Operations Research, Spring 2024 (112-2)

Pre-lecture Problems for Lecture 11: Convex Analysis

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Note. The deadline of submitting the pre-lecture problem is **9:30, April 29**. Please submit a hard copy of your work in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

1. (0 point) Determine whether the following sets and functions are convex:

   (a) $\{(x_1, x_2) \in \mathbb{R}^2 | x_1 + x_2 \geq 4, x_1 \geq 0, x_2 \leq 0\}$.

   (b) $\{(x_1, x_2) \in \mathbb{R}^2 | x_1^2 + x_2^2 \geq 4, x_1 \geq 0, x_2 \leq 0\}$.

   (c) $f(x) = 2x^3 - x^2 - 2x + 1$ for $x \in \mathbb{R}$.

   (d) $f(x) = \begin{cases} -x & \text{if } x < 1 \\ -1 & \text{if } x \geq 1 \end{cases}$.

2. (0 point) Analytically find a global minimum for the following functions:

   (a) $f(x) = 3x^2 + 2x + 1$ for $x \in \mathbb{R}$.

   (b) $f(x) = 2x^3 - x^2 - 2x + 1$ for $x \in [-1, \infty)$.

3. (10 point) For each of the following functions, find the region over which the function is convex (or conclude that the function is nowhere convex).

   (a) $f(x) = 2x^3 - x^2 + x + 2$.

   (b) $f(x_1, x_2) = -x_1^3 + 4x_2^2 + x_1 + 2$.

   (c) $f(x_1, x_2, x_3) = 6x_1^2 x_3 + 2x_2 x_3 + 3x_2^2 + 2$.