GCAD: Anomaly Detection in Multivariate Time Series from the Perspective of Granger Causality

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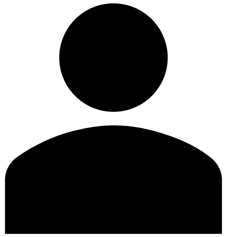
Presented by Xin-Shuang Zhang

Authors



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Contributions

Graph Neural Networks only learn the similarity of variables embedding vectors without exploring the role of the graph structure in the evolution of time series.

Key Challenge

- **Complex**: using nonlinear deep models
- **Time-varying**: using the gradients in deep models

We quantify the Granger causality effect as an integral of deep predictor gradients over the time lag.

How to use the Granger causality for anomaly detection?

Hypothesis: When an anomaly occurs, the Granger causality patterns between variables will change significantly.

Problem Statement

Time series $\{x_1, x_2, ..., x_T\}$, where $x_t \in \mathbb{R}^N$

Anomaly detection task

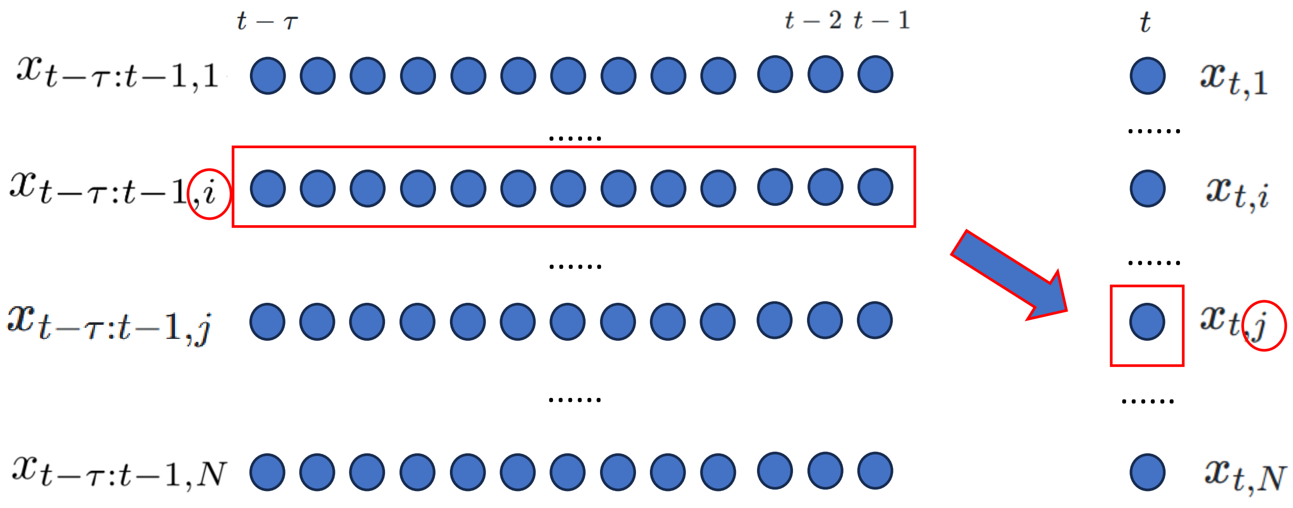
- Input: a sliding window $\{x_{t-\tau}, ..., x_{t-1}, x_t\}$
- Output: a Boolean value for each sliding window

Granger Causality

Definition 1 *Time-series i Granger cause j if and only if there exists* $\boldsymbol{x}'_{t-\tau:t-1,i} \neq \boldsymbol{x}_{t-\tau:t-1,i}$,

$$f_j(\boldsymbol{x}_{t-\tau:t-1,1}, \dots, \boldsymbol{x}'_{t-\tau:t-1,i}, \dots, \boldsymbol{x}_{t-\tau:t-1,N}) \neq f_j(\boldsymbol{x}_{t-\tau:t-1,1}, \dots, \boldsymbol{x}_{t-\tau:t-1,i}, \dots, \boldsymbol{x}_{t-\tau:t-1,N}) \tag{2}$$

i.e., the past data points of time-series i influence the prediction of $x_{t,j}$.



Question:

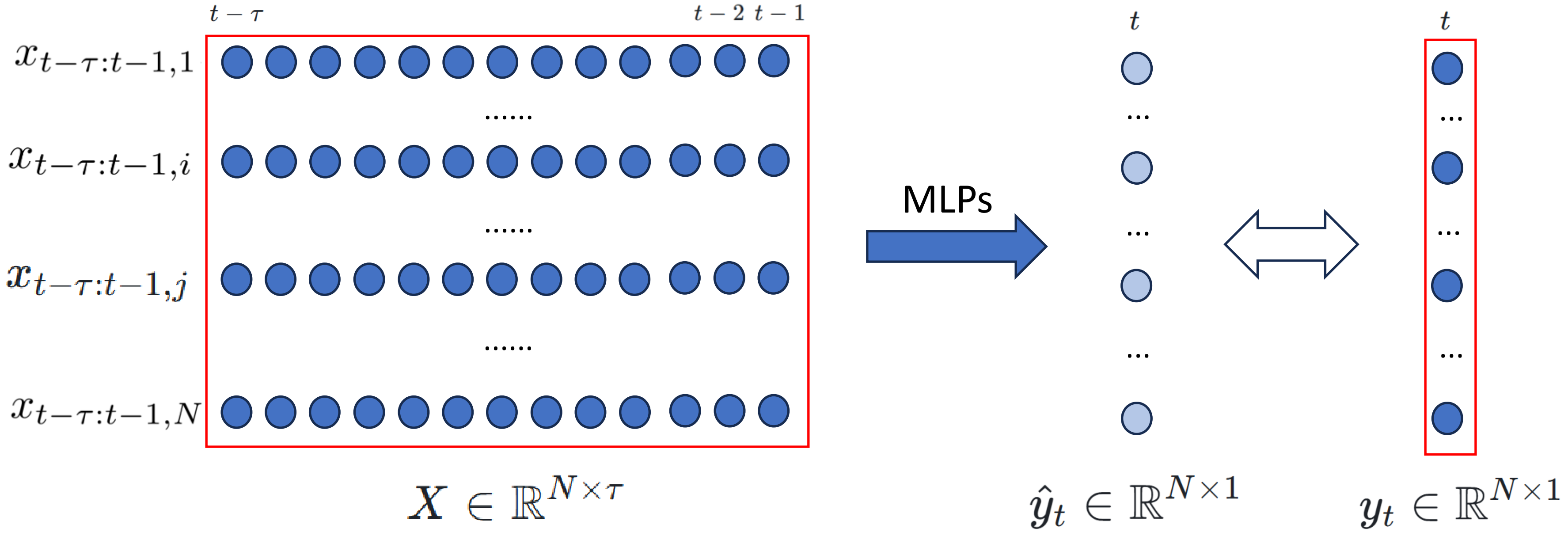
How to quantify the Granger causality?

τ is the maximum time lag and N represents the number of features (or variables).

DNN Predictor

We first train a neural network model for time series prediction.

train data (X, y)

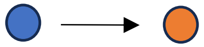


$$L_{train} = MSE(\widehat{y_t}, y_t)$$

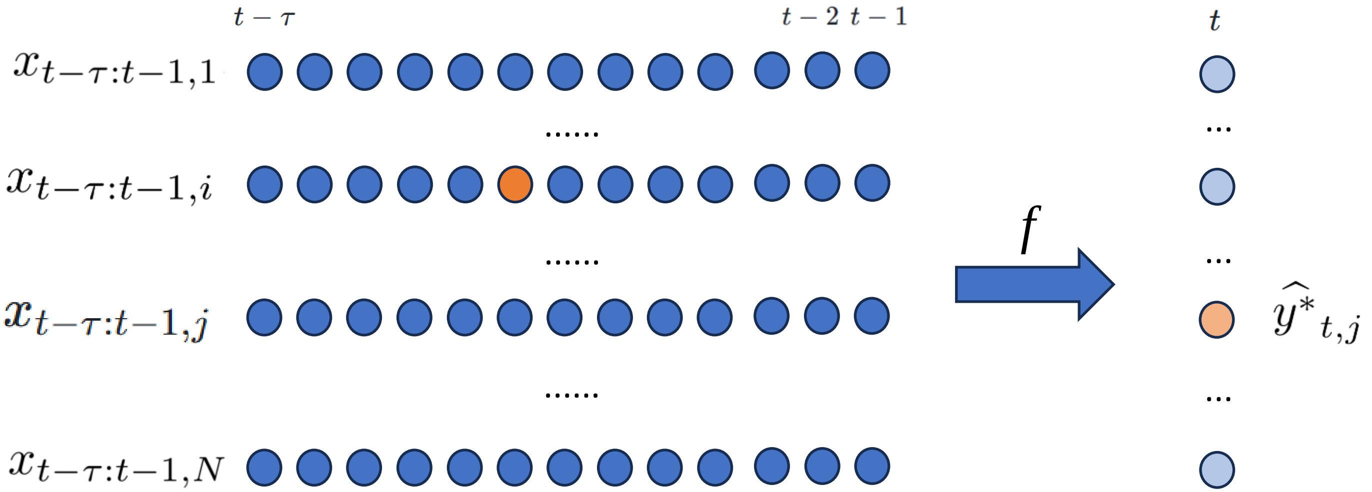
$$L_{t,j} = (\widehat{y}_{t,j} - y_{t,j})^2$$

Intuition

let $t' \in (t-\tau : t-1)$, and let $x^*_{t',i}$ be a perturbation



$$x^*_{t',i} = x_{t',i} + \Delta$$

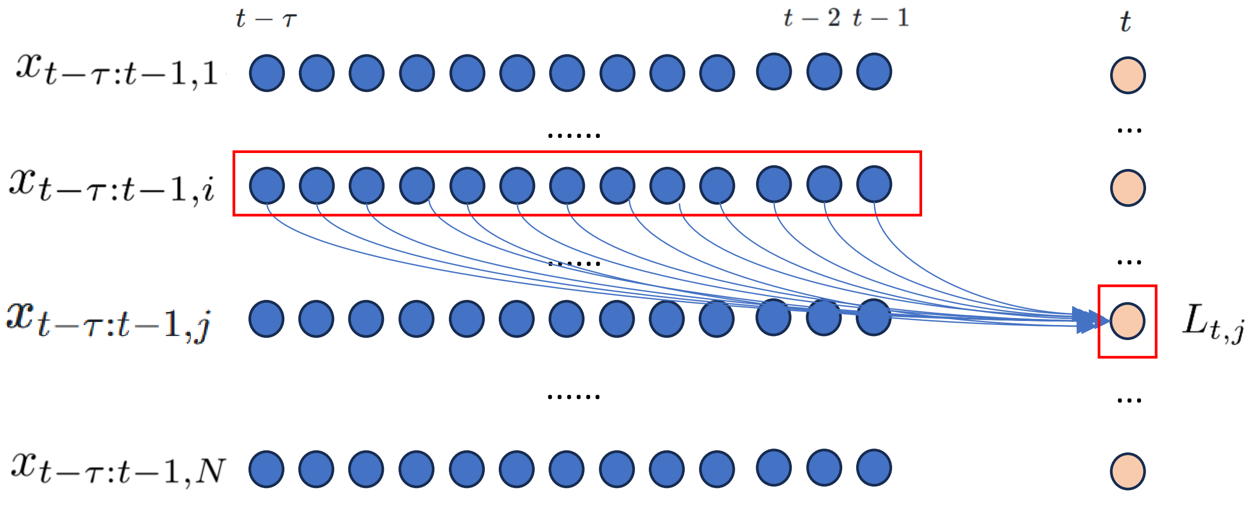


$$L^*_{t,j} = (\widehat{y^*}_{t,j} - y_{t,j})^2$$

$$L_{t,j} = (\widehat{y}_{t,j} - y_{t,j})^2$$

$$\lim_{\Delta \to 0} |L_{t,j} - L^*_{t,j}| = \left| \frac{\partial L_{t,j}}{\partial x_{t',i}} \right| \cdot |\Delta|$$

Quantification of Granger Causality



$$a_{i,j} = \int_{t-\tau}^{t-1} \left| \frac{\partial L_{t,j}}{\partial x_i(T)} \right| P(T)dT \qquad T \in [t-\tau, t-1]$$

The term $a_{i,j}$ represents the degree to which time-series i Granger causes time-series j, parameterized by a distribution of interest P.

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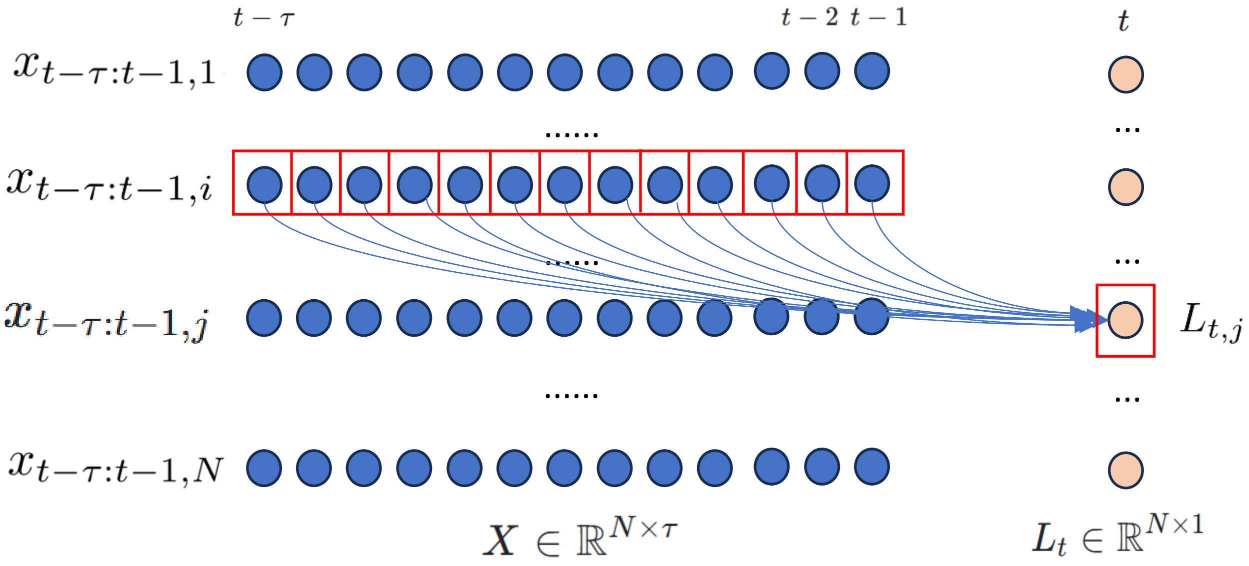
$$f_j(\boldsymbol{x}_{t-\tau:t-1,1}, \ldots, \boldsymbol{x}'_{t-\tau:t-1,i}, \ldots, \boldsymbol{x}_{t-\tau:t-1,N}) \neq \\ f_j(\boldsymbol{x}_{t-\tau:t-1,1}, \ldots, \boldsymbol{x}_{t-\tau:t-1,i}, \ldots, \boldsymbol{x}_{t-\tau:t-1,N}) \tag{2}$$

i.e., the past data points of time-series i influence the prediction of $x_{t,j}$.

Simplified Computation

Integral form $a_{i,j} = \int_{t-\tau}^{t-1} \left| \frac{\partial L_{t,j}}{\partial x_i(T)} \right| P(T)dT \quad T \in [t-\tau, t-1]$

Discretization $a_{i,j} = \sum_{\phi=t-\tau}^{t-1} \left| \frac{\partial L_{t,j}}{\partial x_{\phi,i}} \right| \frac{1}{N} \quad \Delta T = 1$



Gradient matrix

$G_t \in \mathbb{R}^{N\times N\times\tau}$

$\Downarrow$

Causality matrix

$A_t \in \mathbb{R}^{N\times N}$

Causal Deviation Scoring

Guaranteeing the acyclicity: $\tilde{A}_{ij} = \begin{cases} \max(0, A_{i,j} - A_{i,j}^T) & \text{if } i \neq j \\ A_{ij} & \text{otherwise} \end{cases}$

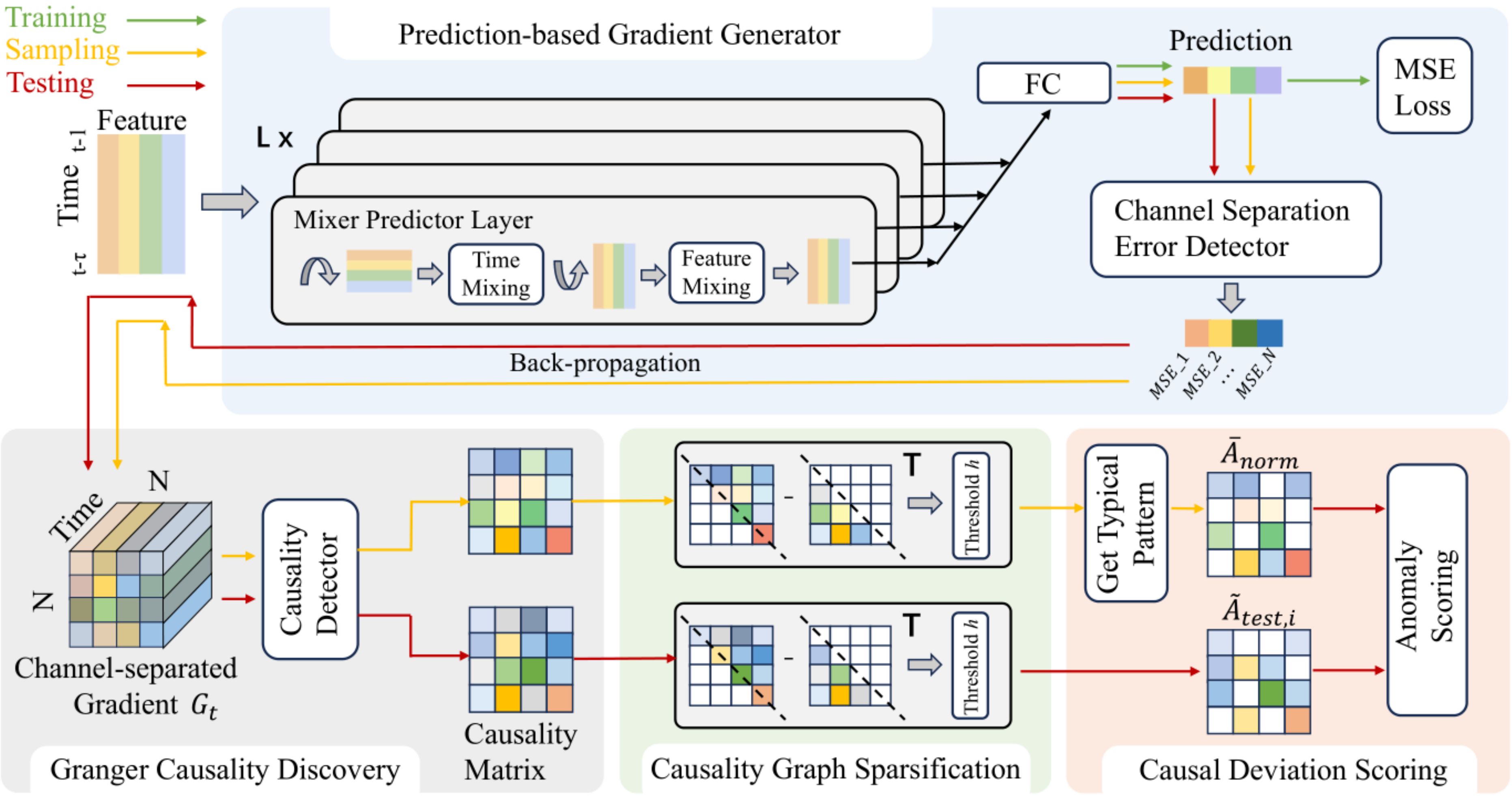
Denoising: $\tilde{A} \leftarrow \tilde{A} \cdot \mathbf{1}_{\tilde{A} > h}$

We sample windows from the training set with *probability p*, and calculate the mean of their corresponding causality graphs.

Define the *causal deviation score* for each test sample as:

$$score_i = \text{mean}\left(\frac{\left|\tilde{A}_{\text{test},i} - \overline{A}_{\text{norm}}\right|}{\overline{A}_{\text{norm}} + \varepsilon}\right)$$

Overall Architecture of GCAD



Experimental Setup

Datasets

Dataset	channels	train	test	anomalies
SWaT	51	47,520	44,991	12.20%
SMD	38	28,479	28,479	9.46%
MSL	55	3,682	2,856	0.74%
SMAP	25	2,876	8,579	2.14%
PSM	25	132,481	87,841	27.76%

Implementation Details

- Train: 80% of the normal data
- Validation: 20% of the normal data
- Testing: the data containing anomalies

Metrics: ROC-AUC and PRC-AUC

All experiments were conducted 10 times and the average results were reported.

Experimental Results

Dataset	SWaT		SMD		MSL		SMAP		PSM	
Metric	ROC	PRC	ROC	PRC	ROC	PRC	ROC	PRC	ROC	PRC
DAGMM	0.7882	0.4955	0.7516	0.3988	0.6209	0.0163	0.5989	0.0380	0.6556	0.3860
USAD	0.8318	<u>0.7173</u>	<u>0.9274</u>	0.5316	0.5601	0.0108	0.5314	0.0279	0.6584	0.4924
GDN	<u>0.8493</u>	0.6076	0.9006	<u>0.5655</u>	0.3846	0.0055	0.5115	0.0338	<u>0.7284</u>	<u>0.4964</u>
AT	0.5117	0.1851	0.1765	0.0576	0.5391	0.0141	0.3467	0.0188	0.5058	0.2902
GANF	0.8112	0.3557	0.6384	0.0017	0.6402	0.0291	<u>0.6926</u>	**0.5259**	0.6335	0.4090
MEMTO	0.7799	0.6067	0.5042	0.1187	<u>0.7271</u>	<u>0.0654</u>	0.4791	0.0176	0.5026	0.2913
GCAD	**0.8690**	**0.7758**	**0.9533**	**0.7502**	**0.7658**	**0.3679**	**0.7273**	<u>0.4555</u>	**0.7618**	**0.6136**