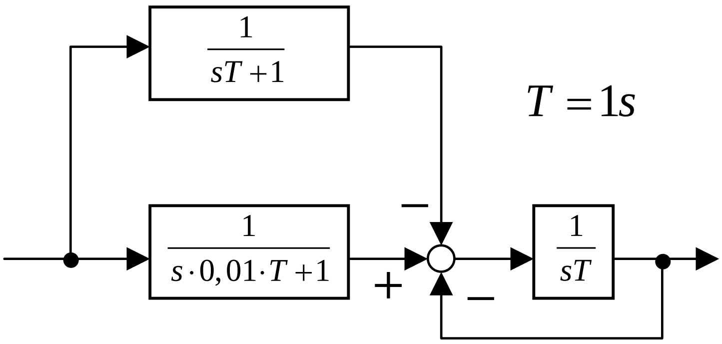
© Copyright by Klaus Janschek 2018 – Restricted use for ZJU course "Modeling and Simulation of Mechatronic Systems"- March 2018

Exercise MS_Ex2 Numerical integration of ODE models

Exercise 2-1 Block oriented ODE integration



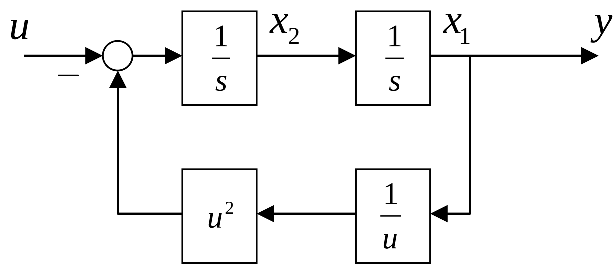
(a) Determine the maximum step size for numerical integration with RUNGE-KUTTA 4th-order.

(b) Give for the step size $h = 10\,\mathrm{s}$ an alternative integration method, which allows high accuracy and sktech the application of this method.

© Copyright by Klaus Janschek 2018 – Restricted use for ZJU course "Modeling and Simulation of Mechatronic Systems"- March 2018

Exercise 2-2 Block oriented ODE integration

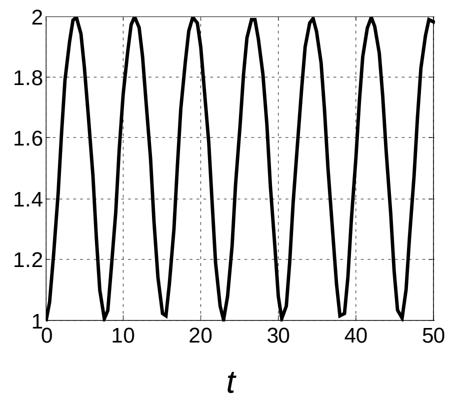
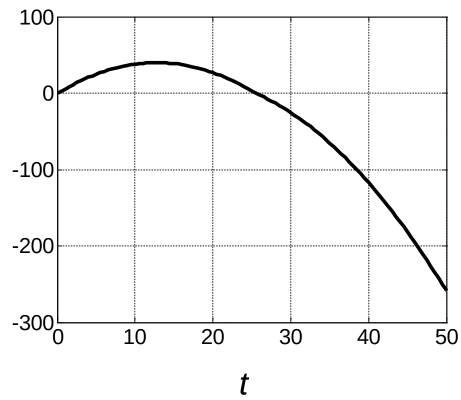
The following model has been realized in a block oriented simulation tool.



(a) Determine an equivalent state space model.

(b) The following simulation results have been obtained from simulation experiments with a RUNGE-KUTTA 4th order integration algorithm with fixed step size h = 0,5 s.

Check the correctness of these results (one result is definitely not correct) and justify your decision.

$\hat{y}(t)$ for $x_1(0) = 1,0;\ x_2(0) = 0$	$\hat{y}(t)$ for $x_1(0) = 0,2;\ x_2(0) = 0$
	

© Copyright by Klaus Janschek 2018 – Restricted use for ZJU course "Modeling and Simulation of Mechatronic Systems"- March 2018

Exercise 2-3 DC-motor simulation

For the current x_1 und die angular velocity x_2 of a DC-motor is given the following ODE model with inputs control voltage u_1 und load torque u_2.

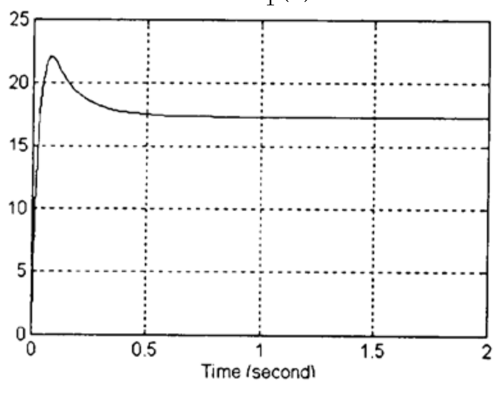
$$\begin{aligned}
\dot{x}_1 &= -45,75x_1 - 0,525x_1x_2 + 1027,5\,u_1 \\
\dot{x}_2 &= x_1^2 - 1500u_2 \\
u_1 &= 1\cdot\sigma\left(t\right) & x_1\left(0\right) = x_2\left(0\right) = 0 \\
u_2 &= 0,2\cdot\sigma\left(t\right) & \left(\sigma\left(t\right) \text{ - unit step}\right)
\end{aligned}$$

(a) Determine the maximum step size numerical integration with RUNGE-KUTTA 4th-order.

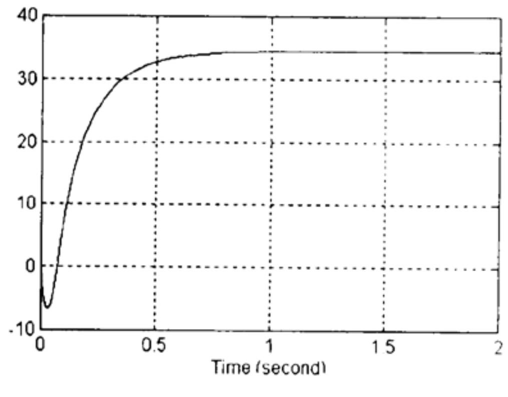
Hint: linearize the system around the stationary point $x_1 = 0,\ x_2 = 0$!

(b) The results of an simulation experiment are shown below. Check the correct implementation of the simulation model. Justify your decision.

$x_1(t)$

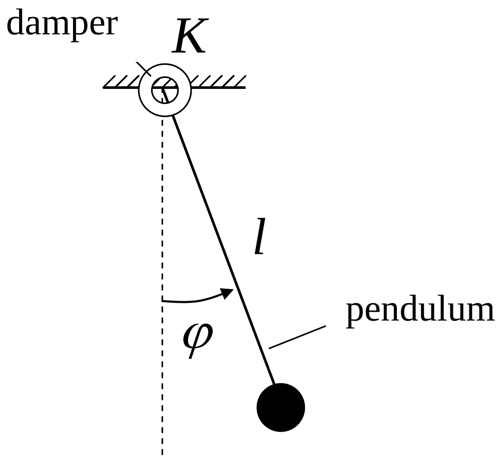


$x_2(t)$



© Copyright by Klaus Janschek 2018 – Restricted use for ZJU course "Modeling and Simulation of Mechatronic Systems"- March 2018

Exercise 2-4 Simulation of a pendulum



The dynamic behavior $\varphi(t)$ of the pendulum shown in the figure above shall be simulated with a RUNGE-KUTTA 4th-order algorithm.
A state space model is given as:

$$\dot{\varphi} = \omega\,,$$

$$\dot{\omega} = -\frac{g}{l}\sin\varphi - K\omega\,.$$

with: ω – angular velocity,
$l = 1\ \mathrm{m}$ – pendulum length,
$K = 0{,}1\ \mathrm{s}^{-1}$ – damping constant
$g = 9{,}81\ \mathrm{m}\cdot\mathrm{s}^{-2}$ – gravitational acceleration.

Determine the maximum possible step size.