Operations Research, Spring 2024 (112-2)

Pre-lecture Problems for Lecture 5: The Simplex Method

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Note. The deadline of submitting the pre-lecture problem is ***09:30, March 18***. Please submit a hard copy of your work to the instructor in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

1. (0 point) Consider the following LP

$$\begin{aligned} \max \quad & 5x_1 + 3x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 16 \\ & x_1 + 4x_2 \leq 20 \\ & x_2 \leq 8 \\ & x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

   (a) Find its standard form.

   (b) Find all the basic solutions of the standard form.

   (c) Show the one-to-one mapping between the basic feasible solutions of the standard form and the extreme points of the feasible region of the original LP.

2. (0 point) Use the simplex method to solve the LP in Problem 1. Depict the route you go through in the solution process.

3. (10 points) Consider the following LP

$$\begin{aligned} \max \quad & x_1 + 2x_2 + x_3 \\ \text{s.t.} \quad & x_1 + x_2 \leq 10 \\ & x_2 + x_3 \leq 8 \\ & x_i \geq 0 \quad \forall i = 1, ..., 3. \end{aligned}$$

   (a) (5 points) Find all the basic solutions and basic feasible solutions for the LP.

   (b) (5 points) Use the simplex method to solve that LP. In the first iteration, enter x_1. Write down all the iterations, an optimal solution, and the associated objective value.