Pre-lecture Problems for Lecture 6: Branch & Bound and Heuristic Algorithms

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3. (10 points; 5 points each) Consider the following IP

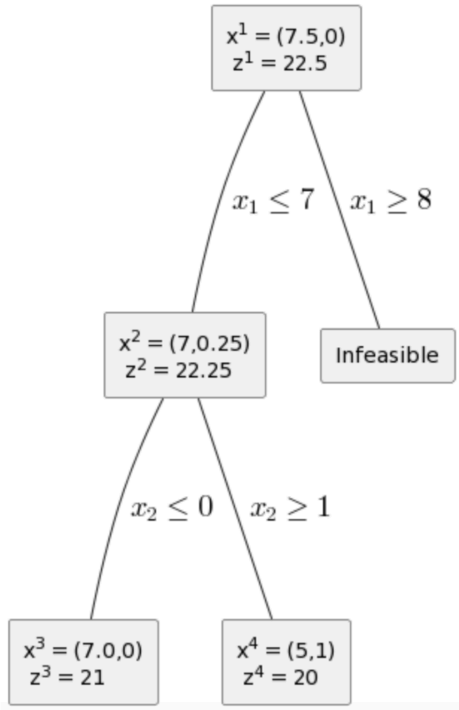
$$
\begin{aligned}
\max\ & 3x_1 + 5x_2 \\
\text{s.t.}\ & x_1 + 3x_2 \leq 8 \\
& 2x_1 + 4x_2 \leq 15 \\
& x_i \in \mathbb{Z}_+ \quad \forall i = 1, 2
\end{aligned}
$$

(a) Use the branch-and-bound algorithm to solve the IP.

Ans. We can first solve the LP relaxation of the problem: Solve the LP relaxation:

$$
\begin{aligned}
\max\ & 3x_1 + 5x_2 \\
\text{s.t.}\ & x_1 + 3x_2 \leq 8 \\
& 2x_1 + 4x_2 \leq 15 \\
& x_i \geq 0 \quad \forall i = 1, 2
\end{aligned}
$$

We can get the optimal solution $(x_1, x_2) = (7.5, 0)$, and the optimal value is 22.5. Then we can branch on the variable x_1.



The optimal solution is $(x_1, x_2) = (7, 0)$, and the optimal value is 21.

(b) Use the following two-step heuristic algorithm to solve the IP: First solve the linear relaxation to obtain an LR-optimal solution (x_1, x_2), and then report $(\lfloor x_1 \rfloor, \lfloor x_2 \rfloor)$ as an IP-feasible solution. Find the solution reported by the heuristic algorithm. Moreover, calculate the optimality gap by comparing the heuristic solution and the LR-optimal solution. Please note that (x_1, x_2) may be fractional in the LR-optimal solution, but $(\lfloor x_1 \rfloor, \lfloor x_2 \rfloor)$ must be integers when reported by the heuristic algorithm.

Ans. The LR-optimal solution is solved in (a.) which is $(x_1, x_2) = (7.5, 0)$, and the optimal value is 22.5. The heuristic solution is $(\lfloor x_1 \rfloor, \lfloor x_2 \rfloor) = (7, 0)$, and the optimal value is 21.

The absolute error is $22.5 - 21 = 1.5$.

The percentage error is $\frac{1.5}{22.5} = 6.67\%$.