1. Disprove the following:

Every animal with wings is a brid.

Bats are not birds

2. Give a direct proof of the following:

if $4x^2 - 16x + 16 = 0$, then $x = 2$

P: $4x^2 - 16x + 16 = 0$
Q: $x = 2$
P $\rightarrow$ Q

Proof:

$4x^2 - 16x + 16 = 0$ (hypothesis)
$4(x^2 - 4x + 4) = 0$ (algebra)
$x^2 - 4x + 4 = 0$ (algebra)
$(x - 2)^2 = 0$ (algebra)
$x - 2 = 0$ (algebra)
$x = 2$ (algebra)

3. Give a proof by contrapositive of the following

if $x - 2 > 0$, then $x > 0$ (be very careful!)

P: $x - 2 > 0$
Q: $x > 0$
P -> Q

Q': $x <= 0$
P': $x - 2 <= 0$
Q' -> P'

x <= 0
x - 2 <= 0 - 2 (algebra)
x - 2 <= - 2 (algebra)
x - 2 <= 0 (algebra)

4. Give a proof by contradiction of the following:

if $2x^2 - 8x + 8 = 0$, then $x \neq 3$

P: $2x^2 - 8x + 8 = 0$
Q: $x \neq 3$
P -> Q

Q': $x = 3$
P $\wedge$ Q' -> 0

Proof by contradiction:

$2x^2 - 8x + 8 = 0$ (hypothesis)
$x = 3$ (hypothesis)
$2 * 3^2 - 8 * 3 + 8 = 0$
$2 * 9 - 8 * 3 + 8 = 0$
$18 - 24 + 8 = 0$
$2 = 0$ (contradiction)

5. Give an exhaustive proof of the following:

if $0 < x < 5$, then $2x^2 + 1 > 2$

x	$2x^2 + 1$	$2x^2 + 1 > 2$
1	3	Yes
2	9	Yes
3	19	Yes
4	33	Yes

6. Prove by mathematical induction that the following is true. Show all work.

$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}, n \geq 2$

Base step:
P(2):
left side: $1^2 + 2^2 = 5$
right side: $\frac{2*(2+1)*(2*2+1)}{6} = \frac{2*3*5}{6} = 5$
P(2) is true.

Induction step: P(k) -> P(k+1)

P(k): $1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$

Proof:

P(k+1): $1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$

$1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$ (hypothesis)

$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$ (algebra)

$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{(k(k+1)(2k+1)+6(k+1)^2)}{6}$ (algebra)

$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)(k(2k+1)+6(k+1))}{6}$ (algebra)

$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)(2k^2+k+6k+6)}{6}$ (algebra)

$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)(2k^2+7k+6)}{6}$ (algebra)

$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$ (algebra)

$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$ (algebra)

P(k) → P(k+1) is true

In [ ]: