Neurosymbolic Object-Centric Learning with Distant Supervision

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Contents

1. Introduction

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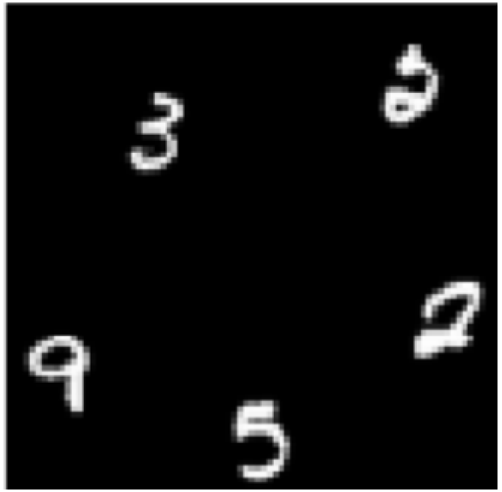
1.2 Task



In Statistical Relational Learning (SRL) and Neurosymbolic AI, many works often make a critical assumption: *The input is already structured .* (e.g. a knowledge graph, a logical database)

This work only uses raw image input.

Object-centric learning tries to learn structured object-centric representation from low-level perceptual features. However, many models often rely on *object-level supervision*, meaning they require a label or annotation for each object in the image (It's difficult for most tasks).

This work only uses distant supervision from tasks.

2. Related Works

2.1 Slot attention

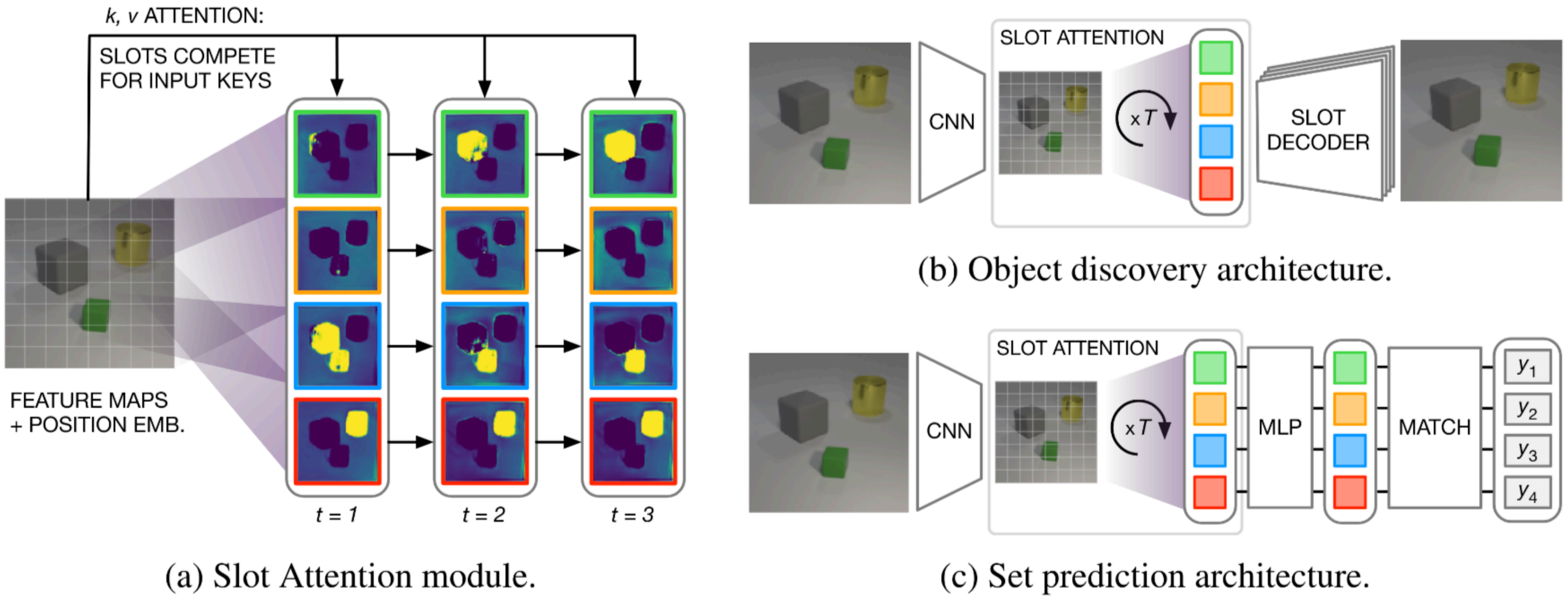


Figure 1: (**a**) Slot Attention module and example applications to (**b**) unsupervised object discovery and (**c**) supervised set prediction with labeled targets y_i. See main text for details.

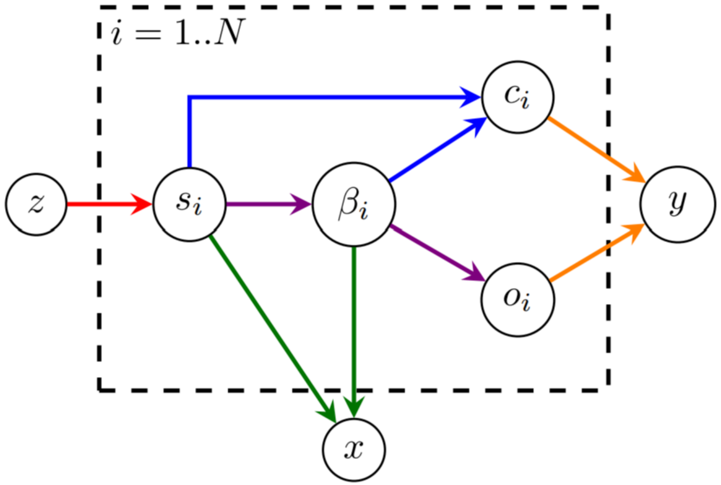
TL;DR: A slot (vector) can store (and bind to) any object in the input.

2.2 Neurosymbolic systems

Neurosymbolic systems

3. Methods

3.1 DeepObjectLog (Highlevel architecture)



- Objects encoder: $z \to s_i (i \in 1, ..., N)$
  z is a global representation.
  s_i describes a potential object in scene.
- Object detector: $s_i \to \beta_i, \beta_i \to o_i$
  β_i is the prob that s_i corresponds to a meaningful object. o_i is an *objectness* flag.
- Object classifier: $s_i, \beta_i \to c_i$, c_i is the class of s_i.
- Probabilistic Logic Reasoning: $c_i, o_i \to y$
  y is the downstream class label computed by reasoning on a logic theory L.
- Image Decoder: $s_i, \beta_i \to x$
  x is a probability distribution over images.

3.2 Probabilistic Graphical Model

$z \in \mathbb{R}^{d_z}$: a latent representation of the image

For $i \in 1, \ldots, N$, each i is a potentially present object.

$$p(y, x, \boldsymbol{s}, \boldsymbol{\beta}, \boldsymbol{o}, \boldsymbol{c}, z) =$$

$$p(z)p_L(y|\boldsymbol{o}, \boldsymbol{c})p(x|\boldsymbol{s}, \boldsymbol{\beta}) \left[\prod_{i=1}^{N} p(s_i|z)p(\beta_i|s_i)p(o_i|\beta_i)p(c_i|\beta_i, s_i)\right]$$

Image decoder

Object detector

Probabilistic logic reasoning

Slot extractor

Object classifier

3.2 Probabilistic Graphical Model

Four inductivbe assumptions:

- Object-level structure is latent and uncertain
- Symbolic abstraction (o_i, c_i, y) and perceptual grounding (x) are complementary flows
- Relational reasoning can be performed over learned representations
- Direct supervision at the object level can be avoided as it can be inferred from distant observations on x and y.

