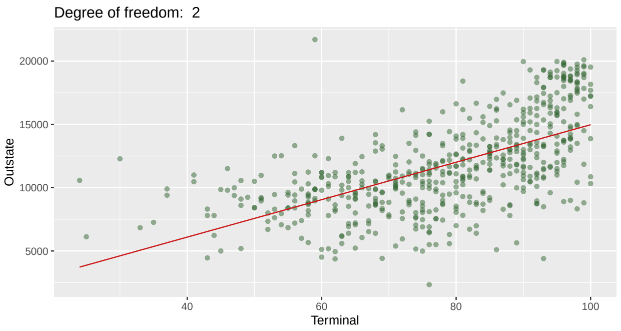
A range of degrees of freedom

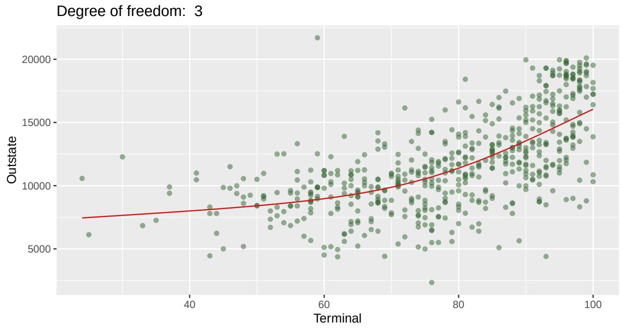
```
spline = function(degree){
    title = paste("Degree of freedom: ", degree)
    spline = smooth.spline(college_df$Terminal,college_df$Outstate, df = degree)
    pred = predict(spline, x = Terminal.grid)
    df_spine_pre = data.frame(Terminal = Terminal.grid,
                              pred = pred$y)

    p = p_0 +
    geom_line(aes(x = Terminal, y = pred),
              data = df_spine_pre,
              color = rgb(.8, .1, .1, 1)
              ) +
    labs(
        title = title
    )
}

p = list()
for (i in 1:10) {
    p[[i]] = spline(i)
}
```

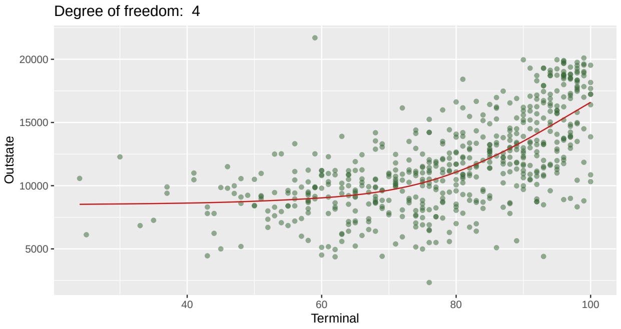
```
## Warning in smooth.spline(college_df$Terminal, college_df$Outstate, df =
## degree): not using invalid df; must have 1 < df <= n := #{unique x} = 65
```

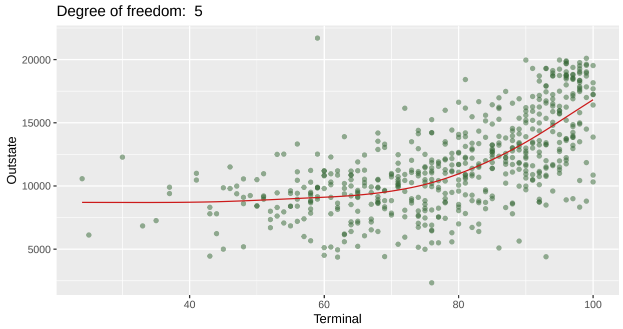
```
p[[1]] + p[[2]] + p[[3]] + p[[4]] + p[[5]] +  p[[6]] + p[[7]] + p[[8]] +
    p[[9]] + p[[10]] +
    plot_layout(ncol = 2, nrow = 5)
```

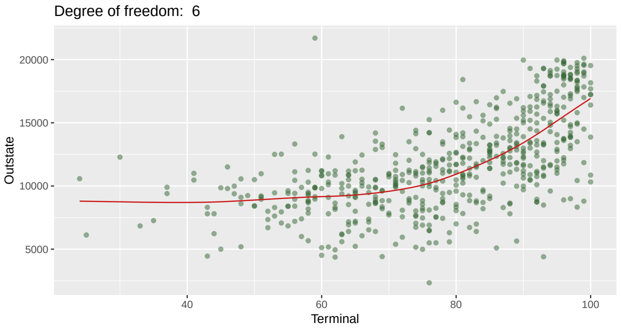


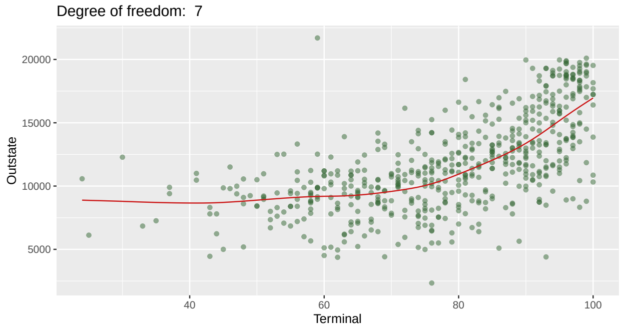


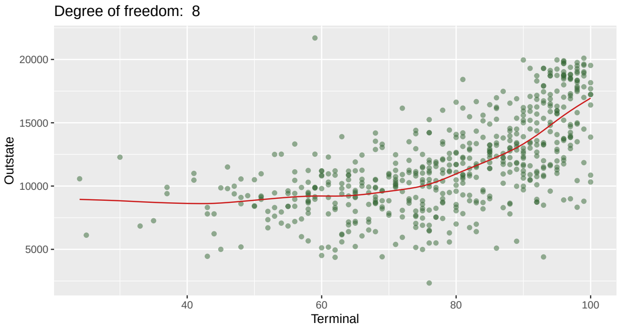


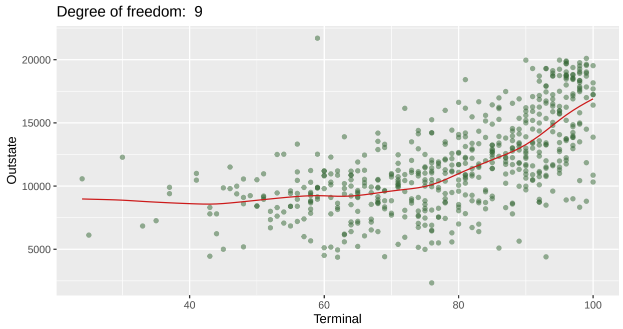


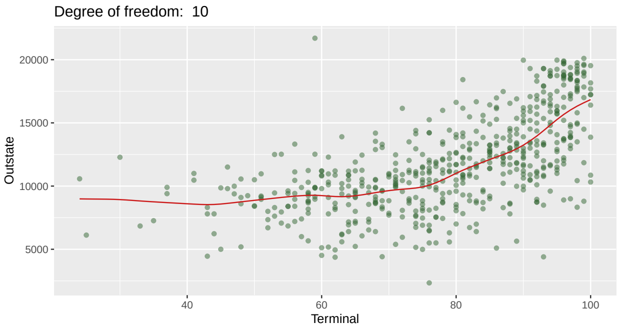












Describe the results obtained:

The out-of-state tuition is a nonlinear function of Terminal. When we only use Terminal as a predictor and use the degree of freedom obtained by generalized cv to fit the data, the fitted curve can be very smoothy.

From the plots we can see that when degree of freedom is bigger than 5 and goes up, the fitted curve is less smoothy.

c) Generalized Additive Model (GAM)

Fit a generalized additive model (GAM) using all the predictors. Plot the results and explain your fndings.

Fit GAM model

```
ctrl1 = trainControl(method = "cv", number = 10)

set.seed(2020)

gam.fit = train(x, y,
                 method = "gam",
                 tuneGrid = data.frame(method = "GCV.Cp",
                                       select = c(TRUE,FALSE)),
                 trControl = ctrl1)

gam.fit$bestTune
```

```
##   select method
## 1  FALSE GCV.Cp
```

```
gam.fit$finalModel
```

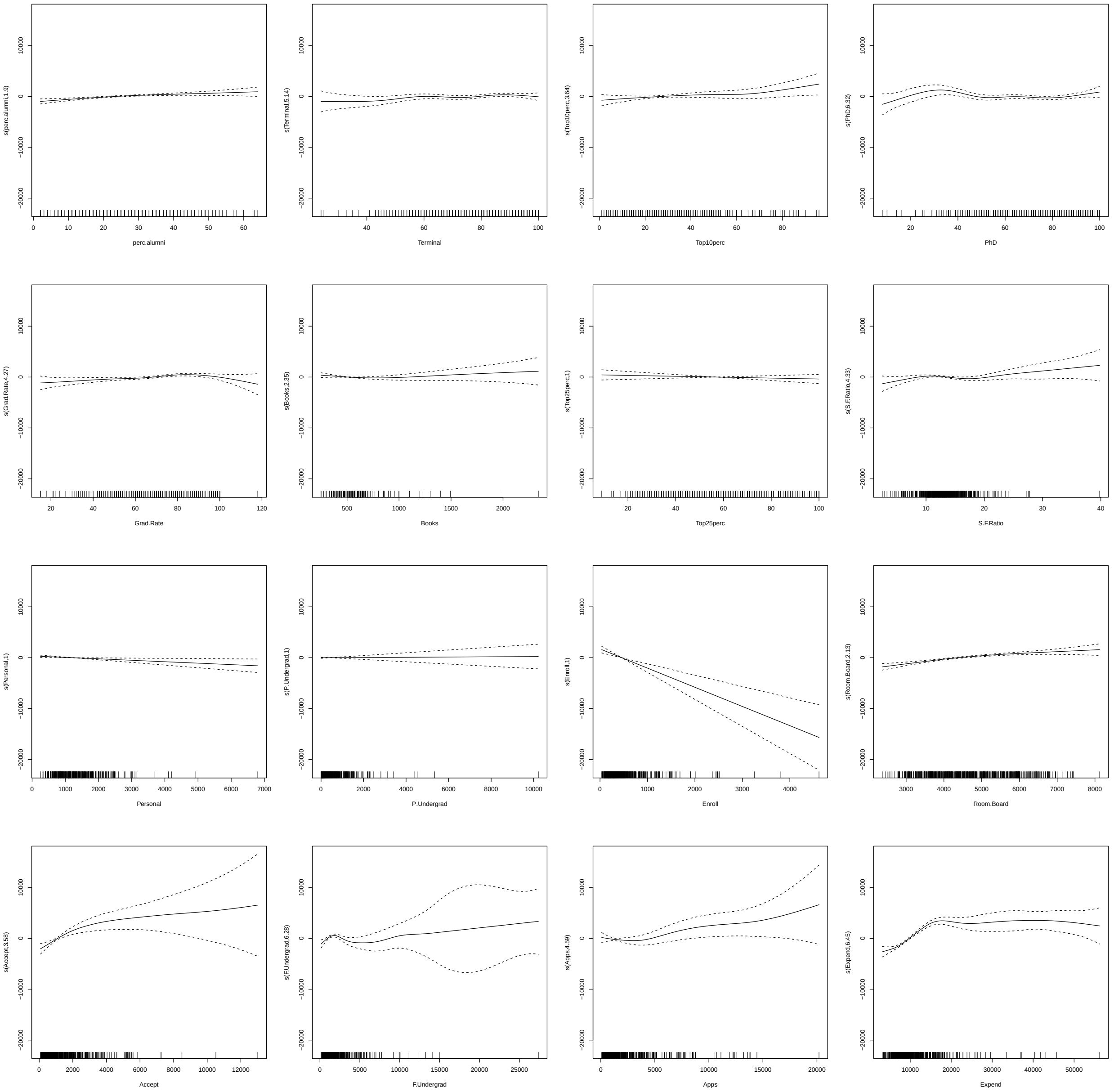
```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## .outcome ~ s(perc.alumni) + s(Terminal) + s(Top10perc) + s(PhD) +
##     s(Grad.Rate) + s(Books) + s(Top25perc) + s(S.F.Ratio) + s(Personal) +
##     s(P.Undergrad) + s(Enroll) + s(Room.Board) + s(Accept) +
##     s(F.Undergrad) + s(Apps) + s(Expend)
##
## Estimated degrees of freedom:
## 1.90 5.14 3.64 6.32 4.27 2.35 1.00
## 4.33 1.00 1.00 1.00 2.13 3.58 6.28
## 4.59 6.45  total = 55.98
##
## GCV score: 2761951
```

According to the MSE of CV, we fit every predictor with smoothing spline.

```
gam = gam(Outstate ~ s(perc.alumni) + s(Terminal) + s(Top10perc) + s(PhD) +
    s(Grad.Rate) + s(Books) + s(Top25perc) + s(S.F.Ratio) + s(Personal) +
    s(P.Undergrad) + s(Enroll) + s(Room.Board) + s(Accept) +
    s(F.Undergrad) + s(Apps) + s(Expend), data = college_df)
```

Plot the results

```r
par(mfrow = c(4,4))
plot(gam)
```



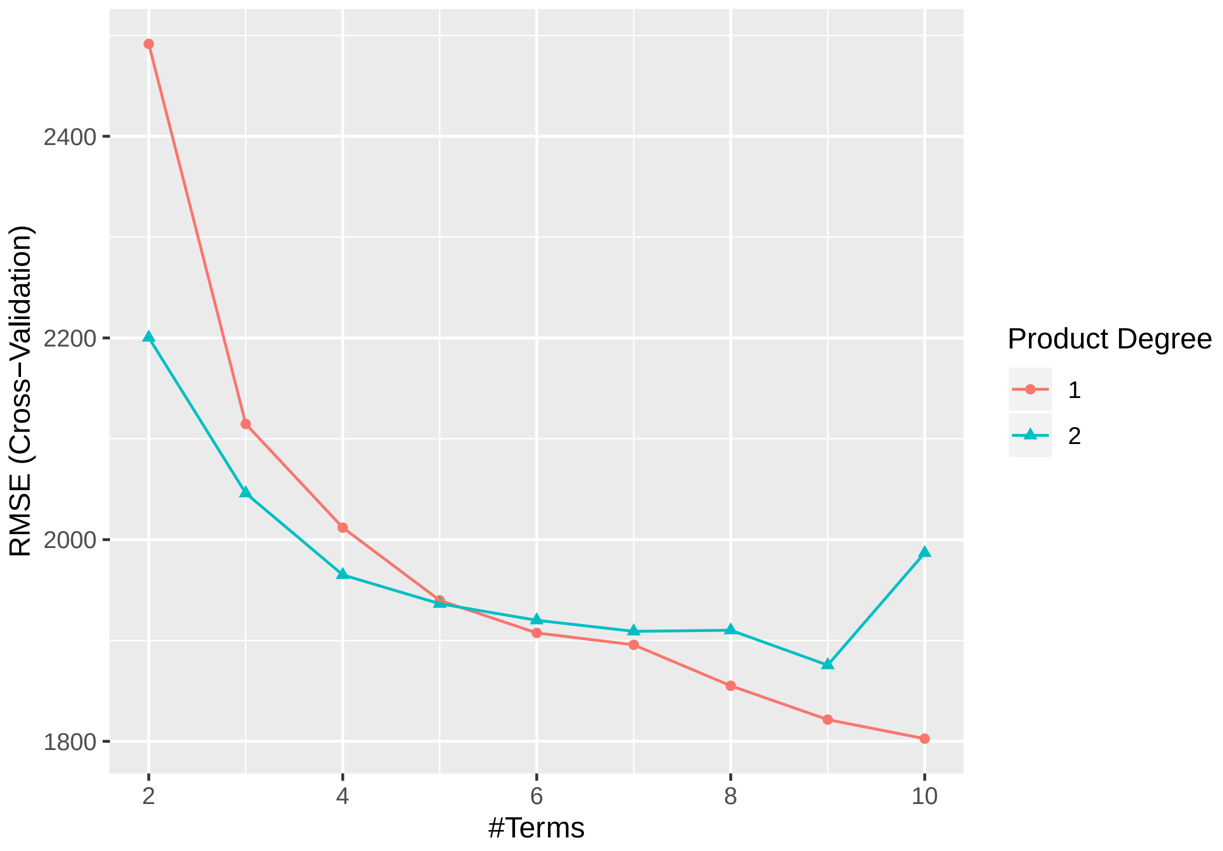
The out-of-state tuition is a nonlinear function of Terminal, Top10perc, PhD, Grad.Rate, Books, S.F.Ratio, Room.Board, Accept, F.Undergrad, Apps, Expend, holding other variable fixed. However, the out-of-state is likely to be a linear function of perc.alumni, Top25perc, Personal, P.Undergrad, Enroll, holding other variable fixed.

d) Multivariate Adaptive Regression Spline (MARS)

Fit a multivariate adaptive regression spline (MARS) model using all the predictors. Report the final model. Present the partial dependence plot of an arbitrary predictor in your final model.

Fit MARS model

```
mars_grid = expand.grid(degree = 1:2,
                        nprune = 2:10)

set.seed(2020)

mars.fit = train(x, y,
                 method = "earth",
                 tuneGrid = mars_grid,
                 trControl = ctrl1)

ggplot(mars.fit)
```



```
mars.fit$bestTune
```

```
##   nprune degree
## 9     10      1
```

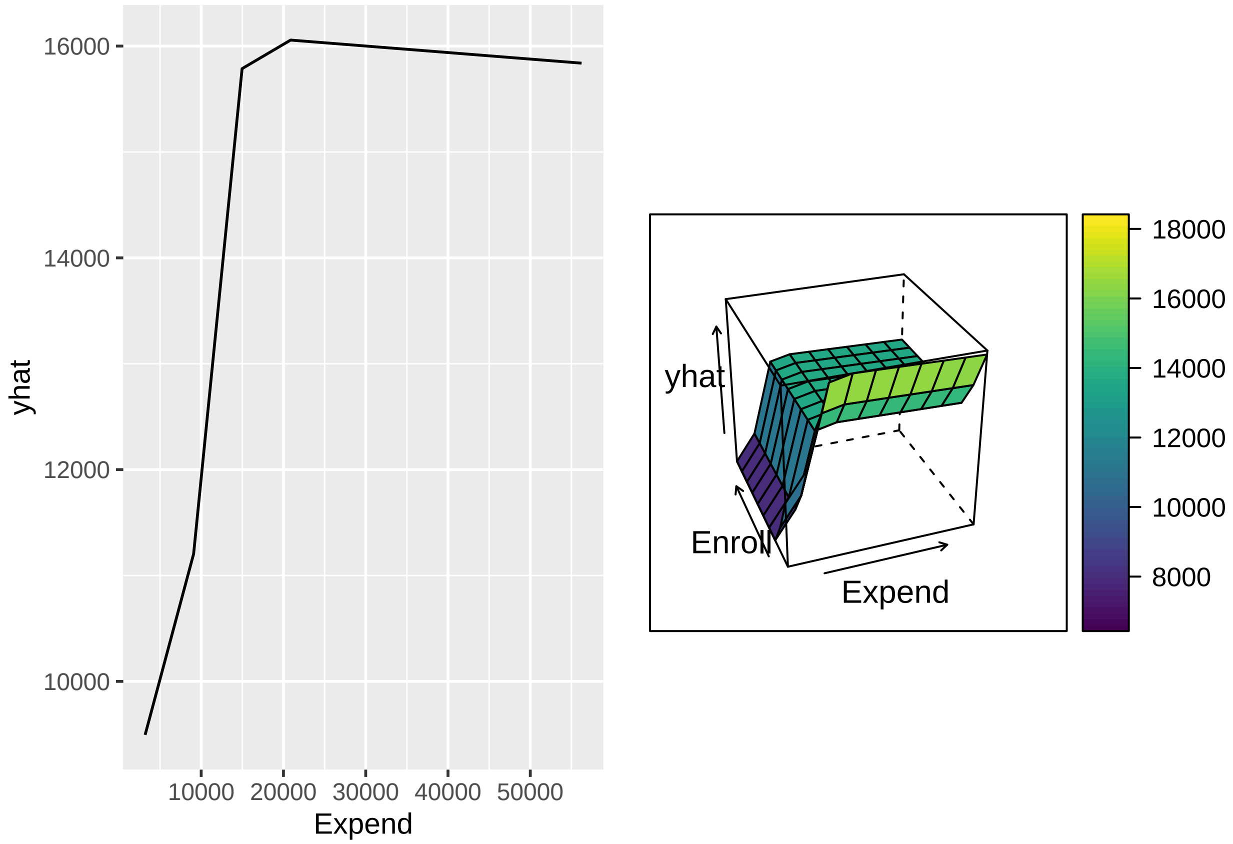
Report the final model

```
coef(mars.fit$finalModel)
```

```
##          (Intercept)     h(Expend-15365)  h(4450-Room.Board)
##        10856.8275542          -0.7836173          -1.4272043
## h(F.Undergrad-1355) h(1355-F.Undergrad)   h(22-perc.alumni)
##           -0.3818847          -1.6799143        -105.5570689
##         h(Apps-3712)       h(913-Enroll)      h(2193-Accept)
##            0.4334737           4.5019587          -1.9769988
##       h(Expend-6881)
##            0.7774546
```

The partial dependence plot (of Expend)

```
p1 = partial(mars.fit, pred.var = c("Expend"), grid.resolution = 10) %>% autoplot()

p2 = partial(mars.fit, pred.var = c("Expend", "Enroll"), grid.resolution = 10) %>%
  plotPartial(levelplot = FALSE, zlab = "yhat", drape = TRUE,
              screen = list(z = 20, x = -60))

grid.arrange(p1, p2, ncol = 2)
```



e) Predict the out-of-state tuition of Columbia University

Based on the above GAM and MARS models, predict the out-of-state tuition of Columbia University.

Predict basing on GAM

```
columbia_u = college_c %>%
  filter(College == "Columbia University")

pred.gam = predict(gam.fit,
                   newdata = columbia_u )

pred.gam
```

```
##        1
## 17728.51
```

The out-of-state tuition of Columbia University is predicted to be 17728.51 by the above GAM model.

Predict basing on MARS

```
pred.mars = predict(mars.fit,
                    newdata = columbia_u)

pred.mars
```

```
##             y
## [1,] 17469.9
```

The out-of-state tuition of Columbia University is predicted to be 17469.9 by the above MARS model.