Middle Tennessee State University
College of Basic and Applied Science
Department of Computer Science

CSCI-3080 Discrete Structures — Test 2

Instructor: Xin Yang

Date: April 18th, 2024 (Thursday)

Total Points: 100 points

Name: _____ **Pipeline ID**: _____

1: Matrix Operation. (5 points each question, 2 questions) Total: 10 points

(a) For r = 2, A = $\begin{pmatrix} 1 & 7 \\ -3 & 4 \\ 5 & 6 \end{pmatrix}$, B = $\begin{pmatrix} 4 & 0 \\ 9 & 2 \\ -1 & 4 \end{pmatrix}$, please find rA + B.

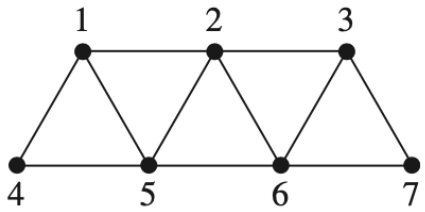
(b) Let A and B be Boolean matrices, A = $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, B = $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$, please find the **Boolean Matrix Multiplication** product A $\times B$.

2: Solve the following system of equations using Gaussian elimination. (Total: 5 points)

Please show all steps.

$$\begin{aligned} 3x - 5y &= 5 \\ 7x + y &= 37 \end{aligned}$$

3: Please answer the following questions refer to the following graph. (2 points each, 4 questions. Total: 8 points)



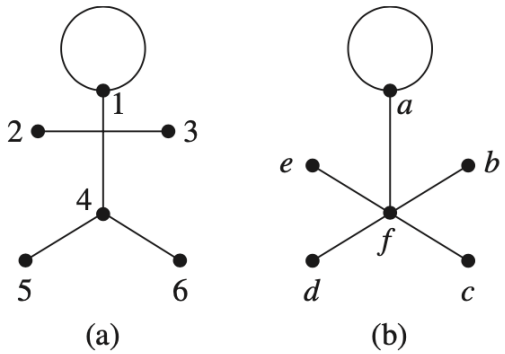
(a) Is the graph simple? Why?

(b) Is the graph connected? Why?

(c) Can you find a Euler Path? If so, please list the Euler Path.

(d) Can you find a Hamiltonian Circuit? If so, please list the Hamiltonian Circuit.

4: Please decide if the two graphs are isomorphic. If so, give the function or functions that establish the isomorphism; if not, explain why. (Total: 2 points)



5: Trees and Representations (5 points each question, 3 questions. Total: 15 points)

(a) Please draw the binary expression tree for (3*4) + [(1+x)*(5 + 3)]

(b) Please write the list of nodes resulting from a **preorder** (prefix) traversal of the above tree.

(c) Please write the list of nodes resulting from a **postorder** (postfix) traversal of the above tree.

6: Huffman tree. (4 questions. Total: 20 points)

(a) Construct the Huffman tree for the following characters and frequencies. (10 points)

Character	a	z	t	e	c
Frequency	27	12	14	31	16

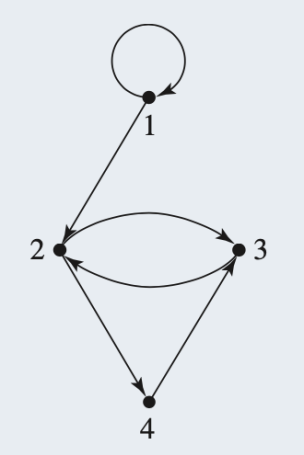
Please show all the steps.

(b) Please find the Huffman codes for these characters. (5 points)

(c) A file consisting of 10,000 instances of these five characters is stored using a **fixed-length** binary encoding scheme. How many bits are required for each code and what is the total number of bits needed? (2 points)
Note: please don't use ASCII code here.

(d) Storing the same file (10,000 instances of these five characters) using the Huffman code of part (b), how many bits are needed? (3 points)

7: Please answer the following questions using the given graph. (5 points each question, 3 questions. Total: 15 points)

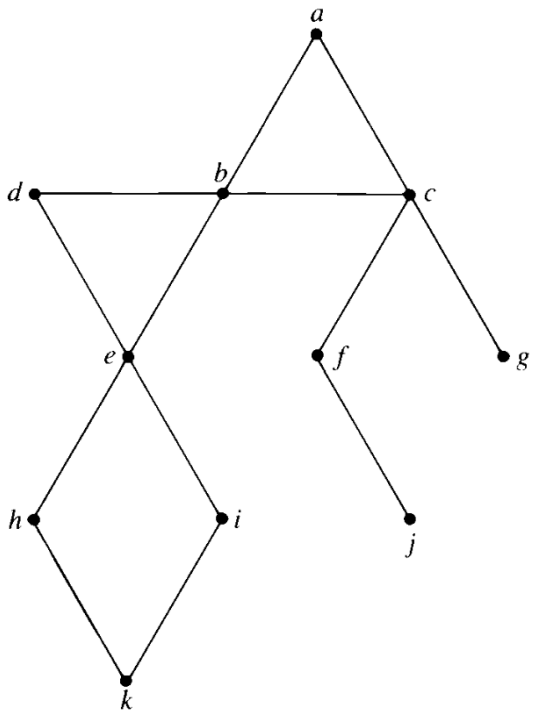


(a) Please write the **adjacency matrix** for the given directed graph.

(b) Please count how many different paths of **length 2** there are using $A^{(2)}$ (Boolean Matrix Multiplication)

(c) What's M_1 using Warshall's algorithm?

8: Traversal Algorithms (5 points each question, 2 questions. Total: 10 points)

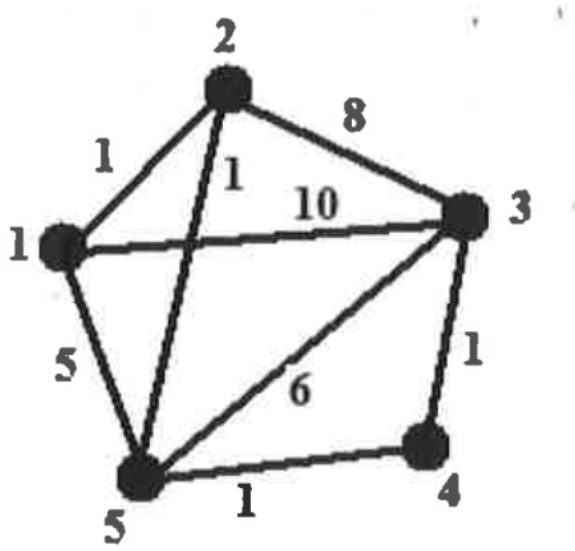


(a) write the nodes in a **depth-first** search of the above graph beginning with the node **d**. **Starting node: d**

(b) write the nodes in a **breadth-first** search of the graph beginning with the node **d**. **Starting node: d**

9: Use the Dijkstra's algorithm to find the shortest path from node 3 to node 1 and the length of that path in the following graph. Show all work. (Total: 15 points)

Starting node: 3
Ending node: 1



Scratch Paper 1

Scratch Paper 2

Scratch Paper 3

Scratch Paper 4