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### CSCI-3080 Discrete Structure
### OLA 2: Chapter 3 -- Recursive Definitions, Recurrence Relations
### Name:
### Student ID:
### Date:
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Exercise 1: Write the first 5 values in the sequence:

$C(1) = 5$
$C(n) = 2C(n-1) + 5$ **for** $n > 1$ ¶

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Exercise 2: Write the first 5 values in the sequence:

$A(1) = 2$
$A(n) = nA(n-1) + n$ **for** $n > 1$

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Exercise 3: An amount of $500 is invested in an account paying 1.2% interest compounded annually (meaning they don't add interest until the end of the year, so the start of the next year you have the money plus interest)

(a) Please write a recursive definition for $P(n)$, the amount in the account at the begining of the n_{th} year.

Hint: P(1) = 500
For n > 1, to calculate the amount in the n_{th} year, you need to use (1+0.012) times the amount in the $(n-1)_{th}$ year.

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(b) After how many years will the account balance exceed $570? (You can write a python program to calculate)

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Exercise 4: Please write the python code of a recursive function to compute the sequence

$$1, 2, 4, 7, 11, 16, 22, \ldots$$

Begin by defining the recursive definition

$S(1) = 1$
$S(n) = S(n-1) + n - 1$ **for** $n > 1$

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Exercise 5: Please find the closed form solution for the recurrence relation subject to the basis step.

Hint: Please use the linear,first-order, constant coefficient formula: $S(n) = c^{n-1}S(1) + \sum_{i=2}^{n} c^{n-i}g(i)$

$B(1) = 5$
$B(n) = 3B(n-1)$ **for** $n > 1$

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Exercise 6: In an account that pays 3% annually (i.e. interest is added at the end of the year to be available at the beginning of the next year), $1000 is deposited. At the end of each year, and additional 100 dollars is deposited into the account.

A. Please write a recurrence relation for the amount in the account at the beginning of year n

Hint: P(1) = 1000
For n > 1, to calculate the amount in the n_{th} year, you need to use (1+0.03) times the amount in the $(n-1)_{th}$ year and plus additonal 100 dollars.

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B. Please find the closed-form solution of the recurrence relation (note it is linear, first order, constant coefficient)

Hint 1: Please use the linear,first-order, constant coefficient formula:

$$S(n) = c^{n-1}S(1) + \sum_{i=2}^{n} c^{n-i} g(i)$$

Hint 2: $\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1} (a \neq 1)$

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C. What is the account worth at the begining of the 8_{th} year?

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