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Robustness of two-qubit and three-qubit states in correlated quantum channels*

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Abstract

We investigate how the correlated actions of quantum channels affect the robustness of entangled states. We consider the Bell-like state and random two-qubit pure states in the correlated depolarizing, bit flip, bit-phase flip, and phase flip channels. It is found that the robustness of two-qubit pure states can be noticeably enhanced due to the correlations between consecutive actions of these noisy channels, and the Bell-like state is always the most robust state. We also consider the robustness of three-qubit pure states in correlated noisy channels. For the correlated bit flip and phase flip channels, the result shows that although the most robust and most fragile states are locally unitary equivalent, they exhibit different robustness in different correlated channels, and the effect of channel correlations on them is also significantly different. However, for the correlated depolarizing and bit-phase flip channels, the robustness of two special three-qubit pure states is exactly the same. Moreover, compared with the random three-qubit pure states, they are neither the most robust states nor the most fragile states.

Keywords: correlated quantum channel, entanglement, concurrence, negativity

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1. Introduction

Quantum entanglement is not only an important feature that distinguishes quantum mechanics from classical physics, but also a key resource for many potential practical applications, such as quantum teleportation, quantum sensing, and quantum cryptography. [1,2] However, in the real world, the inevitable interaction between

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a quantum system and its surrounding environment will lead to decoherence.[3–5] Therefore, it is of fundamental importance to study the robustness of entangled states in the open systems. In the past several years, many studies have been devoted to the robustness of entangled states in different decoherence models.[6–21]

In 1999, Vidal and Tarrach[6] utilized robustness to characterize the endurance of entanglement against noise and jamming, then in 2002, Simon and Kempe[7] studied the robustness of multipartite entanglement under the local decoherence process. They found that the multipartite entanglement could be surprisingly robust under decoherence, and the robustness can increase with the increasing number of parties. In 2008, Aolita *et al.*[8] investigated the decay of entanglement for the generalized N-particle Greenberger-Horne-Zeilinger (GHZ) states and showed that although the entanglement sudden death time increases with an increase in N, the time when the entanglement becomes arbitrarily small is inversely proportional to N. The most robust and most fragile states under different decoherence channels have also been extensively studied.[9–18] In Ref. [16], it is found that the Bell-like states are the most robust states of all two-qubit states under several kinds of decoherence processes. Li *et al.*[17] have found the most robust genuine tripartite entangled states and the most fragile ones during three different decoherence processes and showed that although the two locally unitary equivalent states have the same entanglement properties, their robustness are significantly different.

The previous studies on the robustness of entangled states under different decoherence models are based on the premise that each qubit of the system is coupled to its own environment independently. However, when the time windows for a series of qubits in the channel are partially overlapped, the actions of the channel are no longer independent of each other, but may be correlated. A well-studied correlated Pauli channel is proposed in Refs. [22–24], which considers the memory induced by the classical correlations between multiple actions of the channels. The influence of correlated channels on quantum coherence,[25–28] quantum teleportation,[29] quantum Fisher information,[30] and measurement uncertainty,[31] have been explored recently.

In this paper, the robustness of entanglement for two- and three-qubit pure states in different correlated noisy channels are studied. For the two-qubit states, we use concurrence[32–36] as a measure of entanglement, while for the three-qubit states, we use negativity[37,38] as a measure of entanglement. We use the degree of decoherence noise that induces the entanglement of the time-evolved state decays to $1/c$ $(c > 1)$ of the initial entanglement as a measure of the robustness during decoherence. The main purpose of this paper is to study how the robustness of entangled states changes when one varies the correlation strength of the channels.

This paper is arranged as follows. In section 2, we review briefly the correlated decoherence models. In section 3, we investigate the robustness of entanglement for the Bell-like states and random two-qubit pure states in four typical correlated noisy channels, respectively. Then in section 4, we investigate the robustness of two special three-qubit states and random three-qubit pure states in correlated bit flip and phase flip channels. Finally, section 5 summarizes the main results obtained in this paper.

2. The model of a correlated quantum channel

We start with some preliminaries related to the classically correlated quantum channels, which provide a proper mathematical tool to describe the correlations between consecutive actions of the channel $\mathcal{E}$ on the sequence of qubits.[22–24]

We send N qubits through the channel, and the state of the N qubits is described by the density operator

ρ_0. If the channel acts independently and identically on each qubit, the corresponding output state of the system is given by

$$\mathcal{E}(\rho_0) = \sum_{i_1\cdots i_N} E_{i_1\cdots i_N}\rho_0 E^{\dagger}_{i_1\cdots i_N}, \tag{1}$$

where the Kraus operators $E_{i_1\cdots i_N} = \sqrt{p_{i_1\cdots i_N}}\sigma_{i_1}\otimes\cdots\otimes\sigma_{i_N}$ describe the action of transmission channels, satisfying the normalization condition $\sum_{i_1\cdots i_N} E^{\dagger}_{i_1\cdots i_N}E_{i_1\cdots i_N} = \mathbb{1}$. The joint probability distribution function is given by $p_{i_1\cdots i_N} = p_{i_1}\cdots p_{i_N}$, with $p_{i_n} \geq 0$ and $\sum_{i_n} p_{i_n} = 1$. σ_0 is the 2×2 identity matrix and σ_{i_n} are the Pauli operators in x, y, and z directions. Such a model describes a quantum channel $\mathcal{E}$, which has no memory on the history of its actions on the sequence of qubits.

For two-qubit system, the Kraus operators $E_{ij} = \sqrt{p_{ij}}\sigma_i\otimes\sigma_j$ and the joint probability distribution function is given by $p_{ij} = p_i p_j$. However, for a channel with partial memory between two consecutive uses, the joint probability distribution takes the the following form [22]

$$p_{ij} = (1-\mu)p_i p_j + \mu p_i \delta_{ij}. \tag{2}$$

The channel memory coefficient $0 \leqslant \mu \leqslant 1$ denotes the correlation strength between successive actions of the channel on the two adjacent qubits. In other words, μ can be interpreted as the probability that the same Pauli rotation is applied to the two adjacent qubits. The correlation strength $\mu = 0$ and $\mu = 1$ correspond to the uncorrelated and the fully correlated channels, respectively.

Similarly, for a three-qubit quantum system, it is easy to obtain the following Kraus operators

$$E_{ijk} = \sqrt{p_{ijk}}\sigma_i\otimes\sigma_j\otimes\sigma_k, \tag{3}$$

with $p_{ijk} = p_i p_{j|i} p_{k|j}$, $p_{j|i} = (1-\mu)p_j + \mu\delta_{ji}$, and likewise for $p_{k|j}$.

In the following, we will consider four different correlated noisy channels, including the correlated depolarizing, bit flip, bit-phase flip, and phase flip channels. These four channels have different probability distributions, but they all belong to the class of Pauli channels of Eq. (1). The depolarizing channel has the probability distribution $p_0 = 1 - 3p/4$ and $p_1 = p_2 = p_3 = p/4$, with $p \in [0, 1]$. The bit flip channel has the probability distribution $p_0 = 1-p/2$, $p_1 = p/2$, and $p_2 = p_3 = 0$. The bit-phase flip channel has the probability distribution $p_0 = 1-p/2$, $p_2 = p/2$, and $p_1 = p_3 = 0$. The phase flip channel has the probability distribution $p_0 = 1-p/2$, $p_3 = p/2$, and $p_1 = p_2 = 0$.

3. Robustness of Bell-Like States in Correlated Quantum Channels

We now begin our discussion about how the correlations between consecutive uses of the channel affecting the robustness of entanglement of the two-qubit states. To this end, we use concurrence to quantify the amount of entanglement associated with the density matrix ρ. As stated by Wootters, [32] the concurrence is defined as

$$C(\rho) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \tag{4}$$

where λ_i ($i = 1, 2, 3,$ and 4) are the eigenvalues of the Hermitian operator $R = \rho(\sigma_2\otimes\sigma_2)\rho^*(\sigma_2\otimes\sigma_2)$ in decreasing order and ρ^* is the complex conjugate of the density matrix ρ.

We take the input state of the channels in the following form

$$|\psi\rangle = \cos\theta\,|01\rangle + \sin\theta\,|10\rangle\,, \tag{5}$$

where $\theta \in [0, \pi/2]$ and the initial entanglement of the state $|\psi\rangle$ is $C_0 = \sin(2\theta)$.

3.1. The correlated depolarizing channel

First, we consider the correlated depolarizing channel. For the input state $|\psi\rangle$, the nonzero elements of density matrix for the output state can be obtained as

$$\begin{aligned}
\rho_{11,44} &= \frac{p}{2}\left(1-\frac{p}{2}\right)(1-\mu),\\
\rho_{22} &= \left(\frac{3p}{4}\right)^2 + \left(1-\frac{3p}{2}\right)\cos^2\theta + \frac{3p}{4}\left(1-\frac{3p}{4}\right)\mu,\\
\rho_{33} &= \left(\frac{3p}{4}\right)^2 + \left(1-\frac{3p}{2}\right)\sin^2\theta + \frac{3p}{4}\left(1-\frac{3p}{4}\right)\mu,\\
\rho_{23,32} &= \sin\theta\cos\theta[\mu + (1-\mu)(1-p)^2].
\end{aligned} \tag{6}$$

By substituting the density matrix (6) into Eq. (4), the concurrence for the time-evolved state can be obtained as

$$C = (1-4\alpha)C_0 - 2\alpha, \tag{7}$$

with

$$\alpha = \frac{p}{2}\left(1-\frac{p}{2}\right)(1-\mu). \tag{8}$$

The robustness of entanglement can be understood as a measure of the amount of noise required to disrupt the entanglement of the system. we use the degree of decoherence noise that induces the entanglement decrease to $1/c$ of its initial value to indicate the robustness of the system in the process of decoherence. The degree of depolarizing when the concurrence decreases to C_0/c can be obtained analytically as

$$p(c,\mu) = 1 - \sqrt{\frac{2C_0(1/c-\mu) + (1-\mu)}{(2C_0+1)(1-\mu)}}. \tag{9}$$

To reveal the effect of correlations on the robustness of the two-qubit entangled states in the correlated depolarizing channel, we show the C_0 dependence of the depolarizing degree $p(c,\mu)$ with different correlation strength when the entanglement decays to C_0/c ($c = 2, 4, 8, 16$) in Fig. 1.

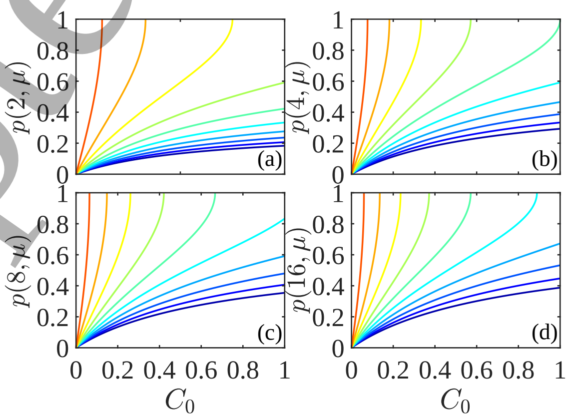

Fig. 1. (color online) Robustness of entanglement for $|\psi\rangle$ in correlated depolarizing channel. We plot the depolarizing degree $p(c,\mu)$ when the entanglement decreases to C_0/c ($c = 2, 4, 8, 16$) in (a), (b), (c) and (d), respectively. The ten color curves (from bottom to top) correspond to the correlation strength $\mu = 0, 0.1, 0.2, ..., 0.9$, respectively.

It can be seen from the plots in Fig. 1 that the depolarizing degree $p(c,\mu)$ has a monotonic relationship with the initial entanglement C_0 and the entanglement of two-qubit system will become more robust as the correlation strength increases. This indicates that the classical correlation in the correlated channels is beneficial to the robustness of entangled states, so it may reduce the speed of entanglement decay. As can be seen from Eq. (9), there exists a critical correlation strength $\mu_{c,1} = (c+2)/3c$. When $\mu < \mu_{c,1}$, $p(c,\mu)$ has a maximum value $p(c,\mu)_m = 1 - \sqrt{[2(1/c-\mu)]/[3(1-\mu)]+1/3}$. When the correlation strength $\mu \geq \mu_{c,1}$, the maximum value of $p(c,\mu)$ is 1. At this time, the maximum value of initial concurrence can only take the value $C_{0,\mu} = (1-\mu)/[2(\mu-1/c)]$.

In order to study the robustness of two-qubit states systematically, we explored the decoherence process of all two-qubit pure states in the correlated depolarizing channel. We generate a large number of randomly distributed two-qubit pure states according to the Haar measure.[39] Both the real and the imaginary parts of each state vector elements are Gaussian random numbers with a zero mean and unit variance. Then we normalize the vectors and calculate their concurrence. We show the depolarizing degree $p(c,\mu)$ for the state $|\psi\rangle$ and 50000 random two-qubit pure states in correlated depolarizing channel with different correlation strength when the concurrence decreases to $C_0/2$, $C_0/4$, $C_0/8$, and $C_0/16$ in Fig. 2, respectively.

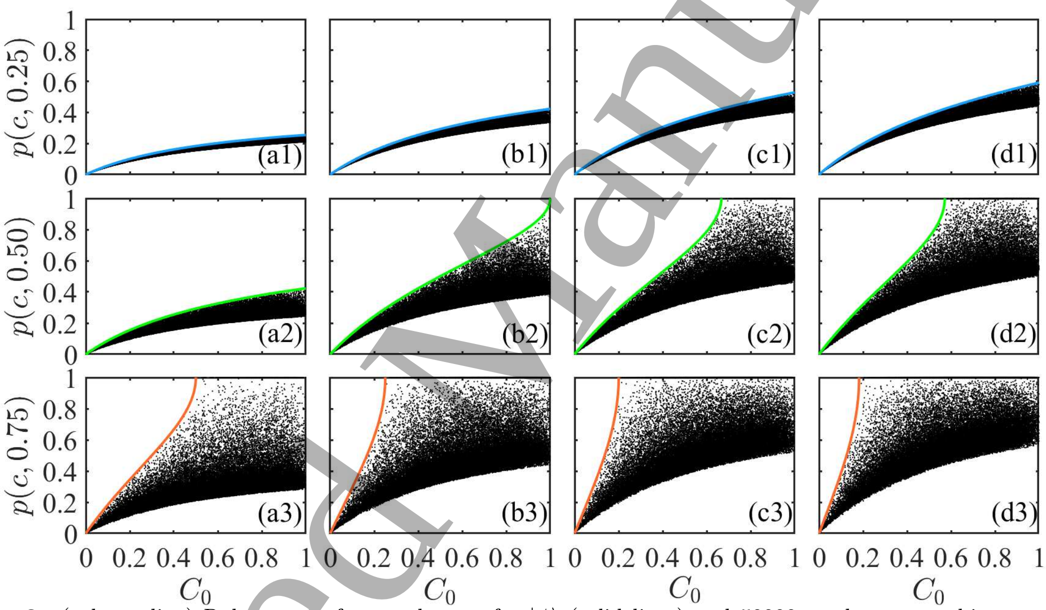

Fig. 2. (color online) Robustness of entanglement for $|\psi\rangle$ (solid lines) and 50000 random two-qubit pure states (black dots) in the correlated depolarizing channel. We plot the depolarizing degree $p(c,\mu)$ for $c = 2, 4, 8$, and 16 corresponding to Fig. 2(ai), (bi), (ci), and (di), respectively. Here $i = 1, 2$, and 3 correspond to correlation strength $\mu = 0.25$, $\mu = 0.50$ and $\mu = 0.75$, respectively.

By comparing with 50000 random two-qubit pure states, as shown in Fig. 2, the degree of depolarizing $p(c,\mu)$ of the Bell-like state is the upper bound of all the two-qubit pure states during decoherence in the correlated depolarizing channel. It is obvious that the Bell-like state is always the most robust state, irrespective of the strength of the classical correlation.

3.2. The correlated bit flip, bit-phase flip, and phase flip channels

Next, we consider the case of the correlated bit flip channel. For the input state $|\psi\rangle$, the nonzero elements of the density matrix of the output state can be obtained as

$$\begin{aligned}
\rho_{11,44} &= \frac{p}{2}\left(1-\frac{p}{2}\right)(1-\mu), \\
\rho_{14,41} &= \frac{p}{2}\left(1-\frac{p}{2}\right)(1-\mu)\sin(2\theta), \\
\rho_{23,32} &= \sin\theta\cos\theta\left[1+(1-\mu)\left(1-\frac{p}{2}\right)^2+\frac{p^2}{4}\right], \\
\rho_{22} &= \frac{p^2}{4}+(1-p)\cos^2\theta+\frac{p}{2}\left(1-\frac{p}{2}\right)\mu, \\
\rho_{33} &= \frac{p^2}{4}+(1-p)\sin^2\theta+\frac{p}{2}\left(1-\frac{p}{2}\right)\mu,
\end{aligned} \tag{10}$$

then by substituting the density matrix (10) into Eq. (4), the entanglement of the output state can be obtained as

$$C = (1-4\alpha)C_0. \tag{11}$$

For the bit-phase flip channel, a further calculation shows that the density matrix and the concurrence of the output state have completely the same form to those for the bit flip channel, so we do not list the corresponding results here. For the correlated phase flip channel, the nonzero elements of the density operator for the output state can be obtained as

$$\begin{aligned}
&\rho_{22} = \cos^2\theta,\ \ \rho_{33} = \sin^2\theta, \\
&\rho_{23,32} = \sin\theta\cos\theta[\mu+(1-\mu)(1-p)^2],
\end{aligned} \tag{12}$$

from which one can get that the concurrence of the output state is exactly the same as Eq. (11). So the influence of correlations between consecutive actions of the three channels on the robustness of two-qubit states is exactly the same. In the following, we take the phase flip channel as an example, and the results also apply to the other two channels.

The phase flip degree when the concurrence C_0 decreases to C_0/c can be easily obtained as

$$p(c,\mu) = 1-\sqrt{\frac{1-c\mu}{c(1-\mu)}}, \tag{13}$$

from which one can see that when the concurrence decreases to C_0/c, there exists a critical correlation strength $\mu_{c,2} = 1/c$. When $\mu > 1/c$, there is no solution to $p(c,\mu)$.

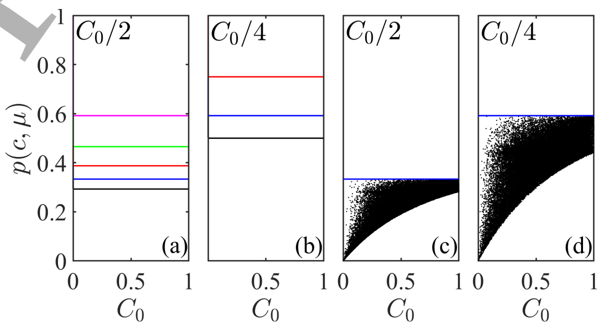

Fig. 3. (color online) Robustness of entanglement for $|\psi\rangle$ (solid lines) and 50000 random two-qubit pure states (black dots) in the correlated phase flip channel. The phase flip degree $p(c, \mu)$ of $|\psi\rangle$ when the entanglement decreases to C_0/c ($c = 2, 4$) with different μ are plotted in (a) and (b), respectively. The solid black, blue, red, green and purple lines (from bottom to top) correspond to $\mu = 0$, 0.1, 0.2, 0.3, and 0.4, respectively. The phase flip degree $p(c, \mu)$ for $|\psi\rangle$ and 50000 random two-qubit pure states with $\mu = 0.1$ when the entanglement decreases to C_0/c ($c = 2, 4$) are shown in (c) and (d), respectively.

We can see from Fig. 3(a) and (b) that the entanglement of two-qubit states can become more robust as the correlation strength increases, and the influence of correlation on robustness is independent of the initial entanglement. In addition, for different c, the upper limit of correlation strength is $1/c$. Similarly, we plot the phase flip degree $p(c, \mu)$ for the Bell-like state $|\psi\rangle$ and 50000 random two-qubit pure states in the correlated phase flip channel with $\mu = 0.1$ when the entanglement decreases to $C_0/2$ and $C_0/4$ in Fig. 3(c) and (d), respectively. It is obvious that the phase flip degree of all the random pure states do not exceed the two blue lines which describe the robustness of the Bell-like states. In the correlated bit flip, bit-phase flip and phase flip channels, the state $|\psi\rangle$ in (5) is the most robust state in the process of decoherence, irrespective of the strength of the classical correlation.

4. Robustness of Three-Qubit States in Correlated Quantum Channels

For a three-qubit state ρ with the three qubits being denoted by A, B, and C, we concentrate on the bipartition $A|BC$ of the system. Then the negativity is defined as twice of the sum of the absolute values of the negative eigenvalues of the partial transpose of ρ^{T_A}, which can be written as [38]

$$N(\rho) = \|\rho^{T_A}\|_1 - 1, \tag{14}$$

where T_A denotes the partial transpose with respect to the subsystem A.

We consider the three-qubit pure state [17]

$$|\Psi\rangle = \frac{\cos\theta}{\sqrt{2}}(|001\rangle + |010\rangle) + \mathbf{i}\frac{\sin\theta}{\sqrt{2}}(|100\rangle + |111\rangle), \tag{15}$$

where $\theta \in [0, \pi/2]$, and the following pure state which is locally unitarily equivalent to $|\Psi\rangle$, i.e.,

$$|\Phi\rangle = \frac{e^{\mathbf{i}\theta}}{2}(|000\rangle + |111\rangle) + \frac{e^{-\mathbf{i}\theta}}{2}(|011\rangle + |100\rangle). \tag{16}$$

In the following, we focus on the robustness of three-qubit states in the correlated phase flip and bit flip channels. For the correlated phase flip channel, the negativity of the output state for the input state $|\Psi\rangle$ can be obtained as

$$N = (1 - 4\alpha)N_0, \tag{17}$$

where α is given in Eq. (8) and $N_0 = \sin(2\theta)$ is the initial negativity of $|\Psi\rangle$. When the negativity decreases to N_0/c ($c \geq 1$), we can obtain the phase flip degree as follows

$$p(c, \mu) = 1 - \sqrt{\frac{1 - c\mu}{c(1 - \mu)}}. \tag{18}$$

We find that Eqs. (17) and (18) have exactly the same form as Eqs. (11) and (13) corresponding to the Bell-like state in the correlated phase flip channel, respectively. Similarly, there exists a critical correlation strength $\mu_c = 1/c$. When the negativity decreases to N_0/c, the upper limit of correlation strength is $1/c$.

We do not list the analytical expression of the negativity of state $|\Phi\rangle$ here due to its complexity. Instead, we numerically solve the entanglement evolution of $|\Psi\rangle$ and $|\Phi\rangle$ and plot the degree of phase flip $p(c,\mu)$ when the negativity decreases to N_0/c ($c = 2, 4$) with different μ in Fig. 4(a) and (b). It can be seen that the influence of correlations on $|\Psi\rangle$ is completely different from that on $|\Phi\rangle$. For the state $|\Psi\rangle$, the phase flip degree $p(c,\mu)$ is independent of N_0 and increases with the increasing correlation strength μ in the range of $\mu \in (0, 1/c)$. For the state $|\Phi\rangle$, $p(c,\mu)$ increases with the increase of N_0 in the whole region of μ. As a comparison, we plot the degree of bit flip $p(c,\mu)$ when the negativity decreases to N_0/c ($c = 2, 4$) with different correlation strength in Fig. 4(c) and (d). Obviously, the effect of correlated correlations on state $|\Phi\rangle$ ($|\Psi\rangle$) in correlated phase flip channel is the same as that of correlated correlations on state $|\Psi\rangle$ ($|\Phi\rangle$) in correlated bit flip channel.

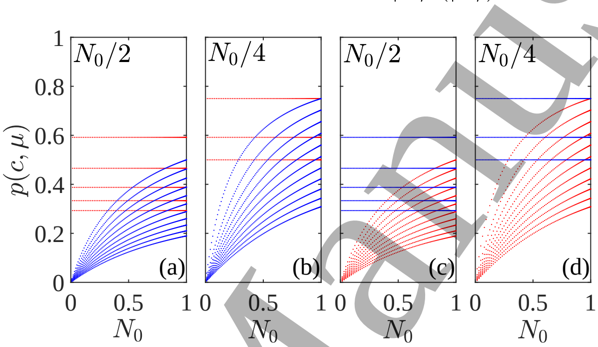

Fig. 4. (color online) For the correlated phase flip [panels (a) and (b)] and bit flip [panels (c) and (d)] channels, we plot the degree of decoherence $p(c,\mu)$ when the negativity decreases to $N_0/2$ and $N_0/4$ for $|\Psi\rangle$ (red dotted lines) and $|\Phi\rangle$ (blue dotted lines), respectively. For each state, the dotted lines from bottom to top correspond to the increase of μ from 0 with an equal interval 0.1.

In order to study the robustness of all three-qubit pure states, we randomly sampled 50000 three-qubit pure states and plot the phase flip degree $p(c,\mu)$ when the negativity decreases to N_0/c ($c = 2, 4$) in Fig. 5(ai) and (bi), respectively. As a contrast, the degree of bit flip $p(c,\mu)$ when the negativity decreases to N_0/c ($c = 2, 4$) are plotted in Fig. 5(ci) and (di), respectively. Here $i = 1$ and $i = 2$ correspond to correlation strength $\mu = 0.1$ and $\mu = 0.2$, respectively.

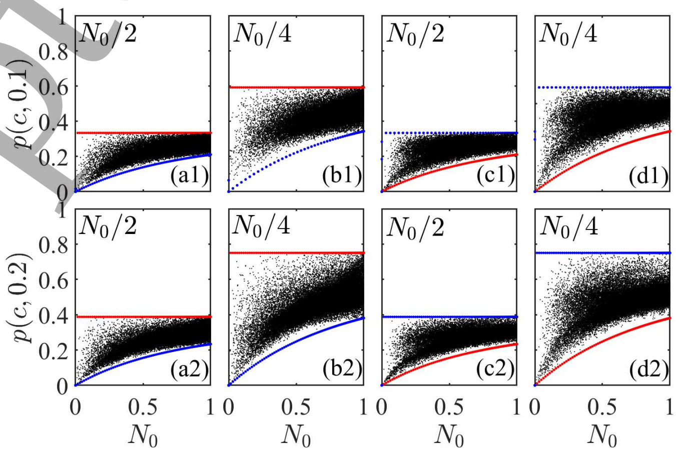

Fig. 5. (color online) Robustness of entanglement for three-qubit pure states in correlated phase flip [panels (ai) and (bi)] and bit flip [panels (ci) and (di)] channels with $i = 1$ and 2 corresponding to $\mu = 0.1$ and 0.2, respectively. We plot the degree of decoherence $p(c, \mu)$ when the negativity decreases to $N_0/2$ and $N_0/4$, respectively. The state $|\Psi\rangle$, $|\Phi\rangle$, and 50000 random sampled pure states are shown with red dotted lines, blue dotted lines, and black dots, respectively.

Obviously, the phase flip degree of all the randomly sampled pure states are between the red dotted lines and the blue dotted lines. Regardless of the value of μ, $|\Psi\rangle$ is always the most robust state, while $|\Phi\rangle$ is always the most fragile state in the correlated phase flip channel. On the contrary, we can see that in the correlated bit flip channel, the state $|\Phi\rangle$ is always the most robust state and the state $|\Psi\rangle$ is always the most fragile state.

For the correlated depolarizing and bit-phase flip channels, the robustness of the state $|\Psi\rangle$ is exactly the same as that of state $|\Phi\rangle$, and their robustness in these two channels is slightly different, as shown in the first row in Fig. 6. The second row in Fig. 6 shows the comparison of the robustness of states $|\Psi\rangle$ and $|\Phi\rangle$ with 50000 random three-qubit pure states in the two correlated channels when the correlation strength $\mu = 0.1$. We can see that state $|\Psi\rangle$ and state $|\Phi\rangle$ are neither the most robust nor the most fragile states.

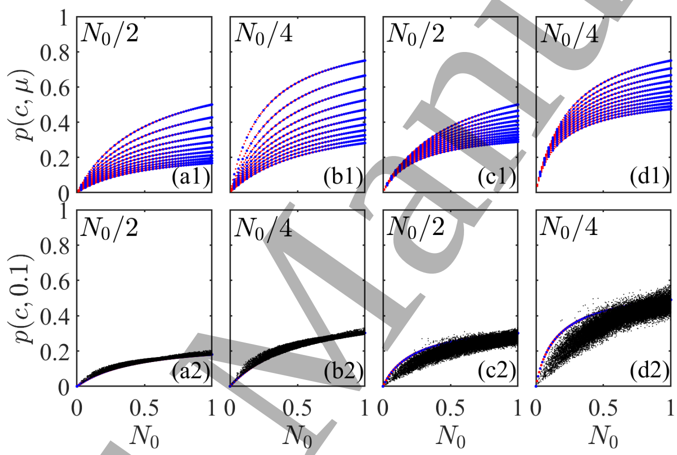

Fig. 6. (color online) Robustness of entanglement for $|\Psi\rangle$ (red dotted lines), $|\Phi\rangle$ (blue dotted lines), and 50000 random sampled pure states (black dots) in correlated depolarizing [panels (ai) and (bi)] and bit-phase flip [panels (ci) and (di)] channels. (a1)-(d1) $p(c, \mu)$ $(c = 2, 4)$ with μ increases from 0 to 1 with an equal interval 0.1. (a2)-(d2) $p(c, \mu)$ $(c = 2, 4)$ for the state $|\Psi\rangle$, $|\Phi\rangle$, and 50000 random three-qubit pure states with $\mu = 0.1$, respectively.

5. Conclusion

In summary, we have explored the robustness of entangled states in different correlated quantum channels. It is found that the robustness of entangled states can be enhanced due to the correlations between consecutive actions of these noisy channels. Moreover, the robustness of the entangled state increases gradually when the correlation strength increases at equal intervals. First, we consider the Bell-like state and random two-qubit pure states in the correlated depolarizing, bit flip, bit-phase flip, and phase flip channels, respectively. We show explicitly that the entanglement depends on the type of quantum channels as well as the decoherence degree and the correlation strength. The Bell-like state is always the most robust state, regardless of the correlation strength, and the increase of correlation strength can enhance the robustness of quantum states. On the other hand, the effects of the correlated actions of the depolarizing channel on the robustness of the two-qubit states is completely different from that in the other three channels: (i) For the correlated depolarizing channel, the decoherence degree $p(c, \mu)$ always behaves as an increasing function of μ and C_0. (ii) For the correlated bit flip, bit-phase flip, and phase flip channels, the decoherence degree $p(c, \mu)$ increases with the increase of μ, and it does not depend on the initial entanglement C_0. Moreover, there exists a limit for the value of correlation strength $\mu_{c,2} = 1/c$.

For the three-qubit states, we numerically study the robustness of entanglement for two locally unitary equivalent states and random three-qubit pure states in four correlated channels. For the correlated phase flip and bit flip channels, we find that the most robust state and the most fragile state during decoherence in these two typical correlated channels are the same as those in the uncorrelated channels in Ref. [18]. Although the two states are locally unitary equivalent, they exhibit different robustness in these two different channels. The effects of the correlated actions on the most robust state is different from that on the most fragile one, while it has the same effect on the most robust states in two different channels, although the most robust states in different channels are different. The same is true for the most fragile state. We see that the robustness of entangled states can be changed by local unitary transformation. We hope that the analysis presented here can help to construct effective schemes to enhance the robustness of entangled states in open quantum systems.

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