In [17]:
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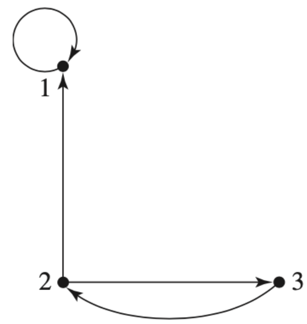
### CSCI-3080 Discrete Structure
### OLA 8: Chapter 7 -- Graphs and Algorithms
### Name:
### Student ID:
### Date:
```

## 1. Find the adjacency matrix and adjacency relation (binary relation) for the following graph.




Adjacency relation: {(1,1),(2,1),(2,3),(3,2)}

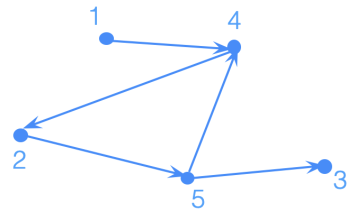
Adjacency matrix: $= \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

## 2. Find the corresponding directed graph and adjacency relation (binary relation) for the following adjacency matrix.

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Adjacency relation: {(1,4),(2,5),(4,2),(5,3),(5,4)}

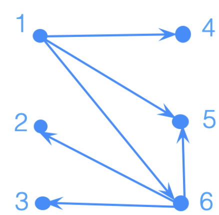
Directed Graph:




**3. Given the adjacency relation ρ = {(1, 4), (1, 5), (1, 6), (6, 2), (6, 3), (6, 5)} on the set of nodes {1, 2, 3, 4, 5, 6,} find the corresponding directed graph and adjacency matrix.**

$$\text{Adjacency matrix:} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Directed Graph:




**4. Let A be the following matrix. Find the products $A^2$ and $A^{(2)}$**

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Matrix Multiplication } A^2 = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

Boolean Matrix Multiplication: $A^{(2)} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

**5. Compute the reachability matrix R using the formula R = A $\vee$ $A^{(2)}$ $\vee \cdots \vee$ $A^{(n)}$ for exercise 2.**

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$A^{(2)} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$A^{(3)} \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A^{(4)} \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$A^{(5)} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

R = A ∨ $A^{(2)}$ ∨ $A^{(3)}$ ∨ $A^{(4)}$ ∨ $A^{(5)}$ = $\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$

### 6. Compute the reachability matrix R using Warshall's algorithm for exercise 2.

$$M_0 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$M_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

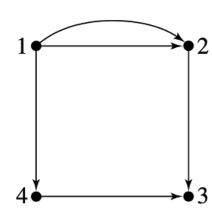
$$M_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$M_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$M_4 = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$R = M_5 \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

### 7. For the following graph, count the number of paths of length 2 from node 1 to node 3. Check by computing $A^2$




In [18]:
```
import numpy as np
A = np.array([[0,2,0,1],[0,0,1,0],[0,0,0,0],[0,0,1,0]])
```

In [19]:
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A2 = np.matmul(A,A)
A2
```
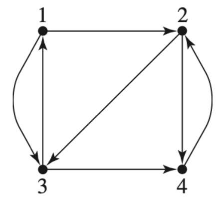

Out[19]:
```
array([[0, 0, 3, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 0]])
```

There are three paths of length 2 from node 1 to node 3.

### 8.

**(1) Determine whether this graph has an Euler path. If so, list the nodes in such a path.**
**(2) Determine whether this graph has a Hamiltonian circuit. If so, list the nodes in such a circuit.**

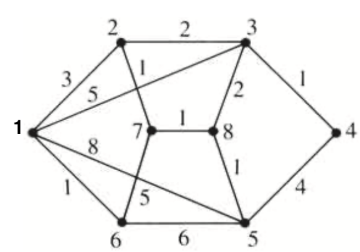



(1) Yes. There are two odd-Node 1 and 4. The euler path starts from Node 1 and ends at Node 4.
1->3->1->2->3->4->2->4

(2) No.

### 9. Apply Dijkstra's algorithm for the following graph. Show the values for p and IN and the d values and s values for each pass. Write out the nodes in the shortest path from 2 to 5 and the distances of the path.

Hint: p is the current node that has the shortest d that you will include in your current IN set.




1. $IN = \{2\}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| d | 3 | 0 | 2 | ∞ | ∞ | ∞ | 1 | ∞ |
| s | 2 | – | 2 | 2 | 2 | 2 | 2 | 2 |

$p = 7, IN = \{2, 7\}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| d | 3 | 0 | 2 | ∞ | ∞ | 6 | 1 | 2 |
| s | 2 | – | 2 | 2 | 2 | 7 | 2 | 7 |

$p = 3, IN = \{2, 7, 3\}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| d | 3 | 0 | 2 | 3 | ∞ | 6 | 1 | 2 |
| s | 2 | – | 2 | 3 | 2 | 7 | 2 | 7 |

$p = 8, IN = \{2, 7, 3, 8\}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| d | 3 | 0 | 2 | 3 | 3 | 6 | 1 | 2 |
| s | 2 | – | 2 | 3 | 8 | 7 | 2 | 7 |

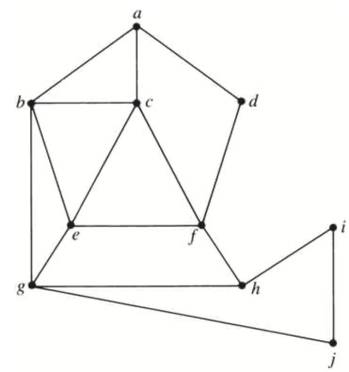
$p = 5, IN = \{2, 7, 3, 8, 5\}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| d | 3 | 0 | 2 | 3 | 3 | 6 | 1 | 2 |
| s | 2 | – | 2 | 3 | 8 | 7 | 2 | 7 |

path: 2, 7, 8, 5 distance = 3

### 10.

**(1) Write the nodes in a depth-first search of the following graph, beginning with the node a.**
**(2) Write the nodes in a breadth-first search of the following graph, beginning with the node a.**




Depth-first search: a,b,c,e,f,d,h,g,j,i

Breadth-first search: a,b,c,d,e,g,f,h,j,i

In [ ]: