



# Tax progressivity and tax incidence of the rich and the poor



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## HIGHLIGHTS

- An OLG model with flexible labor supply is used.
- An increase in tax progressivity increases the tax share of the rich if the tax scheme is moderately progressive.
- An increase in tax progressivity decreases the tax share of the rich if the tax scheme is highly progressive.
- With more rich agents, the rich agent's tax burden declines at a relatively high tax progressivity.

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## ABSTRACT

With the labor supply being flexible, a revenue-neutral increase in the tax progressivity decreases the tax incidence of the rich and increases the tax incidence of the poor if the tax scheme is highly progressive.

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## 1. Introduction

The progressive income tax system is designed to reduce the tax incidence of those with a lower ability to pay and shift the incidence increasingly to those with a higher ability to pay (see, for example, Pechman, 1986). Thus, it is commonly believed that an increase in tax progressivity will increase the tax incidence of the rich and lower the tax incidence of the poor. Stiglitz (2000) argued that, “The analysis has clarified the trade-offs faced as we increase the degree of progressivity. The poorer individuals gain, richer

individuals lose”. Sarte (1997) found a surprising result that an increase in tax progressivity decreases income inequality, and also decreases the tax burden of the wealthier agents since the wealthier agents choose to acquire less wealth facing a more progressive tax structure. He focused on the change in wealth and assumed that labor supply is inelastic.

This paper examines the effect of a revenue-neutral change in labor income tax progressivity on the tax incidence in an overlapping generations model (OLG) with an endogenous labor supply. Prescott (2004) found that the labor supply is highly sensitive to the tax rate based on the data from the United States and some European countries. Koyuncu (2011) developed an endogenous growth model and found that the degree of tax progressivity is a major factor in explaining the patterns of the US and German labor supply over time. This paper shows that, an increase in the labor income tax progressivity increases the tax incidence of the rich and decreases the tax incidence of the poor if the tax scheme is moderately progressive, but decreases the tax incidence of the

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rich and increases the tax incidence of the poor if the tax scheme is highly progressive.

## 2. The model

There are two types of agents in the economy, the rich (type 1) and the poor (type 2). Each individual lives for two periods, working and saving in the first period; and being retired and supplying capital in the second period. In each period,  $N$  individuals are born, and the population does not grow. Following Altig et al. (2001) we assume that each individual is endowed with one unit of time, and the efficiency parameter of the time for  $i$ -type agents is  $e^i$  ( $i = 1, 2$ , hereafter standing for the rich and the poor agent, respectively). Assume that  $e^1 > e^2$ , i.e., the rich agent's time is more efficient due to more human capital. Let  $\alpha^i$  stand for the proportion of type  $i$  individuals in total population, where  $\alpha^1 + \alpha^2 = 1$ . Thus, the number of type  $i$  individuals is  $\alpha^i N$ . The economy produces one good that can be either consumed or invested. Government lives forever, collecting taxes to finance its spending.

Let  $l_t^i$  be the amount of time spent on leisure by a type  $i$  agent in period  $t$  where  $l_t^i \geq 0$ . The amount of labor supplied by a type  $i$  agent in period  $t$  is  $1 - l_t^i$ . Total amount of effective labor supplied by generation  $t$  in period  $t$ ,  $\Phi_t$ , is:

$$\Phi_t = \sum e^i (1 - l_t^i) \alpha^i N$$

where  $e^1 (1 - l_t^1) \alpha^1 N$  and  $e^2 (1 - l_t^2) \alpha^2 N$  are the amount of effective labor supplied by the rich and the poor, respectively. Letting  $L_t \equiv \Phi_t / N$  be the average effective labor supplied per young person in period  $t$ , we obtain:

$$L_t = \sum \alpha^i e^i (1 - l_t^i). \quad (1)$$

Let  $K_t$  be the average capital owned by each person in generation  $t - 1$  (born in period  $t - 1$ ). The production function exhibits constant returns to scale in both capital and labor. The output per person in generation  $t$ ,  $Y_t$ , is given by:  $Y_t = F(K_t, L_t)$ . Letting  $y_t = Y_t / L_t$  be the output-labor ratio, we have:

$$y_t = f(k_t), \quad f'(k_t) > 0, \quad f''(k_t) < 0 \quad (2)$$

where  $k_t = K_t / L_t$  is the ratio of capital to effective labor. Assume that capital is fully depreciated after one period's production. Factor markets are perfectly competitive, and the rate of return to each factor is the marginal product of the factor, i.e.,

$$1 + r_t = f'(k_t) \quad (3)$$

$$w_t = f(k_t) - f'(k_t)k_t \quad (4)$$

where  $1 + r_t$  represents the interest rate in period  $t$ , and  $w_t$  is the wage rate.

Labor income taxation is progressive, i.e., the average tax rate is higher at a higher level of income. The following tax scheme captures the progressive feature of labor income taxation:

$$\pi_t^i = \varepsilon_t + \gamma m_t^i \quad (5)$$

where  $\pi_t^i$  is the average tax rate on labor income,  $m_t^i \equiv e^i (1 - l_t^i) w_t$  is the labor income,  $\varepsilon_t$  is the intercept of the average tax curve, and  $\gamma$  is the slope of the average tax curve. A similar tax scheme has been used by Altig et al. (2001). Taking the first derivative of  $\pi_t^i$  with respect to  $m_t^i$ , we have  $\partial \pi_t^i / \partial m_t^i = \gamma$ . The larger the value of  $\gamma$ , the more progressive the tax system will be.

The marginal tax rate can be obtained by differentiating the tax payment with respect to labor income, i.e.,

$$\frac{\partial [\varepsilon_t + \gamma m_t^i]}{\partial m_t^i} = \varepsilon_t + 2\gamma m_t^i.$$

If  $\gamma = 0$ , then the average tax rate,  $\pi_t^i$ , is equal to the marginal tax rate and equal to  $\varepsilon_t$ , and the tax schedule is flat.

To obtain explicit solutions for savings and other endogenous variables and keep the model tractable, assume that the utility function is of the CES type. The representative agent's optimization problem is:

$$\text{Max } u(c_t^i, c_{t+1}^i) = \frac{(c_t^i)^\delta + (l_t^i)^\delta + \rho^i (c_{t+1}^i)^\delta}{\delta}$$

$$\text{s.t. } c_t^i + \frac{c_{t+1}^i}{1 + r_{t+1}} = e^i (1 - l_t^i) (1 - \pi_t^i) w_t$$

where  $c_{t+j}^i$  is consumption in period  $t + j$  of an agent of type  $i$  ( $i = 1, 2$ ) born in period  $t$  (called generation  $t$ ),  $j = 0, 1$ ;  $l_t^i$  is leisure in period  $t$  of an agent of type  $i$  born in period  $t$ ;  $0 < \delta \leq 1$ ; and  $\rho^i < 1$  represents the pure rate of time preference of type  $i$  agent. Solving the agent's maximization problem yields<sup>2</sup>:

$$c_t^i = \frac{(1 - \pi_t^i) e^i (1 - l_t^i) w_t}{(1 + r_{t+1})^{\delta/(1-\delta)} (\rho^i)^{1/(1-\delta)} + 1} \quad (6)$$

$$l_t^i = \frac{(1 - \pi_t^i) e^i (1 - l_t^i) w_t}{(1 + r_{t+1})^{\delta/(1-\delta)} (\rho^i)^{1/(1-\delta)} + 1} \times [e^i (1 - \varepsilon_t) w_t - 2\gamma (e^i)^2 (1 - l_t^i) w_t^2]^{1/(\delta-1)} \quad (7)$$

$$s_t^i = (1 - \pi_t^i) e^i (1 - l_t^i) w_t - c_t^i = \frac{(1 - \pi_t^i) e^i (1 - l_t^i) w_t}{(1 + r_{t+1})^{\delta/(\delta-1)} (\rho^i)^{1/(\delta-1)} + 1} \quad (8)$$

where  $s_t^i$  represents savings in period  $t$  of an agent born in period  $t$ .

The government finances spending by collecting labor income taxes, and the government budget is balanced. Let  $G_t$  be government consumption. The government budget constraint is:

$$G_t = N \sum \alpha^i \pi_t^i e^i (1 - l_t^i) w_t. \quad (9)$$

Dividing both sides of Eq. (9) by the population,  $N$ , and letting  $g_t \equiv G_t / N$  be government spending per capita, gives:

$$g_t = \sum \alpha^i \pi_t^i e^i (1 - l_t^i) w_t. \quad (10)$$

A competitive equilibrium for the economy is defined as a set of sequences  $\{k_{t+1}, r_t, w_t, l_t^i, \pi_t^i, g_t\}$  satisfying Eqs. (3), (4), (5), (7), (10) and

$$S_t = \sum \alpha^i s_t^i = \sum \alpha^i \frac{(1 - \pi_t^i) e^i (1 - l_t^i) w_t}{(1 + r_{t+1})^{\delta/(\delta-1)} (\rho^i)^{1/(\delta-1)} + 1} = K_{t+1}.$$

Dividing the above equation by  $L_t$  and letting  $s_t \equiv S_t / L_t = S_t / \sum \alpha^i e^i (1 - l_t^i)$ , yields:

$$s_t = \frac{\sum \frac{\alpha^i (1 - \pi_t^i) e^i (1 - l_t^i) w_t}{(1 + r_{t+1})^{\delta/(\delta-1)} (\rho^i)^{1/(\delta-1)} + 1}}{\sum \alpha^i e^i (1 - l_t^i)} = k_{t+1} \frac{L_{t+1}}{L_t} \quad (11)$$

where  $L_t = \sum \alpha^i e^i (1 - l_t^i)$ . Eq. (11) indicates that in equilibrium savings must be equal to investment or capital.

In the steady-state equilibrium, all the endogenous variables are invariant over time, i.e.,  $w_{t+1} = w_t = w$ ,  $r_{t+1} = r_t = r$ ,  $k_{t+1} = k_t = k$ ,  $l_t^i = l_{t+1}^i = l^i$ ,  $\pi_t^i = \pi_{t+1}^i = \pi^i$ ,  $g_{t+1} = g_t = g$ ,  $L_{t+1} = L_t = L$ , and  $s_{t+1} = s_t = s$ . The steady-state equilibrium can be characterized by the following equations:

$$1 + r = f'(k) \quad (12)$$

$$w = f(k) - f'(k)k \quad (13)$$

<sup>2</sup> A detailed derivation is available on request.

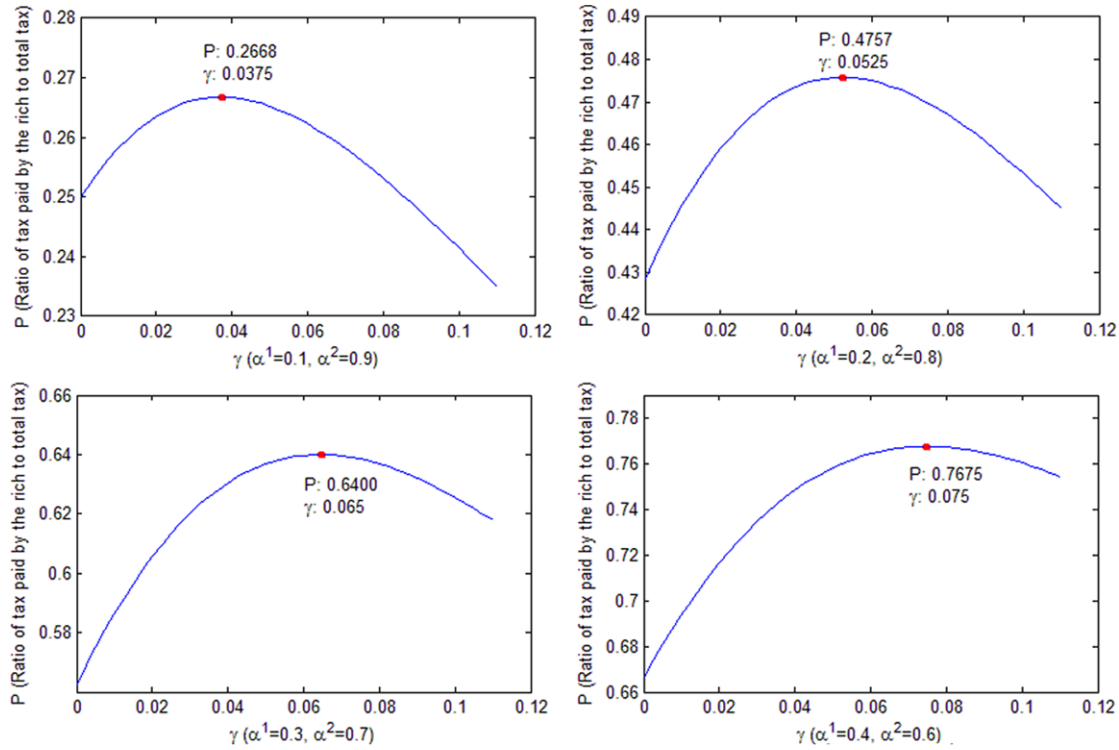


Fig. 1. Tax progressivity and the ratio of tax paid by the rich to total tax.

$$\pi^i = \varepsilon + \gamma e^i(1 - l^i)w \quad (14)$$

$$l^i = \frac{(1 - \pi^i)e^i(1 - l^i)w}{(1 + r)^{\delta/(1-\delta)}(\rho^i)^{1/(1-\delta)} + 1} \times [e^i(1 - \varepsilon)w - 2\gamma(e^i)^2(1 - l^i)w^2]^{1/(\delta-1)} \quad (15)$$

$$g = \sum \alpha^i \pi^i e^i(1 - l^i)w \quad (16)$$

$$\frac{\sum \frac{\alpha^i(1 - \pi^i)e^i(1 - l^i)w}{(1 + r)^{\delta/(1-\delta)}(\rho^i)^{1/(1-\delta)} + 1}}{\sum \alpha^i e^i(1 - l^i)} = k. \quad (17)$$

Eqs. (12)–(17) are steady-state versions of Eqs. (3), (4), (5), (7), (10), and (11), respectively. Letting  $\Gamma^i$  be taxes paid by an individual of type  $i$  ( $i = 1, 2$ ), we have:

$$\Gamma^i \equiv \pi^i e^i(1 - l^i)w = [\varepsilon + \gamma e^i(1 - l^i)w]e^i(1 - l^i)w, \quad i = 1, 2. \quad (18)$$

### 3. Simulating the effects of an increase in tax progressivity

Assume that  $y_t = f(k_t) = Ak_t^\varphi$ ,  $f'(k_t) > 0$ ,  $f''(k_t) < 0$ , and  $\delta = 0.5$ ,  $e^1 = 3$ ,  $e^2 = 1$ ,  $\varphi = 0.33$ ,  $A = 2$ ,  $\alpha^1 = 0.3$ ,  $\alpha^2 = 0.7$ , and  $g = 0.3$ . By varying  $\gamma$ , we can solve the steady-state equilibrium under alternative values of  $\rho^1$  and  $\rho^2$ . The results concerning the effect of an increase in the tax progressivity are reported in Table 1.<sup>3</sup>

Table 1 shows the effect of an increase in tax progressivity on the interest rate, capital-labor ratio, leisure, and the tax share of the rich and the poor. In Table 1(a),  $\rho^1 = 0.8$  and  $\rho^2 = 0.8$ , i.e., the time preferences are the same for the rich and the poor. The tax share of the rich is 56.25% when  $\gamma = 0$ , increases to 63.97% when  $\gamma = 0.06$ , and decreases to 62.56% when  $\gamma = 0.10$ . In Table 1(b),

$\rho^1 = 0.7$  and  $\rho^2 = 0.9$ , i.e., the rich has a higher rate of time preference than the poor. The tax share of the rich is 54.71% when  $\gamma = 0$ , reaches 61.42% when  $\gamma = 0.06$ , and decreases to 60.02% when  $\gamma = 0.10$ . In Table 1(c),  $\rho^1 = 0.9$  and  $\rho^2 = 0.7$ , i.e., the rich has a lower rate of time preference than the poor. The tax share of the rich is 57.72% when  $\gamma = 0$ , increases to 66.48% when  $\gamma = 0.06$ , and decreases to 65.07% when  $\gamma = 0.10$ .

Hence, in all three cases, an increase in the tax progressivity increases the tax incidence of the rich and decreases the tax incidence of the poor if the tax scheme is moderately progressive, but decreases the tax incidence of the rich and increases the tax incidence of the poor if the tax scheme is highly progressive. The intuition is clear. As the tax scheme becomes highly progressive, labor supply and thus the income of the rich will decrease more than that of the poor, resulting in a decrease in the tax share of the rich.

Fig. 1 illustrates the impact of the proportions of the rich and the poor on the effect of tax progressivity with  $\rho^1 = 0.8$  and  $\rho^2 = 0.8$ . Interestingly, with less rich agents, the rich agent's tax burden declines at a relatively low tax progressivity; while with more rich agents, the rich agent's tax burden declines at a relatively high tax progressivity. Specifically, with  $\alpha^1 = 0.1$  and  $\alpha^2 = 0.9$ , the tax burden of the rich begins to decline at  $\gamma = 0.0375$  (where the marginal tax rate for the rich reaches 41.46%); with  $\alpha^1 = 0.4$  and  $\alpha^2 = 0.6$ , the tax burden of the rich declines at  $\gamma = 0.075$  (where the marginal tax rate for the rich is 41.68%). Intuitively, with more rich agents, there are more capital, and thus, the wage rate is higher. With a higher wage rate, the labor supply, income, and therefore, tax burden of the rich declines at a higher marginal tax rate. Using actual data for a country, one can find the marginal tax rate at which the rich agent's tax burden declines as tax progressivity increases.

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<sup>3</sup> Similar qualitative results can be obtained if the values of the parameters are altered.

**Table 1**

The effect of increasing tax progressivity.

| $\gamma^1$<br>Slope of the average<br>tax curve | $\varepsilon$<br>Intercept of the<br>average tax curve | $l^1$<br>Leisure of<br>the rich | $l^2$<br>Leisure of<br>the poor | $r$<br>Interest<br>rate | $k$<br>Capital-labor<br>ratio | $t^1$<br>Tax paid by<br>the rich | $t^2$<br>Tax paid by<br>the poor | $P$<br>Ratio of tax paid by<br>the rich to total tax (%) |
|---|--|---------------------------------|---------------------------------|-------------------------|-------------------------------|----------------------------------|----------------------------------|--|
| a. $\rho^1 = 0.8, \rho^2 = 0.8$                 |  |                                 |                                 |                         |                               |                                  |                                  |  |
| 0.00  | 0.1691   | 0.3117                          | 0.3117                          | 0.0138                  | 0.5269                        | 0.5627                           | 0.1876                           | 56.25  |
| 0.02  | 0.1335   | 0.3552                          | 0.3202                          | 0.0219                  | 0.5208                        | 0.6056                           | 0.1691                           | 60.55  |
| 0.04  | 0.1062   | 0.4005                          | 0.3304                          | 0.0315                  | 0.5135                        | 0.6305                           | 0.1584                           | 63.05  |
| 0.06  | 0.0869   | 0.4454                          | 0.3424                          | 0.0428                  | 0.5053                        | 0.6397                           | 0.1543                           | 63.97  |
| 0.08  | 0.0747   | 0.4884                          | 0.3562                          | 0.0559                  | 0.4959                        | 0.6369                           | 0.1557                           | 63.68  |
| 0.10  | 0.0685   | 0.5283                          | 0.3713                          | 0.0706                  | 0.4858                        | 0.6256                           | 0.1605                           | 62.56  |
| b. $\rho^1 = 0.7, \rho^2 = 0.9$                 |  |                                 |                                 |                         |                               |                                  |                                  |  |
| 0.00  | 0.1731   | 0.3360                          | 0.2933                          | 0.0336                  | 0.5119                        | 0.5473                           | 0.1942                           | 54.71  |
| 0.02  | 0.1386   | 0.3782                          | 0.3014                          | 0.0348                  | 0.5111                        | 0.5848                           | 0.1781                           | 58.46  |
| 0.04  | 0.1112   | 0.4217                          | 0.3113                          | 0.0376                  | 0.5090                        | 0.6064                           | 0.1687                           | 60.64  |
| 0.06  | 0.0909   | 0.4647                          | 0.3229                          | 0.0424                  | 0.5055                        | 0.6143                           | 0.1654                           | 61.42  |
| 0.08  | 0.0768   | 0.5056                          | 0.3360                          | 0.0490                  | 0.5008                        | 0.6111                           | 0.1667                           | 61.11  |
| 0.10  | 0.0683   | 0.5435                          | 0.3506                          | 0.0577                  | 0.4946                        | 0.6004                           | 0.1714                           | 60.02  |
| c. $\rho^1 = 0.9, \rho^2 = 0.7$                 |  |                                 |                                 |                         |                               |                                  |                                  |  |
| 0.00  | 0.1653   | 0.2885                          | 0.3300                          | −0.0039                 | 0.5389                        | 0.5772                           | 0.1812                           | 57.72  |
| 0.02  | 0.1288   | 0.3328                          | 0.3389                          | 0.0110                  | 0.5291                        | 0.6258                           | 0.1604                           | 62.57  |
| 0.04  | 0.1016   | 0.3793                          | 0.3497                          | 0.0277                  | 0.5164                        | 0.6541                           | 0.1484                           | 65.39  |
| 0.06  | 0.0832   | 0.4259                          | 0.3623                          | 0.0459                  | 0.5030                        | 0.6648                           | 0.1437                           | 66.48  |
| 0.08  | 0.0728   | 0.4706                          | 0.3767                          | 0.0658                  | 0.4890                        | 0.6623                           | 0.1447                           | 66.23  |
| 0.10  | 0.0693   | 0.5123                          | 0.3925                          | 0.0874                  | 0.4746                        | 0.6507                           | 0.1497                           | 65.07  |

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