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### CSCI-3080 Discrete Structure
### OLA 1: Chapter 1--Formal Logic & Chapter 2 -- Proofs, Induction
### Name:
### Student ID:
### Date:
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Part 1: Please use formal logic to finish the following 4 exercises. (60 points)

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Exercise 1: Please find the negation of each statement (12 points)

(1) The processor is fast but the printer is slow. (4 points)
(2) If the processor is fast, then the printer is slow. (4 points)
(3) Either the processor is fast and the printer is slow, or else the file is damaged. (4 points)

Please follow the following three steps:
1. First translate the English words into wffs using the formal symbols.
2. Negate the wffs.
3. Translate the negated wffs back into English words.

Note: (3) and (5) rely on the fact that $A \rightarrow B \leftrightarrow A' \vee B$ (Implication rule).
so $(A \rightarrow B)' \leftrightarrow (A' \vee B)'$, $(A \rightarrow B)' \leftrightarrow A \wedge B'$ by De Morgan's law

A: The processor is fast
B: The printer is slow
C: The file is damaged

(1) $A \wedge B$
$(A \wedge B)' = A' \vee B'$
The processor is slow or the printer is fast.

(2) $A \rightarrow B$
$(A \rightarrow B)' = (A' \vee B)' = A \wedge B'$
The processor is fast and the printer is fast.

(3) $(A \wedge B) \vee C$
$((A \wedge B) \vee C)' = (A \wedge B)' \wedge C' = A' \vee B' \wedge C'$
The processor is slow or the printer is fast, and the file is not damaged.

Exercise 2: Construct truth tables for the following wffs. Note any tautologies or contradictions (20 points)

(a) (A → B) ↔ A' ∨ B
(b) (A ∧ B)' ↔ A' ∨ B'

(a) Truth table for (A → B) ↔ A' ∨ B

A	**B**	**A → B**	**A'**	**A' ∨ B**	**(A → B) ↔ A' ∨ B**
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

(b) Truth table for (A ∧ B)' ↔ A' ∨ B'

A	**B**	**A ∧ B**	**(A ∧ B)'**	**A'**	**B'**	**A' ∨ B'**	**(A ∧ B)' ↔ A' ∨ B'**
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

(a) and (b) are tautologies

Exercise 3: Use a proof sequence to prove that the following argument is valid. (14 points)

(A → B) ∧ [B → (C → D)] ∧ [A → (B → C)] → (A → D)

Proof:

1. A → B (hypothesis)
2. B → (C → D) (hypothesis)
3. A → (B → C) (hypothesis)
4. A (hypothesis)

5. B (1,4 mp)
6. C → D (2,5 mp)
7. B → C (3,4 mp)
8. C (5,7 mp)
9. D (6,8 mp)

Exercise 4: Write the following argument using propositional wffs, then prove that the argument is valid using a proof sequence. (14 points)

If DeWayne is not tall, then Jayden is not DeWayne's brother. If DeWayne is tall, then Trevor is DeWayne's brother. Therefore, if jayden is DeWayne's brother, then Trevor is DeWayne's brother.

Hint: you can use D,J,T as the statement letters
D: DeWayne is tall.
J: Jayden is DeWayne's brother
T: Trevor is DeWayne's brother

Proposition Logic: (D' → J') ∧ (D → T) → (J → T)
Proof:

1. D' → J' (hypothesis)
2. D → T (hypothesis)
3. J (hypothesis)
4. D (1,3 mt)
5. T (2,4 mp)

Part 2: Please use Proofs, Induction to finish the following 6 exercises. (80 points)

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Exercise 1: Provide a counter example to the following statement: (4 points)

The number n is an odd integer if and only if $3n + 5$ is an even integer.

This is actually the statement: (If n is an odd integer then $3n + 5$ is an even integer) AND (If $3n + 5$ is an even integer, then n is an odd integer)

Let 3n + 5 = 6 (note doing a counter example is the ONLY time you can pick a specific test case)
Then 3n = 1 which means n = 1/3 (but 1/3 is NOT an odd integer... it isn't even an integer)

Exercise 2: Please prove the following statement using exhaustive proof. (10 points)

For 2 ≤ n ≤ 4 , $n^2 \geq 2^n$

n	n^2	2^n	$n^2 \geq 2^n$
2	**4**	**4**	**yes**
3	**9**	**8**	**yes**

4	**16**	**16**	**yes**

Exercise 3: Please prove the following statement using direct proof (10 points)

The sum of an even integer and an odd integer is odd.

Rewrite mathematically: If x is even and y is odd, then $x + y$ is odd.

1. x = 2m (hypothesis)
2. y = 2n + 1 (hypothesis)
3. x + y = 2m + 2n + 1 (substitution)
4. x + y = 2(m+n) + 1 (algebra)

Exercise 4: Please prove the following statement using proof by contradiction. (10 points)

If $x^2 + 2x - 3 = 0$, then x ≠ 2

1. $x^2 + 2x - 3 = 0$ (hypothesis)
2. x = 2 (hypothesis)
3. $2^2 + 2 \times 2 - 3 = 0$ (substitution)
4. 4 + 4 - 3 = 0 (contradiction)

Exercise 5: For all positive integers, let $P(n)$ be the equation: (16 points)

$$2 + 4 + 6 + \cdots + 2n = n(n + 1)$$

a. Write the equation for the base case $P(1)$ and verify that it is true.
b. Write the inductive hypothesis $P(k)$
c. Write the equation for $P(k + 1)$
d. Prove that $P(k + 1)$ is true given b

(a) LHS: P(1) = $2 \times 1 = 2$; RHS: P(1) = $1 \times (1 + 1) = 2$, since that LHS == RHS, it's true
(b) P(k) = 2 + 4 + 6 + ... + 2k = k(k+1) (hypothesis)
(c) P(k+1) = 2 + 4 + 6 + ... + 2k + 2(k+1) = (k+1)((k+1)+1)
(d)2 + 4 + 6 + ... + 2k + 2(k+1) = k(k+1) + 2(k+1) (1, algebra)
2 + 4 + 6 + ... + 2k + 2(k+1) = (k+1)(k+2) = (k+1)((k+1)+1) (2, algebra)

Exercise 6: Please prove the following statements are true for every

positive integer n using mathematical induction. (30 points)

(1) $2+6+18+\cdots+2\times 3^{n-1} = 3^n - 1$ **(15 points)**

(2) $1\times 2^1 + 2\times 2^2 + 3\times 2^3 + \cdots + n\times 2^n = (n-1)\times 2^{n+1} + 2$ **(15 points)**

(1)

(1) Basis: Show P(1) is true
on the left side the end number in the series is $2\times 3^{1-1}$ which is $2\times 3^0 = 2\times 1 = 2$
on the right side: $3^1 - 1 = 2$
Since LHS == RHS, we prove that P(1) is true

Induction: Show P(k) -> P(k+1) (16 points)

1. P(k) : 2 + 6 + 18 + ... + $2\times 3^{k-1} = 3^k - 1$ (Hypothesis)
2. P(k+1): 2 + 6 + 18 + ... + $2\times 3^{k-1}$ + 2×3^k = $3^{k+1} - 1$ (We need to prove)
3. 2 + 6 + 18 + ... + $2\times 3^{k-1}$ + 2×3^k = 3^k - 1 + 2×3^k (1, algebra)
4. 2 + 6 + 18 + ... + $2\times 3^{k-1}$ + 2×3^k = 3 $\times$ 3^k - 1 (2, algebra) because x + 2x is 3x and here x is 3^k
5. 2 + 6 + 18 + ... + $2\times 3^{k-1}$ + 2×3^k = 3^{k+1} - 1(3, algebra)

(2)

(2) Basis: Show P(1) is true
on the left side the first number in the series is $1\times 2^1 = 2$
on the right side: $(1-1)\times 2^{1+1} + 2 = 2$
Since LHS == RHS, we prove that P(1) is true

Induction: Show P(k) -> P(k+1) (16 points)

1. P(k): $1\times 2^1 + 2\times 2^2 + 3\times 2^3 + \ldots + k\times 2^k = (k-1)\times 2^{k+1} + 2$ (Hypothesis)
2. P(k+1): $1\times 2^1 + 2\times 2^2 + 3\times 2^3 + \ldots + k\times 2^k + (k+1)\times 2^{k+1} = (k+1-1)\times 2^{k+1+1} + 2$ (We need to prove)
3. $1\times 2^1 + 2\times 2^2 + 3\times 2^3 + \ldots + k\times 2^k + (k+1)\times 2^{k+1} = (k-1)\times 2^{k+1} + 2 + (k+1)\times$ (1, algebra)
4. $1\times 2^1 + 2\times 2^2 + 3\times 2^3 + \ldots + k\times 2^k + (k+1)\times 2^{k+1} = k\times 2^{k+1} - 2^{k+1} + 2 + k\times 2^{k+}$ (2, algebra)

$= 2 \times k \times 2^{k+1} + 2$
$= k \times 2^{k+1+1} + 2$
$= (k + 1 - 1) \times 2^{k+1+1} + 2$

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