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In [1]: ### CSCI-3080 Discrete Structure
        ### OLA 3: Chapter 3 -- Recursive Definitions, Recurrence Relations
        ### Name:
        ### Student ID:
        ### Date:
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Please use **Recursive Definitions, Recurrence Relations** to finish the

following 5 exercises. (50 points)

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Exercise 1: Write the first 5 values in the following recursive sequence: (5 points)

$$C(1) = 5$$
$$C(n) = 2C(n - 1) + 5 \text{ for } n > 1$$

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Exercise 2: Write the first 5 values in the following recursive sequence: (5 points)

$$A(1) = 2$$
$$A(n) = nA(n - 1) + n \text{ for } n > 1$$

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Exercise 3: An amount of \$500 is invested in an account paying 1.2% interest compounded annually (meaning they don't add interest until the end of the year, so the start of the next year you have the money plus interest) (10 points)

(a) Please write a recurrence relation for $P(n)$, the amount in the account at the beginning of the n_{th} year.

Hint: $P(1) = 500$

For $n > 1$, to calculate the amount in the n_{th} year, you need to use $(1+0.012)$ times the amount in the $(n-1)_{th}$ year.

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(b) After how many years will the account balance exceed \$570? (You can write a C++ program to calculate)

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Exercise 4: Please find the closed form solution for the recurrence relation subject to the basis step. (10 points)

Hint: Please use the linear, first-order, constant coefficient formula.

$$S(n) = c^{n-1}S(1) + \sum_{i=2}^n c^{n-i}g(i) \quad (8)$$

$$B(1) = 5$$

$$B(n) = 3B(n-1) \text{ for } n > 1$$

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Exercise 5: In an account that pays 3% annually (i.e. interest is added at the end of the year to be available at the beginning of the next year), \$1000 is deposited. At the end of each year, and additional 100 dollars is deposited into the account. (20 points)

A. Please write a recurrence relation for the amount in the account at the beginning of year n

Hint: $P(1) = 1000$

For $n > 1$, to calculate the amount in the n_{th} year, you need to use $(1+0.03)$ times the amount in the $(n-1)_{th}$ year and plus additional 100 dollars.

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B. Please find the closed-form solution of the recurrence relation (note it is linear, first order, constant coefficient)

Hint: $\sum_{i=0}^n a^i = \frac{a^{n+1}-1}{a-1} (a \neq 1)$

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C. What is the account worth at the beginning of the 8_{th} year ($P(8)$)?

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