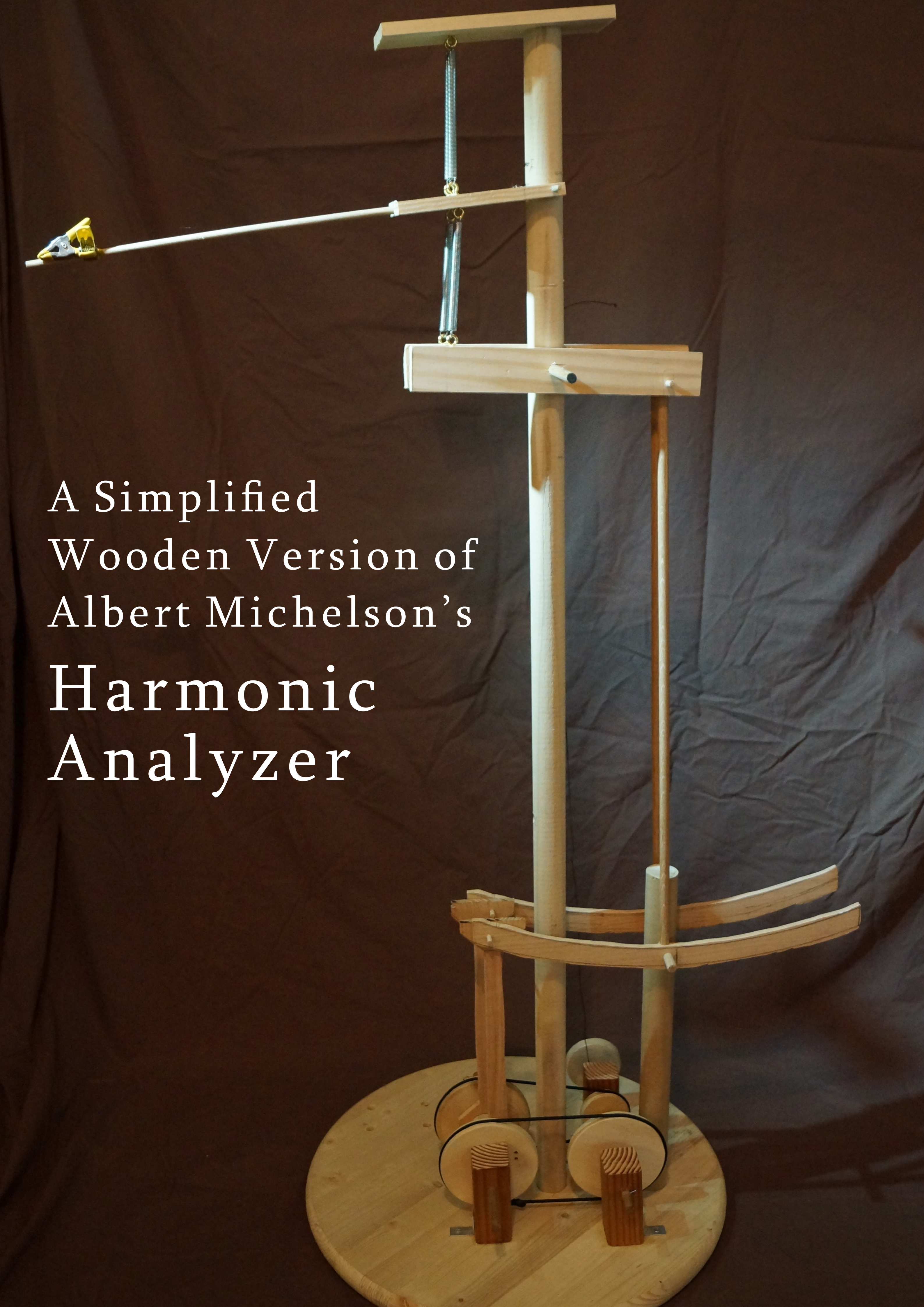
A Simplified Wooden Version of Albert Michelson's

Harmonic Analyzer



A Simplified Wooden Version of Albert Michelson's Harmonic Analyzer

by

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Preface

Albert Michelson's mechanical marvel, the "harmonic analyzer," is truly an amazing machine. Using springs, levers, gears, and chains, the machine can, given a set of twenty numbers, graph the sum of twenty weighted sines and cosines (mathematically speaking, it calculates and graphs an approximation of the function $\sum_{n=1}^{20} a_n \cos nx$). It can even perform the reverse operation, called analysis, by taking a curve and extracting the coefficients that produce it. Built about a hundred years ago, the harmonic analyzer is a functional piece of art and history, captivating in its complexity and yet astounding in its simplicity, and will never lose its ability to excite the imagination and stir the creative spirit.

At the close of 2012, Professor Bill Hammack and others of the University of Illinois began the production of a series of insightful videos and a beautiful book detailing the history and operation of Michelson's harmonic analyzer. The series, available on Professor Hammack's YouTube channel "EngineerGuy," was sent to the authors by a friend in the winter of 2015, and we decided to attempt to make a greatly simplified version of the machine out of wood using the tools we had at hand. Our efforts are chronicled herein, and we hope that this document brings attention to the engineering genius of the analyzer and Professor Hammack's excellent expository work. We highly recommend that the reader watch his videos to gain a better understanding of the workings of the machine; links are available at the end of this paper.

—Xinbo Chen, Madilyne Clark, and Zachary Clark

Construction

"Every one who has had occasion to calculate or to construct graphically the resultant of a large number of simple harmonic motions has felt the need of some simple and fairly accurate machine which would save the considerable time and labor involved in such computations."

—Albert Michelson
A New Harmonic Analyzer, 1898

All construction materials were purchased at Lowe's Home Improvement store, save the metal brackets which strangely could not be located by the staff and had to be purchased at Wal-Mart.

We ended up returning to Lowe's four times and the materials cost about $75 USD.

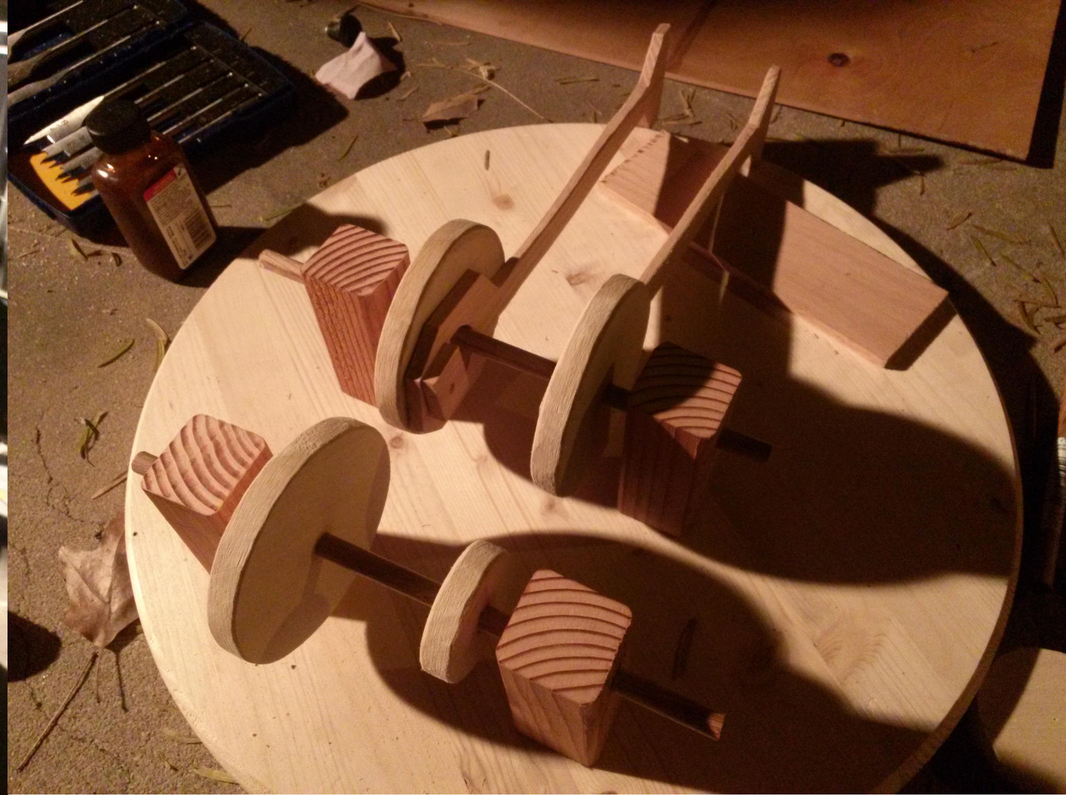
The cart in the picture is not as heavy as the driver is making it out to be.

