

# 1 Numerical Methods

## 2 Confusion problem

Given a waveform under specetiem with non-zero deformation parameter, we need to decide which waveform under Kerr specetime is most similar to it.

If we restrict ourselves to equatorial motion, set the initial  $t$  and  $\phi$  to 0 taking advantage of symmetry and set initial  $r = r_{max}$  imposing the phase to match, orbital eccentricity  $e$ , semilatus rectum  $p$ , BH mass  $M$  and BH spin  $a$  are the parameters that determine the motion.

According to , orbits with same orbital frequency  $\omega_r$  and  $\omega_\phi$  can generate most similar gravitational waveforms. Here we check this result by lookinf at overlaps between waveforms with  $(\delta_1, a, M, e, p) = (0.2, 0.5, , 0.5, 6)$ .

First we look at overlap distribution on a relatively large range of  $(e, p)$ .

Then take a closer look at the distribution near  $(e, p)$  with same orbital frequency. Note that the same orbital frequency with respect to  $t$  can result in largest overlap while the same orbital frequency with respect to  $\tau$  cannot.

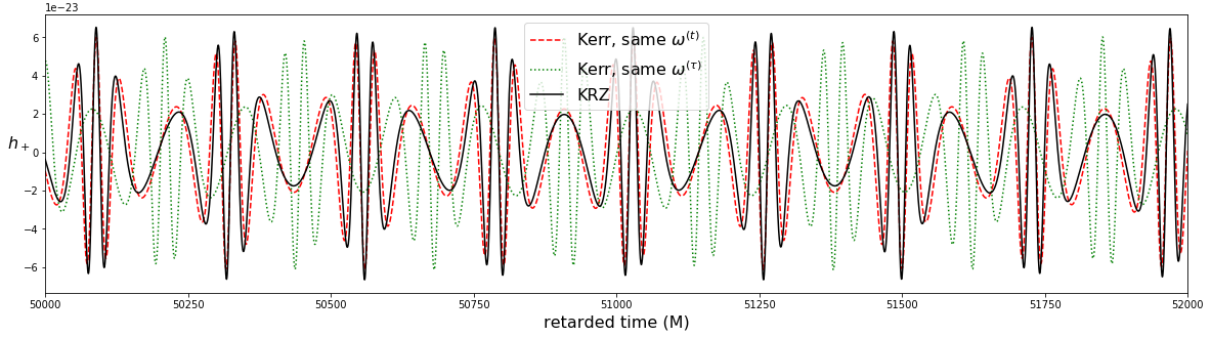


Figure 1: Comparison between waveforms of  $h_+$  with respect to retarded time in units of central black hole mass  $M$ . The black solid line is the waveform under  $\delta_1 = 0.2$ ,  $e=0.5$ ,  $p=6$ . The red dashed line is the waveform under  $\delta_1 = 0$  and  $e, p$  adapted so that the orbital frequencies with respect to  $t$   $\omega_r^{(t)}$  and  $\omega_\phi^{(t)}$  are the same as that of the orbit under  $d1=0.2$ ,  $e=0.5$ ,  $p=6$ . The green dotted line is the waveform under  $\delta_1 = 0$  and  $e, p$  adopted so that  $\omega^{(\tau)}$ s are the same . The spin of the central black hole is  $0.5M$ .

Therefore we regard waveforms in Kerr spacetime with same orbital frequencies as best matches to waveforms in non-Kerr space time under KRZ parametrization.

Fig. shows the overlap distrobution...

As Fig. suggest, the confusion problem still exists in KRZ parametrization. The deformation parameter  $\delta_1$  is kind of degenerated with in Kerr spacetime. This resulted can also be found by looking at covariance matrix as discussed in next section

## 3 Restriction on deformation parameter by future LISA task