1. 卷积相关过程

1) $z[n] = x[n] * y[n] = \sum_{m=0}^{N-1} x[n]\, y[n-m]$

$x[n]$: 1 2 3 4 3 2 1

−2 0 2   −2 0 2   长度 $= M+m-1 = 7+3-1 = 9$

$z[n]$: 2 4 4 4 0 −4 −4 −4 −2

2) $f[m,n] = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}$ $g[m,n] = \begin{pmatrix} 1 & 3 & 2 & 0 & 4 \\ 1 & 0 & 3 & 2 & 3 \\ 0 & 4 & 1 & 0 & 5 \\ 2 & 3 & 2 & 1 & 4 \\ 3 & 1 & 0 & 4 & 2 \end{pmatrix}$

$$f[m,n] * g[m,n] = \begin{pmatrix} -1 & -3 & -1 & 3 & -2 & 0 & 4 \\ -3 & -6 & -4 & 4 & -4 & 2 & 11 \\ -3 & -7 & -6 & 3 & -6 & 4 & 15 \\ -3 & -11 & -4 & 8 & -10 & 3 & 17 \\ -7 & -11 & 2 & 5 & -10 & 6 & 15 \\ -8 & -5 & 6 & -4 & -6 & 9 & 8 \\ -3 & -1 & 3 & -3 & -2 & 4 & 2 \end{pmatrix}$$

"full convolution"

3) Matlab 验证见代码.

4) 1D: $x[n]$: L $y[n]$: ℓ 卷积结果为 $L+\ell-1$

2D: f $m \times n$ g $x \times y$ 卷积结果 $(m+x-1,\ n+y-1)$

(以上为 no padding 的情况)

2. 线性时不变系统

1) $H[f(t)] = H[f(t-\tau)]$ τ为时间延迟

2) $x[n] = [1\ 2\ 3\ 4\ 3\ 2\ 1]$

$$z = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \end{bmatrix} \begin{array}{c} 0 \\ \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{bmatrix} \\ 0 \end{array} = \begin{array}{c} 2 \\ \begin{bmatrix} 2 \\ 4 \\ 4 \\ 4 \\ 0 \\ -4 \\ -4 \\ -4 \\ -2 \end{bmatrix} \\ -2 \end{array}$$

3) 该矩阵为稀疏矩阵，且参数复用

4) 对于一个线性时不变离散系统，单位脉冲的响应为

$$h[n] * \delta[n] = h[n]$$

对于任意信号 $x[n]$

$$h[n] * x[n] = h[n] * \delta[n] * x[n]$$

$\delta[n] * x[n]$ 为 $x[n]$ 本身，即可视作~~成~~一组具有权重的 $\delta[n]$ 的线性组合。 $x[n] = \sum_{k=a}^{b} w_k \delta[n]$

$$h[n] * \omega_1 \delta[n] = \omega_1 h[n]$$

$$\Rightarrow h[n] * x[n] = h[n] * \left(\sum_{k=a}^{b} \omega_k \delta[n]\right)$$

通过卷积的分配律和线性性质.

易证 $h[n] * x[n] = \sum_{k=a}^{b} \omega_k \left(h[n] * \delta[n]\right)$

$$= \sum_{k=a}^{b} \omega_k h[n]$$

3. Laplace 算子

(x, y) 经过旋转, 即乘以一个 Rotation matrix (2D).

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$x = x'\cos\theta - y'\sin\theta$$

$$y = x'\sin\theta + y'\cos\theta$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$= \frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial x}\frac{\partial x}{\partial x'} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial x'}\right) + \frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\frac{\partial x}{\partial y'} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial y'}\right)$$

$$= \frac{\partial}{\partial x'}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right) + \frac{\partial}{\partial y'}\left(\frac{\partial f}{\partial x} - \sin\theta + \cos\theta \cdot \frac{\partial f}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right)\frac{\partial x}{\partial x'} + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right)\frac{\partial y}{\partial x'}$$

$$+ \frac{\partial}{\partial x}\left(-\sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial y}\right)\frac{\partial x}{\partial y'} + \frac{\partial}{\partial y}\left(-\sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial y}\right)\frac{\partial y}{\partial y'}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right)\cos\theta + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right)\sin\theta$$

$$+ \frac{\partial}{\partial x}\left(-\sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial y}\right)(-\sin\theta) + \frac{\partial}{\partial y}\left(-\sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial y}\right)\cos\theta$$

$$= \frac{\partial}{\partial x}\frac{\partial f}{\partial x} + \frac{\partial}{\partial y}\frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

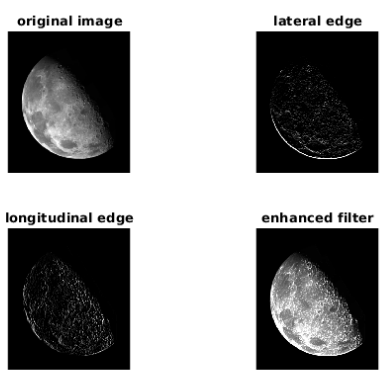
4. Sobel Y filter　　　　Sobel X filter

① $f = \begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix}$　　　$f = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}$

Laplacian

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

② $g(x, y) = f(x, y) - \nabla^2 f(x, y)$



可见锐化后边缘对比度增强

Matlab code.

5. ① $m, n = size(img)$

$$th = \frac{mn}{2}$$

② 将窗口移至一个新行的开始，对其内容排序。

建立窗口像素直方图 H，确定中值 m，记下亮度小于或等于 m 的像素个数 n_m

③ 对于最左列亮度 p_g 的每个像素 p

$$H[p_g] = H[p_g] - 1$$

进一步，if $p_g < m$，$n_m = n_m - 1$

④ 将窗口右移一列，对于最右列亮度 p_g 的像素 p，做

$$H[p_g] = H[p_g] + 1$$

if $p_g < m$，$n_m = n_m + 1$

⑤ if $n_m = t$，结束

⑥ if $n_m > t$，跳至 ⑦

重复 $m = m + 1$；$n_m = n_m + H[m]$

直至 $n_m \geq t$，跳至 ⑧

⑦ 重复

$m = m - 1$；$n_m = n_m - H[m]$

2. (1) · 线性： $\begin{cases} \text{叠加性} & H[f(t)+g(t)] = H[f(t)] + H[g(t)] \\ \text{齐次性} & H[af(t)] = aH[f(t)] \end{cases}$

时不变： $H[f(t)] = g(t)$ $\quad H[f(t-\tau)] = g(t-\tau)$

(2)

$$\begin{bmatrix} -2 & 0 & 2 & & & & \\ & -2 & 0 & 2 & & & \\ & & -2 & 0 & 2 & & \\ & & & \ddots & & & \\ & & & & -2 & 0 & 2 \\ & & & & & -2 & 0 \; 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 3 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \\ 4 \\ 4 \\ 0 \\ -4 \\ -4 \\ -4 \\ -2 \\ 2 \end{bmatrix}$$

(3) 先将信号补0.

确定矩阵维度：$(M+N-1)\times(M+N-1)$ $\quad M=7 \quad N=3$

让 filter 的第1个数作为矩阵的对角线.

补上后面的值.

矩阵构造完毕.

4. 在原题改的