## Pre-lecture Problems for Lecture 4: Integer Programming

3. (10 points) A firm is considering importing some products to sell in a local market. The unit price and cost for product i are p<sub>i</sub> and C<sub>i</sub>, respectively. While the prices may be determined by the firm, the costs are given and fixed. For product i, the demand volume is A<sub>i</sub> - B<sub>i</sub>p<sub>i</sub>, where A<sub>i</sub> and B<sub>i</sub> are all given. The fixed cost of importing product i is K<sub>i</sub>. In other words, the cost K<sub>i</sub> is incurred if and only if a positive amount of product i is imported. Note that given the existence of the fixed cost, to maximize the firm's profit it may be good to import only a subset of products. Formulate a mathematical program that can find a purchasing plan that maximizes the firm's profit. Determine whether your program is a linear program, a linear integer program, a nonlinear program, or a nonlinear integer program.

## Ans.

The profit of importing product i is  $(p_i - C_i)(A_i - B_i p_i) - K_i$ . Let  $z_i$  be the binary variable that indicates whether the firm imports product i. I is the set of products. The mathematical program is:

$$\max \sum_{i \in I} (p_i - C_i)(A_i - B_i p_i) z_i - K_i z_i$$
s.t. 
$$A_i - B_i p_i \ge 0 \quad \forall i \in I$$

$$p_i \ge C_i \quad \forall i \in I$$

$$z_i \in \{0, 1\} \quad \forall i \in I$$

This is a nonlinear integer program.