Operations Research, Spring 2024 (112-2)

Pre-lecture Problems for Lecture 6: Branch & Bound and Heuristic Algorithms

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

Note. The deadline of submitting the pre-lecture problem is ***9:30, March 25***. Please submit a hard copy of your work to the instructor in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

1. (0 point) In this problem, we will use the branch-and-bound algorithm to solve the following IP

$$\begin{aligned} \max \quad & 3x_1 + 5x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 16 \\ & x_2 \leq 7.5 \\ & x_i \in \mathbb{Z}_+ \quad \forall i = 1, 2. \end{aligned}$$

   Solve the linear relaxation of the given IP. Show that both x_1 and x_2 are fractional in the optimal solution.

2. (0 point) Continue from the fractional solution you obtained in the previous problem.

   (a) Branch on x_1 and continue until the IP is fully solved. Depict the branching tree.

   (b) Instead of branch on x_1, branch on x_2 and continue until the IP is fully solved. Depict the branching tree. Compare the result with branching on x_1.

3. (10 points; 5 points each) Consider the following IP

$$\begin{aligned} \max \quad & 3x_1 + 5x_2 \\ \text{s.t.} \quad & x_1 + 3x_2 \leq 8 \\ & 2x_1 + 4x_2 \leq 15 \\ & x_i \in \mathbb{Z}_+ \quad \forall i = 1, 2. \end{aligned}$$

   (a) Use the branch-and-bound algorithm to solve the IP.

   (b) Use the following two-step heuristic algorithm to solve the IP: First solve the linear relaxation to obtain an LR-optimal solution (x_1, x_2), and then report $(\lfloor x_1 \rfloor, \lfloor x_2 \rfloor)$ as an IP-feasible solution. Find the solution reported by the heuristic algorithm. Moreover, calculate the optimality gap by comparing the heuristic solution and the LR-optimal solution. Please note that (x_1, x_2) may be fractional in the LR-optimal solution, but $(\lfloor x_1 \rfloor, \lfloor x_2 \rfloor)$ must be integers when reported by the heuristic algorithm.