1 Numerical Methods

2 Confusion problem

Given a waveform under specetiem with non-zero deformation parameter, we need to decide which waveform under Kerr specetime is most similar to it.

If we restrict ourselves to equatorial motion, set the initial t and ϕ to 0 taking advantage of symmetry and set initial $r = r_{max}$ imposing the phase to match, orbital eccentricity e, semilatus rectum p, BH mass M and BH spin a are the parameters that determine the motion.

According to Ref. [1], orbits with same orbital frequency ω_r and ω_ϕ can generate most similar gravitational waveforms. Here we check this result by lookinf at overlaps between waveforms with $(\delta_1, a, M, e, p) = (0.2, 0.5, , 0.5, 6)$. First we look at overlap distribution on a relatively large range of (e, p). Then we search near (e_{Kerr}, p_{Kerr}) with same orbital frequency, as shown in Fig. 1.

Note that the difference between equating $\omega^{(t)}$, orbital frequency with respect to coordiante time, and $\omega^{(\tau)}$, orbital frequency with respect to proper time, can be significant. From Fig. 1 it is explicit that the same orbital frequency with respect to t can result in almost the largest overlap while the same orbital frequency with respect to τ cannot. In Kerr spacetime, the expression for $\omega^{(t)}$ and $\omega^{(\tau)}$ are given in [2] and [3]

Therefore we regard waveforms in Kerr spacetime with same orbital frequencies as best matches to waveforms in non-Kerr space time under KRZ parametrization.

Fig. shows the overlap distrobution...

As Fig. suggest, the confusion problem still exists in KRZ parametrization. The deformation parameter δ_1 is kind of degenerated with in Kerr spacetime. This resulted can also be found by looking at covariance matrix as discussed in next section

3 Constraints on deformation parameter by future LISA task

References

[1] Kostas Glampedakis and Stanislav Babak. Mapping spacetimes with lisa: inspiral of a test body in a 'quasi-kerr' field. *Classical and Quantum Gravity*, 23(12):4167, 2006.

[2] Kostas Glampedakis and Daniel Kennefick. Zoom and whirl: Eccentric equatorial orbits around spinning black holes and their evolution under gravitational radiation reaction. *Phys. Rev. D*, 66:044002, Aug 2002.

[3] W Schmidt. Celestial mechanics in kerr spacetime. *Classical and Quantum Gravity*, 19(10):2743, 2002.

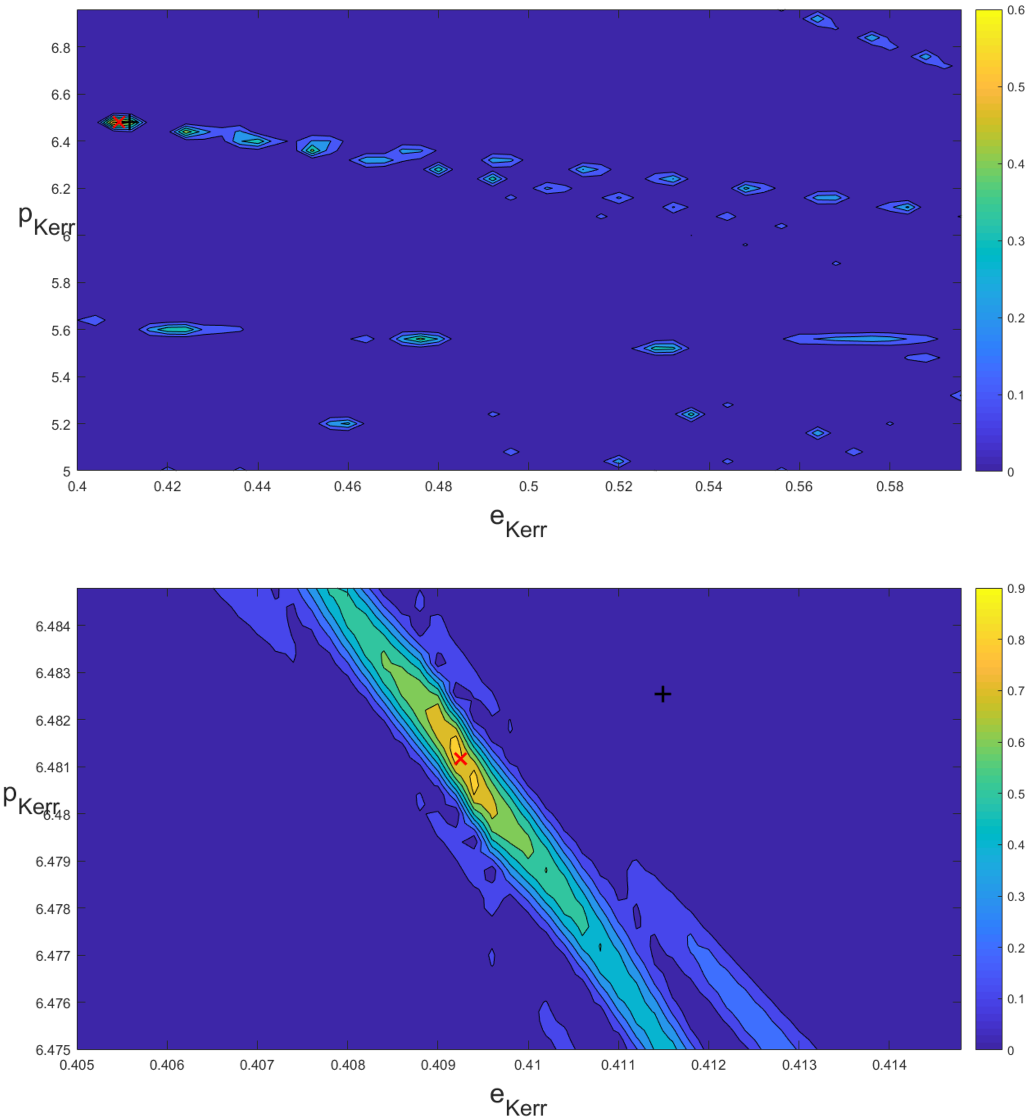

Figure 1: Distribution of overlap between waveform of $(\delta_1, a, M, e, p) = (0.2, 0.5, , 0.5, 6)$ and waveforms of (δ_1, a, M, e, p) =(0.2, 0.5, , e_{Kerr}, p_{Kerr}) on (e_{Kerr}, p_{Kerr}) plane. The original data are both 50*50 grid. Red cross mark: same $\omega^{(t)}$ at $(e_{Kerr}, p_{Kerr}) = (0.409248, 6.481170)$, overlap is 0.8731. Black plus mark: same $\omega^{(\tau)}$ at $(e_{Kerr}, p_{Kerr}) = (0.411495, 6.482549)$, overlap is 0.0507

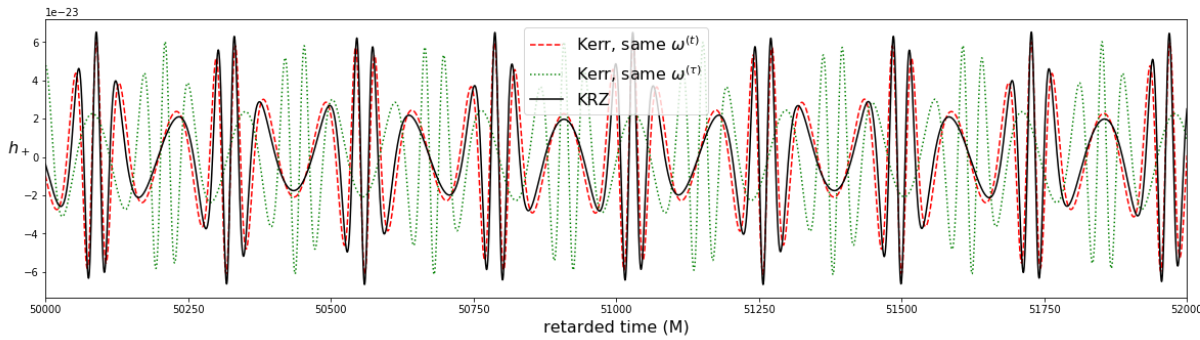

Figure 2: Comparison between waveforms of h_+ with respect to retarded time in units of central black hole mass M. The black solid line is the waveform under $\delta_1 = 0.2$, e=0.5,p=6. The red dashed line is the waveform under $\delta_1 = 0$ and e, p adapted so that the orbital frequencies with respect to t $\omega_r^{(t)}$ and $\omega_\phi^{(t)}$ are the same as that of the orbit under d1=0.2, e=0.5, p=6. The green dotted line is the waveform under $\delta_1 = 0$ and e, p adopted so that $\omega^{(\tau)}$s are the same . The spin of the central black hole is 0.5M.