Final Project

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Abstract

The interest rate is significant in the economy because it connects to people's behaviors. The lower interest rate can boom the economy because the liquidity of the market increase which more people purchase goods and investors make loan to invest. The high interest rate makes economy worse and people hold more cash on hand. So, the interest rate is important for the government and customers. Using the prediction to have the over view of interest rate so that it lets the government have the action on it so that people's purchasing power does not affect too much. In this study, we take the quarterly interest rate of Treasury bills from first quarter of 1953 to second quarter of 1980 in the Unites States. Fit the data to the ARIMA model and make 8 quarters predictions of interest rate. The final result is in general, the interest rate decline especially between the third quarter and fifth quarter. The a large part of the reason is the government of the Unites States control the interest rate so that the rate decrease to the appropriate range. Also, based on the 95% confidence intervals for the identified first three predominant periods, we cannot establish the significance of all three peaks since they lie in the others' peak confidence intervals.

Keywords :

Treasury bills, **interest rate**, **ARIMA model**, **dominant frequencies**, **prediction**

Introduction

Interest rate is closely related to our life, the higher interest rate suffers everyone especially the investors, they have to pay more money to bank on borrowing money. In 1980, Chairman Paul Volcker kept increase interest rate makes economy and inflation to a standstill (Volcker, 2009). So, the interest rate can affects the country's economy and people' purchasing power. The interest rate is connect to everyone and even the whole world, investigating the interest rate is very important. It helps us to predict and prevent the future interest rate. The data is collected from the Treasury bills which is the government debt of the United States. And the interest rates are recorded quarterly from first quarter 1953 to second quarter of 1980 with the sample size 110. For the statistical methods, make transformation so that time series is stable. For the result section, comparing two models to fit the ARIMA model, then, make prediction to analysis interest rate and using spectrum analysis to check whether peaks are significant.

Statistical Methods

Figure 1 shows the quarterly interest rate from 1953 to 1980. As the time goes by, the interest rate rising fast and the variance is not stable especially the range between 1975 and 1980.

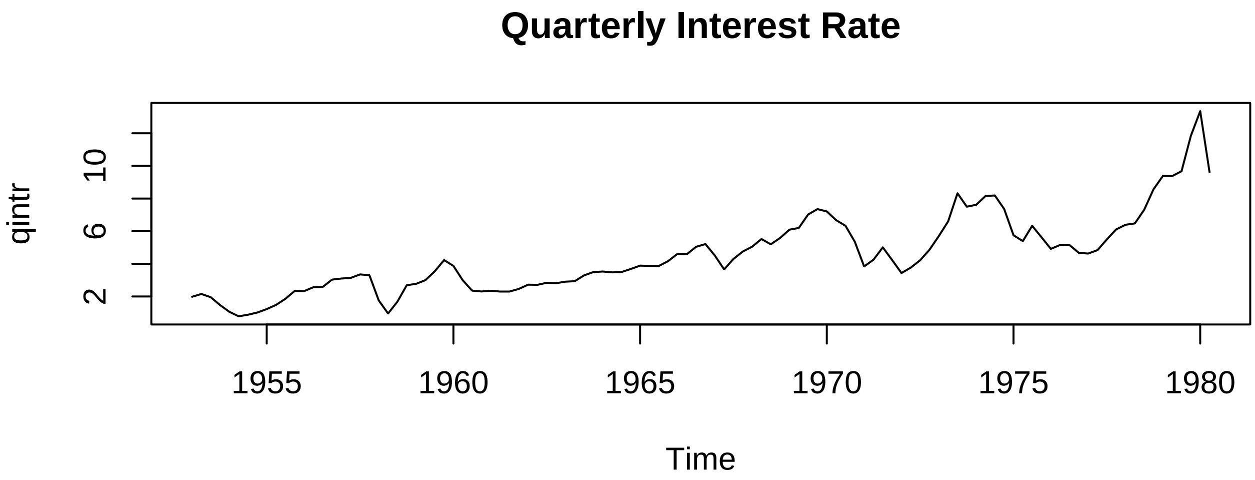

Figure 1: quanter time series about the interest rate in the U.S.

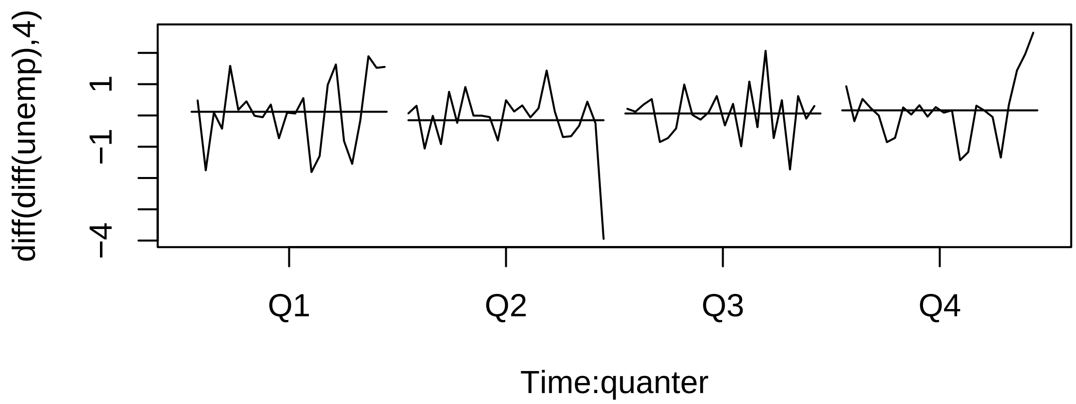

Figure 2: transformaed quanter time series

Since based on figure 1, the time series data is not stationary, we need to transform the quarterly interest rate data with sample size 110. Firstly, X_t is the time series of interest rate I want to study and W_t is the white noise with the variance is σ^2. Now, transform data which difference the data (∇X_t). After that, the upward trend is removed. In addition, based on the seasonal with 4 quarters, make the four order difference based on the difference of the data ($\nabla_4 \nabla X_t$). Figure 2 shows the transformation and $\triangledown_4\ \triangledown log x_t$ becomes stable which the time series fluctuates round 0.

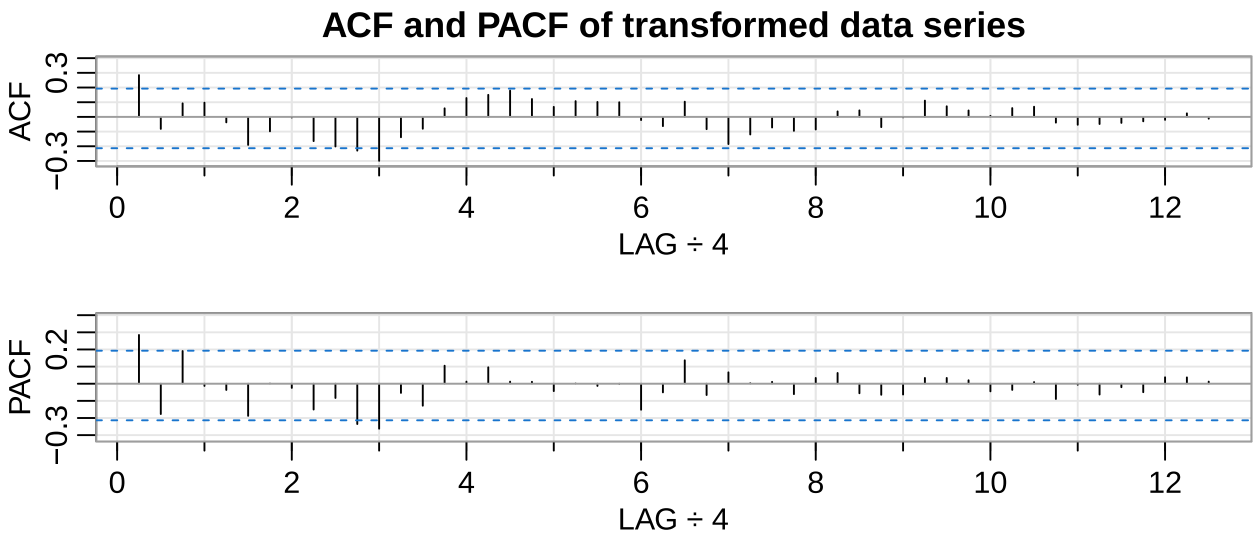

Figure 3: ACF and PACF of transformed data series

Based on the figure 3, it shows the ACF and PACF of the transformed data series. Fix it to the SARIMA model. There are two parts we need to consider which are non-seasonal component and seasonal component. For the non-seasonal component (p,d,q), the PACF cut off lag at 2, then, p=2. The cut off lag of ACF is 3 or 4, q can be 3 or 4. The difference of data is 1, so, d=1. The non-seasonal component can be (2,1,3) and (2,1,4). Now, consider the seasonal component (P, D, Q). With the seasonal of 4 (s=4), the lag of PACF cut off is 0s which P = 0. The seasonal difference is 1 and D is 1. The lag of ACF cut off is 1s, so, Q = 1. The seasonal component is (0,1,1). Therefore, there are two SARIMA models to fit this series data which are $SARIMA(2,1,3)$x$(0,1,1)_4$ and $SARIMA(2,1,4)$x$(0,1,1)_4$.

Result

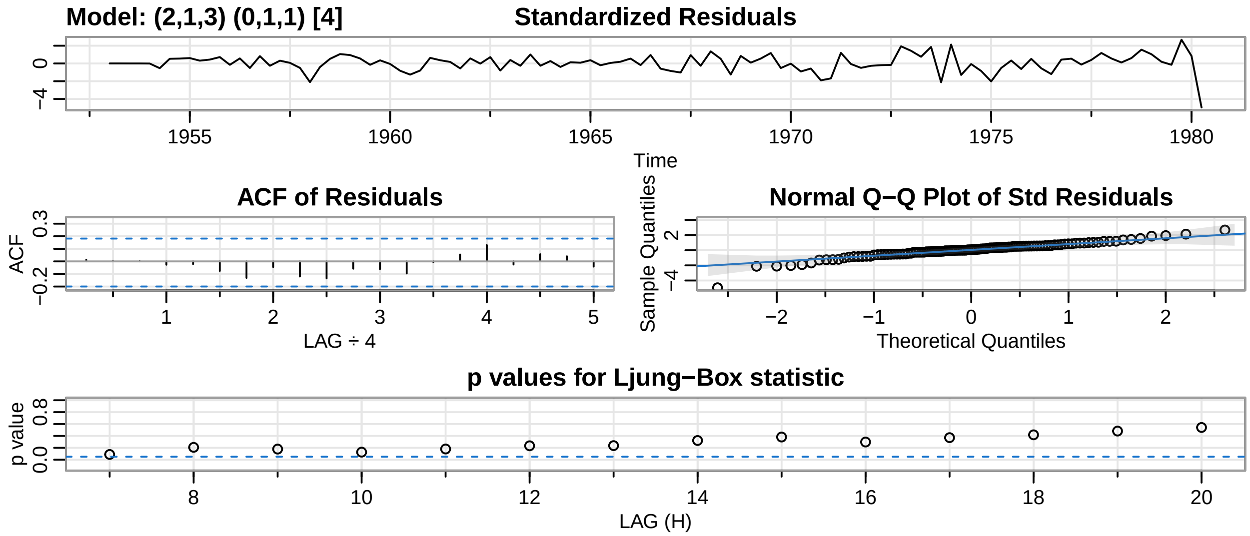

Figure 4: residual analysis of SARIMA(2,1,3)x(0,1,1)4

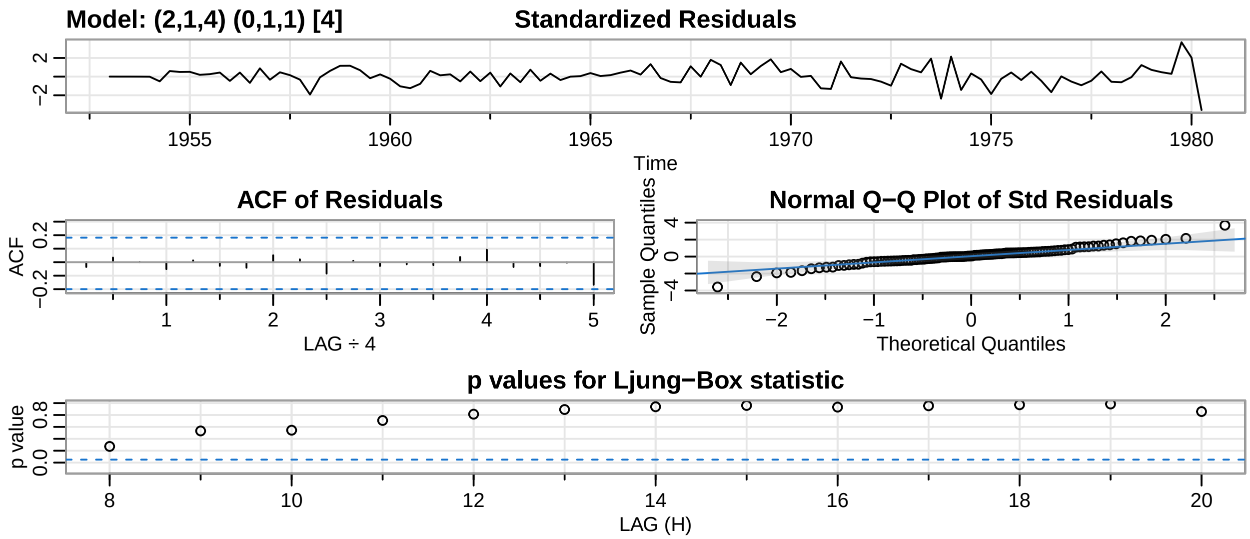

Figure 5: residual analysis of SARIMA(2,1,4)x(0,1,1)4

According to figure 4 and 5, the residual analysis of $SARIMA(2,1,3)\text{x}(0,1,1)_4$ and $SARIMA(2,1,4)\text{x}(0,1,1)_4$, compare the standardized residuals plot, both time series fluctuate around 0 and it does not have pattern, so, it means the variance (σ^2) is a constant. For the plot of ACF of residuals, all lines fall in the cut off line for the both models. It means residuals are uncorrelated. Now, comparing the normal Q-Q plot of standard residuals, almost all points line on the straight line. It means they follow the normal distribution. Lastly, compare the p-value of Ljung_box statistics. We let the null hypothesis (H_0) is the correlation between residuals is 0. Then, the alternative hypothesis (H_a) is the correlation is not 0. By comparing, all points of both models lie above the dashed line. It means p-value are greater than 0.05 which we have no evidence to against the null hypothesis. Then, the residuals are uncorrelated. So, based on the residual analysis, we cannot conclude which model is better. Then we crate a table about AIC, BIC and AICc.

Tbale 1 :Summary table compare two models

model	AIC	AICc	BIC
$ARIMA(2,1,3)$x$(0,1,1)_4$	2.068311	2.076474	2.245242
$ARIMA(2,1,4)$x$(0,1,1)_4$	1.936402	1.947398	2.138608

Based on the summary table 1, compare the AIC, AICc and BIC. The smaller vlaue of them, the better model is. Since the number of AIC, AICc and BIC for $ARIMA(2,1,4)$x$(0,1,1)_4$ are smaller than $ARIMA(2,1,3)$x$(0,1,1)_4$, the final model we would select to fit the time series data is $ARIMA(2,1,4)$x$(0,1,1)_4$.

The formula for the final model is:

$$(1-\phi_1 B-\phi_2 B^2)\nabla_4\nabla\hat{X}_t=(1+\theta_1 B+\theta_2 B^2+\theta_3 B^3+\theta_4 B^4)(1+\Theta_1 B^4)W_t$$

- X_t is the time series of interest rate I want to study.
- W_t is the white noise with the mean is 0 and variance is ${\sigma_w}^2$.
- ∇X_t is taking difference of the data.
- $\nabla_4\nabla X_t$: Make the four order difference based on the difference of the data.

Table 2: Summary table for final model

Parameter	Estimate	p.value
ϕ_1	1.8308	0.0000
ϕ_2	-0.9163	0.0000
θ_1	-1.5002	0.0023
θ_2	-0.3711	0.2094
θ_3	1.2465	0.0067
θ_4	-0.3747	0.1089
Θ_1	-0.3450	0.0107

Based on the summary table 2 for $ARIMA(2,1,4)$x$(0,1,1)_4$, we need to check the p-value, if the p-value is less than $\alpha = 0.05$, w can conclude these parameter are significant, otherwise, they are not significant to interest rate. The p-value of θ_2 and θ_4 are greater than 0.05, it means they are not very significant. For the other parameters, the p-values are less than 0.05 which means these parameters are significant to the interest rate. Then, put the estimate to the final model $ARIMA(2,1,4) * (0,1,1)_4$, then we have:

$$(1 - 1.8308B + 0.9163B^2)\nabla_4\nabla X_t = (1 - 1.5002B + 1.2465B^3)(1 - 0.3450B^4)W_t$$

From the table 2, it shows the absolute value of ϕ_1, θ_1 and θ_3 are large among parameters with the p-value less than 0.05. So, it means they are quite significant parameters for the interest rate and they affect the interest rate heavily. In addition, ϕ_1 and θ_3 are 1.8308 and 1.2465 separately which are the positive number, it would let interest rate becomes higher. For the negative value of θ_1 (-1.5002), it lets the interest rate becomes smaller. So, the interest rate changes in a cycle of four quarters.

