Pre-lecture Problems for Lecture 5: The Simplex Method

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3. (10 points) Consider the following LP

$$
\begin{aligned}
\max\ & x_1 + 2x_2 + x_3 \\
\text{s.t.}\ & x_1 + x_2 \leq 10 \\
& x_2 + x_3 \leq 8 \\
& x_i \geq 0 \quad \forall i = 1, ..., 3.
\end{aligned}
$$

(a) (5 points) Find all the basic solutions and basic feasible solutions for the LP.

Ans.

We can first change the LP into standard form:

$$
\begin{aligned}
\max\ & x_1 + 2x_2 + x_3 \\
\text{s.t.}\ & x_1 + x_2 + x_4 = 10 \\
& x_2 + x_3 + x_5 = 8 \\
& x_i \geq 0 \quad \forall i = 1, ..., 5.
\end{aligned}
$$

Then we can find all the basic solutions and basic feasible solutions for the LP.

Basis	feasible	x_1	x_2	x_3	x_4	x_5
(x_1, x_2)	Yes	2	8	0	0	0
(x_1, x_3)	Yes	10	0	8	0	0
(x_1, x_5)	Yes	10	0	0	0	8
(x_2, x_3)	No	0	10	-2	0	0
(x_2, x_4)	Yes	0	8	0	2	0
(x_2, x_5)	No	0	10	0	0	-2
(x_3, x_4)	Yes	0	0	8	10	0
(x_4, x_5)	Yes	0	0	0	10	8

表 1: All the basic solutions and basic feasible solutions for the LP.

(b) (5 points) Use the simplex method to solve that LP. In the first iteration, enter x1. Write down all the iterations, an optimal solution, and the associated objective value.

Ans.

Let $z = x_1 + 2x_2 + x_3$, we can write the LP in the following form:

$$\begin{aligned} z - x_1 - 2x_2 - x_3 \qquad\qquad &= 0 \\ x_1 + x_2 \qquad + x_4 \qquad &= 10 \\ x_2 + x_3 \qquad + x_5 &= 8 \end{aligned}$$

Then we can write the initial tableau:

-1	-2	-1	0	0	0
1	1	0	1	0	$x_4 = 10$
0	1	1	0	1	$x_5 = 8$

表 2: Initial Tableau

We can see that the coefficient of x_1 in the objective row is -1, so we can enter x_1 into the basis. Then we can write the next tableau:

0	-1	-1	1	0	10
1	1	0	1	0	$x_1 = 10$
0	1	1	0	1	$x_5 = 8$

表 3: Second Tableau

We can see that the coefficient of x_2 in the objective row is -1, so we can enter x_2 into the basis. Then we can write the next tableau:

0	0	0	1	1	18
1	0	-1	1	-1	$x_1 = 2$
0	1	1	0	1	$x_2 = 8$

表 4: Third Tableau

We can see that all the coefficients in the objective row are non-negative, so the current solution is optimal. The optimal solution is $x_1 = 2$, $x_2 = 8$, $x_3 = 0$, and the associated objective value is $z = 2 + 2 * 8 + 0 = 18$.