1 Introduction

2 Analysis and Simulation

2.1 non-relativistic system

A sketch graph is shown at Fig.1,as discussed in [],the non-relativistic equations of motion are:

$$\boldsymbol{F} = \int_S 2\frac{P(\boldsymbol{x'})\hat{\boldsymbol{b}} \cdot \hat{\boldsymbol{n}}(\boldsymbol{x'})}{c} \cdot \hat{\boldsymbol{n}} dS \tag{1}$$

$$\boldsymbol{F} = m\ddot{\boldsymbol{x}} \tag{2}$$

where $\boldsymbol{F}$ is total force of sail, m is mass of sail, S is surface of sail, P is power flux of laser beam, $\boldsymbol{x'}$ is position of laser reflection point, $\boldsymbol{x}$ is position of mass center of sail, $\hat{\boldsymbol{b}}$ is unit vector along the beam, $\hat{\boldsymbol{n}}$ is unit vector perpendicular to the reflection surface
in the coordinate $O'\rho\gamma h$,the value of $\hat{\boldsymbol{n}}$ is:

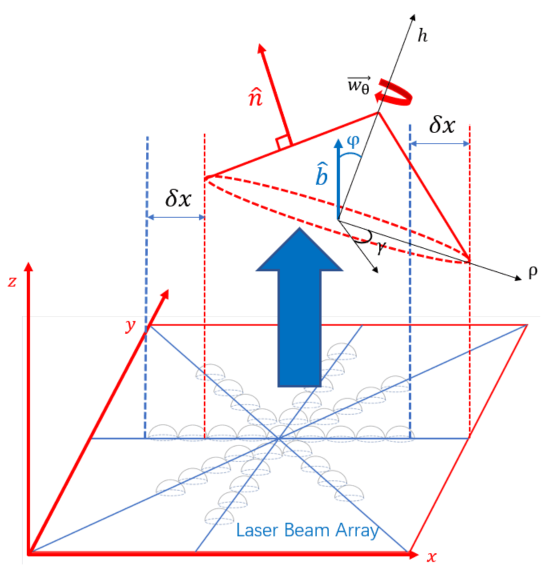

Figure 1: sketch graph of sail,coordinate $Oxyz$ is static with laser beam and $O'\rho\gamma h$ is along with sail

$$\hat{\boldsymbol{n}} = \begin{pmatrix} cos\alpha sin\phi + sin\alpha cos\gamma cos\phi \\ sin\alpha sin\gamma \\ cos\alpha cos\gamma - sin\alpha cos\gamma sin\phi \end{pmatrix} \tag{3}$$

By some geometric tricks, the reflection point is(along with $Oxyz$):

$$\boldsymbol{x} = \boldsymbol{x'} + \begin{pmatrix} \rho sin\alpha \\ \rho cos\gamma \cos\phi - (\frac{H}{2} - \rho tan\alpha) sin\phi \\ \rho cos\alpha sin\phi + (\frac{H}{2} - \rho tan\alpha)\cos\phi \end{pmatrix} \tag{4}$$

where:α is inclinate angle of sail and$H$ is height of sail

2.2 relativistic system

Taking relativity into consideration, given the momentum $\vec{k}$, the forces are given by (see Appendix for the derivation) :

$$F_{//} = \int_S \frac{P(x) + P_f \cos(2\beta)}{c} dS \tag{5}$$

$$F_{\perp} = \int_S \frac{P_f \sin(2\beta)}{c} dS \tag{6}$$

$F_{//}$ is the component parallel to the momentum and $F_{\perp}$ is the component perpendicular to the momentum. P_f is given by:

$$P_f = \frac{P(x)\sqrt{m^2c^4 + |\vec{k}|^2c^2} - P(x)|\vec{k}|c\cos(\alpha - \beta)}{P(x)dt(1 - \cos(1\beta)) + \sqrt{m^2c^4 + |\vec{k}|^2c^2} + |\vec{k}|c\cos(\alpha - \beta)} \tag{7}$$

where α is the angle between $\vec{k}$ and $\hat{n}$, β is the angle between $\hat{b}$ and $\hat{n}$ and $P(\vec{x})$ is the power flux at location $\vec{x}$

Then with the relativistic definition of momentum:

$$\vec{k} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} \tag{8}$$

$$\vec{F} = \frac{d\vec{k}}{dt} \tag{9}$$

we have the relativistic equation of motion.

2.3 simulation result

We use relativistic equations to perform numerical simulations, as shown in Fig 2. From the data, the craft could reach desired velocity in 819.1 seconds. Parameters of this system are set as listed in Table. 2.3, some of the parameters are defined in Chapter 3.

Table 1: Parameters set in Simulation 1

m	S	α	x_0	y_0	σ_P	σ_b
10g	$10m^2$	0	0	0	1.25m	0

3 Marginal Error

In order for the craft to hit Proxima b within earth-moon distance, the deviation of velocity should satisfy following criteria:

