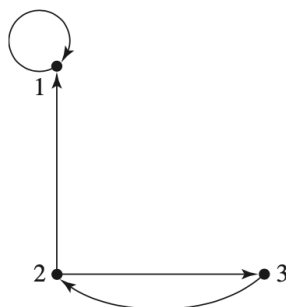


```
In [2]: ### CSCI-3080 Discrete Structure
### OLA 5: Chapter 7 -- Graphs and Algorithms
### Name:
### Student ID:
### Date:
### Total: 100 Points
```

1. Find the adjacency matrix and adjacency relation (**binary relation**) for the following graph. (8 points)



In [ ]:

2. Find the corresponding **directed graph** and **adjacency relation** (binary relation) for the following adjacency matrix.(10 points)

directed graph (5 points)  
binary relation(5 points)

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

In [ ]:

3. Given the adjacency relation  $\rho = \{(1, 4), (1, 5), (1, 6), (6, 2), (6, 3), (6, 5)\}$  on the set of nodes  $\{1, 2, 3, 4, 5, 6\}$  find the corresponding **directed graph** and **adjacency matrix**.(10 points)

directed graph (5 points)  
adjacency matrix(5 points)

In [ ]:

4. Let  $A$  be the following matrix. Find the products  $A^2$  and  $A^{(2)}$  (10 points)

$A^2 = A \cdot A$  (regular matrix multiplication) (5 points)

$A^{(2)} = A \times A$  (boolean matrix multiplication) (5 points)

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

In [ ]:

5. Compute the reachability matrix  $R$  using the formula  $R = A \vee A^{(2)} \vee \dots \vee A^{(n)}$  for the following adjacency matrix: (12 points)

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

**Hint:** Please show the matrix  $A$ ,  $A^{(2)}$ ,  $A^{(3)}$ ,  $A^{(4)}$ , and the final result  $R$ . You don't need to show the steps for boolean matrix multiplication of  $A^{(2)}$ ,  $A^{(3)}$ ,  $A^{(4)}$ .

In [ ]:

6. Compute the reachability matrix  $R$  using **Warshall's algorithm** for exercise 5. (12 points)

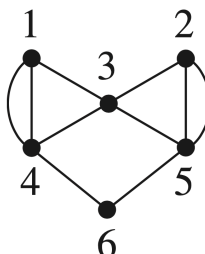
**Hint:** Please show the matrix  $M_0$ ,  $M_1$ ,  $M_2$ ,  $M_3$ ,  $M_4$  and the final result  $R = M_4$ .

In [ ]:

7.

(1) Determine whether this graph has an **Euler path**. If so, list the nodes in such a path. (4 points)

(2) Determine whether this graph has a **Hamiltonian circuit**. If so, list the nodes in such a circuit. (4 points)



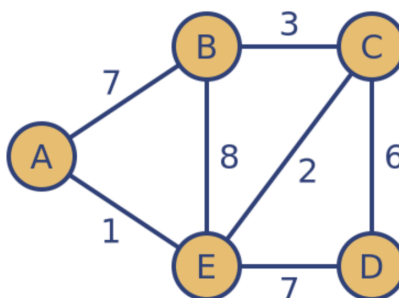
In [ ]:

8.

Apply **Dijkstra's algorithm** for the following graph. Show the values for **p** and **IN** and the **d** values and **s** values for each pass. Write out the nodes in the **shortest path** from **A** to **D** and the distances of the path. (20 points)

Hint:

- (1) p is the current node that has the shortest d that you will include in your current IN set.
- (2) Node **A** is the **starting node**.
- (3) Node **D** is the **ending node**.
- (4) Please show all steps like we did during the class: draw the table for each step, calculate the new distance for each step, update the table, repeat until the ending node is included in the IN set, find the **shortest path** and the **shortest length**.

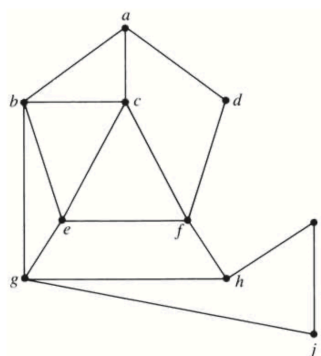


In [ ]:

9.

(1) Please traverse the nodes in a **depth-first search** of the following graph, beginning with the node **a**. (5 points)

(2) Please traverse the nodes in a **breadth-first search** of the following graph, beginning with the node **a**. (5 points)



In [ ]: