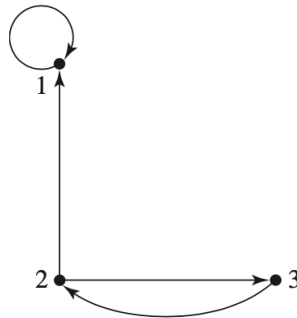


```
In [1]: ### CSCI-3080 Discrete Structure
### OLA 5: Chapter 7 -- Graphs and Algorithms
### Name:
### Student ID:
### Date:
```

**1. Find the adjacency matrix and adjacency relation (binary relation) for the following graph.**



Adjacency relation:  $\{(1,1), (2,1), (2,3), (3,2)\}$

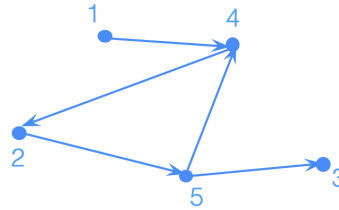
Adjacency matrix:  $= \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

**2. Find the corresponding directed graph and adjacency relation (binary relation) for the following adjacency matrix.**

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Adjacency relation:  $\{(1,4), (2,5), (4,2), (5,3), (5,4)\}$

Directed Graph:

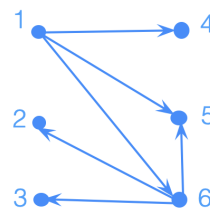


3. Given the adjacency relation  $\rho = \{(1, 4), (1, 5), (1, 6), (6, 2), (6, 3), (6, 5)\}$  on the set of nodes  $\{1, 2, 3, 4, 5, 6\}$  find the corresponding directed graph and adjacency matrix.

Adjacency matrix: =

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Directed Graph:



4. Let  $\mathbf{A}$  be the following matrix. Find the products  $\mathbf{A}^2$  and  $\mathbf{A}^{(2)}$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix Multiplication  $\mathbf{A} \cdot \mathbf{A} = \mathbf{A}^2 =$

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

Boolean Matrix Multiplication:  $A \times A = A^{(2)} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

**5. Compute the reachability matrix  $R$  using the formula  $R = A \vee A^{(2)} \vee \dots \vee A^{(n)}$  for exercise 2.**

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$A^{(2)} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$A^{(3)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A^{(4)} = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$A^{(5)} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$R = A \vee A^{(2)} \vee A^{(3)} \vee A^{(4)} \vee A^{(5)} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

**6. Compute the reachability matrix  $R$  using [Warshall's algorithm](#) for exercise 2.**

$$M_0 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$M_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

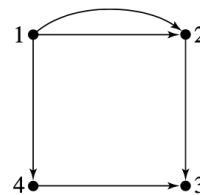
$$M_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$M_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$M_4 = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$R = M_5 \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

7. For the following graph, count the number of paths of length 2 from node 1 to node 3. Check by computing  $A^2$



```
In [18]: import numpy as np
A = np.array([[0,2,0,1],[0,0,1,0],[0,0,0,0],[0,0,1,0]])
```

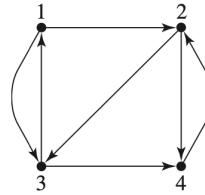
```
In [19]: A2 = np.matmul(A,A)
A2
```

```
Out[19]: array([[0, 0, 3, 0],
               [0, 0, 0, 0],
               [0, 0, 0, 0],
               [0, 0, 0, 0]])
```

There are three paths of length 2 from node 1 to node 3.

8.

- (1) Determine whether this graph has an **Euler path**. If so, list the nodes in such a path.
- (2) Determine whether this graph has a **Hamiltonian circuit**. If so, list the nodes in such a circuit.



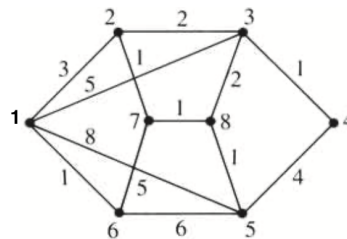
(1) Yes. There are two odd-degree nodes: Node 1 and Node 4. The Euler path starts from Node 1 and ends at Node 4.

1 → 3 → 1 → 2 → 3 → 4 → 2 → 4

(2) No.

9. Apply **Dijkstra's algorithm** for the following graph. Show the values for **p** and **IN** and the **d** values and **s** values for each pass. Write out the nodes in the **shortest path** from **2** to **5** and the distances of the path.

Hint:  $p$  is the current node that has the shortest  $d$  that you will include in your current  $IN$  set.



1.  $IN = \{2\}$

	1	2	3	4	5	6	7	8
$d$	3	0	2	$\infty$	$\infty$	$\infty$	1	$\infty$
$s$	2	—	2	2	2	2	2	2

$p = 7, IN = \{2, 7\}$

	1	2	3	4	5	6	7	8
$d$	3	0	2	$\infty$	$\infty$	6	1	2
$s$	2	—	2	2	2	7	2	7

$p = 3, IN = \{2, 7, 3\}$

	1	2	3	4	5	6	7	8
$d$	3	0	2	3	$\infty$	6	1	2
$s$	2	—	2	3	2	7	2	7

$p = 8, IN = \{2, 7, 3, 8\}$

	1	2	3	4	5	6	7	8
$d$	3	0	2	3	3	6	1	2
$s$	2	—	2	3	8	7	2	7

$p = 5, IN = \{2, 7, 3, 8, 5\}$

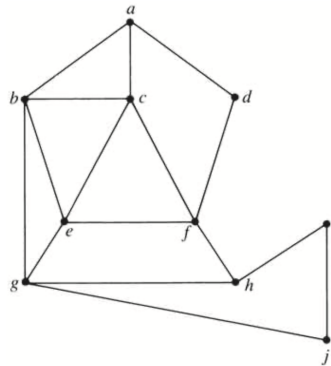
	1	2	3	4	5	6	7	8
$d$	3	0	2	3	3	6	1	2
$s$	2	—	2	3	8	7	2	7

path: 2, 7, 8, 5

distance = 3

10.

- (1) Write the nodes in a **depth-first search** of the following graph, beginning with the node **a**.
- (2) Write the nodes in a **breadth-first search** of the following graph, beginning with the node **a**.



Depth-first search: a,b,c,e,f,d,h,g,j,i

Breadth-first search: a,b,c,d,e,g,f,h,j,i

In [ ]: