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### CSCI-3080 Discrete Structure
### OLA 5: Chapter 7 -- Graphs and Algorithms
### Name:
### Student ID:
### Date:
### Total: 100 Points
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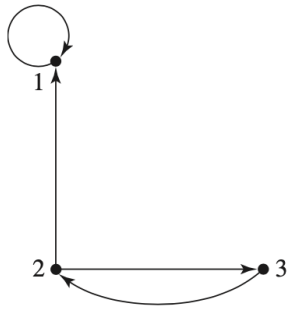
1. Find the adjacency matrix and adjacency relation (binary relation) for the following graph. (8 points)



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2. Find the corresponding directed graph and adjacency relation (binary relation) for the following adjacency matrix.(10 points)

directed graph (5 points)
binary relation(5 points)

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

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3. Given the adjacency relation ρ = {(1, 4), (1, 5), (1, 6), (6, 2), (6, 3), (6, 5)} on the set of nodes {1, 2, 3, 4, 5, 6,} find the corresponding directed graph and adjacency matrix.(10 points)

directed graph (5 points)
adjacency matrix(5 points)

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4. Let A be the following matrix. Find the products A^2 and $A^{(2)}$(10 points)

$A^2 = A.A$ (regular matrix multiplication)(5 points)
$A^{(2)} = A \times A$ (boolean matrix multiplication) (5 points)

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

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5. Compute the reachability matrix R using the formula R = A $\vee$ $A^{(2)}$ $\vee$ $\cdots$ $\vee$ $A^{(n)}$ for the following adjacency matrix:(12 points)

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Hint: Please show the matrix A, $A^{(2)}$, $A^{(3)}$, $A^{(4)}$, and the final result R. You don't need to show the steps for boolean matrix multiplication of $A^{(2)}$, $A^{(3)}$, $A^{(4)}$.

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6. Compute the reachability matrix R using Warshall's algorithm for exercise 5. (12 points)

Hint: Please show the matrix M_0, M_1, M_2, M_3, M_4 and the final result $R = M_4$.

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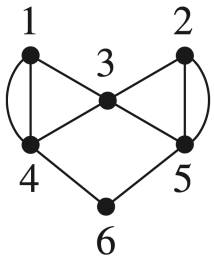
7.

(1) Determine whether this graph has an Euler path. If so, list the nodes in such a path. (4 points)
(2) Determine whether this graph has a Hamiltonian circuit. If so, list the nodes in such a circuit. (4 points)



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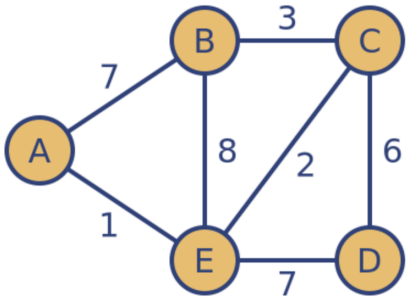
8.

Apply Dijkstra's algorithm for the following graph. Show the values for p and IN and the d values and s values for each pass. Write out the nodes in the shortest path from 2 to 5 and the distances of the path. (20 points)

Hint:

(1) p is the current node that has the shortest d that you will include in your current IN set.
(2) Node **A** is the **starting node**.
(3) Node **D** is the **ending node**.
(4) Please show all steps like we did during the class: draw the table for each step, calculate the new distance for each step, update the table, repeat until the ending node is included in the IN set, find the **shortest path** and the **shortest length**.



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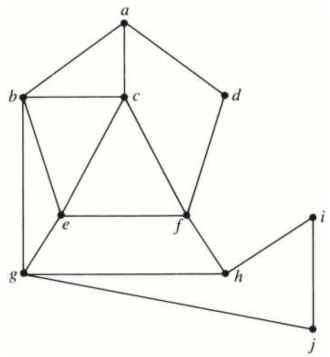
9.

(1) Please traverse the nodes in a depth-first search of the following graph, beginning with the node a. (5 points)
(2) Please traverse the nodes in a breadth-first search of the following graph, beginning with the node a. (5 points)



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