1 Introduction

It is impossible to measure physical quantities without errors. In most cases errors result from deviations and inaccuracies caused by the measuring apparatus or from the inaccurate reading of the displaying device, but also with optimal instruments and digital displays there are always fluctuations in the measured data. Ultimately there is random thermal noise affecting all quantities that are determined at a finite temperature. Any experimentally determined quantity therefore has a certain inaccuracy. If the experiment were to be repeated, the result would be (slightly) different. One could say that the result of a particular experiment is no more than a *random sample* from a probability distribution. When reporting the result of an experiment, it is important to also report the extent of the uncertainty, e.g. in terms of the best estimate of some measure of the *width* of the probability distribution. When experimental data are processed and conclusions are drawn from them, knowledge of the experimental uncertainties is essential to assess the reliability of the conclusion.

Ideally, you should specify the probability distribution from which the reported experimental value is supposed to be a random sample. The problem is that you have only one experiment; even if your experiment consists of many observations of which you report the average, you have only one average to report. So you have only one sample of the reported item and you could naively conclude that you have no knowledge at all about the underlying probability distribution of that sample. Fortunately, there is the science of statistics that tells us differently. When your experiment consists of a series of repeated observations of a variable x, with outcomes $x_1, x_2, \ldots, x_n$, and you report the result of the total experiment as the average of the x_i's, statistics tells you how to *estimate* certain properties of the probability distribution of which the reported result is supposed to be a random sample. Thus you can estimate the mean of the distribution or – if you prefer – the most probable value of the distribution, which then is the result of your measurement. You can also estimate the width of the distribution, which indicates the random uncertainty in the result.

The result of an experiment is generally not equal to a directly measured quantity, but is derived from measured quantities by some functional relation.

For example, the area of a rectangle is the product of the measured length and width of two sides. Each measurement has its estimated value and random error and these errors *propagate* through the functional relation (here a product) to the final result. The contributing errors must be properly combined to one error estimate in the result.

The purpose of this book is to indicate how one can arrive at the best estimates of both the value(s) and the random error(s) in the result, based on the measurements from which the result is derived. In order to maintain its usefulness as a practical guide, the main part of this book simply states the equations and procedures, without proper derivations. Thus the practical applicant is not bothered by unnecessary detail. However, several appendices are included that provide further details and give a proper background in statistics with derivations of the equations used. For further reading many textbooks are available.[1]

Chapter 2 describes the proper presentation of results of measurements with their accuracies and with their units. Chapter 3 classifies the various types of error and describes how contributing errors will propagate and combine into a more complex result. Chapter 4 describes a number of common probability distributions from which experimental errors may be sampled. In Chapter 5 it is shown how the characteristics of a *data series* can be defined and then be used to arrive at estimates of the best value and accuracy of the result. Chapter 6 is concerned with simple graphic treatment of data, while Chapter 7 treats the more accurate *least-squares* fitting of model parameters to experimental data. Chapter 8, finally, discusses the philosophical basis of statistical methods, confronting traditional hypothesis testing with the more intuitive but powerful *Bayesian* method to determine the probability distribution of model parameters.

[1] Most textbooks aim at a wider audience and are therefore less useful for physical scientists and engineers. For the latter interest group see Bevington and Robinson (2003), Taylor (1997), Barlow (1989) and Petruccelli *et al.* (1999).