Operations Research, Spring 2024 (112-2)

Pre-lecture Problems for Lecture 8: Linear Programming Duality

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The deadline of submitting the pre-lecture problem is ***9:30, April 15***. Please submit a hard copy of your work to the instructor in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

1. (0 point) Find the dual for the following LP:

$$\begin{array}{rrcrcrcr}
\max & 4x_1 & - & 2x_2 & + & x_3 & & \\
\text{s.t.} & 2x_1 & + & x_2 & & & \leq & 10 \\
 & & & x_2 & + & x_3 & \geq & 16 \\
 & x_1 & + & 3x_2 & - & 3x_3 & = & 14 \\
 & \multicolumn{7}{l}{x_1 \geq 0,\ x_2 \leq 0,\ x_3 \text{ urs.}}
\end{array}$$

2. (0 point) Consider a primal LP

$$\begin{array}{rl}
\max & 3x_1 + 5x_2 \\
\text{s.t.} & x_1 + x_2 \leq 8 \\
 & x_1 + 2x_2 \leq 12 \\
 & x_1 \geq 0, x_2 \geq 0.
\end{array}$$

   (a) Find a primal optimal solution $\bar{x}$.
   (b) Formulate the dual LP.
   (c) Solve the dual LP to get a dual optimal solution $\bar{y}$. Show that $c^{\mathrm{T}}\bar{x} = \bar{y}^{\mathrm{T}}b$, where c and b are the primal and dual objective function.

3. (10 points; 2 points each) Consider a primal LP

$$\begin{array}{rl}
\max & 5x_1 + 3x_2 \\
\text{s.t.} & x_1 + x_2 \leq 8 \\
 & x_1 + 2x_2 \leq 10 \\
 & x_1 \geq 0, x_2 \geq 0.
\end{array}$$

   (a) Find a primal optimal solution $\bar{x}$ by the simplex method.
   (b) Formulate the dual LP.
   (c) Solve the dual LP in any way you like to get a dual optimal solution $\bar{y}$. Show that $c^{\mathrm{T}}\bar{x} = \bar{y}^{\mathrm{T}}b$, where c and b represent the primal and dual objective coefficients.
   (d) Use the primal optimal basis B you found in Part (a) to verify that $c_B^{\mathrm{T}}A_B^{-1} = \bar{y}^{\mathrm{T}}$.
   (e) Find the shadow prices for the two primal constraints.[1]

[1] If you are applying the correct concept, you may need no calculation for finding them. But if you prefer, you may solve two modified primal LPs to see whether you may get the same conclusion. In any case, there is no need to submit the process of solving the two modified primal LPs.