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In [1]: ### CSCI-3080 Discrete Structure
### OLA 3: Chapter 5 -- Matrices
### Name:
### Student ID:
### Date:
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Exercise 1: Find x and y if

$$\begin{pmatrix} 1 & 3 \\ x & x+y \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$$

$$x = 2$$

$$y = 4$$

Exercise 2: Compute $A + rD$

$$A + rD = \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 4 \end{pmatrix} + 3 \begin{pmatrix} 4 & -6 \\ 1 & 3 \\ 2 & -1 \end{pmatrix}$$

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In [7]: import numpy as np
A = np.array([[2,1],[-1,0],[3,4]])
B = np.array([[4,-6],[1,3],[2,-1]])
r = 3
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In [10]: A + r*B
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Out[10]: array([[ 14, -17],
                [  2,   9],
                [  9,   1]])
```

Exercise 3: Compute $B \cdot D$

$$B \cdot D = \begin{pmatrix} 4 & 1 & 2 \\ 6 & -1 & 5 \\ 1 & 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 4 & -6 \\ 1 & 3 \\ 2 & -1 \end{pmatrix}$$

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In [12]: B = np.array([[4,1,2],[6,-1,5],[1,3,2]])  
D = np.array([[4,-6],[1,3],[2,-1]])
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In [13]: np.matmul(B,D)
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Out[13]: array([[ 21, -23],  
               [ 33, -44],  
               [ 11,   1]])
```

Exercise 4: Solve the system of equations using Gaussian Elimination
(Please show all the steps)

$$\begin{aligned} -x + 2y + z &= -1 \\ 3x - 5y - z &= 5 \\ 2x - y + 3z &= 22 \end{aligned}$$

We want:

$$\begin{pmatrix} -1 & 2 & 1 & -1 \\ 3 & -5 & -1 & 5 \\ 2 & -1 & 3 & 22 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \end{pmatrix}$$

1) Multiply row 1 by 3 and add to row 2

$$\begin{pmatrix} -1 & 2 & 1 & -1 \\ 0 & 1 & 2 & 2 \\ 2 & -1 & 3 & 22 \end{pmatrix}$$

2) Multiply row 1 by 2 and add to row 3

$$\begin{pmatrix} -1 & 2 & 1 & -1 \\ 0 & 1 & 2 & 2 \\ 0 & 3 & 5 & 20 \end{pmatrix}$$

3) Multiply row 1 by -1 scalar

$$\begin{pmatrix} 1 & -2 & -1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 3 & 5 & 20 \end{pmatrix}$$

4) Multiply row 2 by 2 and add to row 1

$$\begin{pmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 2 & 2 \\ 0 & 3 & 5 & 20 \end{pmatrix}$$

5) Multiply row 2 by -3 and add to row 3

$$\begin{pmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -1 & 14 \end{pmatrix}$$

6) Multiply row 3 by 2 and add to row 2

$$\begin{pmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & -1 & 14 \end{pmatrix}$$

7) Multiply row 3 by 3 and add to row 1

$$\begin{pmatrix} 1 & 0 & 0 & 47 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & -1 & 14 \end{pmatrix}$$

8) Multiply row 3 by -1 scalar

$$\begin{pmatrix} 1 & 0 & 0 & 47 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 1 & -14 \end{pmatrix}$$

Therefore: $x = 47$
 $y = 30$
 $z = -14$

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In [19]: x = 47
         y = 30
         z = -14
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Exercise 5: For the given Boolean matrices, calculate $A \wedge B$, $A \vee B$, $A \times B$, and $B \times A$

(Don't need to show all steps, the final result is fine.)

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$A \wedge B$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$A \vee B$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$A \times B$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

B × A

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

In []: