Middle Tennessee State University
College of Basic and Applied Science
Department of Computer Science

CSCI-3080 Discrete Structures — Test 2

Instructor: Xin Yang

Date: April 22nd, 2022 (Friday)

Total Points: 100 points

98

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1: Matrix Operation. (5 points each question, 2 questions) Total: 10 points

(a) For r = 2, $A = \begin{pmatrix} 1 & 7 \\ -3 & 4 \\ 5 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 0 \\ 9 & 2 \\ -1 & 4 \end{pmatrix}$, please find rA + B.

$$2\begin{bmatrix} 1 & 7 \\ -3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 9 & 2 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 14 \\ -6 & 8 \\ 10 & 12 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 9 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 14 \\ 3 & 10 \\ 9 & 16 \end{bmatrix}$$

(b) Let A and B be Boolean matrices, $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$, please find the Boolean Matrix Multiplication product $A \times B$.

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

2: Solve the following system of equations using Gaussian elimination. (Total: 5 points)

$$\begin{aligned} 3x - 5y &= 5 \\ 7x + y &= 37 \end{aligned}$$

$$= \begin{bmatrix} 3 & -5 & 5 \\ 7 & 1 & 37 \end{bmatrix}$$

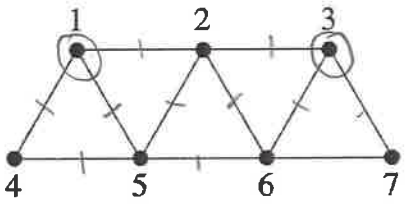
$$= \begin{bmatrix} 1 & -5/3 & 5/3 \\ 0 & 38/3 & 76/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \end{bmatrix}$$

$X = 5$

$Y = 2$

3: Please answer the following questions refer to the following graph. (2 points each, 4 questions. Total: 8 points)



(a) Is the graph simple? Why?
yes, it's undirected, unweighted, + has no loops or parallel arcs

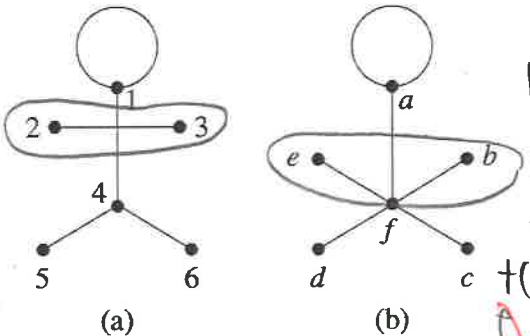
(b) Is the graph connected? Why?
yes. b/c you can get to any/every node starting at any node.

(c) Can you find a Euler Path? If so, please list the Euler Path.
yes. 1, 4, 5, 1, 2, 5, 6, 2, 3, 6, 7, 3

(d) Can you find a Hamiltonian Circuit? If so, please list the Hamiltonian Circuit. NO b/c you cannot start at a node and end at same node w/o repeating arcs/nodes.

1 → 2 → 3 → 7 → 6 → 5 → 4 → 1 -2

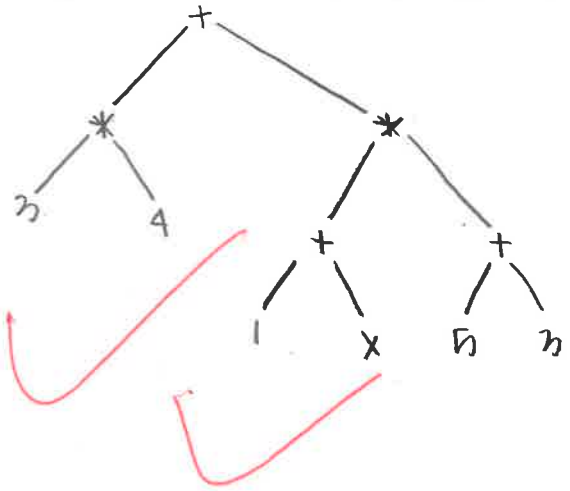
4: Please decide if the two graphs are isomorphic. If so, give the function or functions that establish the isomorphism; if not, explain why. (Total: 2 points)



Not isomorphic b/c nodes 2 and 3 are not connected to the graph, they're connected just to each other, while e and b are connected to other nodes. The 2 graphs have different relations

5: Trees and Representations (5 points each question, 3 questions. Total: 15 points)

(a) Please draw the expression tree for (3*4) + [(1+x)*(5 + 3)]



(b) Please write the list of nodes resulting from a preorder (prefix) traversal of the above tree.

+, *, 3, 4, *, +, 1, x, +, 5, 3

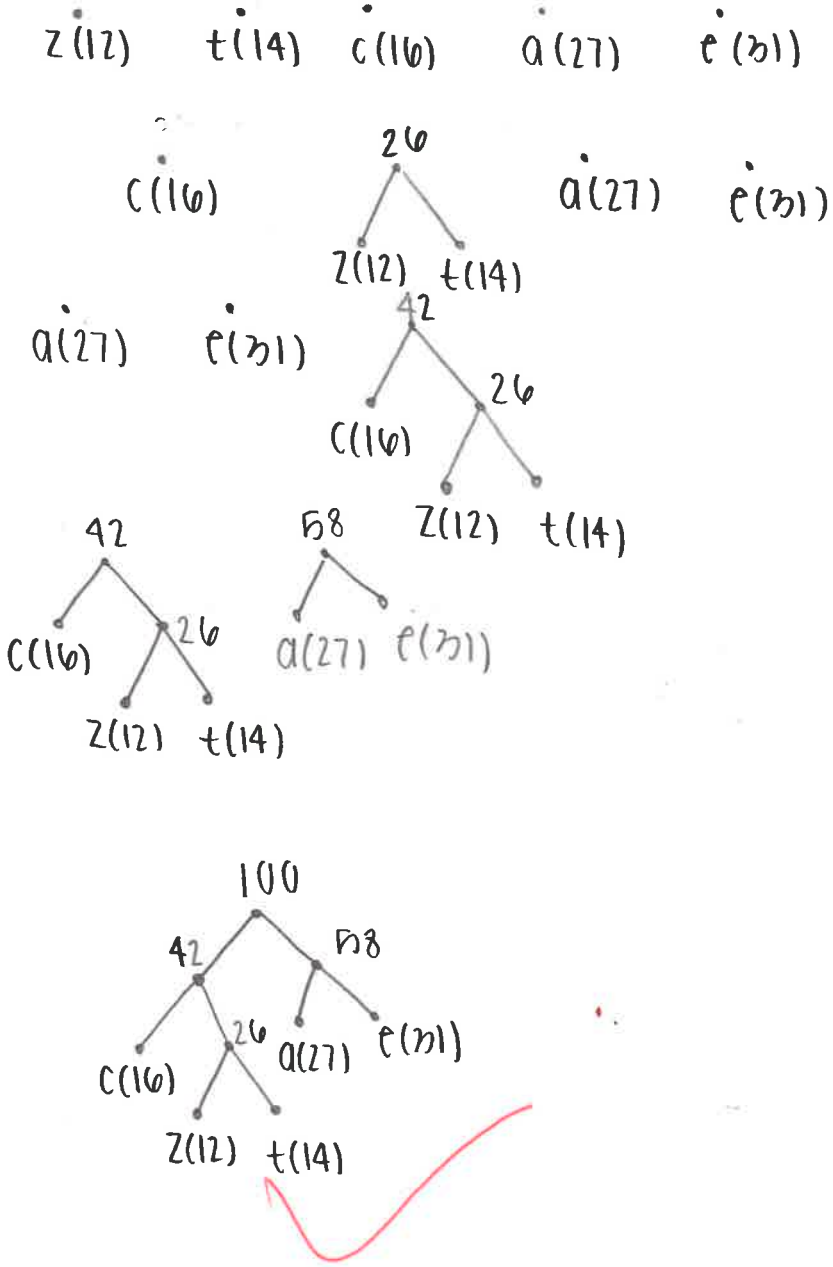
(c) Please write the list of nodes resulting from a postorder (postfix) traversal of the above tree.

3, 4, *, 1, x, +, 5, 3, +, *, +

6: Huffman tree. (4 questions. Total: 20 points)

(a) Construct the Huffman tree for the following characters and frequencies. (10 points)

Character	a	z	t	e	c
Frequency	27	12	14	31	16



(b) Please find the Huffman codes for these characters. (5 points)

a: 10
z: 010
t: 011
e: 11
c: 00

(c) A file consisting of 100,000 instances of these five characters is stored using a fixed-length binary encoding scheme. How many bits are required for each code and what is the total number of bits needed? (2.5 points)

$2^1 = 2$, $2^2 = 4$

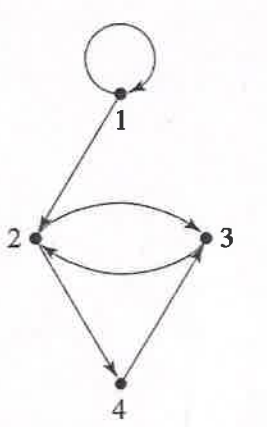
$3(100,000) = 300,000$ bits

(d) Storing the same file (100,000 instances of these five characters) using the Huffman code of part (b), how many bits are needed? (2.5 points)

$100000[(0.27*2) + (0.12*3) + (0.14*3) + (0.31*2) + (0.16*2)]$

$= 226,000$ bits

7: Please answer the following questions using the given graph. (5 points each question, 3 questions. Total: 15 points)



(a) Please write the adjacency matrix for the given directed graph.

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(b) Please count how many different paths of length 2 there are using $A^{(2)}$ (Boolean Matrix Multiplication)

$$A^{(2)} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

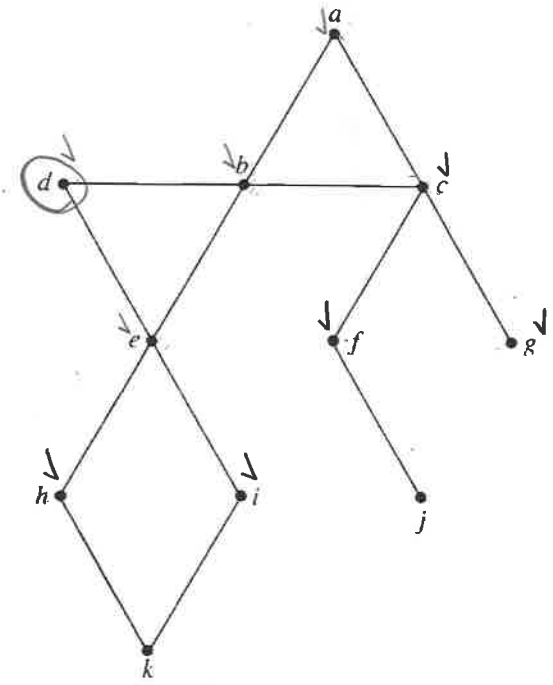
9 paths

(c) What's M_1 using Warshall's algorithm?

$$M_0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

8: Traversal Algorithms (5 points each question, 2 questions. Total: 10 points)



(a) write the nodes in a depth-first search of the above graph beginning with the node **d**.

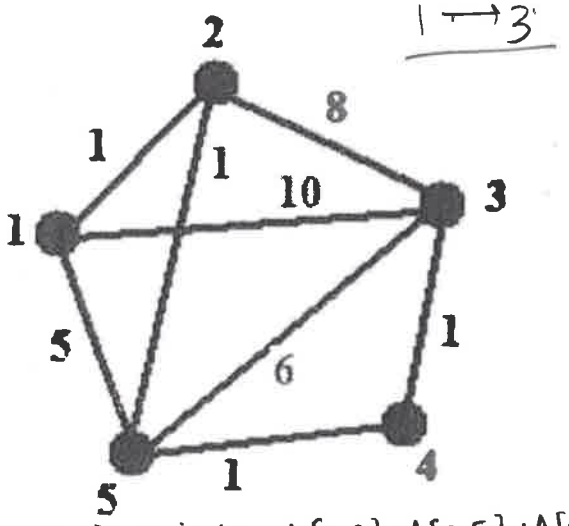
d, b, a, c, f, j, g, e, h, k, i

(b) write the nodes in a breadth-first search of the graph beginning with the node **d**.

d, b, e, a, c, h, i, f, g, k, j

9: Use the Dijkstra's algorithm to find the shortest path from node 1 to node 3 and the length of that path in the following graph. Show all work. (Total: 15 points)

	1	2	3	4	5
1	0	1	10	∞	5
2	1	0	8	∞	1
3	10	8	0	1	6
4	∞	∞	1	0	1
5	5	1	6	1	0



IN = {1}

	1	2	3	4	5
d	0	1	10	∞	5
S	-	1	1	1	1

p = 2 IN = {1,2}

	1	2	3	4	5
d	0	1	9	∞	2
S	-	1	2	1	2

d[3] = min(10, A[1,2]+A[2,3])
= min(10, 1+8) = 9
d[4] = min(∞, A[1,2]+A[2,4])
= min(∞, 1+∞) = ∞
d[5] = min(5, A[1,2]+A[2,5])
= min(5, 1+1) = 2

p = 5 IN = {1,2,5}

	1	2	3	4	5
d	0	1	8	3	2
S	-	1	5	5	2

d[3] = min(9, A[1,2]+A[2,5]+A[5,3])
= min(9, 2+6) = 8

d[4] = min(∞, A[1,2]+A[2,5]+A[5,4])
= min(∞, 2+1) = 3

p = 4 IN = {1,2,5,4}

	1	2	3	4	5
d	0	1	4	3	2
S	-	1	4	5	2

d[3] = min(8, A[1,2]+A[2,5]+A[5,4]+A[4,3])
= min(8, 3+1) = 4

p = 3 IN = {1,2,5,4,3}

	1	2	3	4	5
d	0	1	4	3	2
S	-	1	4	5	2

1 → 2 → 5 → 4 → 3

length 4