Make 10 quarters predictions

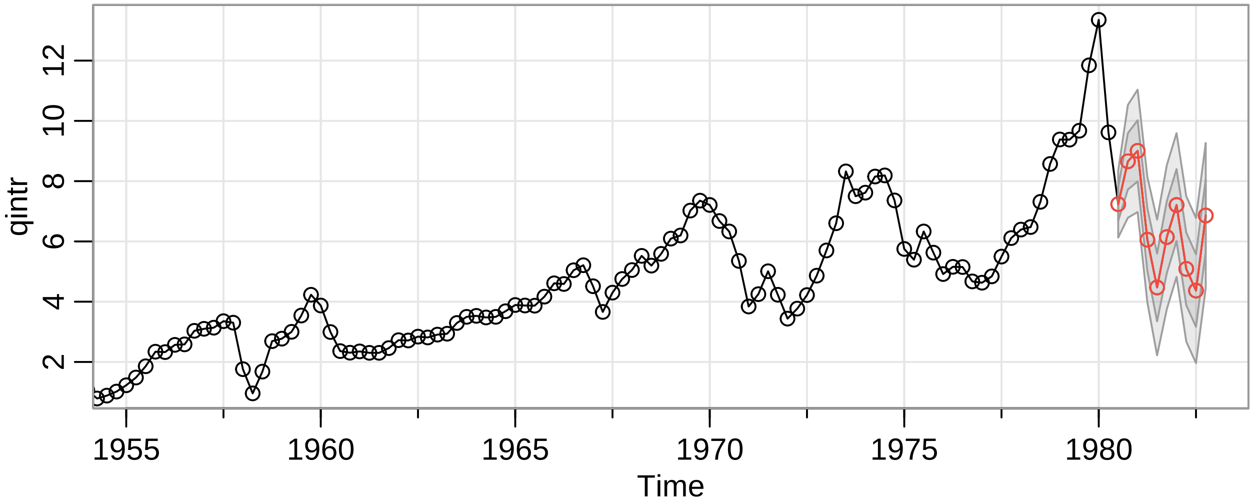

Figure 6: predict 8 quarters of interest rate

Table 3: summary table for prediction of 8 quanters interest rate

Quarter	Prediction	Lower bond	Upper bond
1st quarter	7.237880	6.148751	8.327009
2nd quarter	8.657856	6.823144	10.492568
3rd quarter	9.005575	7.016452	10.994699
4th quarter	6.056190	4.016709	8.095670
5th quarter	4.472473	2.267127	6.677819
6th quarter	6.144104	3.804452	8.483755
7th quarter	7.208434	4.870789	9.546079
8th quarter	5.089267	2.727534	7.451001
9th quarter	4.368902	2.009411	6.728393
10th quarter	6.860444	4.503167	9.217721

As we get the final model, predict next two and half years interest rate. Based on the figure 6, it shows the prediction of 10 quarters of interest rate. There are some seasonal fluctuation, and in general, in the next ten periods, the trend is downward. The interest rate decreases a lot which about from 9 percent to 5 percent between 3rd quarter and 5th quarter. After predict one year, the interest rate becomes stable which fluctuate between 4.3 to 7 percent. So, it means the economy in the United States are becomes better since businesses and consumers can make more loan from banks.

The table 3 shows the predictions and 95% prediction interval of next 8 quarters based on the $ARIMA(2,1,4)\text{x}(0,1,1)_4$ model. Let upper bond of 95% prediction interval minus the lower bond of 95% prediction interval, we get the range. By calculating, the range becomes larger which from 2.178258 to 4.714554. But the prediction still very accurate after we comparing the predicted estimates of each quarter.

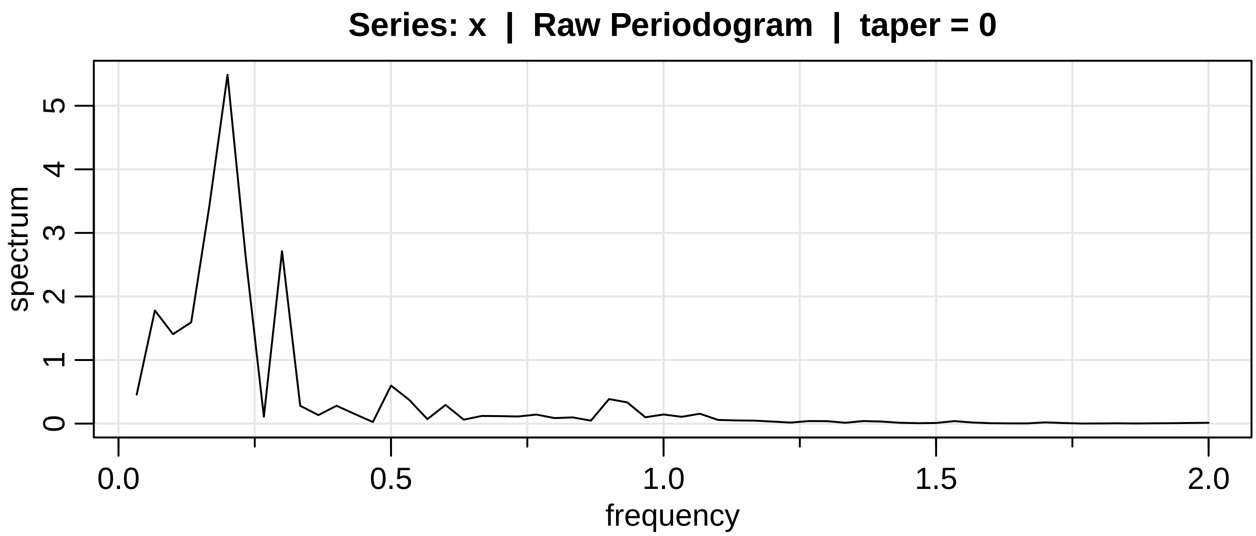

Figure 7: first three dominant frequencies

The first three peaks of figure 7 are about at 0.16, 0.2 and 0.3.

Table 4: 95% confidence interval for the first three dominant frequencies

Period	**Frequency**	**Spectrum**	**Lower bond**	**Upper bond**
Period 1	0.2000	5.4886	1.4878773	216.7881
Period 2	0.1667	3.4232	0.9279783	135.2092
Period 3	0.3000	2.7133	0.7355350	107.1696

Based on the table 4, it represents the 95% confidence intervals for the first three dominant frequencies. The three frequencies at 0.2000, 0.1667 and 0.3000. If the periodogram ordinate only lies in its own confidant interval, we can conclude it is significant, or we cannot say the peak is significant. For the first peak, we cannot establish the significance of the first peak. Because the periodogram ordinate is 5.4886 which lies in the confidence intervals of the second and third peak. For the second peak, we cannot establish the significance of the second peak as the periodogram ordinate is 3.4232 which lies in the confidence intervals of the first and third peak. For the third peak, we cannot establish the significance of the third peak. Because the periodogram ordinate is 2.7133 which lies in the confidence intervals of the first and second peak. Therefore, we cannot establish the significance of all three peaks.

Discussion

Based on the prediction of 8 quarters interest rate, we find out that the overall interest rate declined. The government of the United States can control the range of interest rates. Since 1980, the interest rate was extremely high, people decrease their consumption of products and investors do not loan money anymore, so, the economy of the U.S becomes worse. In order to recover the economy, the government has to take action to decrease the interest rate to the normal range. So, in the prediction, the interest rate decrease and it lets more money flows into the market so that consumers start the increase money spending and more firms loan money from the bank with lower interest rate. The prediction is reasonable. However, there are some limitations of the model. Firstly, there are some outliers in the data and we can notice them from the Q-Q plot. Then, it may cause the result not very accurate. Another limitation is the year we choose. If we select the year close to the recent years, the prediction would be more useful and helpful to the government and people. Moreover, this model is not the best to fit this data since we only use two models to compare, so, in the future study, I would like to try more models to fit more data to predict.

Reference

Volcker, P. (2009, May 29). What led to the high interest rates of the 1980s? PBS. Retrieved April 15, 2022, from https://www.pbs.org/newshour/economy/what-led-to-the-high-interest