

Please answer questions 1 - 5 refer to the following recurrence relation.

$$B(1) = 3$$

$$B(n) = 2B(n - 1) \text{ for all } n > 1$$

1. Please write the first five terms in the sequence

$$B(1) = 3$$

$$B(2) = 2B(1) = 6$$

$$B(3) = 2B(2) = 12$$

$$B(4) = 2B(3) = 24$$

$$B(5) = 2B(4) = 48$$

2. Write the C++ code of a recursive function to solve the relation above

```
In [ ]: int Brecur(int n)
{
    if(n == 1)
        return 3;
    else{
        return 2*Brecur(n-1);
    }
}
```

```
In [ ]: for(int i = 1; i <= 5; i++):
        cout << "B(" << i << ")" << Brecur(i) << endl;
```

3. Write a C++ for loop to solve the relation above

```
In [ ]: int Bloop(int n)
{
    int B = 3;
    if(n == 1)
        return 3;
    else{
        for(int i=2; i <=n; i++)
            B = 2*B;
        return B;
    }
}
```

```
In [2]: for(int i = 1; i <= 5; i++):
        cout << "B(" << i << ")" << Bloop(i) << endl;
```

```
B( 1 ) = 3
B( 2 ) = 6
B( 3 ) = 12
B( 4 ) = 24
B( 5 ) = 48
```

4. Please find the closed form solution using the linear, first-order recurrence relation with constant coefficients formula:

$$S(n) = c^{n-1}S(1) + \sum_{i=2}^n c^{n-i}g(i)$$

```
In [ ]: B(1) = 3
        B(n) = 2*B(n-1)

        S(n) = cS(n-1) + g(n)
        c = 2
        g(n) = 0
```

$$S(n) = 2^{n-1} * 3 + 0$$

Please answer questions 5 - 6 refer to the following recurrence relation.

$$S(1) = 3$$

$$S(n) = S(n-1) + n \text{ for all } n > 1$$

In []:

5. Using the formula in Q5, write the formula for the given recurrence relation.

In []:

$S(1) = 3$
 $c = 1$
 $g(n) = n$

$$1^{n-1} * 3 + \sum_{i=2}^n 1^{n-i} * i$$

$$3 + \sum_{i=2}^n i$$

6. Please simplify the formula you got in Q5 using summation facts

$$3 + (2 + 3 + \dots + n)$$

$$3 - 1 + (1 + 2 + 3 + \dots + n)$$

$$2 + \frac{n(n+1)}{2}$$

Summation Facts

$$(1) \sum_{i=m}^n c = (n - m + 1)c$$

$$(2) \sum_{i=m}^n ca_i = c \sum_{i=m}^n a_i$$

$$(3) \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$(4) \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

In []: