The argument says that *if* the hypotheses are true, then the conclusion will be true. The validity of the argument is a function only of its logical form and has nothing to do with the actual truth of any of its components. We still have no idea about whether the diary is really missing. Furthermore, the argument "Skooses are pink, but if Gingoos does not like perskees, then skooses are not pink; therefore Gingoos does like perskees," which has the same logical form, is also valid, even though it does not make sense. ●

PRACTICE 14 Use propositional logic to prove that the following argument is valid. Use statement letters S, R, and B.

If security is a problem, then regulation will be increased. If security is not a problem, then business on the Web will grow. Therefore if regulation is not increased, then business on the Web will grow. ■

Formal logic is not necessary to prove the validity of propositional arguments. A valid argument is represented by a tautology, and truth tables provide a mechanical test for whether a wff is a tautology. So, what was the point of all of this? In the next section we will see that propositional wffs are not sufficient to represent everything we would like to say, and we will devise new wffs called *predicate wffs*. There is no mechanical test for the predicate wff analogue of tautology, and in the absence of such a test, we will have to rely on formal logic to justify arguments. We have developed formal logic for propositional arguments as a sort of dry run for the predicate case.

In addition, the sort of reasoning we have used in propositional logic carries over into everyday life. It is the foundation for logical thinking in computer science, mathematics, the courtroom, the marketplace, and the laboratory. Although we have approached logic as a mechanical system of applying rules, enough practice should ingrain this way of thinking so that you no longer need to consult tables of rules, but can draw logical conclusions and recognize invalid arguments on your own.

SECTION 1.2 REVIEW

TECHNIQUES

- Apply derivation rules for propositional logic.
- Use propositional logic to prove the validity of a verbal argument.

MAIN IDEAS

- A valid argument can be represented by a wff of the form $P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n \rightarrow Q$ that is a tautology.
- A proof sequence in a formal logic system is a sequence of wffs that are either hypotheses or derived from earlier wffs in the sequence by the derivation rules of the system.
- The propositional logic system is complete and correct; valid arguments and only valid arguments are provable.

EXERCISES 1.2

For Exercises 1–4, what inference rule is illustrated by the argument given?

1. If Martina is the author, then the book is fiction. But the book is nonfiction. Therefore Martina is not the author.
2. If the business declares bankruptcy, then all assets must be confiscated. The business declared bankruptcy. It follows that all assets must be confiscated.

3. The dog has a shiny coat and loves to bark. Consequently, the dog loves to bark.
4. If Paul is a good swimmer, then he is a good runner. If Paul is a good runner, then he is a good biker. Therefore if Paul is a good swimmer, then he is a good biker.

For Exercises 5–8, decide what conclusion, if any, can be reached from the given hypotheses and justify your answer.

5. If the car was involved in the hit-and-run, then the paint would be chipped. But the paint is not chipped.
6. Either the weather will turn bad or we will leave on time. If the weather turns bad, then the flight will be canceled.
7. If the bill was sent today, then you will be paid tomorrow. You will be paid tomorrow.
8. The grass needs mowing and the trees need trimming. If the grass needs mowing, then we need to rake the leaves.
9. Justify each step in the proof sequence of

$$A \wedge (B \to C) \to (B \to (A \wedge C))$$

1. A
2. $B \to C$
3. B
4. C
5. $A \wedge C$

10. Justify each step in the proof sequence of

$$B \wedge [(B \wedge C) \to A'] \wedge (B \to C) \to A'$$

1. B
2. $(B \wedge C) \to A'$
3. $B \to C$
4. C
5. $B \wedge C$
6. A'

11. Justify each step in the proof sequence of

$$[A \to (B \vee C)] \wedge B' \wedge C' \to A'$$

1. $A \to (B \vee C)$
2. B'
3. C'
4. $B' \wedge C'$
5. $(B \vee C)'$
6. A'

12. Justify each step in the proof sequence of

$$A' \wedge B \wedge [B \to (A \vee C)] \to C$$

1. A'
2. B
3. $B \to (A \vee C)$
4. $A \vee C$
5. $(A')' \vee C$
6. $A' \to C$
7. C

In Exercises 13–24, use propositional logic to prove that the argument is valid.

13. $(A \vee B')' \wedge (B \to C) \to (A' \wedge C)$
14. $A' \wedge (B \to A) \to B'$
15. $(A \to B) \wedge [A \to (B \to C)] \to (A \to C)$
16. $[(C \to D) \to C] \to [(C \to D) \to D]$
17. $A' \wedge (A \vee B) \to B$

18. $[A \to (B \to C)] \wedge (A \vee D') \wedge B \to (D \to C)$
19. $(A' \to B') \wedge B \wedge (A \to C) \to C$
20. $(A \to B) \wedge [B \to (C \to D)] \wedge [A \to (B \to C)] \to (A \to D)$
21. $[A \to (B \to C)] \to [B \to (A \to C)]$
22. $(A \wedge B) \to (A \to B')'$
23. $(A \to C) \wedge (C \to B') \wedge B \to A'$
24. $[A \to (B \vee C)] \wedge C' \to (A \to B)$

Use propositional logic to prove the validity of the arguments in Exercises 25–33. These will become additional derivation rules for propositional logic, summarized in Table 1.14.

25. $(P \vee Q) \wedge P' \to Q$
26. $(P \to Q) \to (Q' \to P')$
27. $(Q' \to P') \to (P \to Q)$
28. $P \to P \wedge P$
29. $P \vee P \to P$ (*Hint:* Instead of assuming the hypothesis, begin with a version of Exercise 28; also make use of Exercise 27.)
30. $[(P \wedge Q) \to R] \to [P \to (Q \to R)]$
31. $P \wedge P' \to Q$
32. $P \wedge (Q \vee R) \to (P \wedge Q) \vee (P \wedge R)$ (*Hint:* First rewrite the conclusion.)
33. $P \vee (Q \wedge R) \to (P \vee Q) \wedge (P \vee R)$ (*Hint:* Prove both $P \vee (Q \wedge R) \to (P \vee Q)$ and $P \vee (Q \wedge R) \to (P \vee R)$; for each proof, first rewrite the conclusion.)

TABLE 1.14

More Inference Rules

From	Can Derive	Name/Abbreviation for Rule
$P \to Q$, $Q \to R$	$P \to R$ [Example 16]	Hypothetical syllogism—hs
$P \vee Q$, P'	Q [Exercise 25]	Disjunctive syllogism—ds
$P \to Q$	$Q' \to P'$ [Exercise 26]	Contraposition—cont
$Q' \to P'$	$P \to Q$ [Exercise 27]	Contraposition—cont
P	$P \wedge P$ [Exercise 28]	Self-reference—self
$P \vee P$	P [Exercise 29]	Self-reference—self
$(P \wedge Q) \to R$	$P \to (Q \to R)$ [Exercise 30]	Exportation—exp
P, P'	Q [Exercise 31]	Inconsistency—inc
$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$ [Exercise 32]	Distributive—dist
$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$ [Exercise 33]	Distributive—dist

For Exercises 34–42, use propositional logic to prove the arguments valid; you may use any of the rules in Table 1.14 or any previously proved exercise.

34. $A' \to (A \to B)$

35. $(P \to Q) \wedge (P' \to Q) \to Q$
36. $(A' \to B') \wedge (A \to C) \to (B \to C)$
37. $(A' \to B) \wedge (B \to C) \wedge (C \to D) \to (A' \to D)$
38. $(A \vee B) \wedge (A \to C) \wedge (B \to C) \to C$
39. $(Y \to Z') \wedge (X' \to Y) \wedge [Y \to (X \to W)] \wedge (Y \to Z) \to (Y \to W)$
40. $(A \wedge B) \wedge (B \to A') \to (C \wedge B')$
41. $(A \wedge B)' \wedge (C' \wedge A)' \wedge (C \wedge B')' \to A'$
42. $(P \vee (Q \wedge R)) \wedge (R' \vee S) \wedge (S \to T') \to (T \to P)$

In Exercises 43–54, write the argument using propositional wffs (use the statement letters shown). Then, using propositional logic, including the rules in Table 1.14, prove that the argument is valid.

43. If the program is efficient, it executes quickly. Either the program is efficient, or it has a bug. However, the program does not execute quickly. Therefore, it has a bug. E, Q, B
44. If Jane is more popular, then she will be elected. If Jane is more popular, then Craig will resign. Therefore, if Jane is more popular, she will be elected and Craig will resign. J, E, C
45. If chicken is on the menu, then don't order fish, but you should have either fish or salad. So, if chicken is on the menu, have salad. C, F, S
46. The crop is good, but there is not enough water. If there is a lot of rain or not a lot of sun, then there is enough water. Therefore, the crop is good and there is a lot of sun. C, W, R, S
47. If the ad is successful, then the sales volume will go up. Either the ad is successful or the store will close. The sales volume will not go up. Therefore, the store will close. A, S, C
48. If DeWayne is not tall then Jayden is not DeWayne's brother. If DeWayne is tall then Trevor is DeWayne's brother. Therefore, if Jayden is DeWayne's brother, then Trevor is DeWayne's brother. D, J, T
49. Russia was a superior power, and either France was not strong or Napoleon made an error. Napoleon did not make an error, but if the army did not fail, then France was strong. Hence, the army failed and Russia was a superior power. R, F, N, A
50. It is not the case that if electric rates go up, then usage will go down, nor is it true that either new power plants will be built or bills will not be late. Therefore, usage will not go down and bills will be late. R, U, P, B
51. If Jose took the jewelry or Mrs. Krasov lied, then a crime was committed. Mr. Krasov was not in town. If a crime was committed, then Mr. Krasov was in town. Therefore, Jose did not take the jewelry. J, L, C, T
52. If the birds are flying south and the leaves are turning, then it must be fall. Fall brings cold weather. The leaves are turning but the weather is not cold. Therefore, the birds are not flying south. B, L, F, C
53. If a Democrat is elected then taxes will go up. Either a Democrat will be elected or the bill will pass. Therefore, if taxes do not go up, then the bill will pass. D, T, B
54. Either Emily was not home or if Pat did not leave the tomatoes, then Sophie was ill. Also, if Emily was not home, then Olivia left the peppers. But it is not true that either Sophie was ill or Olivia left the peppers. Therefore, Pat left the tomatoes and Olivia did not leave the peppers. E, P, S, O
55. a. Use a truth table to verify that $A \to (B \to C) \leftrightarrow (A \wedge B) \to C$ is a tautology.
    b. Prove that $A \to (B \to C) \Leftrightarrow (A \wedge B) \to C$ by using a series of equivalences.
    c. Explain how this equivalence justifies the deduction method that says: to prove $P_1 \wedge P_2 \wedge \ldots \wedge P_n \to (R \to S)$, deduce S from $P_1, P_2, \ldots, P_n$, and R.
56. The argument of the defense attorney at the beginning of this chapter was

    If my client is guilty, then the knife was in the drawer. Either the knife was not in the drawer or Jason Pritchard saw the knife. If the knife was not there on October 10, it follows that Jason Pritchard didn't see the knife. Furthermore, if the knife was there on October 10, then the knife was in the drawer and also the hammer was in the barn. But we all know that the hammer was not in the barn. Therefore, ladies and gentlemen of the jury, my client is innocent.

Use propositional logic to prove that this is a valid argument.

SECTION 1.3 QUANTIFIERS, PREDICATES, AND VALIDITY

Quantifiers and Predicates

Propositional wffs have rather limited expressive power. For example, we would consider the sentence "For every x, $x > 0$" to be a true statement about the positive integers, yet it cannot be adequately symbolized using only statement letters, parentheses, and logical connectives. It contains two new features, a *quantifier* and a *predicate.* Quantifiers are phrases such as "for every" or "for each" or "for some" that tell in some sense *how many* objects have a certain property. The **universal quantifier** is symbolized by an upside down A, $\forall$, and is read "for all," "for every," "for each," or "for any." Thus the example sentence can be symbolized by

$$(\forall x)(x > 0)$$

A quantifier and its named variable are always placed in parentheses. The second set of parentheses shows that the quantifier acts on the enclosed expression, which in this case is "$x > 0$."

The phrase "$x > 0$" describes a property of the variable x, that of being positive. A property is also called a **predicate**; the notation $P(x)$ is used to represent some unspecified predicate or property that x may have. Thus, our original sentence is an example of the more general form

$$(\forall x)P(x)$$

The truth value of the expression $(\forall x)(x > 0)$ depends on the domain of objects in which we are "interpreting" this expression, that is, the collection of objects from which x may be chosen. This collection of objects is called the *domain of interpretation.* We have already agreed that if the domain of interpretation consists of the positive integers, the expression has the truth value true because every possible value for x has the required property of being greater than zero. If the domain of interpretation consists of all the integers, the expression has the truth value false, because not every x has the required property. We impose the condition that the domain of interpretation contain at least one object so that we are not talking about a trivial case.

An interpretation of the expression $(\forall x)P(x)$ would consist of not only the collection of objects from which x could take its value but also the particular property that $P(x)$ represents in this domain. Thus an interpretation for $(\forall x)P(x)$ could be the following: The domain consists of all the books in your local library, and $P(x)$ is the property that x has a red cover. In this interpretation, $(\forall x)P(x)$ says that every book in your local library has a red cover. The truth value of this expression, in this interpretation, is undoubtedly false.