

- i) Switch any two rows
- ii) Multiply row by non-zero scalar
- iii) Add a scalar multiple of any one row to another row

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## Quiz 4 #3 Solution

3/21/22

If  $A$  is a  $n \times n$  invertible matrix, use Gaussian elimination method to find  $A^{-1}$ .

$$A = \begin{pmatrix} 2 & w & u \\ 7 & 0 & v \\ 1 & -3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 7 & 1 \\ 7 & 0 & -3 \\ 1 & -3 & 4 \end{pmatrix} \begin{pmatrix} \text{from} \\ \#2 \end{pmatrix}$$

1) Append identity matrix of same dimensions

Augmented Matrix =  $\begin{pmatrix} 2 & 7 & 1 & 1 & 0 & 0 \\ 7 & 0 & -3 & 0 & 1 & 0 \\ 1 & -3 & 4 & 0 & 0 & 1 \end{pmatrix}$

2) Multiply row 1 by  $1/2$  scalar

$$\begin{pmatrix} 1 & 7/2 & 1/2 & 1/2 & 0 & 0 \\ 7 & 0 & -3 & 0 & 1 & 0 \\ 1 & -3 & 4 & 0 & 0 & 1 \end{pmatrix}$$

3) Multiply row 1 by  $-1$  and add to row 3

$$\begin{pmatrix} 1 & 7/2 & 1/2 & 1/2 & 0 & 0 \\ 7 & 0 & -3 & 0 & 1 & 0 \\ 0 & -13/2 & 7/2 & -1/2 & 0 & 1 \end{pmatrix}$$

4) Multiply row 1 by  $-7$  and add to row 2

$$\begin{pmatrix} 1 & 7/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & -49/2 & -13/2 & -7/2 & 1 & 0 \\ 0 & -13/2 & 7/2 & -1/2 & 0 & 1 \end{pmatrix}$$

5) Multiply row 2 by  $-2/49$  scalar

$$\begin{pmatrix} 1 & 7/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 13/49 & 1/7 & -2/49 & 0 \\ 0 & -13/2 & 7/2 & -1/2 & 0 & 1 \end{pmatrix}$$

6) Multiply row 2 by  $-7/2$  and add to row 1

$$\begin{pmatrix} 1 & 0 & -3/7 & 0 & 1/7 & 0 \\ 0 & 1 & 13/49 & 1/7 & -2/49 & 0 \\ 0 & -13/2 & 7/2 & -1/2 & 0 & 1 \end{pmatrix}$$

7) Multiply row 2 by  $13/49$  and add to row 3

$$\begin{pmatrix} 1 & 0 & -3/7 & 0 & 1/7 & 0 \\ 0 & 1 & 13/49 & 1/7 & -2/49 & 0 \\ 0 & 0 & 256/49 & 3/7 & -13/49 & 1 \end{pmatrix}$$

8) Multiply row 3 by  $49/256$

$$\begin{pmatrix} 1 & 0 & -3/7 & 0 & 1/7 & 0 \\ 0 & 1 & 13/49 & 1/7 & -2/49 & 0 \\ 0 & 0 & 1 & 21/256 & -13/256 & 49/256 \end{pmatrix}$$

9) Multiply row 3 by  $-13/49$  and add to row 2

$$\begin{pmatrix} 1 & 0 & -3/7 & 0 & 1/7 & 0 \\ 0 & 1 & 0 & 31/256 & -7/256 & -13/256 \\ 0 & 0 & 1 & 21/256 & -13/256 & 49/256 \end{pmatrix}$$

10) Multiply row 3 by  $3/7$  and add to row 1

$$\begin{pmatrix} 1 & 0 & 0 & 9/256 & 31/256 & 21/256 \\ 0 & 1 & 0 & 31/256 & -7/256 & -13/256 \\ 0 & 0 & 1 & 21/256 & -13/256 & 49/256 \end{pmatrix}$$

Therefore:

$$A^{-1} = \begin{pmatrix} 9/256 & 31/256 & 21/256 \\ 31/256 & -7/256 & -13/256 \\ 21/256 & -13/256 & 49/256 \end{pmatrix}$$