SECI1013-03 (DISCRETE STRUCTURE)

ASSIGNMENT 1 (CHAPTER 1 & 2)

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QUESTIONS 1, 2, 3

Assignment 1

Chapter 1

Question 1

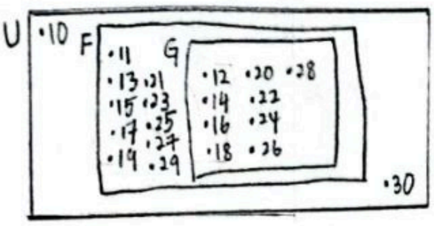
a. $F = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$

$|F| = 19$

b. $G = \{12, 14, 16, 18, 20, 22, 24, 26, 28\}$

$|G| = 9$

c.



d. $F - G = \{11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$

$|F-G| = 10$

Question 2

a) $|P(A)| = 2^3$

$= 8$

b) $A \cap B = \{s\}$

$A \cap B \cap C = \{s, n, e, t\}$

c) $A - B = \{u, b\}$

d) $B = \{s, e, t\}$, $C = \{n, e, t\}$

$B \times C = \{(s,n), (s,e), (s,t), (e,n), (e,e), (e,t), (t,n), (t,e), (t,t)\}$

Question 3

a. Proposition, true

b. Proposition, true

c. Proposition, true

d. Proposition, false

e. Proposition, true

QUESTION 4

Question 4

a.

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg p \leftrightarrow \neg q$	$(p \to q) \wedge (\neg p \leftrightarrow \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

b.

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$	$\neg p \to \neg q$	$(p \leftrightarrow q) \vee (\neg p \to \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	F	F	F
F	F	T	T	T	T	T

QUESTIONS 5, 6, 7

Question 5

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg q \vee \neg r$	$q \wedge r$	A	B
T	T	T	F	F	F	F	T	F	T
T	T	F	F	F	T	T	F	F	T
T	F	T	F	T	F	T	F	F	T
T	F	F	F	T	T	T	F	F	T
F	T	T	T	F	F	F	T	F	T
F	T	F	T	F	T	T	F	T	F
F	F	T	T	T	F	T	F	T	F
F	F	F	T	T	T	T	F	T	F

$\therefore A \not\equiv B.$

Question 6

p	q	$p \vee q$	$p \wedge q$	A	B
T	T	T	T	T	T
T	F	T	F	T	T
F	T	T	F	F	F
F	F	F	F	F	F

$\therefore A \equiv B.$

Question 7

(a) $\exists(x)(P(x) \wedge R(x))$

(b) $\forall(x)\ (Q(x) \rightarrow \neg R(x))$

QUESTION 8

Question 8

For all negative integers x, x^2 is positive.

let $-a$ is a negative integer:

$$(-a)^2 = (-a).(-a)$$
$$= a^2$$

The multiplication of two same signs, either positive or negative, brings positive sign as its outcome.

Therefore, $x = -a$.

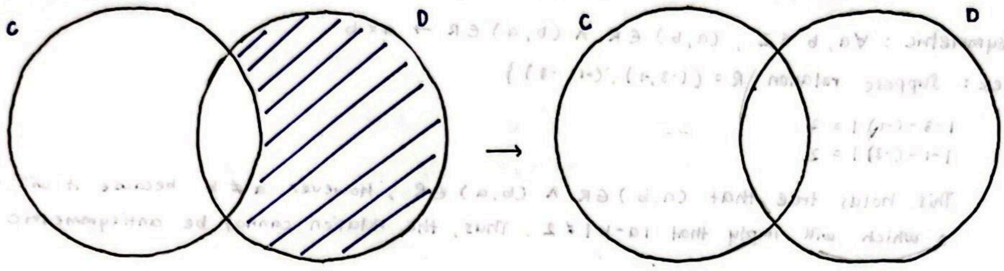
Since $x<0$ and $x^2>0$, we can conclude that $\forall(x)\ (x<0 \rightarrow x^2>0)$.

Hence, proven ※

QUESTION 9

9. Give a proof by contradiction to show if C and D are sets, then $C \cap (D \cap C') = \{\}$

∴ Supposition : Suppose C & D are sets, $C \cap (D \cap C') \neq \{\}$



$(D \cap C')$ $\qquad$ $C \cap (D \cap C')$

Thus, $C \cap (D \cap C')$ is an empty set, a contradiction. Which proves that the original supposition is false.

∴ Therefore if C&D are sets, then $C \cap (D \cap C') = \{\}$

Contradiction : if C & D are sets, then $C \cap (D \cap C') \neq \{\}$

if $C \cap (D \cap C') \neq \{\}$, then there exists an element x such that

$$x \in C \cap (D \cap C')$$

This implies :

1. $x \in C$
2. $x \in (D \cap C')$

From ② : x must exist and x must exist C' for ② to be true

However since $x \in C$, x cannot exist in C' at the same time.

∴ Thus $C \cap (D \cap C') = \{\}$

QUESTION 10

Chapter 2 : (2.1 & 2.2) Relations and Functions

10. Determine whether the relation R on set Z (set of integer) is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive.

$$\boxed{a\ R\ b \text{ if and only if } |a-b| = 2}$$

$Z = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$

Reflexive : $\forall a \in Z$, $(a,a) \in R$

Check : $|-1-(-1)| = 0$

$|0-0| = 0$

$|1-1| = 0$

$\rightarrow$ R is not reflexive.

Irreflexive : $\forall a \in Z$, $(a,a) \notin R$

Check : since the relation doesn't contain any pair of the form (a,a), thus the relation will be irreflexive.

Symmetric : $\forall a,b \in Z$, $(a,b) \in R \rightarrow (b,a) \in R$

Check : Suppose R is symmetric, $R = \{(-3,-1), (-1,-3), (0,2), (2,0), (2,4), (4,2)\}$

$|-3-(-1)| = 2$

$|-1-(-3)| = 2$

$|0-2| = 2$

$|2-0| = 2$

$|2-4| = 2$

$|4-2| = 2$

$$M_R = \begin{array}{c} \\ -3 \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccccc} -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

The relation is symmetric

Asymmetric : $\forall a, b \in Z$, $(a, b) \in R \rightarrow (b, a) \notin R$

Check : Since the relation is symmetric , thus the relation cannot be asymmetric because $\forall a, b \in Z$, $(a, b) \in R \rightarrow (b, a) \notin R$ does not hold true.

Antisymmetric : $\forall a, b \in Z$, $(a, b) \in R \wedge (b, a) \in R \rightarrow a = b$

Check : Suppose relation $R = \{(-3, -1), (-1, -3)\}$

$|-3-(-1)| = 2$

$|-1-(-3)| = 2$

This holds true that $(a, b) \in R \wedge (b, a) \in R$, however $a \neq b$ because it will yield 0 which will imply that $|a-b| \neq 2$. Thus, the relation cannot be antisymmetric .

Transitive : $\forall a, b, c \in Z$, $(a, b) \in R \wedge (b, c) \in R \rightarrow (a, c) \in R$

Check : Suppose $a = 1$, $b = 3$, $c = 5$

Method #1

$|1-3| = 2$ and $|3-5| = 2$ However, $|1-5| \neq 2$

Thus, $(1, 5) \notin R$ and the relation is not transitive

Method #2

Suppose : $R = \{(-1, 3), (3, -1), (0, 2), (2, 0)\}$

$$M_R = \begin{array}{c} \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} -1 & 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_R \otimes M_R = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore M_R \otimes M_R \neq M_R$. Thus, the relation is not transitive .

QUESTION 11

11. Given a relation, R on $A = \{a, b, c, d\}$ on as follows:

$$\boxed{R = \{(a,a), (a,b), (a,d), (b,b), (b,c), (c,c), (c,d), (d,a), (d,d)\}}$$

Show that the matrix of relation, M_R and determine whether the relation, R is an equivalence relation.

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

* Relation is reflexive

Symmetric: if $M_R = M_R^T$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$M_R \neq M_R^T$ * Relation is not symmetric

Transitive: $M_R \otimes M_R = M_R$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$M_R \otimes M_R \neq M_R$ * Relation is not transitive

∴ Relation, R is reflexive, but not symmetric nor transitive. Thus relation R is not an equivalent relation.

QUESTION 12

12. Let $f(x, y) = (2x - y, x - 2y)$; $(x, y) \in R \times R$, (R is the set of real numbers)

a) Show that f is one-to-one

If f is one-to-one, $f(x_1, y_1) = f(x_2, y_2)$ then $x_1 = x_2$ and $y_1 = y_2$

$$f(x_1, y_1) = f(x_2, y_2)$$

$f(x_1) = f(x_2)$ $\qquad$ $f(y_1) = f(y_2)$

$2x_1 - y_1 = 2x_2 - y_2 \cdots ①$ $\qquad$ $x_1 - 2y_1 = x_2 - 2y_2 \cdots ②$

From ①: $y_1 = 2x_1 - 2x_2 + y_2 \cdots ③$

③ → ②: $x_1 - 2(2x_1 - 2x_2 + y_2) = x_2 - 2y_2$

$$x_1 - 4x_1 + 4x_2 - \cancel{2y_2} = x_2 - \cancel{2y_2}$$

$$-3x_1 = -3x_2$$

$$x_1 = x_2$$

From ②: $x_1 = x_2 - 2y_2 + 2y_1 \cdots ④$

④ → ①: $2(x_2 - 2y_2 + 2y_1) - y_1 = 2x_2 - y_2$

$$\cancel{2x_2} - 4y_2 + 4y_1 - y_1 = \cancel{2x_2} - y_2$$

$$3y_1 = 3y_2$$

$$y_1 = y_2$$

$\therefore$ This shows that f is a one-to-one function.

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b) Find f^{-1}

$f(x, y) = (2x - y, x - 2y)$

$(x, y) = f^{-1}(2x - y, x - 2y) \rightarrow (x, y) = f^{-1}(a, b)$

let $a = 2x - y$, $b = x - 2y$

$a = 2x - y \cdots ①$

$b = x - 2y \cdots ②$

From ①: $x = \dfrac{a + y}{2} \cdots ③$

③ → ②: $b = \dfrac{a + y}{2} - 2y$

$$2b = a + y - 4y$$

$$y = \frac{a - 2b}{3}$$

From ②: $y = \dfrac{x - b}{2} \cdots ④$

④ → ①: $a = 2x - \dfrac{(x - b)}{2}$

$$2a = 4x - x + b$$

$$x = \frac{2a - b}{3}$$

$\therefore f^{-1}(a, b) = (x, y)$

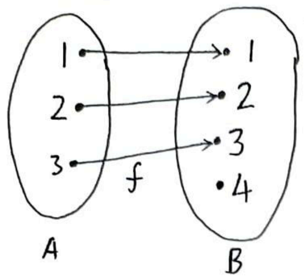
$$f^{-1}(a, b) = \left(\frac{2a - b}{3}, \frac{a - 2b}{3}\right)$$

$$f^{-1}(x, y) = \left(\frac{2x - y}{3}, \frac{x - 2y}{3}\right)$$

QUESTIONS 13, 14, 15

Question 13

a)



b) $g = \{(1,1), (2,2), (3,2)\}$

Question 14

i) $f(x) = x^3$

$g(x) = x - 1$

$g \circ f = g[f(x)]$

$= x^3 - 1$

$f \circ g = f[g(x)]$

$= (x-1)^3$

ii) $g[f(x)] = x^3 - 1$

$f[g(x)] = (x-1)^3$

$= (x-1)^2(x-1)$

$= (x^2 - 2x + 1)(x-1)$

$= x^3 - x^2 - 2x^2 + 2x + x - 1$

$= x^3 - 3x^2 + 3x - 1$

Thus, $g[f(x)] \neq f[g(x)]$

Question 15

only "000...." "111...." "...1100...." allowed

$n = 1$ = 0 , 1 $\quad a_1 = 2$

$n = 2$ = 00 , 11 , 10 $\quad a_2 = 3$

$n = 3$ = 000 , 111 , 100 , 110 $\quad a_3 = 4$

$n = 4$ = 0000 , 1111 , 1000 , 1100 , 1110 $\quad a_4 = 5$

$n = 5$ = $\underbrace{00000, 11111}_{2}, \underbrace{10000, 11000, 11100, 11110}_{a_{n-2}}$ $\quad a_5 = 6$

$\therefore a_n = a_{n-2} + 2$, $n \geq 2$, $a_1 = 2$, $a_2 = 3$

QUESTION 16

Question 16

```
Input = n
Output = Cn
Cn {
 if (n = 1)
    return 0
else if (n=2 or n=3)
   return 1
else
  return Cn-2 + Cn-3
  }
```