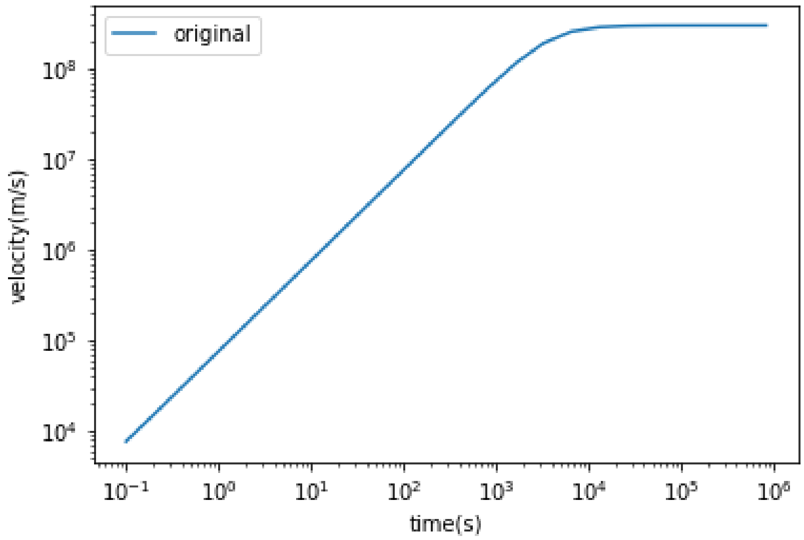

Figure 2: velocity evolution. The velocity reaches 60860120.67 m/s at 819.1 seconds

- Direction: the direction should be within angle that 2 earth-moon distance has at 4.365 light year (distance from earth to Proxima).
- Amplitude: the time required to reach Proxima b at the speed should not lag or lead the desired value to much so that the craft misses the planet due to revolution around Promixa.

Therefore, we have :

$$\Delta\theta \leq \frac{2 * d_{earth-moon}}{4.365 l.y.} \tag{10}$$

$$\frac{4.365 l.y.}{v_0} - \frac{4.365 l.y.}{v_0 + \Delta v} \leq \frac{2 d_{earth-moon}}{2\pi a} T \tag{11}$$

where $d_{earth-moon}$ is the distance between earth and the moon, v_0 is 0.2 light speed as desired, a is the semi latus rectum of the Proxima b's orbit and T is the period of the orbit.

So the marginal error in the velocity is:

$$\Delta\theta \leq 1.862 \times 10^{-8} rad \quad \Delta v \leq 4.58 \times 10^{-7} c \tag{12}$$

where c is light speed.

In order to satisfy this strict constraint, the system should be carefully calibrated. Here we list the parameters we choose to quantify the system:

- Accuracy of beam array: central position of power flux distribution (x, y)
- Precision of beam array: Gaussian variance of the power flux distribution σ

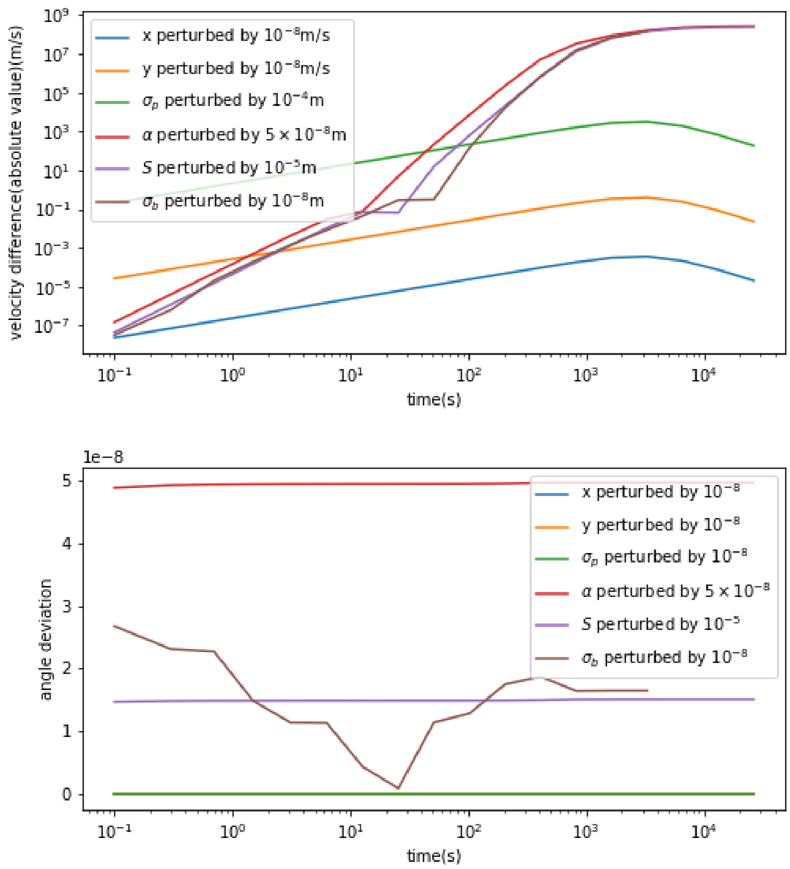

Figure 3: evolution of velocity amplitude and angle after different perturbations

- Fabrication of the light sail: The area S and the conic angle α
- Uniformity of the laser beam: the Gaussian variance of $\vec{b}$, σ_b

In the simulation discussed in last section, we perturb the parameters listed above by a small amount and observe the resulted velocity, which are then used to calculated partial derivatives. The velocity evolution of several perturbed cases are shown in Figure. 3. The results of derivations and marginal errors are listed in Table. 3

Table 2: Partial derivatives and marginal errors

$\frac{\partial v}{\partial x}$	$\frac{\partial v}{\partial y}$	$\frac{\partial v}{\partial \sigma}$	$\frac{\partial v}{\partial \alpha}$	$\frac{\partial v}{\partial S}$	$\frac{\partial v}{\partial \sigma_b}$	$\frac{\partial \theta}{\partial x}$	$\frac{\partial \theta}{\partial y}$	$\frac{\partial \theta}{\partial \sigma}$	$\frac{\partial \theta}{\partial \alpha}$	$\frac{\partial \theta}{\partial S}$	$\frac{\partial \theta}{\partial \sigma_b}$
1.8e4	-2e7	1.6e11	3.3e15	1.2e15	1.5e15	0	0	0	4.96	1.5	1.8