

**1. Disprove the following:**

Every animal with wings is a bird.

Bats are not birds

**2. Give a direct proof of the following:**

if  $4x^2 - 16x + 16 = 0$ , then  $x = 2$

P:  $4x^2 - 16x + 16 = 0$

Q:  $x = 2$

$P \rightarrow Q$

Proof:

$4x^2 - 16x + 16 = 0$  (hypothesis)

$4(x^2 - 4x + 4) = 0$  (algebra)

$x^2 - 4x + 4 = 0$  (algebra)

$(x - 2)^2 = 0$  (algebra)

$x - 2 = 0$  (algebra)

$x = 2$  (algebra)

**3. Give a proof by contrapositive of the following**

if  $x - 2 > 0$ , then  $x > 0$  (be very careful!)

P:  $x - 2 > 0$

Q:  $x > 0$

$P \rightarrow Q$

Q':  $x \leq 0$

P':  $x - 2 \leq 0$

$Q' \rightarrow P'$

$x \leq 0$

$x - 2 \leq 0 - 2$  (algebra)

$x - 2 \leq -2$  (algebra)

$x - 2 \leq 0$  (algebra)

**4. Give a proof by contradiction of the following:**

if  $2x^2 - 8x + 8 = 0$ , then  $x \neq 3$

$$P: 2x^2 - 8x + 8 = 0$$

$$Q: x \neq 3$$

$$P \rightarrow Q$$

$$Q': x = 3$$

$$P \wedge Q' \rightarrow 0$$

Proof by contradiction:

$$2x^2 - 8x + 8 = 0 \text{ (hypothesis)}$$

$$x = 3 \text{ (hypothesis)}$$

$$2 * 3^2 - 8 * 3 + 8 = 0$$

$$2 * 9 - 8 * 3 + 8 = 0$$

$$18 - 24 + 8 = 0$$

$$2 = 0 \text{ (contradiction)}$$

## 5. Give an exhaustive proof of the following:

if  $0 < x < 5$ , then  $2x^2 + 1 > 2$

$x$	$2x^2 + 1$	$2x^2 + 1 > 2$
1	3	Yes
2	9	Yes
3	19	Yes
4	33	Yes

## 6. Prove by mathematical induction that the following is true. Show all work.

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}, n \geq 2$$

Base step:

P(2):

$$\text{left side: } 1^2 + 2^2 = 5$$

$$\text{right side: } \frac{2*(2+1)*(2*2+1)}{6} = \frac{2*3*5}{6} = 5$$

P(2) is true.

Induction step:  $P(k) \rightarrow P(k+1)$

$$P(k): 1^2 + 2^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

Proof:

$$P(k+1): 1^2 + 2^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$1^2 + 2^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6} \text{ (hypothesis)}$$

$$1^2 + 2^2 + \dots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \text{ (algebra)}$$

$$1^2 + 2^2 + \dots + k^2 + (k+1)^2 = \frac{(k(k+1)(2k+1) + 6(k+1)^2)}{6} \text{ (algebra)}$$

$$1^2 + 2^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1)(k(2k+1) + 6(k+1))}{6} \text{ (algebra)}$$

$$1^2 + 2^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1)(2k^2 + k + 6k + 6)}{6} \text{ (algebra)}$$

$$1^2 + 2^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1)(2k^2 + 7k + 6)}{6} \text{ (algebra)}$$

$$1^2 + 2^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6} \text{ (algebra)}$$

$$1^2 + 2^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6} \text{ (algebra)}$$

$P(k) \rightarrow P(k+1)$  is true

In [ ]: