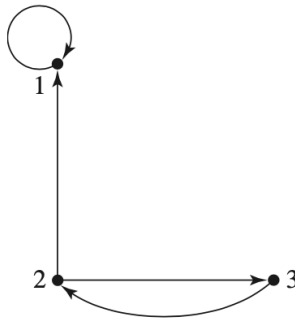


```
In [2]: ### CSCI-3080 Discrete Structure
### OLA 5: Chapter 7 -- Graphs and Algorithms
### Name:
### Student ID:
### Date:
```

1. Find the adjacency matrix and adjacency relation (binary relation) for the following graph.



In [ ]:

2. Find the corresponding directed graph and adjacency relation (binary relation) for the following adjacency matrix.

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

In [ ]:

3. Given the adjacency relation  $\rho = \{(1, 4), (1, 5), (1, 6), (6, 2), (6, 3), (6, 5)\}$  on the set of nodes  $\{1, 2, 3, 4, 5, 6\}$  find the corresponding directed graph and adjacency matrix.

In [ ]:

4. Let  $A$  be the following matrix. Find the products  $A^2$  and  $A^{(2)}$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

In [ ]:

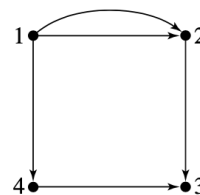
5. Compute the reachability matrix  $\mathbf{R}$  using the formula  $\mathbf{R} = \mathbf{A} \vee \mathbf{A}^{(2)} \vee \dots \vee \mathbf{A}^{(n)}$  for exercise 2.

In [ ]:

6. Compute the reachability matrix  $\mathbf{R}$  using **Warshall's algorithm** for exercise 2.

In [ ]:

7. For the following graph, count the number of paths of length 2 from node 1 to node 3. Check by computing  $A^2$  (**The solution is given**)



```
In [6]: import numpy as np
A = np.array([[0, 2, 0, 1], [0, 0, 1, 0], [0, 0, 0, 0], [0, 0, 1, 0]])
A
```

```
Out[6]: array([[0, 2, 0, 1],
               [0, 0, 1, 0],
               [0, 0, 0, 0],
               [0, 0, 1, 0]])
```

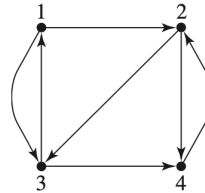
```
In [4]: A2 = np.matmul(A,A)
A2
```

```
Out[4]: array([[0, 0, 3, 0],
               [0, 0, 0, 0],
               [0, 0, 0, 0],
               [0, 0, 0, 0]])
```

There are three paths of length 2 from node 1 to node 3.

8.

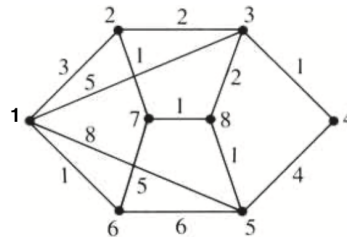
- (1) Determine whether this graph has an **Euler path**. If so, list the nodes in such a path.
- (2) Determine whether this graph has a **Hamiltonian circuit**. If so, list the nodes in such a circuit.



In [ ]:

9. Apply **Dijkstra's algorithm** for the following graph. Show the values for **p** and **IN** and the **d** values and **s** values for each pass. Write out the nodes in the **shortest path** from **2** to **5** and the distances of the path.

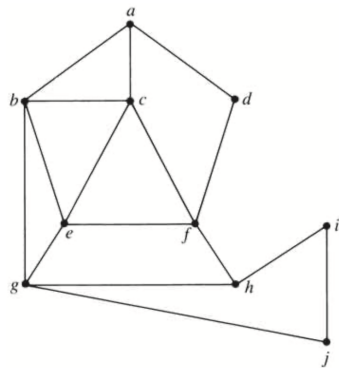
Hint: p is the current node that has the shortest d that you will include in your current IN set.



In [ ]:

10.

- (1) Write the nodes in a **depth-first search** of the following graph, beginning with the node **a**.
- (2) Write the nodes in a **breadth-first search** of the following graph, beginning with the node **a**.



In [ ]: