Chapter 3 (3.5): Probability

QUESTION 1

1. (i) Let $P(C)$ = Probability of chemistry books
Let $P(M)$ = Probability of math books
Let $P(B)$ = Probability of biology books
Let $P(P)$ = Probability of physics books

Assume $P(B)$ and $P(P) = x$
Since $P(C)$ is 2 times $P(B)$; $P(C) = 2x$
Since $P(M)$ is 3 times $P(C)$; $P(M) = 3(2x) = 6x$

$$P(C)+P(M)+P(B)+P(P) = 1$$
$$2x + 6x + x + x = 1$$
$$10x = 1$$
$$x = \frac{1}{10}$$
$$x = 0.1$$

$P(B) = 0.1$
$P(P) = 0.1$
$P(C) = 2(0.1)$
$= 0.2$
$P(M) = 3(2(0.1))$
$= 3(0.2)$
$= 0.6$

$\therefore P(B) = 0.1$, $P(P) = 0.1$, $P(C) = 0.2$, $P(M) = 0.6$

1. (ii) $P(M \cup B) = P(M) + P(B)$
$= 0.6 + 0.1$
$= 0.7$

QUESTION 2

2. (i) $P(A \cup B) = P(A) + P(B)$
$= 0.4 + 0.5$
$= 0.9$

(ii) $P(A^c) = 1 - P(A)$
$= 1 - 0.4$
$= 0.6$

(iii) $P(A^c \cap B)$ - Since A and B are mutually exclusive, $P(A^c \cap B) = P(B) = 0.5$

QUESTION 3

3. Total participants = 100
Total prize = 3

T_1: Anis win grand prize $= \frac{1}{100}$
$= 0.01$

T_2: Anis win second prize $= \frac{1}{100}$
$= 0.01$

T_3: Anis win third prize $= \frac{1}{100}$
$= 0.01$

Probability of Anis winning either one of the prize $= T_1 + T_2 + T_3$
$= 0.01 + 0.01 + 0.01$
$= 0.03$

QUESTION 4

Question 4

Let $P(P)$ = Probability a randomly chosen male has pneumonia = 0.40

$P(S)$ = Probability a male is smoker

$P(S|P)$ = Probability a male is smoker given he ~~have~~ has pneumonia = 0.80

$P(S|\neg P)$ = Probability a male is smoker given he doesn't have pneumonia = 0.30

i) $P(\neg P) = 1 - 0.40$

$= 0.60$

ii) $P(P|S) = \frac{P(P \cap S)}{P(S)}$

$P(S|P) = \frac{P(S \cap P)}{P(P)} = 0.80$

$\frac{P(S \cap P)}{0.40} = 0.80$

$P(S \cap P) = 0.80 \times 0.40$

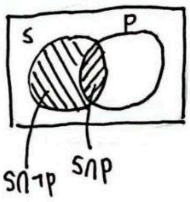
$= 0.32$

$P(S|\neg P) = \frac{P(S \cap \neg P)}{P(\neg P)} = 0.30$

$\frac{P(S \cap \neg P)}{0.60} = 0.30$

$P(S \cap \neg P) = 0.60 \times 0.30$

$= 0.18$



$P(S) = P(S \cap P) + P(S \cap \neg P)$

$= 0.18 + 0.32$

$= 0.50$

$P(P|S) = \frac{P(P \cap S)}{P(S)}$

$= \frac{0.32}{0.50}$

$= 0.64$

QUESTION 5

Question 5

$P(Black) = \frac{1}{3}$

$\underline{B}$ $\underline{B}$

Probability choosing black boots both time:

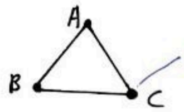
$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Chapter 4 (4.1 to 4.6): Graph Theory
QUESTION 1

Chapter 4

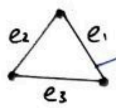
Question 1

1a) Vertices are the nodes or dots in a graph.



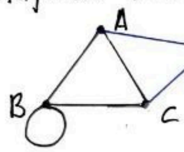
A, B, C are vertices

b) Edges are the line connecting two vertices in a graph.



e_1, e_2, e_3 are edges

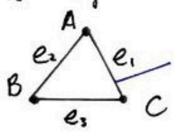
c) Adjacent vertices are two vertices (dots) connected by an edge (line).



A and C are adjacent

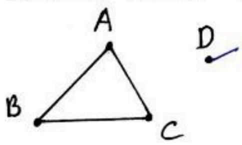
* B is adjacent to itself

d) An edge is said to be incident to a vertex if it connects to that vertex



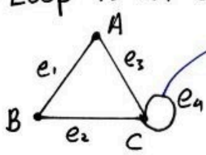
e_1 is incident on A and C

e) Isolated vertex is a vertex with no edges (lines) connected to it.



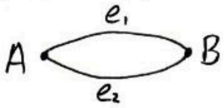
D is an isolated vertex

f) Loop is an edge (line) that starts and ends at the same vertex.



e_4 is a loop

g) Parallel edges are two or more edges that connect the same pair of vertices.

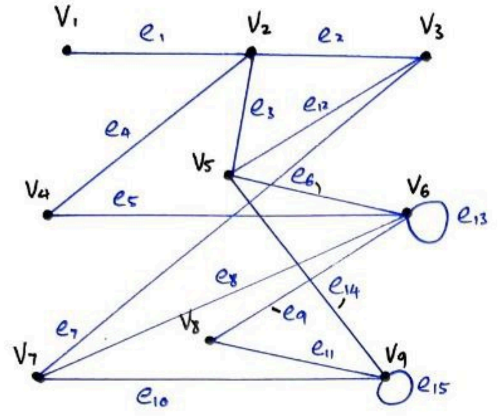


* e_1 and e_2 are parallel edges.

QUESTION 2

Question 2

a)



i)

Vertex	Degree
V_1	1
V_2	4
V_3	3
V_4	2
V_5	4
V_6	6
V_7	3
V_8	2
V_9	5

ii) Adjacent

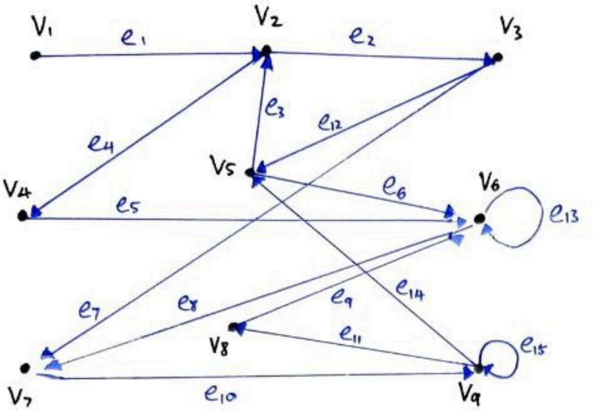
$$A_G = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & V_7 & V_8 & V_9 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

ii) Incident

$$I_G = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{array} \begin{array}{c} \begin{array}{ccccccccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} & e_{11} & e_{12} & e_{13} & e_{14} & e_{15} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 2 \end{bmatrix} \end{array}$$

Question 2

b)



i)

Vertex	In degree	Out degree
V_1	0	1
V_2	2	2
V_3	1	2
V_4	1	1
V_5	2	2
V_6	4	2
V_7	2	1
V_8	1	1
V_9	2	3

ii) Adjacent

$$A_G = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & V_7 & V_8 & V_9 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

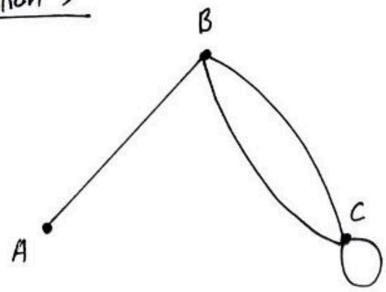
ii) Incident

$$I_G = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{array} \begin{array}{c} \begin{array}{ccccccccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} & e_{11} & e_{12} & e_{13} & e_{14} & e_{15} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

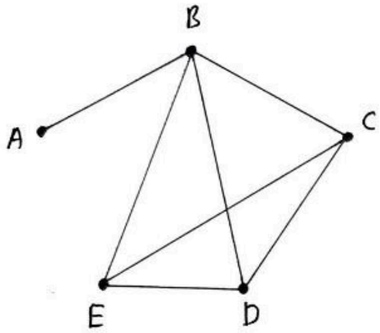
QUESTION 3

Question 3

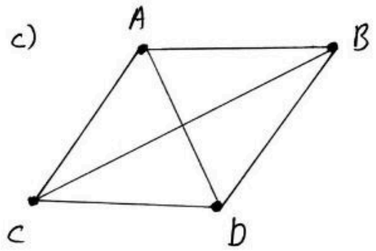
a)



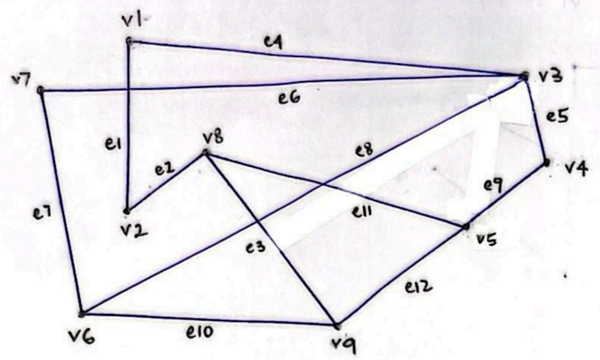
b)



c)



QUESTION 4



i. Find all possible path from v1 to v9?

Path #1 : (v1, e1, v2, e2, v8, e3, v9)

Path #2 : (v1, e1, v2, e2, v8, e11, v5, e12, v9)

Path #3 : (v1, e1, v2, e2, v8, e11, v5, e9, v4, e5, v3, e6, v7, e7, v6, e10, v9)

Path #4 : (v1, e1, v2, e2, v8, e11, v5, e9, v4, e5, v3, e8, v6, e10, v9)

Path #5 : (v1, e4, v3, e6, v7, e7, v6, e10, v9)

Path #6 : (v1, e4, v3, e8, v6, e10, v9)

Path #7 : (v1, e4, v3, e5, v4, e9, v5, e12, v9)

Path #8 : (v1, e4, v3, e5, v4, e9, v5, e11, v8, e3, v9)

ii Find all possible trails from v1 to v9?

Trail #1 : (v1, e1, v2, e2, v8, e3, v9)

Trail #2 : (v1, e1, v2, e2, v8, e11, v5, e12, v9)

Trail #3 : (v1, e1, v2, e2, v8, e11, v5, e9, v4, e5, v3, e6, v7, e7, v6, e10, v9)

Trail #4 : (v1, e1, v2, e2, v8, e11, v5, e9, v4, e5, v3, e8, v6, e10, v9)

Trail #5 : (v1, e4, v3, e6, v7, e7, v6, e10, v9)

Trail #6 : (v1, e4, v3, e8, v6, e10, v9)

Trail #7 : (v1, e4, v3, e5, v4, e9, v5, e12, v9)

Trail #8 : (v1, e4, v3, e5, v4, e9, v5, e11, v8, e3, v9)

Trail #9 : (v1, e4, v3, e8, v6, e7, v7, e6, v3, e5, v4, e9, v5, e12, v9)

Trail #10 : (v1, e4, v3, e6, v7, e7, v6, e8, v3, e5, v4, e9, v5, e12, v9)

iii. Find the longest path and the shortest path from v1 to v9?

Shortest path : (v1, e1, v2, e2, v8, e3, v9) / (v1, e4, v3, e8, v6, e10, v9)

Longest path : (v1, e1, v2, e2, v8, e11, v5, e9, v4, e5, v3, e6, v7, e7, v6, e10, v9)

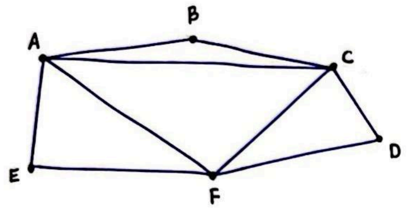
iv. Find the shortest and logest trail from v1 to v9?

Shortest : (v1, e1, v2, e2, v8, e3, v9) / (v1, e4, v3, e8, v6, e10, v9)

Longest : (v1, e1, v2, e2, v8, e11, v5, e9, v4, e5, v3, e6, v7, e7, v6, e10, v9)

QUESTION 5

5.



i. Find the possible Euler path for the path?

↳ (E, A, B, C, A, F, C, D, F, E)

ii. Find the possible Euler Circuit for the map?

↳ (F, E, A, B, C, D, F, A, C, F)

iii. Find the Hamiltonian Circuit for this map?

↳ (D, C, B, A, E, F, D)

iv. Differences between Euler & Hamiltonian Circuit.

Euler Circuit	Hamiltonian Circuit
- Visits every edge exactly once	- Visit every vertex exactly once.
- Every vertex must be visited at least once.	- Every edge doesn't need to be visted at least once.