Pre-lecture Problems for Lecture 9: Sensitivity Analysis and Dual Simplex Method

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1. (10 point) For each of the following functions, find the region over which the function is convex (or conclude that the function is nowhere convex.)

   (a) $f(x) = 2x^3 - x^2 + x + 2$.

   Ans.

   $\nabla^2 f(x) = 12x - 2$. So the fuction is convex when $x \geq \frac{1}{6}$.

   (b) $f(x_1, x_2) = -x_1^3 + 4x_2^2 + x_1 + 2$.

   Ans.

   The Hessian matrix of $f(x_1, x_2)$ is $\begin{bmatrix} -6x_1 & 0 \\ 0 & 8 \end{bmatrix}$. So the function is convex when $x_1 \leq 0$.

   (c) $f(x_1, x_2, x_3) = 6x_1^2 x_3 + 2x_2 x_3 + 3x_2^2 + 2$.

   Ans.

   The Hessian matrix of $f(x_1, x_2, x_3)$ is $\begin{bmatrix} 12x_3 & 0 & 12x_1 \\ 0 & 6 & 2 \\ 12x_1 & 2 & 0 \end{bmatrix}$.

   Let the first leading priciple minor $12x_3 \geq 0$, and the third leading priciple minor $-864x_1^2 - 48x_3 \geq 0$, which is impossible. So the function is nowhere convex.