

ASSIGNMENT 2 DISCRETE STRUCTURE
CHAPTER 2.3 (MICHELLE)

Chapter 2 (2.3 Recurrence Relation)

Question 1

i) * assume RM 50 is day 0 (before day 1 starts)

$$50 \times 1.02 \times 0.98 = 49.98$$

$$a_n = a_{n-1} (1.02 \times 0.98)$$

$$a_n = a_{n-1} (0.9996), a_0 = 50, n \geq 1$$

ii) day 4, $n = 4$

$$a_1 = a_0 (0.9996) = 50 (0.9996) = 49.98$$

$$a_2 = a_1 (0.9996) = 49.98 (0.9996) \approx 49.96$$

$$a_3 = a_2 (0.9996) = 49.96 (0.9996) \approx 49.94$$

$$a_4 = a_3 (0.9996) = 49.94 (0.9996) \approx 49.92$$

\therefore Stock price at the end of the day 4, a_4 is RM 49.92.

Question 2

$$a) \begin{matrix} n_1 & n_2 & n_3 & n_4 \\ 5 & \frac{37}{7} & \frac{39}{7} & \frac{41}{7} \end{matrix}$$

$\xrightarrow{+\frac{2}{7}} \quad \xrightarrow{+\frac{2}{7}}$

$$a_1 = 5 \quad \left| \quad d = \frac{37}{7} - 5 \right.$$

$$a_2 = \frac{37}{7} \quad \left| \quad = \frac{2}{7} \right.$$

$$\therefore a_n = a_{n-1} + \frac{2}{7}, a_1 = 5, n \geq 2$$

b) ~~Input~~

input = n

output = a_n

$a_n \{$

if ($n = 1$)

return 5

else

return $a_{n-1} + \frac{2}{7}$

}

CHAPTER 3.1 (JAY)

Assignment 2 - Chapter 3.2 and 3.3

1. (i) Choosing 3 letters = $P(26, 3)$

Choosing 5 digits = $P(10, 5)$

$$\begin{aligned}\text{Subject codes possible} &= P(26, 3) \cdot P(10, 5) \\ &= (26^3)(10^5) \\ &= 1\,757\,600\,000 \text{ codes}\end{aligned}$$

(ii) Begin with CS = $P(1, 1) \cdot P(1, 1)$

Choose 1 letter = $P(26, 1)$

Choose 4 digits = $P(10, 4)$

Choose 3 or 2 = $P(2, 1)$

$$\begin{aligned}\text{Total subject codes} &= P(1, 1) \cdot P(1, 1) \cdot P(26, 1) \cdot P(10, 4) \cdot P(2, 1) \\ &= (1)(1)(26)(10^4)(2) \\ &= 520\,000 \text{ codes}\end{aligned}$$

(iii) All letters and digits are distinct:

$$\begin{aligned}\text{Total subject codes} &= P(26, 1) \cdot P(25, 1) \cdot P(24, 1) \cdot P(10, 1) \cdot P(9, 1) \cdot P(8, 1) \cdot P(7, 1) \cdot P(6, 1) \\ &= (26)(25)(24)(10)(9)(8)(7)(6) \\ &= 471\,744\,000 \text{ codes}\end{aligned}$$

CHAPTER 3.2 (KAVI)

2. (i) $C(10, 3) = 120$ ways

(ii) $C(15, 9) = 5005$ ways

(iii) Total strings = $P(8, 5)$
 $= 8^5$
 $= 32768$ strings.

(iv) Select 2 girls: $C(10, 2) = 45$ ways

Select 2 boys: $C(7, 2) = 21$ ways

Select 2 girls and 2 boys = $(45)(21)$
 $= 945$ ways

3. Select 3 permutation questions: $C(20, 3) = 1140$ ways

Select 2 combination questions: $C(15, 2) = 105$ ways

Select 3 permutation questions and 2 combination questions

$= (1140)(105)$

$= 119700$ ways

Chapter 3 (3.4): Pigeonhole Principle

1. $n = 40$ (Pigeon - People)
 $k = 12$ (Pigeonhole - month)

$$m = \left\lceil \frac{n}{k} \right\rceil$$

$$m = \left\lceil \frac{40}{12} \right\rceil = 4$$

\therefore at least 4 people

2. Possible score : 90 to 100
= 11 numbers

(Pigeon = number of students) $n = 35$

(Pigeonhole = possible score) $k = 11$

$$m = \left\lceil \frac{n}{k} \right\rceil$$

$$= \left\lceil \frac{35}{11} \right\rceil$$

$$= 4$$

3. To make sum of 2 numbers is 11, possible combinations are :
 $\{1, 10\}, \{2, 9\}, \{3, 8\}, \{4, 7\}, \{5, 6\}$

There is 5 possible combinations (Pigeonhole).

If we had to pick 6 numbers (Pigeon), by the pigeonhole principle, at least one of the above combination must be chosen.

Hence, at least 2 numbers will sum to 11.

4. Pigeonhole = time periods that can be schedule, $k = 53$.

Pigeon = number of classes to be schedule, $n = 115$.

To prevent overlapping of classes, minimum number of classroom

required, $m = \left\lceil \frac{n}{k} \right\rceil$

$$= \left\lceil \frac{115}{53} \right\rceil$$

$$= 3$$

\therefore at least 3 rooms will be needed.

5. Pigeon : number of computer, $n = 25$

Pigeonhole : numbers of connection each computer can made
= 1 to 24 connections (a computer can't connect to itself)

$$k = 24$$

$$m = \left\lceil \frac{n}{k} \right\rceil$$

$$= \left\lceil \frac{25}{24} \right\rceil$$

$$= 2$$

\therefore hence, there are at least 2 computers in the network that are directly connected to same number of other computers according to Pigeonhole Principle.
Since there are 25 computers but only 24 possible connections.