The first step was to create the “gears” for the cylindrical and conical gear sets.

We contemplated undertaking the difficult task of cutting toothed wooden gears with our imprecise equipment, but decided that we could use smooth wheels and coat the outer surface with rubber cement. This idea was eventually scrapped in favor of using pulleys, but the smooth wheels remained and the name “gear” stuck.

We cut the rough shape with a band-saw and smoothed the edges with a table sander.

The two free-spinning gears also had cam pegs installed so that they create the up-and-down motion of the connecting arms as they rotate.

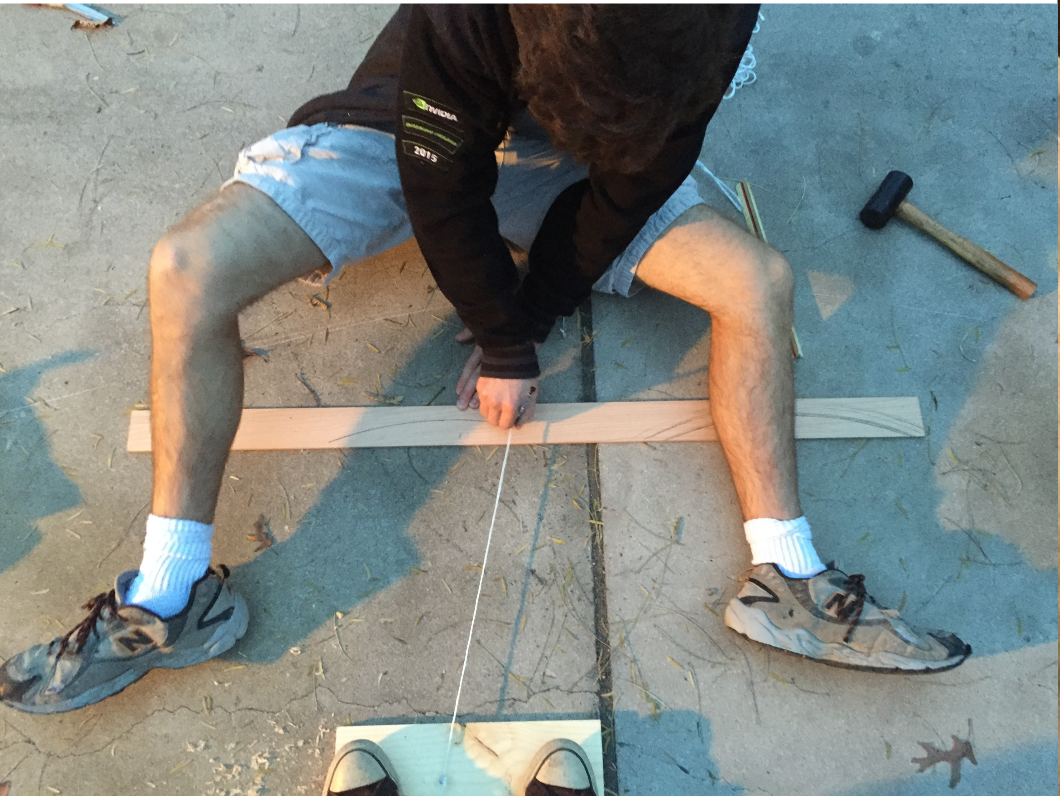


After shaping the gears, we drilled holes for the axes and mounted them on small stands.

The fixed gears were glued to their axis so that they rotate with a soon-to-be handle. The free-spinning gears are held in place by the stand and a separator (not visible). The connecting arms were cut from small planks and fitted around the axes and cam pegs.

On many occasions we worked long into the night.

We made the rocker arms by tracing the outline of a circle and cutting first on the band-saw, following up with the sander as with the gears.



We were able to compact the design of the machine due to the fact that we supply only two sizes of gear, but these optimizations are not applicable if more gears are to be supported.

The image to the left shows one such instance: we were able to avoid having to build a surrounding frame by placing a post in the center. A frame would have been difficult to precisely position and bore.

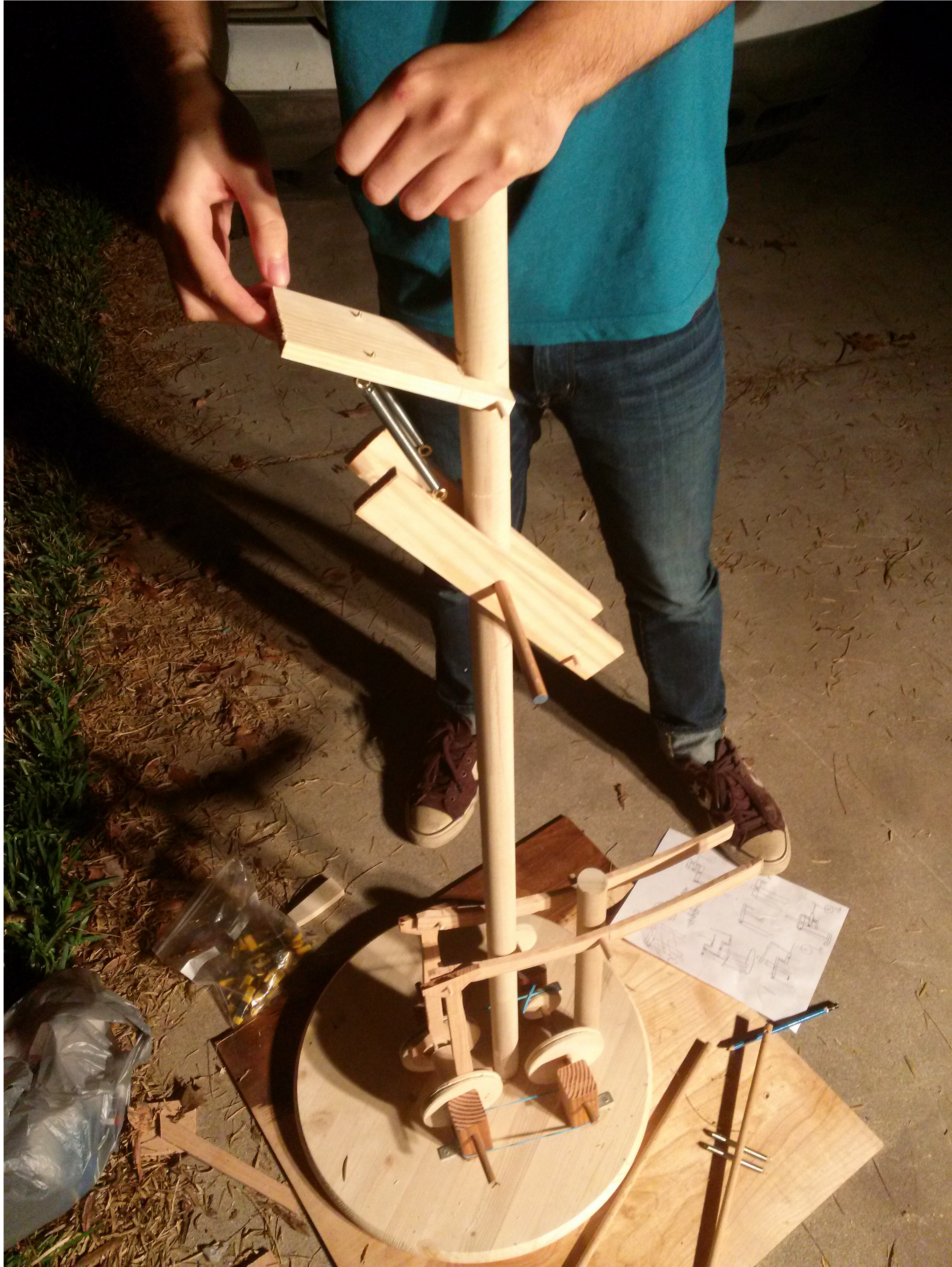
We also attached a wheel to the axis of the fixed gears to serve as a handle.

