Pre-lecture Problems for Lecture 2: Linear Programming

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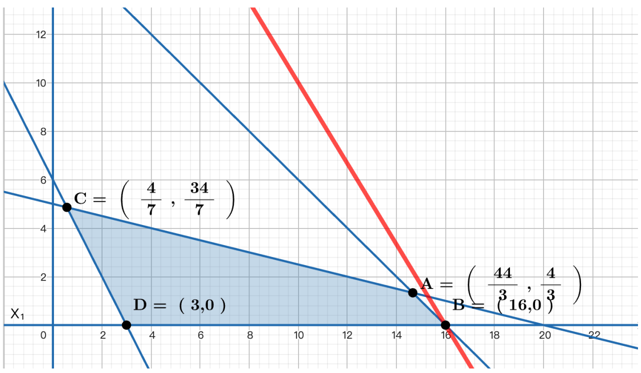
1. (0 point) Graphically solve the following LP:

$$
\begin{aligned}
\max\ & 5x_1 + 3x_2 \\
\text{s.t. } & x_1 + x_2 \leq 16 \\
& x_1 + 4x_2 \leq 20 \\
& 2x_1 + x_2 \geq 6 \\
& x_1 \geq 0, x_2 \geq 0.
\end{aligned}
$$

Ans.



2. (0 point) Bob is the owner of a furniture shop. He uses woods to make tables and chairs. Each day, he buys woods from his supplier at a cost of \$50 per unit. Each table requires 2 units of woods while each chair requires 1 unit. He, as well as his employees, needs to spend time on making these products. He can make 1 chair or 0.5 table in 1 hour. Each of his two employees, who are not as experiences as him, can make 0.8 chair or 0.3 tables in 1 hour. The outputs are always proportional to the amount of time they spend. Each of the two employees works 8 hours per day. Bob can work 12 hours per

day. A table can be sold at 200 and a chair can be sold at 80. Formulate an LP that can find a production plan for Bob to maximize his daily profit.

Ans. We can formulate the LP as follows:

Let x_1 be the number of tables and x_2 be the number of chairs. Then the LP is

$$\begin{aligned} \max\ & 100x_1 + 30x_2 \\ \text{s.t. } & 2x_1 + 1x_2 \leq 12 \\ & \frac{10}{3}x_1 + \frac{5}{4}x_2 \leq 16 \\ & x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

3. (10 points) Tom is the owner of a furniture shop and makes n products. Each day, he buys woods from his supplier at a cost of C dollars per unit. The maximum amount of woods that may be purchased is K units. Each unit of product j requires Rj units of woods, $j = 1, ..., n$. He also needs to spend time on making these products. He can make Tj units of product j per hour, $j = 1, ..., n$. The outputs are always proportional to the amount of time they spend. Tom can work for H hours per day. Each unit of product j can be sold at Pj dollars. All produced products will be sold.

   (a) (5 points) Formulate an LP that can help make a production plan for Tom to maximize his average daily profit.

   Note. You are required to formulate a "linear program." In particular, there should be no integer constraints on your variables. Therefore, please set your production quantities as fractional variables rather than integer one. If you wonder why 4.7 tables or 15.2 chairs are reasonable, you may consider these quantities as the average production quantities per day.

   Ans. We can formulate the LP as follows:

   Let x_j be the number of product j. Then the LP is

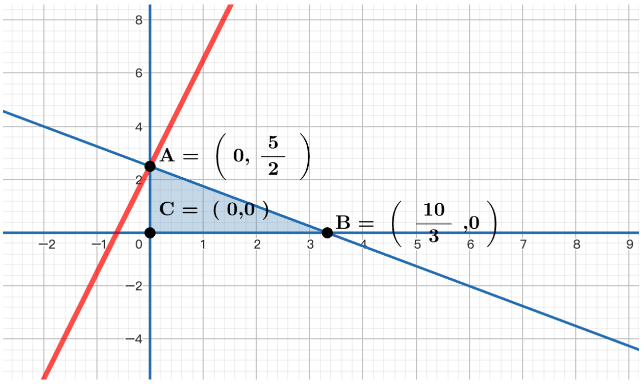
$$\begin{aligned} \max\ & \sum_{j=1}^{n}(P_j - C \cdot R_j)x_j \\ \text{s.t. } & \sum_{j=1}^{n} R_j x_j \leq K \\ & \sum_{j=1}^{n} T_j x_j \leq H \\ & x_j \geq 0, j = 1, ..., n. \end{aligned}$$

   (b) (5 points) Let $n = 2,\ C = 15,\ K = 300,\ R_1 = 8,\ R_2 = 5,\ T_1 = 3,\ T_2 = 4,\ H = 10,\ P_1 = 100$, and $P_2 = 80$. Graphically solve the LP. Interpret your solution to make a suggestion to Tom.

Ans. We can formulate the LP as follows:

Let x_1 be the number of product 1 and x_2 be the number of product 2. Then the LP is

$$\begin{aligned} \max \ & -20x_1 + 5x_2 \\ \text{s.t. } & 8x_1 + 5x_2 \leq 300 \\ & 3x_1 + 4x_2 \leq 10 \\ & x_1 \geq 0, x_2 \geq 0. \end{aligned}$$



As the graph shows, the optimal solution is $(0, \frac{5}{2})$. I suggest that Tom should spend all his time on product 2.