3.3 Object detector & classifier

Object detector:

$s_i \to \beta_i$, $\beta_i \in [0, 1]$ is the uncertainty scores. β_i is a determinstic value (the output of a neural network $\beta_i^* = f_\beta(s_i \mid \theta_\beta)$)

$\beta_i \to o_i$, $o_i \in \{0, 1\}$ is the objectness flag. (a Bernoulli distribution)

The classifier $s_i, \beta_i \to c_i$: a Bernoulli or a Categorical distribution. This distribution is parameterized by a NN $f_c(\beta_i s_i; \theta_c)$

s_i multiplies the uncertainty β_i elementwise, allows the model to downweigh uncertain or irrelevant representations.

3.4 Probabilistic logic programming (ProbLog)

Example:

```
% Probabilistic
facts:
0.5::heads1.
0.6::heads2.
% Rules:
someHeads :- heads1.
someHeads :- heads2.
% Queries:
query(someHeads).
% Result: 0.8
```

A ProbLog program L is defined over two sets of syntactic constructs: probabilistic facts and rules.

$$p(y) = \sum_{\boldsymbol{f}} \boxed{p(y|\boldsymbol{f}; R)} \prod_{f_i \in \boldsymbol{f}} p(f_i)$$

probabilistic facts **f**

a deterministic distribution stating whether y can be obtained from f applying the rules in R.

3.5 ProbLog for object-centric learning

Using ProbLog to model 3 components of the DeepObjectLog:

1. The object detectors $p(o_i|\beta_i)$: `object(i)`
2. The object classifiers $p(c_i|s_i, \beta_i)$: `class(i)` (binary classifier) or `class(i,k)` (categorical classifier)
3. The conditional task distribution $p_L(y| \boldsymbol{c}, \boldsymbol{o})$: Given the objectnesses `object(i)` and the classes of all objects `class(i, k)`, use all the logic rules to obtain the task y.

3.5 ProbLog for object-centric learning

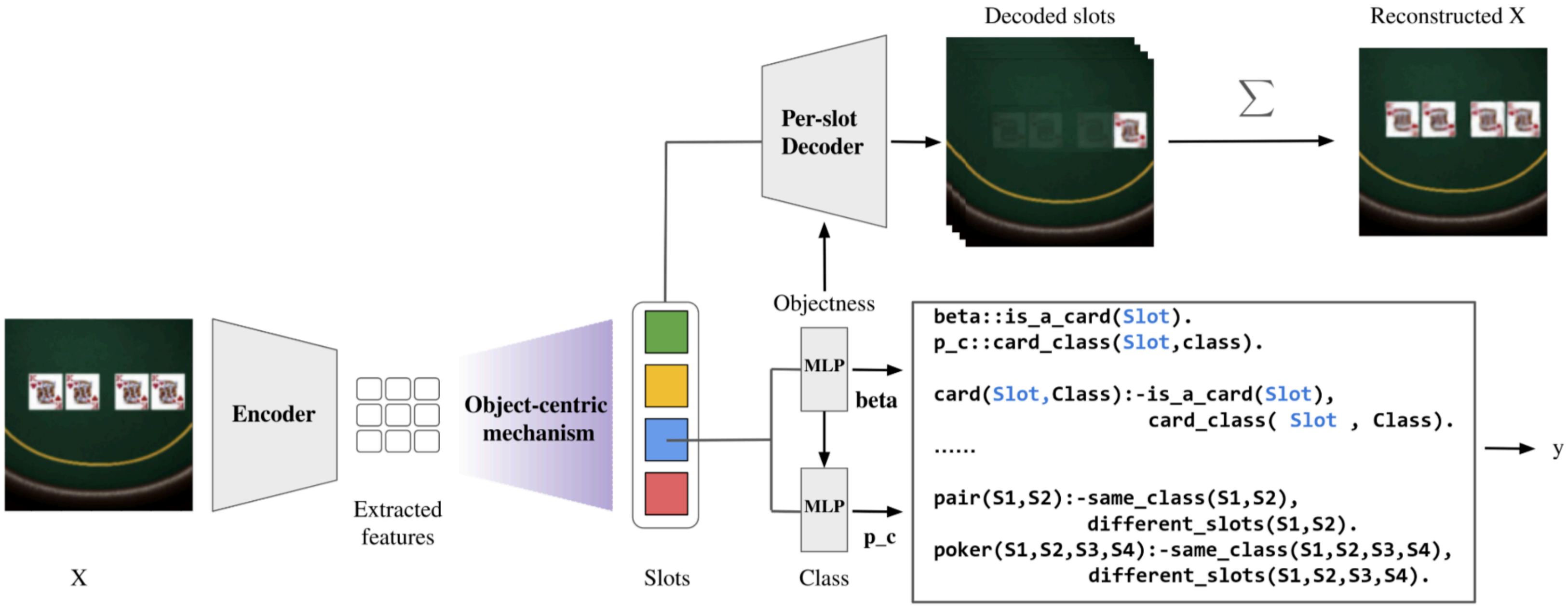
Example. Addition of at most two digits.

An image contains zero tw two digits that can only be 0 or 1. The task is to get the sum of digits in the image. We can encode this task as follows:

```
p_o_id :: object(ID) % distribution of objectness of the slot ID
p_c_id :: class(ID, C) % distribution of corresponding classes of the slot ID
digit(ID, Val) :- object(ID), class(ID, Val).
digit(ID, 0) :- \+ object(ID)
add(Z) :- digit(1, Y1), digit(2, Y2), Z is Y1 + Y2.
query(add(2))
```

3.6 Masked detector

The image decoder $p(x|\boldsymbol{s}, \boldsymbol{\beta})$ model the image as a mixture of Multivariate Gaussian distributions. Each slot can be decoded as $\mathcal{N}(\mu, I)$ that is parameterized with a NN $f_x(s_i\beta_i; \theta_x)$. The mixing weights w of slots are parameterized by a NN $f_w(s_i\beta_i; \theta_w)$.



3.7 Learning

$D = \{x, y\}_K$ is a dataset of pairs (x, y). $\theta_p = \{\theta_s, \theta_\beta, \theta_c, \theta_x, \theta_w\}$

$$\max_{\theta_p} \sum_{x, y \in D} \log p(x, y; \theta_p)$$

$$p(x, y) = \sum_{\boldsymbol{o}, \boldsymbol{c}} \int dz d\boldsymbol{s} d\boldsymbol{\beta} p(x, y, \boldsymbol{s}, \boldsymbol{\beta}, \boldsymbol{o}, \boldsymbol{c}, z)$$

For $\boldsymbol{o}, \boldsymbol{c}$, we use ProbLog to solve. And s, β are determinstic. So:

$$p(x, y) = \int dz p(x, y, z) = \int dz p(z) p(x | \boldsymbol{s}^*(z), \boldsymbol{\beta}^*(z)) p(y | \boldsymbol{s}^*(z))$$

z is global representation. We can't consider every z. How to solve it?

3.7 Learning

Since we are interested in a discriminative setting only, we can approximate the MAP state $z^* = \arg\max_z \log p(x, y, z)$ by a NN encoder $z^* \simeq q(x; \phi)$, where ϕ is set of parameters of the NN.

$$\max_{\theta_p, \phi} \sum_{(x,y)\in D} \log p\left(y | \boldsymbol{s}^*(q(x)), \boldsymbol{\beta}^*(q(x))\right)[1] +$$

$$\log p\left(x | \boldsymbol{s}^*(q(x)), \boldsymbol{\beta}^*(q(x))\right)[2] + \log p(q(x))[3]$$

[1]: A maximum likelihood term for the observed label y.

[2]: A standard reconstruction term of the observed x.

[3]: An activation regularization of $q(x)$.

4. Experiments

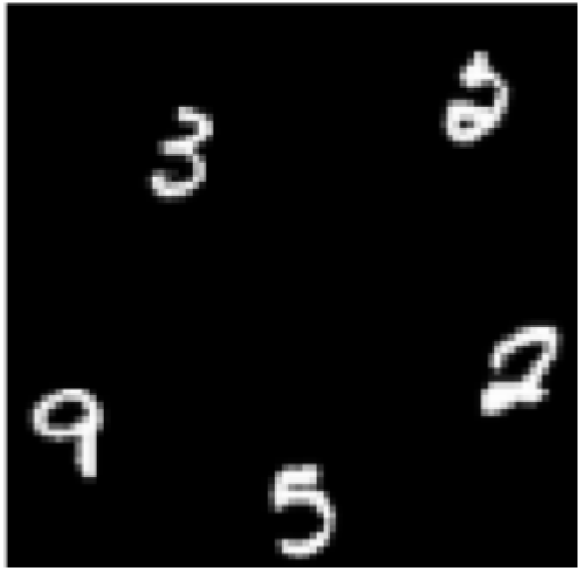
4.1 Three rearch questions

1. **Compositional generalization**: Can the model recombine familiar objects in novel ways beyond those encountered during training?
2. **Task generalization**: Can the model generalize to new tasks or output classes not present in the training data?
3. **Object count generalization**: Can the model detect, classify, and reason over a varying number of objects, including configurations larger (extrapolation) or smaller (interpolation) than those seen during training?

4.2 Datasets

MultiMNIST-Addition Dataset

Input: an image containing sevel digits. **Output**: the sum of the digits image contains.



PokerRules Dataset

Input: an image containing sevel poker cards. **Output**: The poker combination form (Nothing, pair, straight, etc.).

4.3 Results

Task Accuracy and Concept Accuracy (digit-level decomposition) for models evaluated on MultiMNIST-Addition.
The model is trained on images containing at most 3 digits and evaluated on images containing 4 and 5 digits (extrapolation).

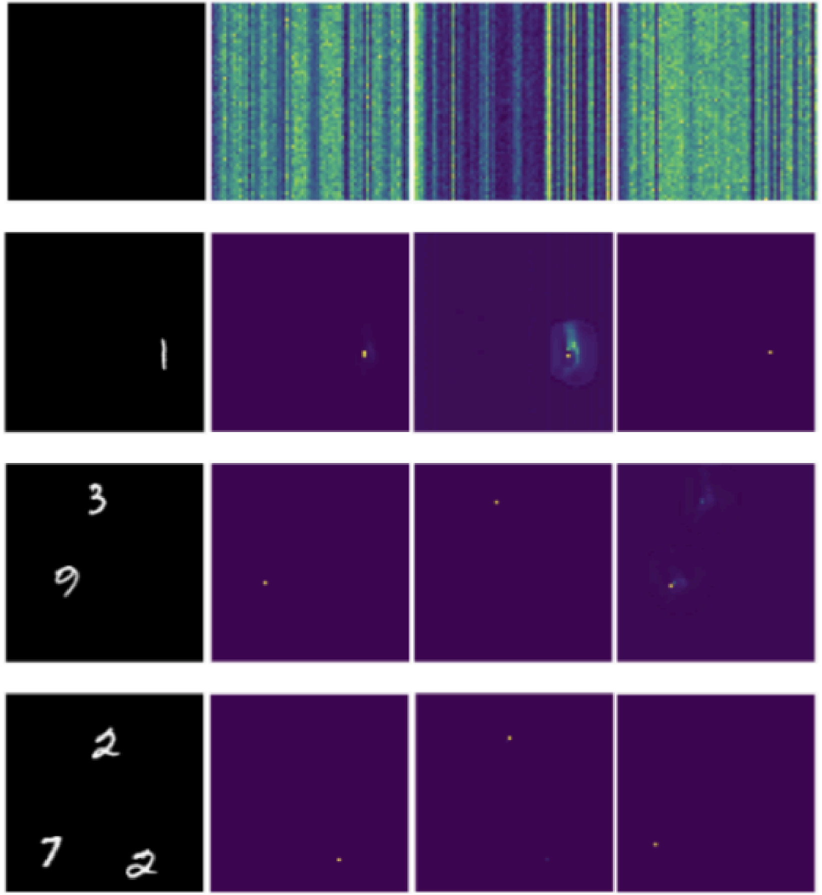
	Task Acc.		Concept Acc.		Extrapolation	
	Test	OOD	Test	OOD	4 digits	5 digits
CNN	79.90±0.17	3.43±0.15	-	-	22.16±0.90	13.16±0.96
SA	**98.90**±0.34	13.30±3.14	-	-	36.23±4.44	9.76±6.11
SA-MESH	98.86±0.47	18.26±1.33	-	-	37.50±1.15	12.00±0.51
CoSA	93.00±1.92	36.2±15.10	-	-	52.20±14.01	25.06±12.82
NeSy	90.70±4.02	35.76±6.24	55.43±26.40	23.83±3.82	-	-
Ours	94.26±2.00	**90.00**±3.01	**85.16**±2.45	**65.46**±5.70	**69.73**±10.74	**44.06**±3.85

4.3 Results

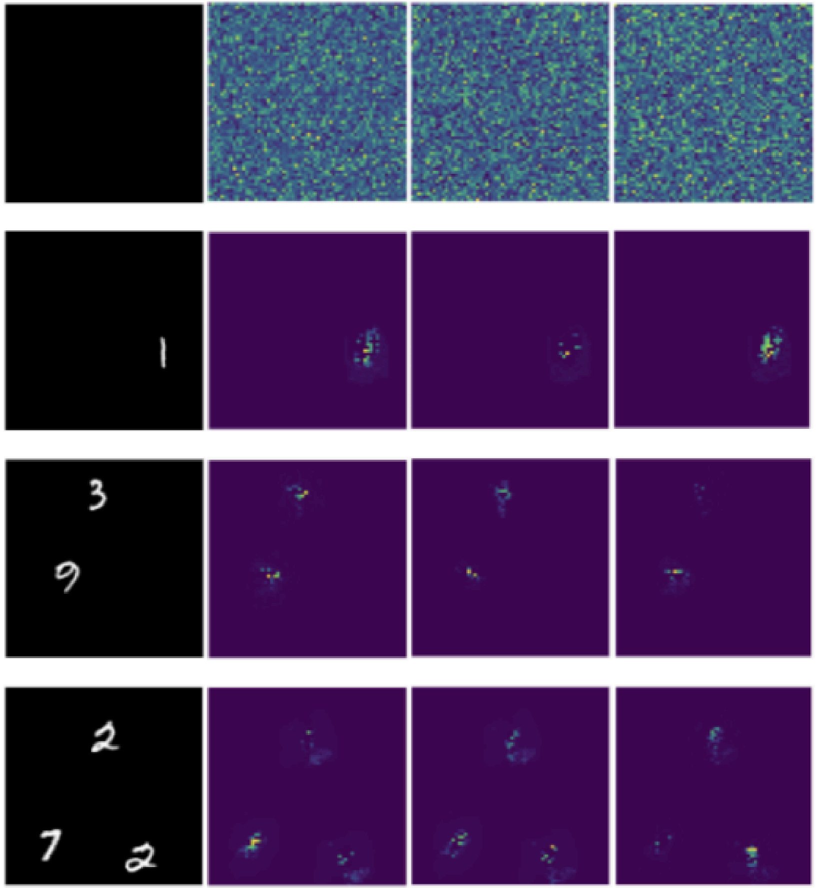
Task Accuracy for models evaluated on PokerRules:

			Extrapolation: 5 cards	
	Test	OOD class	In-distribution class	OOD class
CNN	81.96±1.10	-	22.03±5.27	-
SA	99.30±1.21	-	35.86±8.72	-
SA-MESH	**99.93**±0.11	-	37.80±4.91	-
CoSA	95.46±3.90	-	44.26±17.16	-
NeSy	80.23±2.11	0.46±0.05	-	-
Ours	97.90±1.17	**72.23**±16.72	**78.53**±4.68	**78.23**±19.24

4.3 Results



Ours



SA-MESH

4.3 Results