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### CSCI-3080 Discrete Structure
### OLA 5: Chapter 7 -- Graphs and Algorithms
### Name:
### Student ID:
### Date:
### Total: 100 Points
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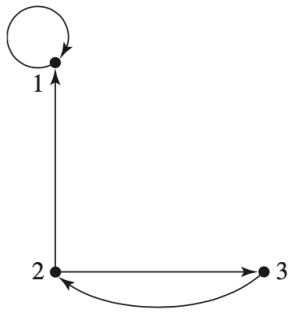
1. Find the adjacency matrix and adjacency relation (binary relation) for the following graph. (8 points)



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2. Find the corresponding directed graph and adjacency relation (binary relation) for the following adjacency matrix.(10 points)

directed graph (5 points)
binary relation(5 points)

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

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3. Given the adjacency relation ρ = {(1, 4), (1, 5), (1, 6), (6, 2), (6, 3), (6, 5)} on the set of nodes {1, 2, 3, 4, 5, 6,} find the corresponding directed graph and adjacency matrix.(10 points)

directed graph (5 points)
adjacency matrix(5 points)

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4. Let A be the following matrix. Find the products A^2 and $A^{(2)}$ (10 points)

A^2 (regular matrix multiplication)(5 points)
$A^{(2)}$ (boolean matrix multiplication) (5 points)

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

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5. Compute the reachability matrix R using the formula R = A ∨ $A^{(2)}$ ∨ ⋯ ∨ $A^{(n)}$ for exercise 2. (12 points)

Hint: Please show the matrix A, $A^{(2)}$, $A^{(3)}$, $A^{(4)}$, $A^{(5)}$, and the final result R. You don't need to show the steps for boolean matrix multiplication of $A^{(2)}$, $A^{(3)}$, $A^{(4)}$, $A^{(5)}$.

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6. Compute the reachability matrix R using Warshall's algorithm for exercise 2. (12 points)

Hint: Please show the matrix M_0, M_1, M_2, M_3, M_4, M_5 and the final result $R = M_5$.

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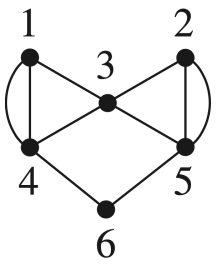
7.

(1) Determine whether this graph has an Euler path. If so, list the nodes in such a path. (4 points)
(2) Determine whether this graph has a Hamiltonian circuit. If so, list the nodes in such a circuit. (4 points)



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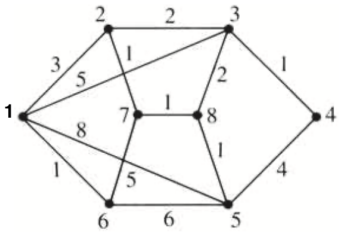
8.
Apply Dijkstra's algorithm for the following graph. Show the values for p and IN and the d values and s values for each pass. Write out the nodes in the shortest path from 2 to 5 and the distances of the path. (20 points)

Hint:

(1) p is the current node that has the shortest d that you will include in your current IN set.
(2) Node **2** is the **starting node**
(3) Node **5** is the **ending node**
(4) Please show all steps like we did during the class: draw the table for each step, calculate the new distance for each step, update the table, repeat until the ending node is included in the IN set, find the shortest path and the shortest length



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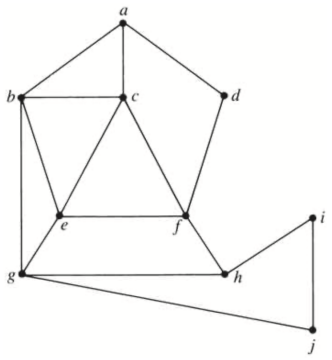
9.

(1) Write the nodes in a depth-first search of the following graph, beginning with the node a. (5 points)
(2) Write the nodes in a breadth-first search of the following graph, beginning with the node a. (5 points)



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