Towards Learning to Reason: Comparing LLMs with Neuro-Symbolic on Arithmetic Relations in Abstract Reasoning

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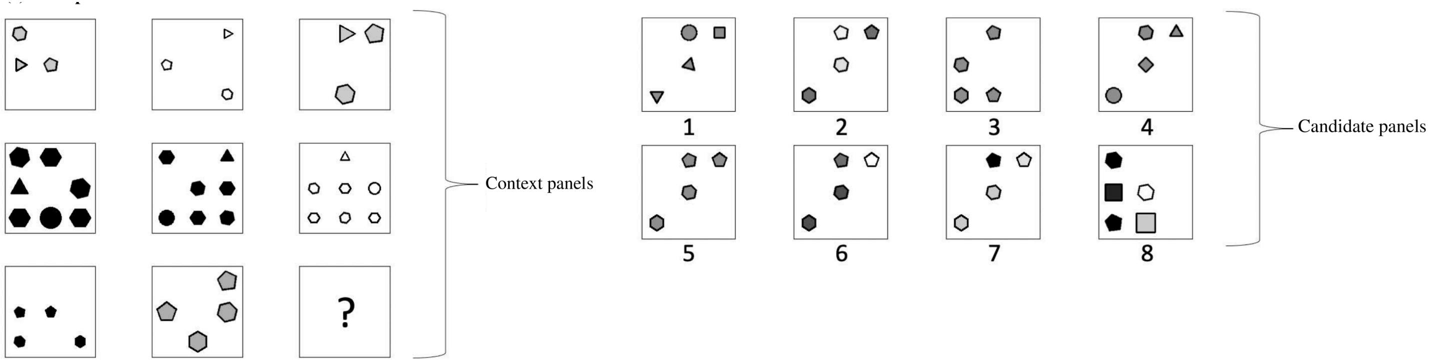
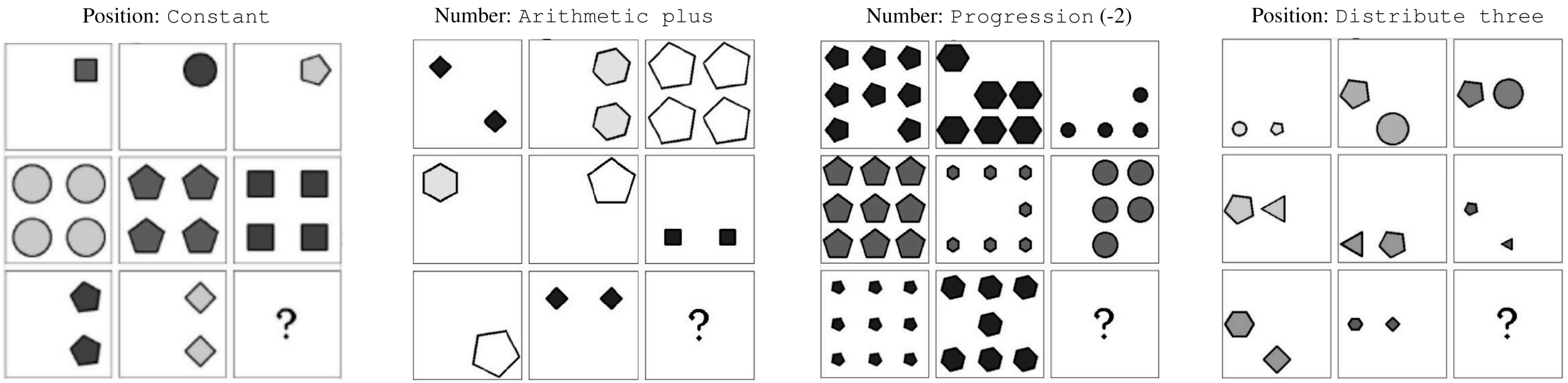
- Abu Sebastian(In-memory computing, Brain-inspired computing),
- Abbas Rahimi(NeSy AI, Knowledge and Reasoning),

who mainly focus on machine learning and symbolic reasoning.

RPM: Task Introduction

Raven’s Progressive Matrices

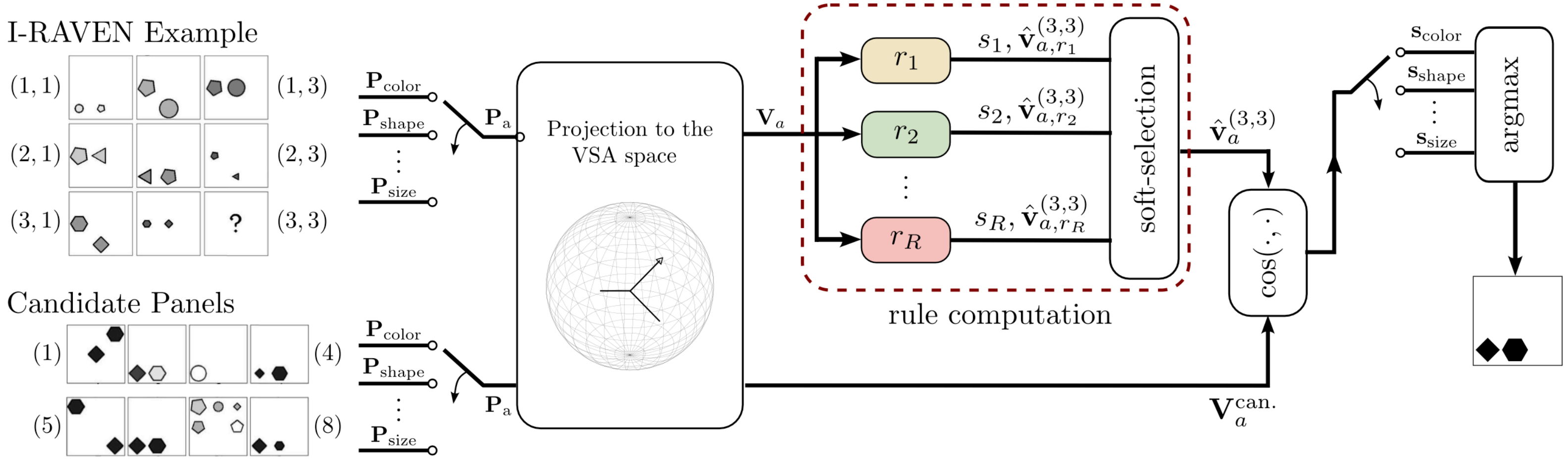




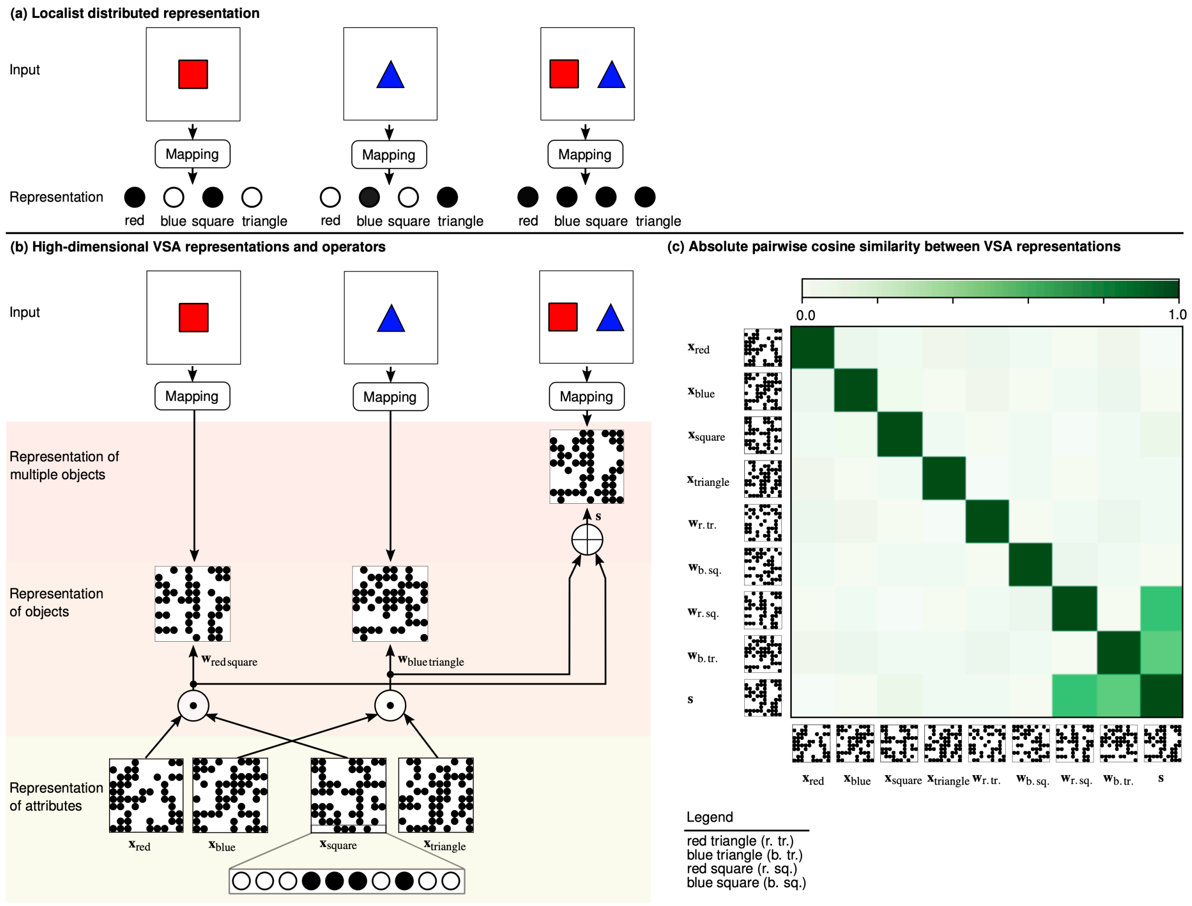
ARLC: An Overview

Abductive Rule Learner with Context-awareness

Panel $\rightarrow$ Probability Mass Function(PMF) $\rightarrow$ Vector-Symbolic Architectures(VSA) $\rightarrow$ Rule Computation $\rightarrow$ Soft Selection



Vector-Symbolic Architectures(VSA): Intuition



Perception: Panel → PMF → VSA

- For every attribute a, each panel's label is translated to a probability mass function (PMF) $\mathbf{p}_a^{(i,j)}$, (i,j) is the coordinate of the panel.
- The PMF is projected into the VSA space as

$$\mathbf{v}_a^{(i,j)} = \sum_{k=1}^{N} \mathbf{p}_a^{(i,j)}[k] \cdot \mathbf{b}[k]$$

$\mathbf{b}$ is a *Dictionary* in the form of binary generalized sparse block codes, specifically, a 1024 dimensional vector divided into 4 blocks of equal length, where only one randomly selected element per block is set to 1.

N is number of possible values that the attribute a can take.

Operation	Binary GSBCs	Equivalent in $\mathbb{R}$
Binding ($\otimes$)	Block-wise circular convolution ($\circledast$)	Addition $+$
Unbinding ($\oslash$)	Block-wise circular correlation ($\circledcirc$)	Subtraction $-$
Bundling ($\oplus$)	Sum & normalization	—
Similarity ($\odot$)	Cosine similarity ($\cos(\cdot,\cdot)$)	—

Learning Context-Augmented RPM Rules

Rules can be generalized

Rule	VSA formulation
Constant	$\hat{\mathbf{a}}^{(3,3)} = \mathbf{a}^{(3,2)} = \mathbf{a}^{(3,1)}$
Arithmetic plus	$\hat{\mathbf{a}}^{(3,3)} = \mathbf{a}^{(3,1)} \circledast \mathbf{a}^{(3,2)}$
Progression positive	$\hat{\mathbf{a}}^{(3,3)} = \mathbf{a}^{(3,2)} \circledast (\mathbf{a}^{(3,2)} \otimes \mathbf{a}^{(3,1)})$
Distribute three	$\hat{\mathbf{a}}^{(3,3)} = (\mathbf{a}^{(1,1)} \circledast \mathbf{a}^{(1,2)} \circledast \mathbf{a}^{(1,3)}) \otimes (\mathbf{a}^{(3,1)} \circledast \mathbf{a}^{(3,2)})$

$\rightarrow \quad \hat{\mathbf{a}}^{(3,3)} = (\mathbf{c}_1 \circledast \mathbf{c}_2 \circledast \mathbf{c}_3) \otimes (\mathbf{c}_4 \circledast \mathbf{c}_5 \circledast \mathbf{c}_6)$

$$r = (\mathbf{c}_1 \circledast \mathbf{c}_2 \circledast \mathbf{c}_3 \circledast \mathbf{c}_4 \circledast \mathbf{c}_5 \circledast \mathbf{c}_6) \otimes (\mathbf{c}_7 \circledast \mathbf{c}_8 \circledast \mathbf{c}_9 \circledast \mathbf{c}_{10} \circledast \mathbf{c}_{11} \circledast \mathbf{c}_{12})$$

$$\mathbf{c}_k = \sum_{i=1}^{I} w_k^i \cdot \mathbf{x}_i + \sum_{j=1}^{J} u_k^j \cdot \mathbf{o}_j + v_k \cdot \mathbf{e}$$

$$\text{s.t.} \ \sum_{i=1}^{I} w_k^i + \sum_{j=1}^{J} u_k^j + v_k = 1; \ w_k^i, u_k^j, v_k \geqslant 0$$

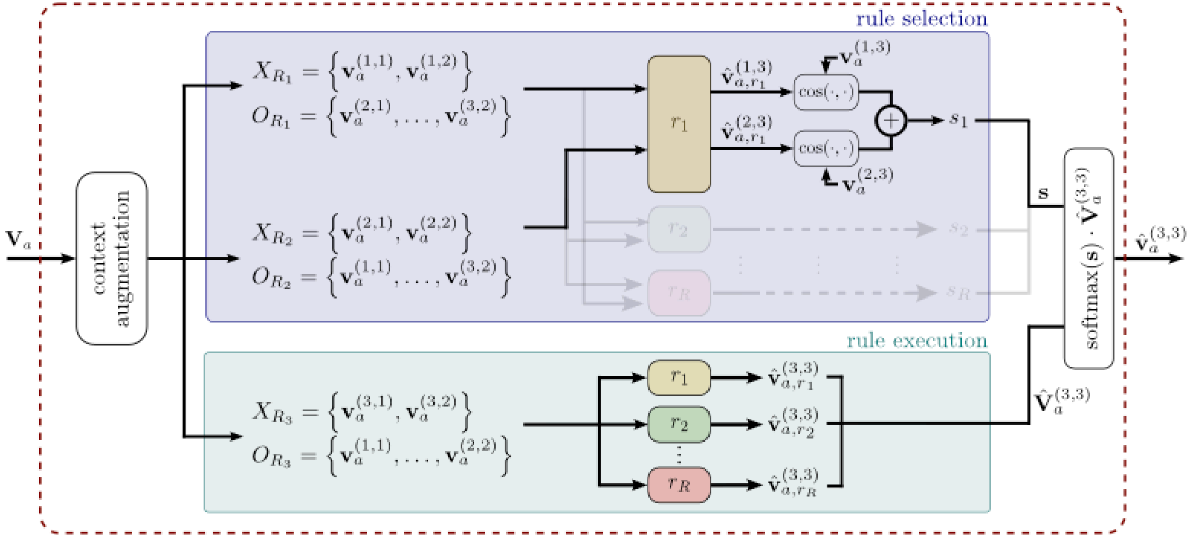
Training & Reasoning(Inference)

Learning & Executing of the Rules

Rule confidence: $s_r = \sum_{i=1}^{3} \cos(\mathbf{v}_a^{(i,3)}, \hat{\mathbf{v}}_{a,r}^{(i,3)})$

Softmax based on rule confidence is in use to enable *soft selection* of the final VSA.

Training Loss: $\mathcal{L} = 1 - \sum_{i=1}^{3} \cos(\mathbf{v}_a^{(i,3)}, \hat{\mathbf{v}}_a^{(i,3)})$



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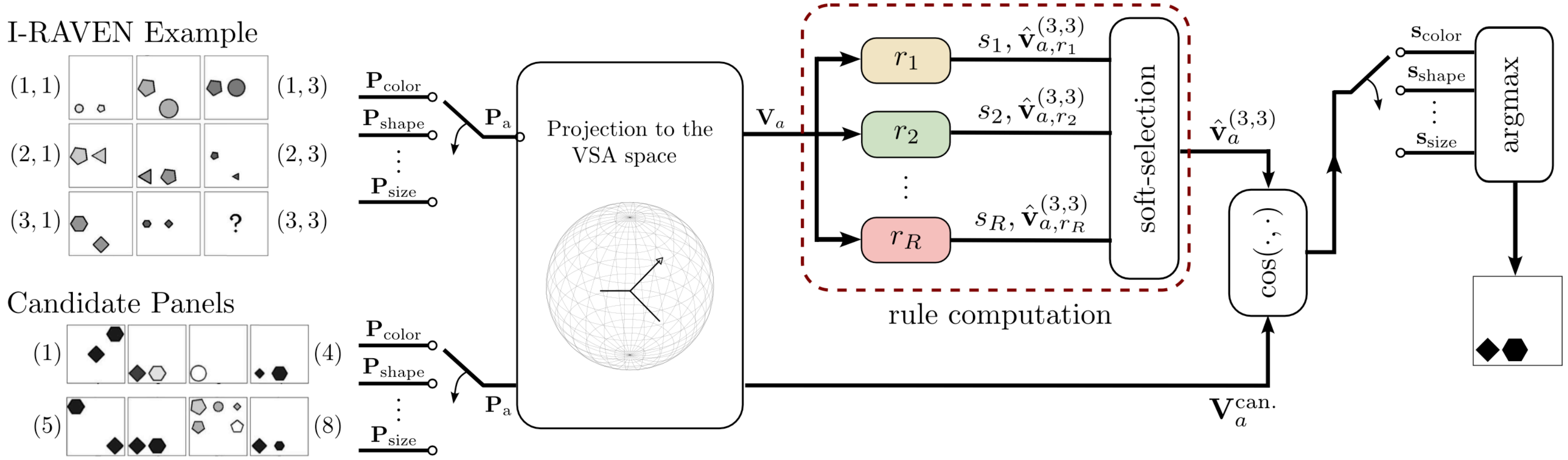

LLM-based RPM solving & Results

prompts used for different datasets

a) LLM prompts for I-RAVEN

```
System: Complete the Raven's progressive matrix:
User:   Only return the missing numbers!
        row 1: 5, 5, 5;
        row 2: 3, 3, 3;
        row 3: 6, 6,
Output: 6
```

Attribute: **shape**
Rule: **constant**
Correct answer: **6**

```
System: Complete the Raven's progressive matrix:
User:   Only return the missing numbers!
        row 1: 6, 6, 6;
        row 2: 4, 4, 4;
        row 3: 2, 2,
Output: 2
```

Attribute: **size**
Rule: **constant**
Correct answer: **2**

```
System: Complete the Raven's progressive matrix:
User:   Only return the missing numbers!
        row 1: 8, 2, 6;
        row 2: 1, 0, 1;
        row 3: 8, 7,
Output: 6
```

Attribute: **color**
Rule: **arithmetic -**
Correct answer: **1**

b) LLM prompts for our new I-RAVEN-X

```
System: Complete the Raven's progressive matrix:
User:   Only return the missing numbers!
        row 1: 320, 322, 324, 326, 328, 330, 332, 334, 336, 338;
        row 2: 718, 720, 722, 724, 726, 728, 730, 732, 734, 736;
        row 3: 224, 226, 228, 230, 232, 234, 236, 238, 240,
Output: 242
```

Attribute: **shape**
Rule: **progression**
Correct answer: **242**

```
System: Complete the Raven's progressive matrix:
User:   Only return the missing numbers!
        row 1: 73,  73,  73,  73,  73,  73,  73,  73,  73,  73;
        row 2: 677, 677, 677, 677, 677, 677, 677, 677, 677, 677;
        row 3: 695, 695, 695, 695, 695, 695, 695, 695, 695,
Output: 695
```

Attribute: **size**
Rule: **constant**
Correct answer: **695**

```
System: Complete the Raven's progressive matrix:
User:   Only return the missing numbers!
        row 1: 769, 667, 0, 4, 2,   20, 63,  3,   5,   5;
        row 2: 848, 0,   0, 0, 387, 2,  106, 7,   308, 38;
        row 3: 611, 2,   0, 0, 0,   0,  0,   551, 0,
Output: 352
```

Attribute: **color**
Rule: **arithmetic -**
Correct answer: **58**

Method	Parameters	Accuracy
MLP [8]	300 k	97.6
SCL [5]	961 k	$99.9^{\pm 0.0}$
PrAE [6]	n.a.	$83.8^{\pm 3.4}$
NVSA [7]	n.a.	$99.8^{\pm 0.2}$
Learn-VRF [8]	20 k	$97.7^{\pm 4.1}$
GPT-3 [14]	175 b	86.4
Llama-3	70 b	85.0
GPT-4	unk.	93.2
$\text{ARLC}_{\text{progr}}$	n.a.	$97.2^{\pm 0.0}$
$\text{ARLC}_{\text{p}\mapsto 1}$	480	$97.6^{\pm 0.0}$
$\text{ARLC}_{\text{learn}}$	480	$98.4^{\pm 1.5}$

Discussion

A novel symbolic representation ~~at least to me~~

works both semantically and for rules

Introducing Differentiability

Possible Drawbacks

Might fail on non-toy tasks

The nature of this task demands minimal "reasoning", and I doubt its application in more complex tasks due to the **limited expressiveness** of arithmetic operations.

Not very explict with the symbolic expression
Only center constellation?

Thank you for your attention

References to be updated.

Quesions are welcome.

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