Operations Research, Spring 2024 (112-2)

Pre-lecture Problems for Lecture 2: Linear Programming

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

Note 1. The deadline of submitting the pre-lecture problem is ***9:30, February 26***. Please submit a hard copy of your work to the instructor in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

Note 2. Pre-lecture problems are the only assignments that require hard-copy submissions. If you cannot make it to the classroom, please feel free to ask a friend to submit your work for you. You may either type or write down your answers. Nevertheless, if you write down your answers, please make sure that they are legible. Unrecognizable answers will receive no point.

1. (0 point) Graphically solve the following LP:

$$\begin{array}{rl} \max & 5x_1 + 3x_2 \\ \text{s.t.} & x_1 + x_2 \leq 16 \\ & x_1 + 4x_2 \leq 20 \\ & 2x_1 + x_2 \geq 6 \\ & x_1 \geq 0, x_2 \geq 0. \end{array}$$

2. (0 point) Bob is the owner of a furniture shop. He uses woods to make tables and chairs. Each day, he buys woods from his supplier at a cost of \$50 per unit. Each table requires 2 units of woods while each chair requires 1 unit. He, as well as his employees, needs to spend time on making these products. He can make 1 chair or 0.5 table in 1 hour. Each of his two employees, who are not as experiences as him, can make 0.8 chair or 0.3 tables in 1 hour. The outputs are always proportional to the amount of time they spend. Each of the two employees works 8 hours per day. Bob can work 12 hours per day. A table can be sold at \$200 and a chair can be sold at \$80. Formulate an LP that can find a production plan for Bob to maximize his daily profit.

3. (10 points) Tom is the owner of a furniture shop and makes n products. Each day, he buys woods from his supplier at a cost of C dollars per unit. The maximum amount of woods that may be purchased is K units. Each unit of product j requires R_j units of woods, $j = 1, ..., n$. He also needs to spend time on making these products. He can make T_j units of product j per hour, $j = 1, ..., n$. The outputs are always proportional to the amount of time they spend. Tom can work for H hours per day. Each unit of product j can be sold at P_j dollars. All produced products will be sold.

   (a) (5 points) Formulate an LP that can help make a production plan for Tom to maximize his average daily profit.

   Note. You are required to formulate a "linear program." In particular, there should be no integer constraints on your variables. Therefore, please set your production quantities as fractional variables rather than integer one. If you wonder why 4.7 tables or 15.2 chairs are reasonable, you may consider these quantities as the *average* production quantities per day.

   (b) (5 points) Let $n = 2$, $C = 15$, $K = 300$, $R_1 = 8$, $R_2 = 5$, $T_1 = 3$, $T_2 = 4$, $H = 10$, $P_1 = 100$, and $P_2 = 80$. Graphically solve the LP. Interpret your solution to make a suggestion to Tom.