In [1]:
```
### CSCI-3080 Discrete Structure
### OLA 2: Chapter 3 -- Recursive Definitions, Recurrence Relations
### Name:
### Student ID:
### Date:
```

Exercise 1: Write the first 5 values in the sequence:

$$C(1) = 5$$
$$C(n) = 2C(n-1) + 5 \text{ for } n > 1$$ ¶

In [3]:
```
def C(n):
    if n == 1:
        return 5
    else:
        return 2*C(n-1) + 5
```

In [2]:
```
for i in range (1,6):
    print("C(",i,") = ", C(i))
```

```
C( 1 ) =  5
C( 2 ) =  15
C( 3 ) =  35
C( 4 ) =  75
C( 5 ) =  155
```

Exercise 2: Write the first 5 values in the sequence:

$$A(1) = 2$$
$$A(n) = nA(n-1) + n \text{ for } n > 1$$

In [4]:
```
def A(n):
    if n == 1:
        return 2
    else:
        return n*A(n-1) + n
```

In [5]:
```
for i in range (1,6):
    print("A(",i,") = ", A(i))
```

```
A( 1 ) =  2
A( 2 ) =  6
A( 3 ) =  21
A( 4 ) =  88
A( 5 ) =  445
```

Exercise 3: An amount of $500 is invested in an account paying 1.2% interest compounded annually (meaning they don't add interest until the end of the year, so the start of the next year you have the money plus interest)

(a) Please write a recursive definition for $P(n)$, the amount in the account at the begining of the n_{th} year.

Hint: P(1) = 500
For n > 1, to calculate the amount in the n_{th} year, you need to use (1+0.012) times the amount in the $(n-1)_{th}$ year.

(a)
P(1) = 500
P(n) = 1.012*P(n-1) for n > 1

(b) After how many years will the account balance exceed $570? (You can write a python program to calculate)

(b) P(12) is over 570, so after 11 years is the answer.

In [7]:
```
def P(n):
    if n == 1:
        return 500
    else:
        return 1.012*P(n-1)
```

In [8]:
```
for i in range (1,13):
    print("P(",i,") = ", P(i))
```

```
P( 1 ) =  500
P( 2 ) =  506.0
P( 3 ) =  512.072
P( 4 ) =  518.216864
P( 5 ) =  524.435466368
P( 6 ) =  530.728691964416
P( 7 ) =  537.097436267989
P( 8 ) =  543.5426055032049
P( 9 ) =  550.0651167692433
P( 10 ) =  556.6658981704743
P( 11 ) =  563.34588894852
P( 12 ) =  570.1060396159023
```

Exercise 4: Please write the python code of a recursive function to compute the sequence

$$1, 2, 4, 7, 11, 16, 22, \ldots$$

Begin by defining the recursive definition

$$S(1) = 1$$
$$S(n) = S(n-1) + n - 1 \text{ for } n > 1$$

In [9]:
```
def S(n):
    if n == 1:
        return 1
    else:
        return S(n-1) + n - 1
```

In [10]:
```
for i in range (1,10):
    print("S(",i,") = ", S(i))
```

```
S( 1 ) =  1
S( 2 ) =  2
S( 3 ) =  4
S( 4 ) =  7
S( 5 ) =  11
S( 6 ) =  16
S( 7 ) =  22
S( 8 ) =  29
S( 9 ) =  37
```

Exercise 5: Please find the closed form solution for the recurrence relation subject to the basis step.

Hint: Please use the linear,first-order, constant coefficient formula: $S(n) = c^{n-1}S(1) + \sum_{i=2}^{n} c^{n-i}g(i)$

$$B(1) = 5$$
$$B(n) = 3B(n-1) \text{ for } n > 1$$

c = 3
g(n) = 0
P(1) = 5

$$B(n) = 3^{n-1} \times 5 + 0 = 3^{n-1} \times 5$$

Exercise 6: In an account that pays 3% annually (i.e. interest is added at the end of the year to be available at the beginning of the next year), $1000 is deposited. At the end of each year, and additional 100 dollars is deposited into the account.

A. Please write a recurrence relation for the amount in the account at the beginning of year n

Hint: P(1) = 1000
For n > 1, to calculate the amount in the n_{th} year, you need to use (1+0.03) times the amount in the $(n-1)_{th}$ year and plus additonal 100 dollars.

A.
P(1) = 1000
P(n) = 1.03*P(n-1) + 100 for n > 1

B. Please find the closed-form solution of the recurrence relation (note it is linear, first order, constant coefficient)

Hint 1: Please use the linear,first-order, constant coefficient formula:

$S(n) = c^{n-1}S(1) + \sum_{i=2}^{n} c^{n-i}g(i)$

Hint 2: $\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1}(a \neq 1)$

B.
c = 1.03
g(n) = 100
P(1) = 1000

$$P(n) = 1.03^{n-1} * 1000 + \sum_{i=2}^{n} 1.03^{n-i} * 100$$

$P(n) = 1.03^{n-1} * (100 + 900) + 1.03^{n-2} * 100 + 1.03^{n-3} * 100 + \cdots + 1.03^{n-n} * 100$
$P(n) = 1.03^{n-1} * 900 + 1.03^{n-1} * 100 + 1.03^{n-2} * 100 + 1.03^{n-3} * 100 + \cdots + 1.03^{n-n} * 100$
$P(n) = 1.03^{n-1} * 900 + (1.03^{n-1} + 1.03^{n-2} + 1.03^{n-3} + \cdots + 1.03^{n-n}) * 100$
$P(n) = 1.03^{n-1} * 900 + (1.03^{n-n} + 1.03^{n-(n-1)} + \cdots + 1.03^{n-3} + 1.03^{n-2} + 1.03^{n-1}) * 100$
$P(n) = 1.03^{n-1} * 900 + (1.03^{0} + 1.03^{1} + \cdots + 1.03^{n-3} + 1.03^{n-2} + 1.03^{n-1}) * 100$
$P(n) = 1.03^{n-1} * 900 + 100 * \sum_{i=0}^{n-1} 1.03^i$ $P(n) = 1.03^{n-1} * 900 + 100 * \frac{1.03^n-1}{1.03-1}$

C. What is the account worth at the begining of the 8_{th} year?

$P(8) = 1.03^{8-1} * 900 + 100 * \frac{1.03^8-1}{1.03-1}$

In [15]:
```
900*1.03**(8-1) + 100*(1.03**8-1)/(1.03-1)
```

Out[15]:
```
1996.1200835077702
```