

# Symbolic metaprogram search improves learning efficiency and explains rule learning in humans

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# Joshua S. Rule

- Postdoctoral scholar with Steven Piantadosi and Alison Gopnik at UC Berkeley.
- Completed PhD with Josh Tenenbaum at MIT.
- Uses behavioral experiments and computational methods to study how people develop conceptual systems and leverage them to accomplish their goals
- Work revolves around an idea called the child as hacker.



# Rule Learning

## Motivation

Throughout their lives, humans seem to learn a variety of rules for things like applying category labels, following procedures, and explaining causal relationships.

We question: how people acquire **algorithmically rich rules?**

The goal is to propose a model that resembles humans' ability in rule learning, thus providing a valid hypothesis that explains people's conceptual systems.

## Observation/Requirements

- representations should be interpretable
- learning should also be possible from sparse data
- learning should require only moderate amounts of computation and search

# Task Statement

What is the target of our rule-learning?

The task is to find a function that maps a given input to a given output. An example is to find an  $f$  that satisfies

$$[1, 3, 9, 7] \xrightarrow{f} [1, 1, 3, 3, 9, 9, 7, 7]$$

$$[9, 2] \xrightarrow{f} [9, 9, 2, 2]$$

Spoiler alert:  $f = (\lambda \text{xs}(\text{if } (\text{empty xs})\text{xs}[(\text{head xs}), (\text{head xs})|(\text{f}(\text{tail xs}))]))$  clearly is the  $f$  we've been looking for.

## Why is this so difficult?

The  $f$  above is composed using *primitives*, and rule learning, or *program induction*, or *program synthesis*, is faced with the hard problem of searching the program space, which grows **exponentially** according to the program length and number of primitives.

# Why I chose this paper: The parts that deserve our attention

Dr. Dai asked me to.

## Methods and Paradigms proposed in this paper

Model: **MPL**(MetaProgram Learner)

- A novel knowledge representation
- A search scheme to consider: custom MCMC(Markov Chain Monte Carlo)

Don't panic. We'll walk through them.

## Personal Take in Reading this paper

Ignore the human behavioral analysis part.

# MPL: a proper introduction

Instead of using only *object-level primitives*(e.g., head xs), MPL defines and employs *metaprimitives*.

repeat every element two times in order of appearance

```
[1, 3, 9, 7]      → [1, 1, 3, 3, 9, 9, 7, 7]
[6, 9, 2, 8, 0, 5] → [6, 6, 9, 9, 2, 2, 8, 8, 0, 0, 5, 5]
[9, 2]            → [9, 9, 2, 2]
[1, 6, 1, 8, 8, 2] → [1, 1, 6, 6, 1, 1, 8, 8, 8, 8, 2, 2]
[2, 8, 7]          → [2, 2, 8, 8, 7, 7]
```

(A)

$\varepsilon$ .MemorizeAll

(B) .Recurse( $\psi_1$ )  
.AntiUnify

(C)

```
F [4, 6, 8, 7]
≡ F [4 | [6, 8, 7]]
≈ [4, 4 | (F [6, 8, 7])]
≈ [4, 4 | [6, 6 | (F [8, 7])]]
≈ [4, 4 | [6, 6 | [8, 8 | (F [7])]]]
≈ [4, 4 | [6, 6 | [8, 8 | [7, 7 | (F [])]]]]
≈ [4, 4 | [6, 6 | [8, 8 | [7, 7 | []]]]]
≡ [4, 4, 6, 6, 8, 8, 7, 7]
```

```
F [1, 3, 9, 7]      ≈ [1, 1, 3, 3, 9, 9, 7, 7]
F [6, 9, 2, 8, 0, 5] ≈ [6, 6, 9, 9, 2, 2, 8, 8, 0, 0, 5, 5]
F [9, 2]            ≈ [9, 9, 2, 2]
F [1, 6, 1, 8, 8, 2] ≈ [1, 1, 6, 6, 1, 1, 8, 8, 8, 8, 2, 2]
F [2, 8, 7]          ≈ [2, 2, 8, 8, 7, 7]
```

```
F []                ≈ []
F [1, 3, 9, 7]      ≈ [1, 1 | (F [3, 9, 7])]
F [3, 9, 7]          ≈ [3, 3 | (F [9, 7])]
F [9, 7]            ≈ [9, 9 | (F [7])]
F [7]               ≈ [7, 7 | (F [])]
F [6, 9, 2, 8, 0, 5] ≈ [6, 6 | (F [9, 2, 8, 0, 5])]
F [9, 2, 8, 0, 5]   ≈ [9, 9 | (F [2, 8, 0, 5])]
F [2, 8, 0, 5]       ≈ [2, 2 | (F [8, 0, 5])]
F [8, 0, 5]          ≈ [8, 8 | (F [0, 5])]
F [0, 5]            ≈ [0, 0 | (F [5])]
F [5]               ≈ [5, 5 | (F [])]
F [9, 2]            ≈ [9, 9 | (F [2])]
F [2]               ≈ [2, 2 | (F [])]
F [1, 6, 1, 8, 8, 2] ≈ [1, 1 | (F [6, 1, 8, 8, 2])]
F [6, 1, 8, 8, 2]   ≈ [6, 6 | (F [1, 8, 8, 2])]
F [1, 8, 8, 2]       ≈ [1, 1 | (F [8, 8, 2])]
F [8, 8, 2]          ≈ [8, 8 | (F [8, 2])]
F [8, 2]             ≈ [8, 8 | (F [2])]
F [2, 8, 7]          ≈ [2, 2 | (F [8, 7])]
F [8, 7]             ≈ [8, 8 | (F [7])]
```

```
F []                ≈ []
F [x | xs]          ≈ [x, x | (F xs)]
```

# More on Metaprimitives

## Some Other Examples

### MemorizeAll(t)

```
# where datum 1: C [1, 38, 19, 4] = [19]          C [1, 38, 19, 4] = [19]
# where datum 2: C [31, 41, 59, 62, 5] = [59]      C [68, 47, 3, 6, 0, 9, 77] = [3]
# where datum 3: C [68, 47, 3, 6, 0, 9, 77] = [3]   C [31, 41, 59, 62, 5] = [59]
C [31, 41, 59, 62, 5] = [59]                      C [x, y, 19, z] = [19]
C [x, y, 19, z] = [19]
```

### DeleteRule(t, ψ)

```
C [1, 38, 19, 4] = [19]          C [1, 38, 19, 4] = [19]
C [68, 47, 3, 6, 0, 9, 77] = [3]  C [68, 47, 3, 6, 0, 9, 77] = [3]
C [31, 41, 59, 62, 5] = [59]      C [x, y, 19, z] = [19]
C [x, y, 19, z] = [19]
```

### Variabilize(t, ψ)

```
C [w, x, y, z] = [y]          C [w, x, y, z] = [y]
C [91, 12, 63, 42, 35] = [63]  C [91, 12, x, 42, 35] = [x]
```

### AntiUnify(t)

```
C [25] = []                  C [x] = []
C [0] = []                  C [x | y] = [x | (C y)]
C [9, 25] = [9 | (C [25])]  C [x | y] = [x | (C y)]
C [81, 9, 25] = [81 | (C [9, 25])]  C [50, 0] = [50 | (C [0])]
C [50, 0] = [50 | (C [0])]    C [28, 50, 0] = [28 | (C [50, 0])]
C [28, 50, 0] = [28 | (C [50, 0])]  C [37, 28, 50, 0] = [37 | (C [28, 50, 0])]
```

## Comparison with MetaRules

```
metarule([P,Q],[P,A,B],[[Q,A,B]]).  
metarule([P,Q,R],[P,A,B],[[Q,A,B],[R,A,B]]).  
metarule([P,Q,R],[P,A,B],[[Q,A,C],[R,C,B]]).
```

# Discussion on Metaprimitives

## Advantages

- Combines object-level content and structured transformation
- Can leverage the internal structure of data.
- Is interpretable and resembles the way human access concepts to a certain degree.

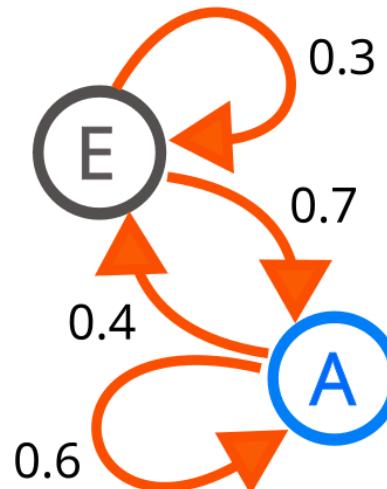
## Cons

- Is domain specific. Can be hard to generalize to other problems.
- The current implementation of metaprimitives can be sub-optimal: more of an intuition or a direction than a already powerful instrument.

# Search Scheme

The paper is super ablate on this part so I'm not crystal clear how the search is done either, yet we can surely explore the basic tactics.

## A quick recap on Markov Chain and Monte Carlo



# Markov Chain Monte Carlo Search

with parallel tempering

## MAP(Maximum A Posteriori) inference in Bayesian posterior

$$p(H|D) \propto p(D|H)p(H)$$

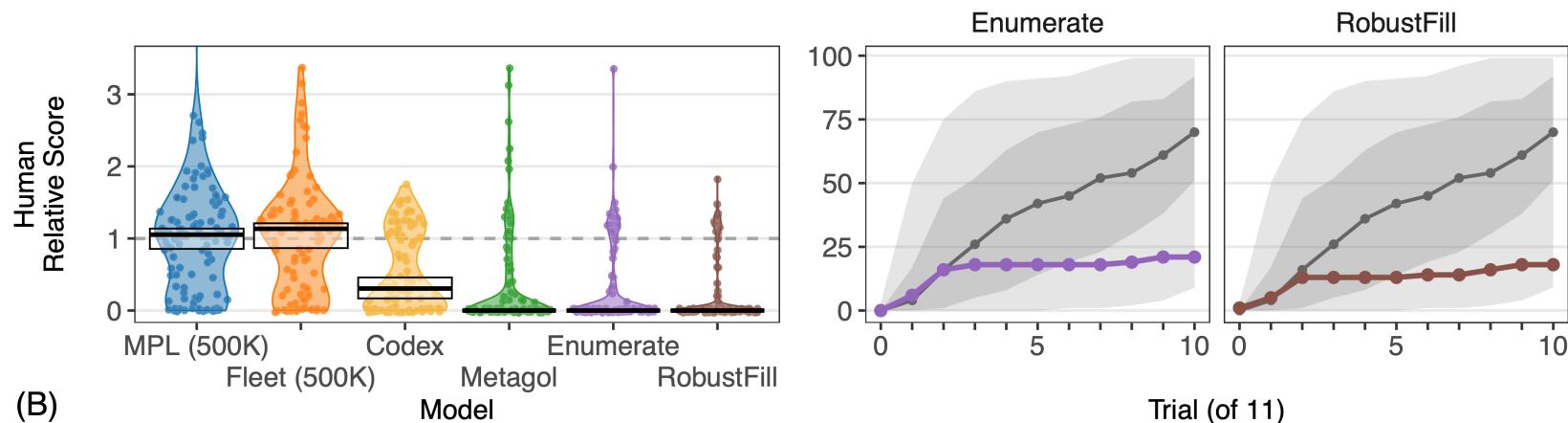
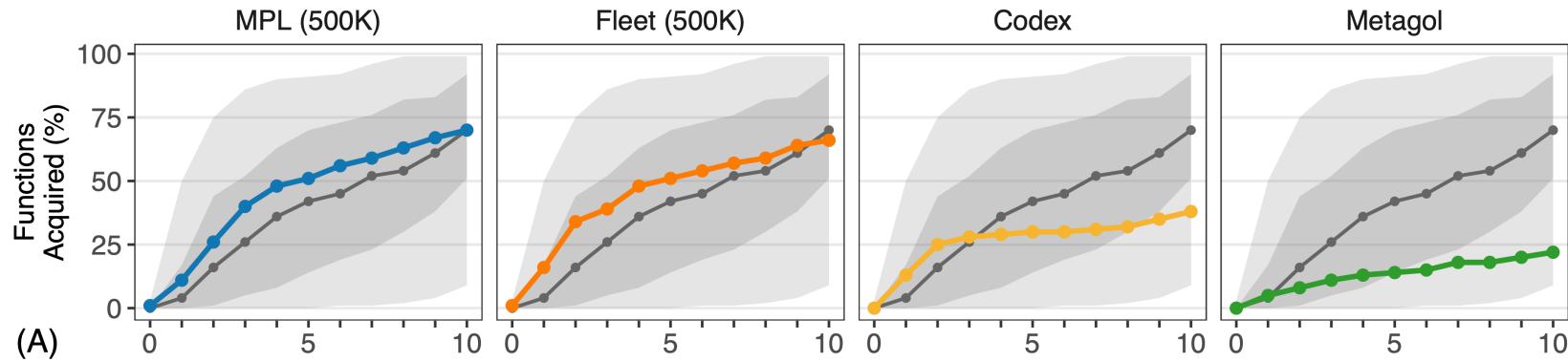
where  $H$  is the prior distribution of metaprograms and  $D$  is the data given, and  $p(D|H)$  is a prefix-based likelihood.

## Parallel Tempering

$$p(H|D) \propto p(D|H)^{\frac{1}{T}}p(H)$$

## Tree Proposal

# Results



# Thanks For Your Attention

Questions are welcome. References will be updated on future versions of the slides.

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