Physica Scripta

PAPER

Robustness of multipartite entangled states for fermionic systems under noisy channels in non-inertial frames

To cite this article: Kwang-Il Kim *et al* 2021 *Phys. Scr.* **96** 125122

IOP Publishing Phys. Scr. 96 (2021) 125122 https://doi.org/10.1088/1402-4896/ac437c

Physica Scripta

CrossMark

RECEIVED
21 October 2021

REVISED
5 December 2021

ACCEPTED FOR PUBLICATION
15 December 2021

PUBLISHED
24 December 2021

PAPER

Robustness of multipartite entangled states for fermionic systems under noisy channels in non-inertial frames

Kwang-Il Kim , Myong Chol Pak, Tae-Hyok Kim, Jong Chol Kim, Yong-Hae Ko and Hak Chol Pak

Department of Physics, Kim Il Sung University, Ryongnam Dong, Taesong District, Pyongyang, Democratic People's Republic of Korea

E-mail: kwangilkim@163.com

Keywords: robustnesses, multipartite entangled states, noisy channels, non-inertial frames

Abstract

We investigate robustness of bipartite and tripartite entangled states for fermionic systems in non-inertial frames, which are under noisy channels. We consider two Bell states and two Greenberger-Horne-Zeilinger (GHZ) states, which possess initially the same amount of entanglement, respectively. By using genuine multipartite (GM) concurrence, we analytically derive the equations that determine the difference between the robustness of these locally unitarily equivalent states under the amplitude-damping channel. We find that tendency of the robustness for two GHZ states evaluated by using three-tangle τ and GM concurrence as measures of genuine tripartite entanglement is equal to each other. We also find that the robustness of two Bell states is equal to each other under the depolarizing, phase damping and bit flip channels, and that the same is true for two GHZ states.

1. Introduction

As a pure quantum mechanical feature, quantum entanglement plays an important role in quantum information. It has been considered as a major resource for quantum information processing [1–5]. However, unavoidable interaction between an entangled quantum system and its environment usually degrades entanglement of the system. This decoherence introduces some disadvantages on the creation and manipulation of multiqubit entanglements in realistic quantum information processing tasks. Especially, under certain environments, entanglement sudden death (ESD) [6] can be induced.

Recently, many studies on the robustness of multiqubit quantum systems under different noisy channels have been reported [7–17]. Vidal and Tarrach [7] have studied the robustness of two-qubit systems by calculating the minimal amount of mixing with locally prepared states. Some studies [8–10] have shown that the robustness of multiqubit entanglement under local decoherence may increase with the number of qubits. Particularly, in [8], Simon and Kempe found that GHZ states are more robust than other generic ones for three and four qubits. Aolita *et al* [11] have studied the robustness of generalized GHZ entanglement for arbitrary number of qubits. In 2009, Aolita *et al* [12] have analytically derived upper bounds for the entanglement of generalized GHZ states coupled to locally depolarizing and dephasing environments. They have also shown that randomly generated initial states tend to violate the bounds and that this discrepancy grows with the number of qubits. In 2010, Zhao and Deng [13] have investigated the relationship between the entanglement and the robustness of multiqubit system under depolarizing noise. They found that the robustness of an arbitrary pure two-qubit state depends completely on its entanglement, but this is not always true for a three-qubit system. By using the Monte Carlo method, we [14] have found that Bell-like states are always the most robust states during decoherence process. Zhang *et al* [15] have systematically investigated the speed of disentanglement for two-qubit arbitrary states and three-qubit pure states under depolarizing channel and found that GHZ-type states are the most robust. Several studies [16–20] have been devoted to the investigation of the robustness of multiqubit states which are locally unitarily equivalent to each other and therefore initially contain the same amount of entanglement. Borras *et al* [16] have investigated the decay of entanglement of multiqubit states that are equivalent to GHZ states under local unitary transformations. They have considered five decoherence models (phase damping, depolarizing, bit flip, phase flip, and bit-phase flip) and studied which models are the most robust for four and six qubit systems. Li and Zhao [17] have done a systematic study on the robustness of three-qubit states under amplitude-damping, dephasing and bit flip channels. In each of these decoherence processes, they proved the most robust genuine tripartite entangled states and the most fragile ones. Li *et al* [18] have investigated the robustness of multiqubit states under dephasing and bit flip channels and showed the difference between the entanglement evolution of the states of the two special forms, which are locally unitarily equivalent to each other. Pan *et al* [19] have investigated the entanglement dynamics of two-qubit systems including two Bell states for various quantum noises and found that the entanglement of state $(|01\rangle + |10\rangle)/\sqrt{2}$ is more robust than the state $(|00\rangle + |11\rangle)/\sqrt{2}$ under the amplitude-damping channel. To study the robustness of genuinely entangled states, $|GHZ_1\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$, $|GHZ_2\rangle = (|001\rangle + |110\rangle)/\sqrt{2}$ and $|W\rangle = (|001\rangle + |010\rangle + |100\rangle)/\sqrt{3}$, Ali [20] has compared the dynamics of three states with that of random pure states and weighted graph states under collective dephasing, and has found that almost every quantum state in this type of dynamics is robust.

On the other hand, many recent experiments for quantum information processing include relativistic particles such as photons and are carried out by observers (or detectors) that are in relative motions. The relativistic quantum information is also needed to further complete quantum information theory and to understand black hole physics and quantum gravity. Unfortunately, by the Unruh effect [21], the entanglement shared between inertial and non-inertial observers will also be degraded. There have been many studies on the dynamics of multipartite entanglement under different noisy channels in non-inertial frames [22–32]. Therefore, studying robustness of multipartite entangled states under noisy channels in non-inertial frames is very important from both theoretical and practical aspects.

This paper is organized as follows. In the next section, by using GM concurrence [33, 34] as a measure of bipartite entanglement, we investigate the robustness of two Bell states of fermionic systems under noisy channels in non-inertial frames and analytically derive equations for determining the difference between the robustness of two Bell states. In section 3, we also investigate the robustness of two GHZ states by using both GM concurrence and three-tangle τ [35]. Finally, we present some conclusions in section 4.

2. Robustness of two Bell states

As states shared initially by Alice and Bob in an inertial frame, we consider two Bell states

$$|\mathrm{Bell}_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B), \tag{1}$$

$$|\mathrm{Bell}_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B). \tag{2}$$

These states are locally unitarily equivalent to each other and thus possess the same amount of entanglement (initially, $C_{GM}(|\mathrm{Bell}_1\rangle) = C_{GM}(|\mathrm{Bell}_2\rangle) = 1$).

Now we assume that each qubit is coupled to its own noisy environment. These processes may be described by using the quantum operator-sum representation approach [1]. In general, under a noisy channel, the final density matrix of a single qubit system becomes

$$\rho' = \varepsilon(\rho) = \sum_{i=1}^{n} E_i(t)\rho E_i^{\dagger}(t), \tag{3}$$

where ρ and ρ' are the initial and the final density matrix of a single qubit system, $E_i(t)(i = 1, 2, \ldots n)$ and $E_i^{\dagger}(t)$ are the Kraus operators [1, 36] of the channel and their complex conjugate operators.

We then let Bob moves with respect to Alice with uniform acceleration a_b. Using the singles-mode approximation, Bob's vacuum and excited states are transformed as follows in Minkowski space [37]:

$$|0\rangle_M = \cos r_b\,|0\rangle_I|0\rangle_{II} + \sin r_b\,|1\rangle_I|1\rangle_{II}, \tag{4}$$

$$|1\rangle_M = |1\rangle_I|0\rangle_{II}, \tag{5}$$

where $\cos r_b = (e^{-2\pi\omega c/a_b} + 1)^{-1/2}$, ω is frequency of the Dirac particle, c is the speed of light in vacuum. In equations (4) and (5), $\{\langle n|_I\}$ and $\{\langle n|_{II}\}$ indicate Rindler modes in regions I and II, respectively.

We first consider the case of the amplitude-damping channel. In this case, Kraus operators are

$$E_0^j = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-P_j} \end{pmatrix},\ E_1^j = \begin{pmatrix} 0 & \sqrt{P_j} \\ 0 & 0 \end{pmatrix}. \tag{6}$$

Here $j = (A, B)$, P_A and P_B are the decay parameters in Alice's and Bob's channel, respectively.

Using equations (3)–(6) as in [22, 32], we can get the density matrix ρ' for Bell_1 state. Nonzero elements of the desity matrix are given by

$$\begin{aligned}
\rho'_{11} &= \frac{1}{2}[\cos^2 r_b + P_B \sin^2 r_b + P_A P_B],\\
\rho'_{22} &= \frac{1}{2}[(1 - P_B)\sin^2 r_b + P_A(1 - P_B)],\\
\rho'_{33} &= \frac{1}{2}P_B(1 - P_A),\\
\rho'_{44} &= \frac{1}{2}(1 - P_A)(1 - P_B),\\
\rho'_{14} &= \rho'_{41} = \frac{1}{2}\cos r_b \sqrt{1 - P_A}\sqrt{1 - P_B}.
\end{aligned} \tag{7}$$

The density matrix ρ' has an X-form [38], therefore we obtain

$$\begin{aligned}
C_{GM}(\mathrm{Bell}_1) &= \max[0, \sqrt{1 - P_A}\cdot\\
&\cdot\sqrt{1 - P_B}(\cos r_b - \sqrt{P_B(P_A + \sin^2 r_b)})].
\end{aligned} \tag{8}$$

Similarly, nonzero elements of the density matrix ρ' for Bell_2 state are given by

$$\begin{aligned}
\rho'_{11} &= \frac{1}{2}[P_A \cos^2 r_b + P_B(P_A \sin^2 r_b + 1)],\\
\rho'_{22} &= \frac{1}{2}(1 - P_B)(P_A \sin^2 r_b + 1),\\
\rho'_{33} &= \frac{1}{2}(1 - P_A)(\cos^2 r_b + P_B \sin^2 r_b),\\
\rho'_{44} &= \frac{1}{2}(1 - P_A)(1 - P_B)\sin^2 r_b,\\
\rho'_{23} &= \rho'_{32} = \frac{1}{2}\cos r_b \sqrt{1 - P_A}\sqrt{1 - P_B}.
\end{aligned} \tag{9}$$

So we obtain

$$\begin{aligned}
C_{GM}(\mathrm{Bell}_2) &= \max[0, \sqrt{1 - P_A}\sqrt{1 - P_B}(\cos r_b -\\
&- \sin r_b \sqrt{P_A \cos^2 r_b + P_B(P_A \sin^2 r_b + 1)})].
\end{aligned} \tag{10}$$

Now let's compare two GM concurrences. From equations (8) and (10) we can see that the relationship between two GM concurrences, $C_{GM}(\mathrm{Bell}_1)$ and $C_{GM}(\mathrm{Bell}_2)$, depends on the values of two decay parameters P_A and P_B, and the acceleration r_b. For any values of r_b, P_A and P_B, in order to $C_{GM}(\mathrm{Bell}_1) \leqslant C_{GM}(\mathrm{Bell}_2)$, the following relation must be satisfied:

$$P_B(P_A + \sin^2 r_b) - \sin^2 r_b(P_A \cos^2 r_b + P_B(P_A \sin^2 r_b + 1)) \geqslant 0. \tag{11}$$

More specifically, if $P_A = 0$, then we get $C_{GM}(\mathrm{Bell}_1) = C_{GM}(\mathrm{Bell}_2)$ regardless of P_B and r_b. In other words, if there is no influence of noisy channel acting on Alice's qubit, the robustness of two Bell states is always equal to each other. If $P_A \neq 0$, instead, then $C_{GM}(\mathrm{Bell}_1) \leqslant C_{GM}(\mathrm{Bell}_2)$ only when

$$P_B \geqslant \frac{\sin^2 r_b}{1 + \sin^2 r_b}. \tag{12}$$

Obviously, one can see that equation (12) does not depend on the state of Alice, but only on the state of Bob, i.e., only on the relationship between noise acting on Bob and his acceleration.

Let us consider the cases where one of the two GM concurrences is always more robust than the other. For example, in the case when $r_b = 0$, $P_A \neq 0$ and $P_B \neq 0$ we get

$$C_{GM}(\mathrm{Bell}_1) = \max[0, \sqrt{1 - P_A}\sqrt{1 - P_B}(1 - \sqrt{P_A P_B})], \tag{13}$$

$$C_{GM}(\mathrm{Bell}_2) = \max[0, \sqrt{1 - P_A}\sqrt{1 - P_B}], \tag{14}$$

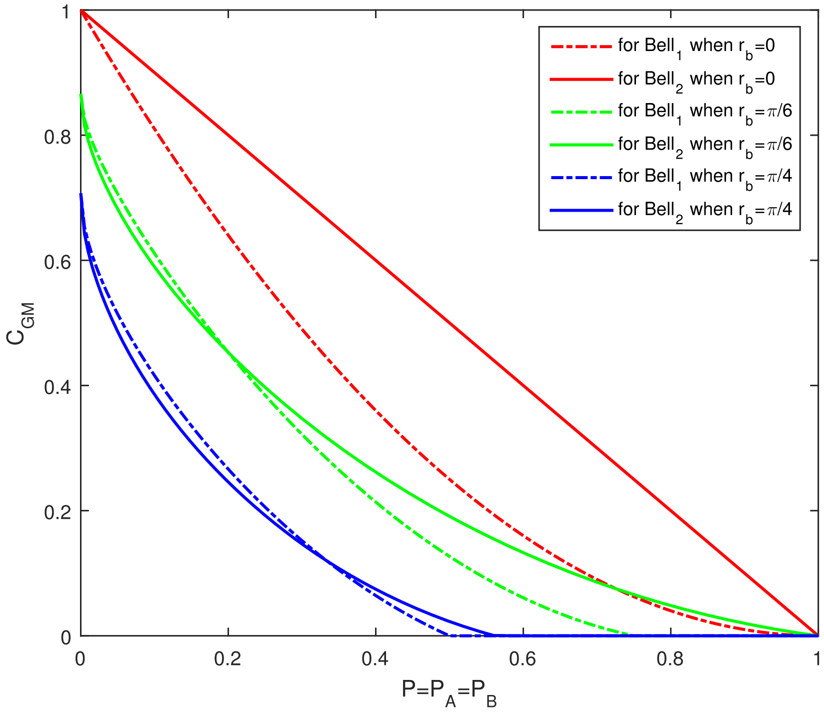

Figure 1. GM concurrences for Bell_1 (dot-dashed line) and Bell_2 (solid line) states as functions of $P(= P_A = P_B)$ under amplitude-damping channel for several values of r_b [$r_b = 0$ (red), $r_b = \pi/6$ (green), $r_b = \pi/4$ (blue)].

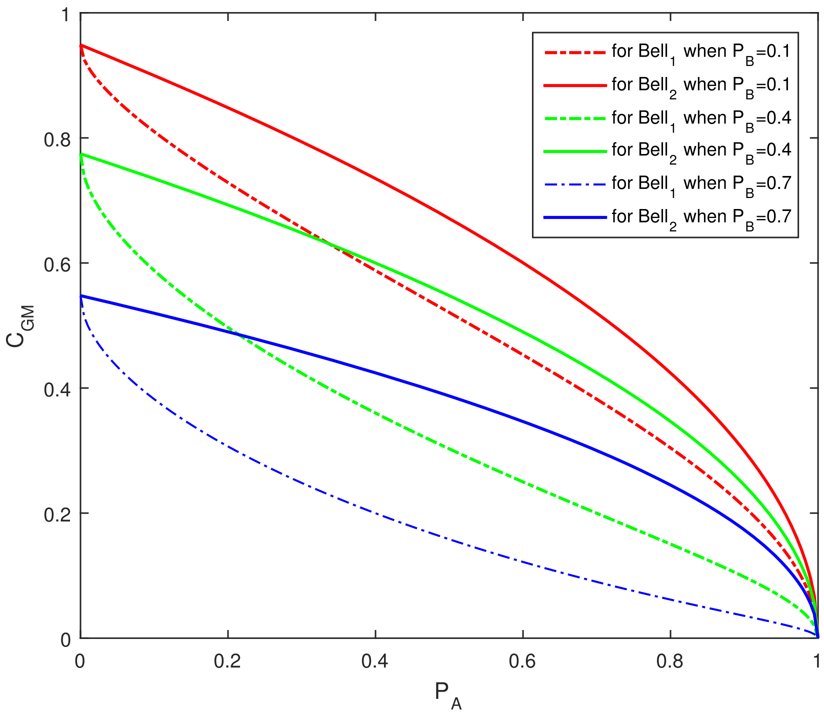

Figure 2. GM concurrences for Bell_1 (dot-dashed line) and Bell_2 (solid line) states as functions of P_A under amplitude-damping channel for several values of P_B [$P_B = 0.1$ (red), $P_B = 0.4$ (green), $P_B = 0.7$ (blue)] when $r_b = 0$.

and therefore always $C_{GM}(\text{Bell}_1) \leqslant C_{GM}(\text{Bell}_2)$. But, in the case when $r_b \neq 0$, $P_A \neq 0$ and $P_B = 0$ we obtain

$$C_{GM}(\text{Bell}_1) = \max[0, \sqrt{1 - P_A}\cos r_b], \tag{15}$$

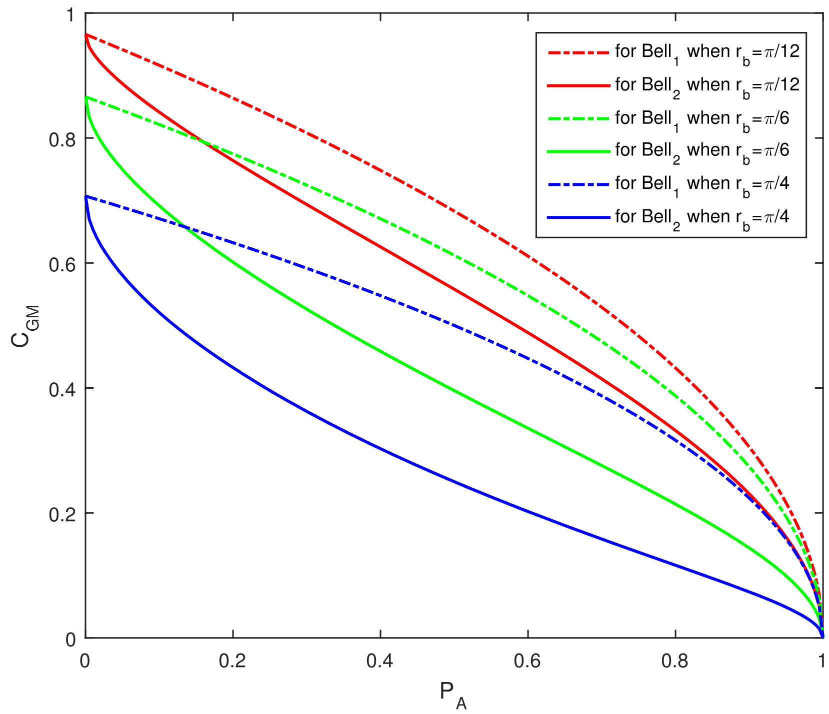

Figure 3. GM concurrences for Bell_1 (dot-dashed line) and Bell_2 (solid line) states as functions of P_A under amplitude-damping channel for several values of r_b [$r_b = \pi/12$ (red), $r_b = \pi/6$ (green), $r_b = \pi/4$ (blue)] when $P_B = 0$.

$$C_{GM}(\mathrm{Bell}_2) = \max[0, \sqrt{1 - P_A}\cos r_b(1 - \sin r_b\sqrt{P_A})], \tag{16}$$

and always $C_{GM}(\mathrm{Bell}_2) \leqslant C_{GM}(\mathrm{Bell}_1)$.

In figure 1 we plot GM concurrences for two Bell states as a function of the decay parameter $P(= P_A = P_B)$ under the amplitude-damping channel for several values of the acceleration parameter r_b. For two special cases, the GM concurrences of two Bell states are shown in figures 2 and 3, respectively. These figures show how Bob's acceleration and the noisy channel acting on his qubit would change the relationship between the robustness of Bell_1 state and the one of Bell_2 state. One can see that in an inertial frame ($r_b = 0$), the GM concurrence of Bell_2 state is always more robust than that of Bell_1 state for all values of decay parameters (see figure 2), which is consistent with the results of [19]. But if there is no influence of noisy channel acting on Bob's qubit in a non-inertial frame ($r_b \neq 0$ and $P_B = 0$), the opposite is true (see figure 3).

In the cases of the depolarizing, phase damping and bit flip channels, we find that two GM concurrences are always equal to each other.

3. Robustness of two GHZ states

We now continue the above discussion to the case of genuine tripartite entanglement states, GHZ states. We assume that three observers, Alice, Bob and Charlie, share two GHZ states:

$$|\mathrm{GHZ}_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B|0\rangle_C + |1\rangle_A|1\rangle_B|1\rangle_C), \tag{17}$$

$$|\mathrm{GHZ}_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B|0\rangle_C + |1\rangle_A|1\rangle_B|1\rangle_C). \tag{18}$$

where $|0\rangle_{A(B,C)}$ and $|1\rangle_{A(B,C)}$ are vacuum and excited states from the perspective of an inertial observer. These two states, again, have the same amount of initial entanglement ($C_{GM}(\mathrm{GHZ}_1) = C_{GM}(\mathrm{GHZ}_2) = 1$).

Let r_b and r_c are Bob's and Charlie's accelerations, P_A, P_B and P_C are the decay parameters in Alice's, Bob's and Charlie's channel, respectively. As in section 2, we first consider the case of the amplitude-damping channel. We can get the density matrix ρ' for GHZ_1 state as in our previous work [32] to obtain its nonzero entries

$$
\begin{aligned}
\rho'_{11} &= \frac{1}{2}(\cos^2 r_b \cos^2 r_c + P_B P_C \sin^2 r_b \sin^2 r_c + \\
&\quad + P_C \cos^2 r_b \sin^2 r_c + P_B \sin^2 r_b \cos^2 r_c + P_A P_B P_C), \\
\rho'_{22} &= \frac{1}{2}(1 - P_C)(\cos^2 r_b \sin^2 r_c + P_B \sin^2 r_b \sin^2 r_c + P_A P_B), \\
\rho'_{33} &= \frac{1}{2}(1 - P_B)(\sin^2 r_b \cos^2 r_c + P_C \sin^2 r_b \sin^2 r_c + P_A P_C), \\
\rho'_{44} &= \frac{1}{2}(1 - P_B)(1 - P_C)(\sin^2 r_b \sin^2 r_c + P_A), \\
\rho'_{55} &= \frac{1}{2}P_B P_C(1 - P_A), \\
\rho'_{66} &= \frac{1}{2}P_B(1 - P_A)(1 - P_C), \\
\rho'_{77} &= \frac{1}{2}P_C(1 - P_A)(1 - P_B), \\
\rho'_{88} &= \frac{1}{2}(1 - P_A)(1 - P_B)(1 - P_C), \\
\rho'_{18} &= \rho'_{81} = \frac{1}{2}\sqrt{1 - P_A}\sqrt{1 - P_B}\sqrt{1 - P_C}\cos r_b \cos r_c,
\end{aligned} \tag{19}
$$

and for GHZ_2 state,

$$
\begin{aligned}
\rho'_{11} &= \frac{1}{2}(P_C \cos^2 r_b + P_B P_C \sin^2 r_b + P_A P_B \cos^2 r_c + P_A P_B P_C \sin^2 r_c), \\
\rho'_{22} &= \frac{1}{2}(1 - P_C)(\cos^2 r_b + P_B \sin^2 r_b + P_A P_B \sin^2 r_c), \\
\rho'_{33} &= \frac{1}{2}(1 - P_B)(P_C \sin^2 r_b + P_A \cos^2 r_c + P_A P_C \sin^2 r_c), \\
\rho'_{44} &= \frac{1}{2}(1 - P_B)(1 - P_C)(\sin^2 r_b + P_A \sin^2 r_c), \\
\rho'_{55} &= \frac{1}{2}(1 - P_A)P_B(\cos^2 r_c + P_C \sin^2 r_c), \\
\rho'_{66} &= \frac{1}{2}(1 - P_A)P_B(1 - P_C)\sin^2 r_c, \\
\rho'_{77} &= \frac{1}{2}(1 - P_A)(1 - P_B)(\cos^2 r_c + P_C \sin^2 r_c), \\
\rho'_{88} &= \frac{1}{2}(1 - P_A)(1 - P_B)(1 - P_C)\sin^2 r_c, \\
\rho'_{27} &= \rho'_{72} = \frac{1}{2}\sqrt{1 - P_A}\sqrt{1 - P_B}\sqrt{1 - P_C}\cos r_b \cos r_c.
\end{aligned} \tag{20}
$$

Therefore we obtain

$$
\begin{aligned}
C_{GM}(\mathrm{GHZ}_1) &= \max[0, \sqrt{(1 - P_A)(1 - P_B)(1 - P_C)}\cdot \\
&\cdot(\cos r_b \cos r_c - \sqrt{\alpha + \delta} - \sqrt{\beta + \delta} - \sqrt{\gamma + \delta})],
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
C_{GM}(\mathrm{GHZ}_2) &= \max[0, \sqrt{(1 - P_A)(1 - P_B)(1 - P_C)}\cdot \\
&\cdot(\cos r_b \cos r_c - \sqrt{\alpha + \theta} - \sqrt{\beta + \theta} - \sqrt{\gamma + \theta})],
\end{aligned} \tag{22}
$$

where

$$
\begin{aligned}
\alpha &= P_C \cos^2 r_b \sin^2 r_c + P_B P_C \sin^2 r_b \sin^2 r_c, \\
\beta &= P_B P_C \sin^2 r_b \sin^2 r_c + P_B \sin^2 r_b \cos^2 r_c, \\
\gamma &= P_B P_C \sin^2 r_b \sin^2 r_c, \\
\delta &= P_A P_B P_C, \\
\theta &= P_A P_B \cos^2 r_c \sin^2 r_c + P_A P_B P_C \sin^4 r_c.
\end{aligned} \tag{23}
$$

Now we compare two GM concurrences, $C_{GM}(\mathrm{GHZ}_1)$ and $C_{GM}(\mathrm{GHZ}_2)$. For any values of r_b, r_c, P_A, P_B and P_C, in order to $C_{GM}(\mathrm{GHZ}_1) \leqslant C_{GM}(\mathrm{GHZ}_2)$, the relation $\delta \geqslant \theta$ must be satisfied. The two GM concurrences becomes equal to each other if at least one of two decay parameters, P_A and P_B, is zero. However, if both of two decay parameters, P_A and P_B, are nonzero, then the required condition for $C_{GM}(\mathrm{GHZ}_1) \leqslant C_{GM}(\mathrm{GHZ}_2)$ is

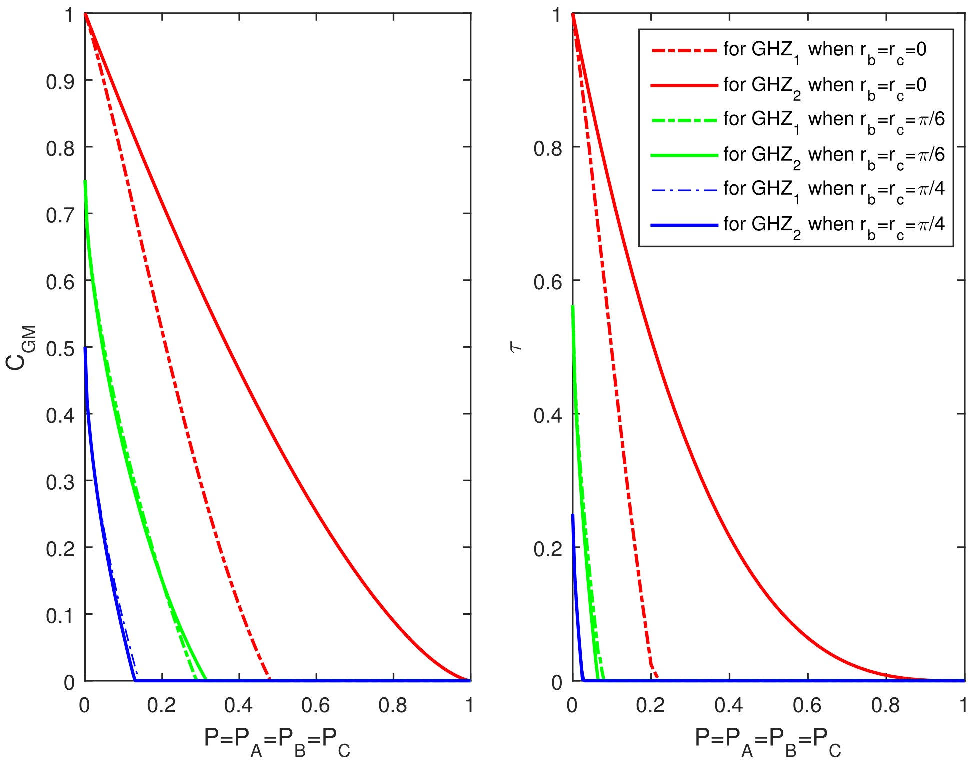

Figure 4. GM concurrences and three-tangle τ for GHZ_1 (dot-dashed line) and GHZ_2 (solid line) states as functions of $P(=P_A=P_B=P_C)$ under amplitude-damping channel for several values of r_b and r_c [$r_b=r_c=0$(red), $r_b=r_c=\pi/6$ (green), $r_b=r_c=\pi/4$ (blue)].

$$P_C \geqslant \frac{\sin^2 r_c}{1+\sin^2 r_c}. \tag{24}$$

One can see that equation (24) does not depend on the states of Alice and Bob, but only on the state of Charlie, i.e., only on the noise acting on Charlie and his acceleration. As in the previous section, we consider the cases where one of the two GM concurrences is always more robust than the other. For example, when $r_b=0$, $r_c=0$, $P_A\neq 0$, $P_B\neq 0$ and $P_C\neq 0$, we get

$$C_{GM}(\mathrm{GHZ}_1) = \max[0, \sqrt{1-P_A}\sqrt{1-P_B}\sqrt{1-P_B}(1-3\sqrt{\delta})], \tag{25}$$

$$C_{GM}(\mathrm{GHZ}_2) = \max[0, \sqrt{1-P_A}\sqrt{1-P_B}\sqrt{1-P_C}], \tag{26}$$

and find that the GM concurrence of GHZ_2 state is more robust than the one of GHZ_1 state. But, in the case when $r_b=0$, $r_c\neq 0$, $P_A\neq 0$, $P_B\neq 0$ and $P_C=0$, we obtain

$$C_{GM}(\mathrm{GHZ}_1) = \max[0, \sqrt{1-P_A}\sqrt{1-P_B}\cos r_c], \tag{27}$$

$$C_{GM}(\mathrm{GHZ}_2) = \max[0, \sqrt{1-P_A}\sqrt{1-P_B}\cdot \\ \cdot\cos r_c(1-3\sqrt{P_A P_B}\sin r_c)]. \tag{28}$$

and find that the GM concurrence of GHZ_1 state is more robust than the one of GHZ_2 state.

On the other hand, given the density matrix for GHZ states, we can also calculate three-tangle τ as a measure of tripartite entanglement. The three-tangle τ is a good measure for tripartite entanglement states but in general, it is difficult to calculate the three-tangle τ analytically for mixed three-qubit states. By using the conjugate gradient method [39] as in [32], we numerically calculate the three-tangle τ of GHZ states.

In figure 4 we plot GM concurrences and three-tangle τ of two GHZ states as functions of the decay parameter $P(=P_A=P_B=P_C)$ under the amplitude-damping channel for several values of the acceleration parameters r_b and r_c. For two special cases, GM concurrences and three-tangle τ are shown in figures 5 and 6, respectively. In figures 4–6, we can see that three-tangle τ of two GHZ states decreases faster than GM concurrences but tendency of the robustness of two GHZ states evaluated by using three-tangle τ and GM concurrence is equal to each other. In other words, equation (24) obtained by using GM concurrence can be applied when evaluating the robustness of two GHZ states by using three-tangle τ. These figures show how

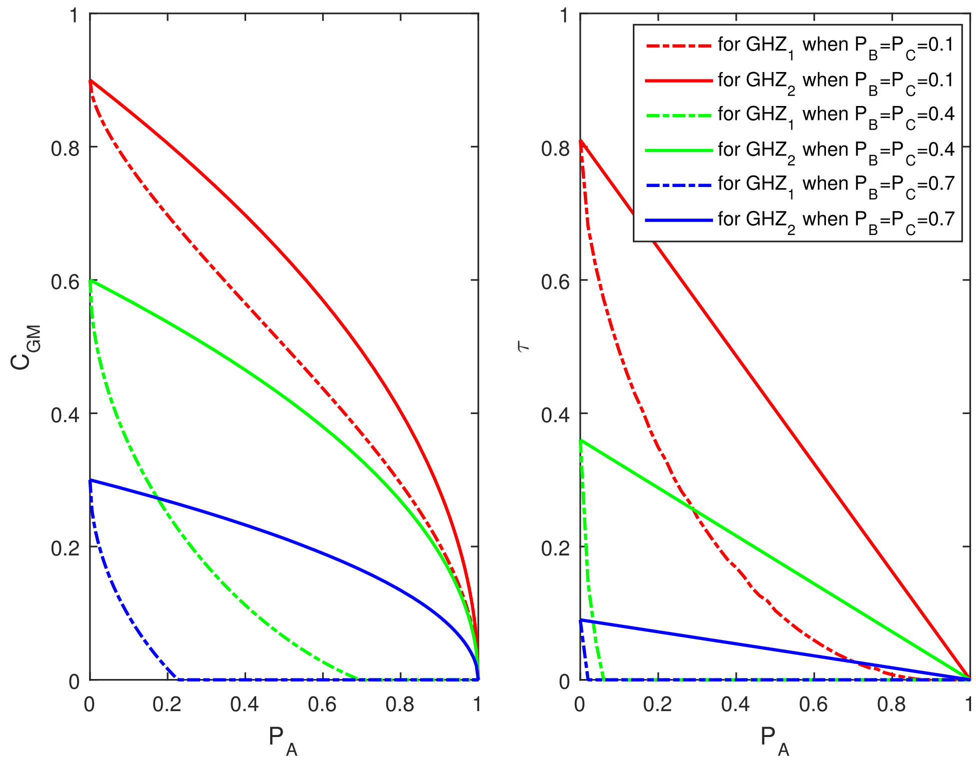

Figure 5. GM concurrences and three-tangle τ for GHZ$_1$ (dot-dashed line) and GHZ$_2$ (solid line) states as functions of P_A under amplitude-damping channel for several values of P_B and P_C [$P_B = P_C = 0.1$ (red), $P_B = P_C = 0.4$ (green), $P_B = P_C = 0.7$ (blue)] when $r_b = r_c = 0$.

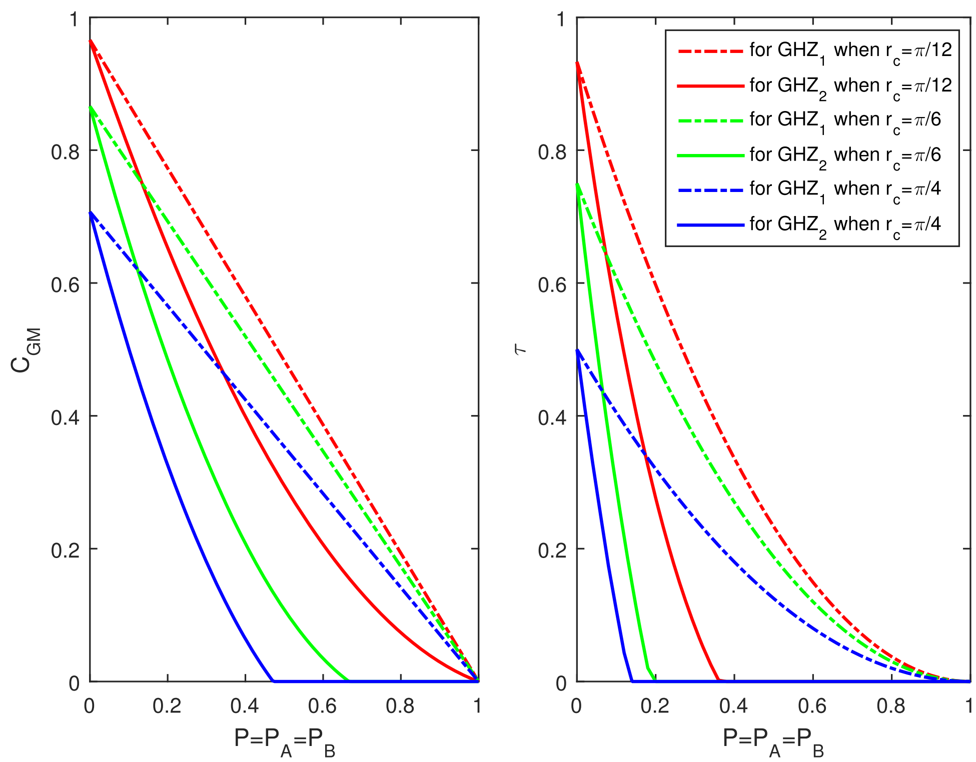

Figure 6. GM concurrences and three-tangle τ for GHZ$_1$ (dot-dashed line) and GHZ$_2$ (solid line) states as functions of $P(= P_A = P_B)$ under amplitude-damping channel for several values of r_c [$r_c = \pi/12$ (red), $r_c = \pi/6$ (green), $r_c = \pi/4$ (blue)] when $P_C = 0$.

Charlie's acceleration and the noisy channel acting on his qubit would change the relationship between the robustness of GHZ_1 and GHZ_2 states.

In the cases of the depolarizing, phase damping and bit flip channels, we again find that two GM concurrences are always equal to each other, and that the same is true for two three-tangles.

4. Conclusion

In this paper we investigated robustness of two Bell states $((|01\rangle + |10\rangle)/\sqrt{2}$ and $(|00\rangle + |11\rangle)/\sqrt{2})$ and two GHZ states $((|000\rangle + |111\rangle)/\sqrt{2}$ and $(|001\rangle + |110\rangle)/\sqrt{2})$ of fermionic systems under different noisy channels in non-inertial frames. The two Bell states are locally, unitarily equivalent to each other and so are the two GHZ states. We found that the robustness of multipartite entangled states is affected not only by noisy channel, but also by acceleration. By using genuine multipartite (GM) concurrence as a measure of multipartite entanglement, we analytically derived explicit equations that determine the difference between the robustness of locally unitarily equivalent states under the amplitude-damping channel in non-inertial frames. In particular, robustness of two GHZ states showed the same tendency for the two different measures of genuine tripartite entanglement used in this study: three-tangle τ and GM concurrence. In contrast with the case of the amplitude-damping channel, robustness of two Bell states and two GHZ states is equal to each other, respectively, under the depolarizing, phase damping and bit flip channels. Our results will further enrich the results of robustness of multipartite entangled states in practical quantum information processing tasks.

Acknowledgments

We thank Ok Song An and Un Gyong Ri for useful discussions. This work was supported by the National Program on Key Science Reseach of Democratic People's Republic of Korea(Grant No.18-1-2).

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

ORCID iDs

Kwang-Il Kim https://orcid.org/0000-0002-6237-9832

References

[1] Nielsen M A and Chuang I L 2010 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[2] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 Quantum entanglement *Rev. Mod. Phys.* **81** 865
[3] Bennett C H, Brassard G and Mermin N D 1992 Quantum cryptography without Bell's theorem *Phys. Rev. Lett.* **68** 557
[4] Long G L and Liu X S 2002 Theoretically efficient high-capacity quantum-key-distribution scheme *Phys. Rev.* A **65** 032302
[5] Deng F G, Long G L and Liu X S 2003 Two-step quantum direct communication protocol using the Einstein-Podolsky-Rosen pair block *Phys. Rev.* A **68** 042317
[6] Yu T and Eberly J H 2004 Finite-time disentanglement via spontaneous emission *Phys. Rev. Lett.* **93** 140404
[7] Vidal G and Tarrach R 1999 Robustness of entanglement *Phys. Rev.* A **59** 141
[8] Simon C and Kempe J 2002 Robustness of multiparty entanglement *Phys. Rev.* A **65** 052327
[9] Hein M, Dür W and Briegel H J 2005 Entanglement properties of multipartite entangled states under the influence of decoherence *Phys. Rev.* A **71** 032350
[10] Liu Z and Fan H 2009 Decay of multiqudit entanglement *Phys. Rev.* A **79** 064305
[11] Aolita L, Chaves R, Cavalcanti D, Acin A and Davidovich L 2008 Scaling laws for the decay of multiqubit entanglement *Phys. Rev. Lett.* **100** 080501
[12] Aolita L, Cavalcanti D, Acin A, Salles A, Tiersch M, Buchleitner A and de Melo F 2009 Scalability of Greenberger-Horne-Zeilinger and random-state entanglement in the presence of decoherence *Phys. Rev.* A **79** 032322
[13] Zhao B K and Deng F G 2010 Residual effect on the robustness of multiqubit entanglement *Phys. Rev.* A **82** 014301
[14] Kim K I, Zhao B K and Li H M 2014 Robustness of entanglement during decoherence *Commun. Theor. Phys.* **62** 667
[15] Zhang F L, Jiang Y and Liang M L 2013 Speed of disentanglement in multiqubit systems under a depolarizing channel *Ann. Phys. (Amsterdam, Neth.)* **333** 136
[16] Borras A, Majtey A P, Plastino A R, Casas M and Plastino A 2009 Robustness of highly entangled multiqubit states under decoherence *Phys. Rev.* A **79** 022108
[17] Li H M and Zhao B K 2018 Most robust and fragile states with the same entanglement properties *Ann. Phys. (Berlin)* **530** 1800053
[18] Li H M, Guo M D, Zhang R and Su X M 2019 Boundary states for entanglement robustness under dephasing and bit flip channels *Chin. Phys.* B **28** 100302
[19] Pan C N, Li F, Fang J S and Fang M F 2011 Entanglement dynamics of two-qubit systems in different quantum noises *Chin. Phys.* B **20** 020304

[20] Ali M 2015 Robustness of genuine tripartite entanglement under collective dephasing *Chin. Phys. Lett.* **32** 060302

[21] Unruh W G 1976 Notes on black-hole evaporation *Phys. Rev.* D **14** 870

[22] Wang J and Jing J 2010 Quantum decoherence in noninertial frames *Phys. Rev.* A **82** 032324

[23] Khan S and Khan M K 2011 Open quantum systems in noninertial frames *J. Phys. A: Math. Theor.* **44** 045305

[24] Wang J and Jing J 2012 System-environment dynamics of X-type states in noninertial frames *Ann. Phys.* **327** 283

[25] Khan S 2012 Entanglement of tripartite states with decoherence in non-inertial frames *J. Mod. Opt.* **59** 250

[26] Piao M Z and Ji X 2012 Quantum decoherence under phase damping in non-inertial frames *J. Mod. Opt.* **59** 21

[27] Ramzan M and Khan M K 2012 Decoherence and entanglement degradation of a qubit-qutrit system in non-inertial frames *Quantum Inf. Process.* **11** 443

[28] Ramzan M 2013 Entanglement dynamics of non-inertial observers in a correlated environment *Quantum Inf. Process.* **12** 83

[29] Zounia M, Shamirzaie M and Ashouri A 2017 Quantum teleportation via noisy bipartite and tripartite accelerating quantum states: beyond the single mode approximation *J. Phys. A : Math. Theor.* **50** 395302

[30] Sun W Y, Wang D, Yang J and Ye L 2017 Enhancement of multipartite entanglement in an open system under non-inertial frames *Quantum Inf. Process.* **16** 90

[31] Chen X and Chan K W C 2019 Quantum teleportation of Dirac fields in noninertial frames with amplitude damping *Phys. Rev.* A **99** 022334

[32] Kim K I, Pak M C and Kim T H 2020 Decoherence of multipartite entanglement states under amplitude-damping environment in non-inertial frames *Eur. Phys. J.* D **74** 124

[33] Wootters W K 1998 Entanglement of formation of an arbitrary state of two qubits *Phys. Rev. Lett.* **80** 2245

[34] Ma Z H, Chen Z H, Chen J L, Spengler C, Gabriel A and Huber M 2011 Measure of genuine multipartite entanglement with computable lower bounds *Phys. Rev.* A **83** 062325

[35] Coffman V, Kundu J and Wootters W K 2000 Distributed entanglement *Phys. Rev.* A **61** 052306

[36] Kraus K 1983 *States, Effect, and Operation: Fundamental Notions in Quantum Theory* (Berlin: Springer)

[37] Alsing P M, Fuentes-Schuller I, Mann R B and Tessier T E 2006 Entanglement of Dirac fields in non-inertial frames *Phys. Rev.* A **74** 032326

[38] Hashemi Rafsanjani S M, Huber M, Broadbent C J and Eberly J H 2012 Genuinely multipartite concurrence of N-qubit X matrices *Phys. Rev.* A **86** 062303

[39] Nocedal J and Wright S J 1999 *Numerical Optimization* (New York: Springer)