

```
In [3]: ### CSCI-3080 Discrete Structure
### OLA 1: Chapter 1--Formal Logic & Chapter 2 -- Proofs, Induction
### Name:
### Student ID:
### Date:
```

Part 1: Please use **formal logic** to finish the following 4 exercises. (60 points)

=====

Exercise 1: Please find the negation of each statement (**12 points**)

- (1) The processor is fast but the printer is slow. (4 points)
- (2) If the processor is fast, then the printer is slow. (4 points)
- (3) Either the processor is fast and the printer is slow, or else the file is damaged. (4 points)

Please follow the following three steps:

1. First translate the English words into wffs using the formal symbols.
2. Negate the wffs.
3. Translate the negated wffs back into English words.

Note: (3) and (5) rely on the fact that $A \rightarrow B \leftrightarrow A' \vee B$ (Implication rule).
so $(A \rightarrow B)' \leftrightarrow (A' \vee B)'$, $(A \rightarrow B)' \leftrightarrow A \wedge B'$ by De Morgan's law

In []:

Exercise 2: Construct truth tables for the following wffs. Note any tautologies or contradictions (**20 points**)

- (a) $(A \rightarrow B) \leftrightarrow A' \vee B$
- (b) $(A \wedge B)' \leftrightarrow A' \vee B'$

(a) Truth table for $(A \rightarrow B) \leftrightarrow A' \vee B$

A	B	$A \rightarrow B$	A'	$A' \vee B$	$(A \rightarrow B) \leftrightarrow A' \vee B$
T	T				
T	F				
F	T				
F	F				

(b) Truth table for $(A \wedge B)' \leftrightarrow A' \vee B'$

A	B	$A \wedge B$	$(A \wedge B)'$	A'	B'	$A' \vee B'$	$(A \wedge B)' \leftrightarrow A' \vee B'$
---	---	--------------	-----------------	------	------	--------------	--

T	T
T	F
F	T
F	F

In []:

Exercise 3: Use a **proof sequence to prove that the following argument is valid. (14 points)**

$(A \rightarrow B) \wedge [B \rightarrow (C \rightarrow D)] \wedge [A \rightarrow (B \rightarrow C)] \rightarrow (A \rightarrow D)$

In []:

Exercise 4: Write the following argument using propositional wffs, then prove that the argument is valid using a proof sequence. (14 points)

If DeWayne is not tall, then Jayden is not DeWayne's brother. If DeWayne is tall, then Trevor is DeWayne's brother. Therefore, if Jayden is DeWayne's brother, then Trevor is DeWayne's brother.

Hint: you can use D,J,T as the statement letters

D: DeWayne is tall.

J: Jayden is DeWayne's brother

T: Trevor is DeWayne's brother

In []:

Part 2: Please use **Proofs, Induction to finish the following 6 exercises. (80 points)**

=====

Exercise 1: Provide a **counter example to the following statement: (4 points)**

The number n is an odd integer if and only if $3n + 5$ is an even integer.

This is actually the statement: (If n is an odd integer then $3n + 5$ is an even integer) AND (If $3n + 5$ is an even integer, then n is an odd integer)

In []:

Exercise 2: Please prove the following statement using **exhaustive proof. (10 points)**

For $2 \leq n \leq 4$, $n^2 \geq 2^n$

In []:

Exercise 3: Please prove the following statement using **direct proof (10 points)**

The sum of an even integer and an odd integer is odd.

Rewrite mathematically: If x is even and y is odd, then $x + y$ is odd.

In []:

Exercise 4: Please prove the following statement using **proof by contradiction. (10 points)**

If $x^2 + 2x - 3 = 0$, then $x \neq 2$

In []:

Exercise 5: For all positive integers, let $P(n)$ be the equation: (16 points)

$$2 + 4 + 6 + \cdots + 2n = n(n + 1)$$

- Write the equation for the base case $P(1)$ and verify that it is true.
- Write the inductive hypothesis $P(k)$
- Write the equation for $P(k + 1)$
- Prove that $P(k + 1)$ is true given b

In []:

Exercise 6: Please prove the following statements are true for every positive integer n using **mathematical induction. (30 points)**

(1) $2 + 6 + 18 + \cdots + 2 \times 3^{n-1} = 3^n - 1$ **(15 points)**

(2) $1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots + n \times 2^n = (n - 1) \times 2^{n+1} + 2$ **(15 points)**

In []: