Middle Tennessee State University
College of Basic and Applied Science
Department of Computer Science

CSCI-3080 Discrete Structures — Test 1

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Date: Feb 29th, 2024 (Thursday)

Total Points: 100 points

Name: _____ **Student ID**: _____

1: Write the negation of each statement. (8 points each question, 2 questions) Total: 16 points

Please first translate the following compound statements into symbolic notation (2 points);

then negate the symbolic notation (4 points);

finally, translate the negation symbolic notation into English (2 points).

$P \to Q \leftrightarrow P' \vee Q$ **(Implication – imp)**

$(P \vee Q)' \leftrightarrow P' \wedge Q'$ **(De Morgan)**

$(P \wedge Q)' \leftrightarrow P' \vee Q'$ **(De Morgan)**

1. If the food is good, then the service is excellent.

2. If the price is high, then the food is good and the service is excellent.

2: Construct truth table for the following wff. Note it is tautology or contradiction, or neither. (Total: 12 points)

$A \wedge B \rightarrow A'$

(1) How many rows will be in the truth table of a wff that contains 2 variables (2 points)?

(2) Construct a truth table. (8 points)

(3) Please note it is a tautology or contradiction, or neither. (2 points)

3: In an account that pays 5% annually (i.e. interest is added at the end of the year to be available at the beginning of the next year), $5000 is deposited. At the end of each year, an additional $500 dollars is deposited into the account. (Total: 12 points)

(a) Please write a **recurrence relation** for the amount in the account at the beginning of year n (2 points)
Hint:

```
P(1) = 5000;
P(n) =
```

(b) Please find the **closed-form solution** of the recurrence relation (note it is linear, first order, constant coefficient) (10 points)
(1) $S(n) = cS(n-1) + g(n)$
(2) $S(n) = c^{n-1}S(1) + \sum_{i=2}^{n} c^{n-i}g(i)$
(3) $\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1} (a \neq 1)$

4: Write the following argument using propositional wff, then prove that the argument is valid using a proposition logic proof sequence. You may use any of the rules in Table 1. (Total: 14 points)

If a Democrat is elected then taxes will go up. Either a Democrat will be elected or the bill will pass. Therefore, if taxes do not go up, then the bill will pass. (**D, T, B**)

Table 1

$P \to Q \leftrightarrow P' \vee Q$ (Implication – imp)
$P \to Q \leftrightarrow Q' \to P'$ (Contraposition – cont)
$P \leftrightarrow (P')'$ (Double negation – dn)
$(P, P \to Q) \to Q$ (Modus ponens – mp)
$(P \to Q) \wedge (Q \to R) \to (P \to R)$ (Hypothetical syllogism – hs)
$(P \to Q, Q') \to P'$ (Modus tollens – mt)

(1) Please write your propositional wff here: (4 points)

(2) Proof Sequence: (10 points)

5: Induction (Total: 15 points)

For all positive integers $n \geq 1$, let $P(n)$ be the equation:

$$2 + 6 + 10 + \ldots + (4n - 2) = 2n^2$$

a. Write the equation for the base case $P(1)$ and verify that it is true. (2 points)

b. Write the inductive hypothesis equation for $P(k)$ (3 points)

c. Write the equation for $P(k + 1)$ (3 points)

d. Prove that $P(k + 1)$ is true. (7 points)

6: Please prove the following statement using exhaustive proof. (Total: 8 points)

For any positive integer less than or equal to 5, the square of the integer is less than or equal to **the sum of 10 plus 5 times the integer (10+5n)**. **Note:** 0 is not included.

7: Give a proof by contradiction of the following: (Total: 8 points)

If $2x^2 - 8x + 8 = 0, \quad then \quad x \neq 3$

8: Please prove the following statement using proof by contraposition ($P \to Q \leftrightarrow Q' \to P'$) (Total: 10 points)

If a number x is positive, so is $x + 1$.

Hint:
(1) Write the correct P (1 point)
(2) Write the correct Q (1 point)
(3) Write the correct Q' (2 points)
(4) Write the correct P' (2 points)
(5) Proof (4 points)

10: Please convert the following recursive algorithm into the iterative algorithm in C++ (Total: 5 points)

```
T(1) = 2
T(n) = nT(n-1) + n for n > 1
```

The following code is a version of the recursive algorithm for the above relation T(n), written in C++:

```
int T_recur(int n)
{
     if(n == 1)
        return 2;
     else
        return n*T_recur(n-1) + n;
}
```