Operations Research, Spring 2024 (112-2)

Pre-lecture Problems for Lecture 3: Integer Programming

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Note. The deadline of submitting the pre-lecture problem is ***9:30, March 4***. Please submit a hard copy of your work in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

1. (0 point) Consider the following integer program

$$
\begin{array}{rl}
\max & x_1 + x_2 \\
\text{s.t.} & 2x_1 + x_2 - 6 \leq M_1 z \\
& x_1 + 2x_2 - 8 \leq M_2(1 - z) \\
& x_1 \leq 10 \\
& x_2 \leq 10 \\
& x_i \geq 0 \quad \forall i = 1, 2 \\
& z \in \{0, 1\},
\end{array}
$$

   where M_1 and M_2 are parameters and x_1, x_2, and z are variables. The binary variable z is to select at least one constraint to be satisfied.

   (a) What values of M_1 and M_2 can enable z to do the "at-least-one" selection?

   (b) Depict the feasible region on the (x_1, x_2) plane. Then graphically solve the IP.
   Hint. The feasible region is nonconvex on the (x_1, x_2) plane. To solve this problem graphically, no branch-and-bound is needed.

2. (0 point) Six towns locate at the following six points on a Cartesian plane: $(0, 60)$, $(20, 50)$, $(30, 20)$, $(40, 80)$, $(50, 50)$, and $(90, 60)$ (in km). Currently a company owns six retail stores, one in each town. The weekly sales of a product in these stores are 10000, 15000, 12000, 8000, 20000, and 3000. The company currently has one distribution center (DC) in town 3. It plans to build some more DCs for the retail stores to replenish from. The construction costs of building a DC in these towns are \$200000, \$180000, \$160000, \$190000, \$150000, and \$200000. A truck can carry 500 units of this product. The shipping cost for a truck to move 1 km is \$1. The existing DC and any newly built DC can be used for 5 years. There is no capacity limit for a DC. Transportation between two locations can be done by traveling through the straight line connecting them.

   (a) Formulate the problem of minimizing the 5-year total construction and shipping costs as an integer program if DCs can only be built in towns.

   (b) Suppose DCs may also be built in the following locations: $(0, 20)$, $(20, 40)$, $(40, 30)$, and $(60, 40)$. Do Part (a) again.

3. (10 point; 5 points each) Ten jobs should be scheduled on one single machine. The processing times for these jobs are 7, 4, 3, 9, 10, 6, 8, 9, 7, and 10 (in hours). The due times for these jobs are 40, 43, 45, 46, 49, 50, 52, 57, 58, and 60 (in hours).

   (a) Suppose that we want to minimize the total ***tardiness*** of all jobs. For a job, its tardiness is its completion time minus its due time if this quantity is positive or zero otherwise. For the schedule $(1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$, which means first processing job 1, then job 2 right after job 1 is done, then job 3 right after job 2 is done, etc., calculate the total tardiness.

   (b) Formulate an integer program that finds a schedule to minimize the total tardiness of all jobs.