We originally used blue rubber-bands to connect the gears. However, the rubber-bands' elasticity caused a variable delay in the propagation of motion through the machine and distorted the graph, so we switched to less elastic material later.

To support the upper mechanisms, we anchored another taller post in center of the four gears. We drilled several holes for the pivots of the term levers and the summing lever, shown right.

We were able to mount the entire third level on this post, again taking advantage of the small number of gears. We traded flexibility of design for build time, and we would need to make several adjustments to this and other tiers of the machine if we had more time to dedicate to the project.

In the many places where moving wood contacts stationary wood, we sanded at least one of the surfaces with a high-grit paper to reduce friction. As there were several different types of wood involved and each has its own grain characteristics, this effort was successful to varying degrees.



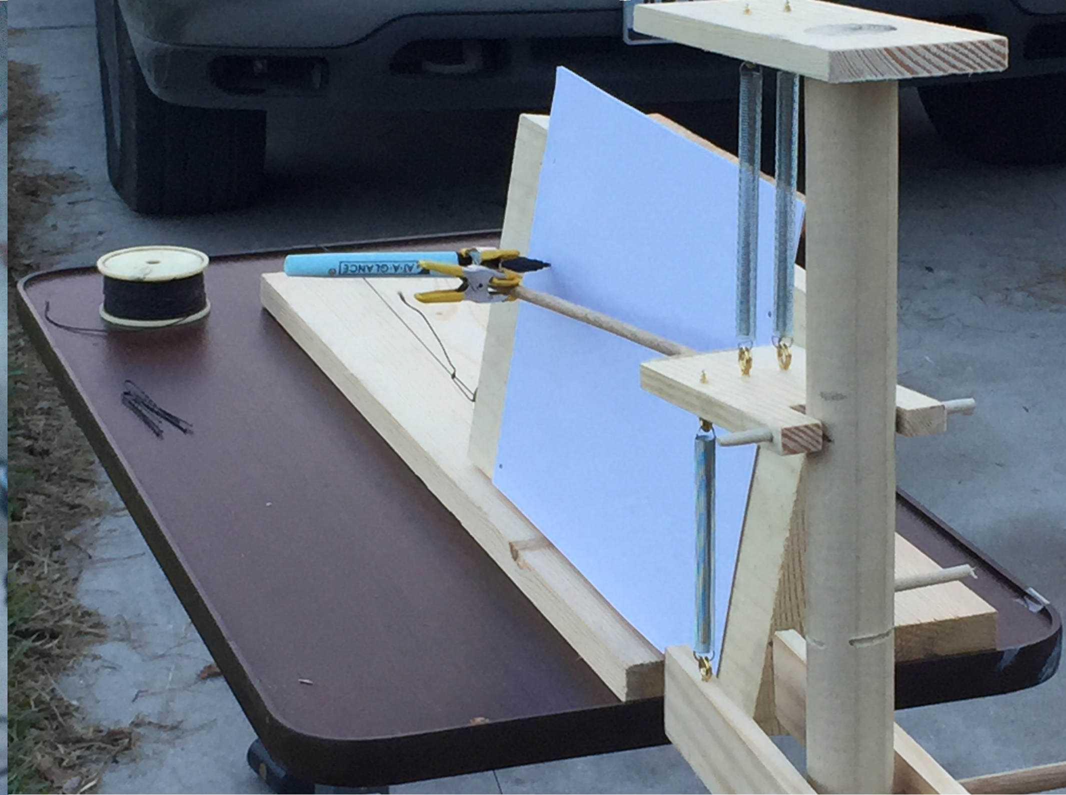
The design of this upper level of the machine departs considerably from Michelson's, but the effect is the same. Here, the summing lever is positioned between a fixed upper plate and the lower term bars. As the rockers undulate, the factor selection bars push on the lower levers of the upper level, which pull downward on the summing lever with more or less force.

Where Michelson used one large spring to balance the force of the twenty smaller springs, we simply used an equal number of small springs, which would have been too cumbersome in his case.

After adding the amplification bar to the end of the summing lever, we were ready for a test.

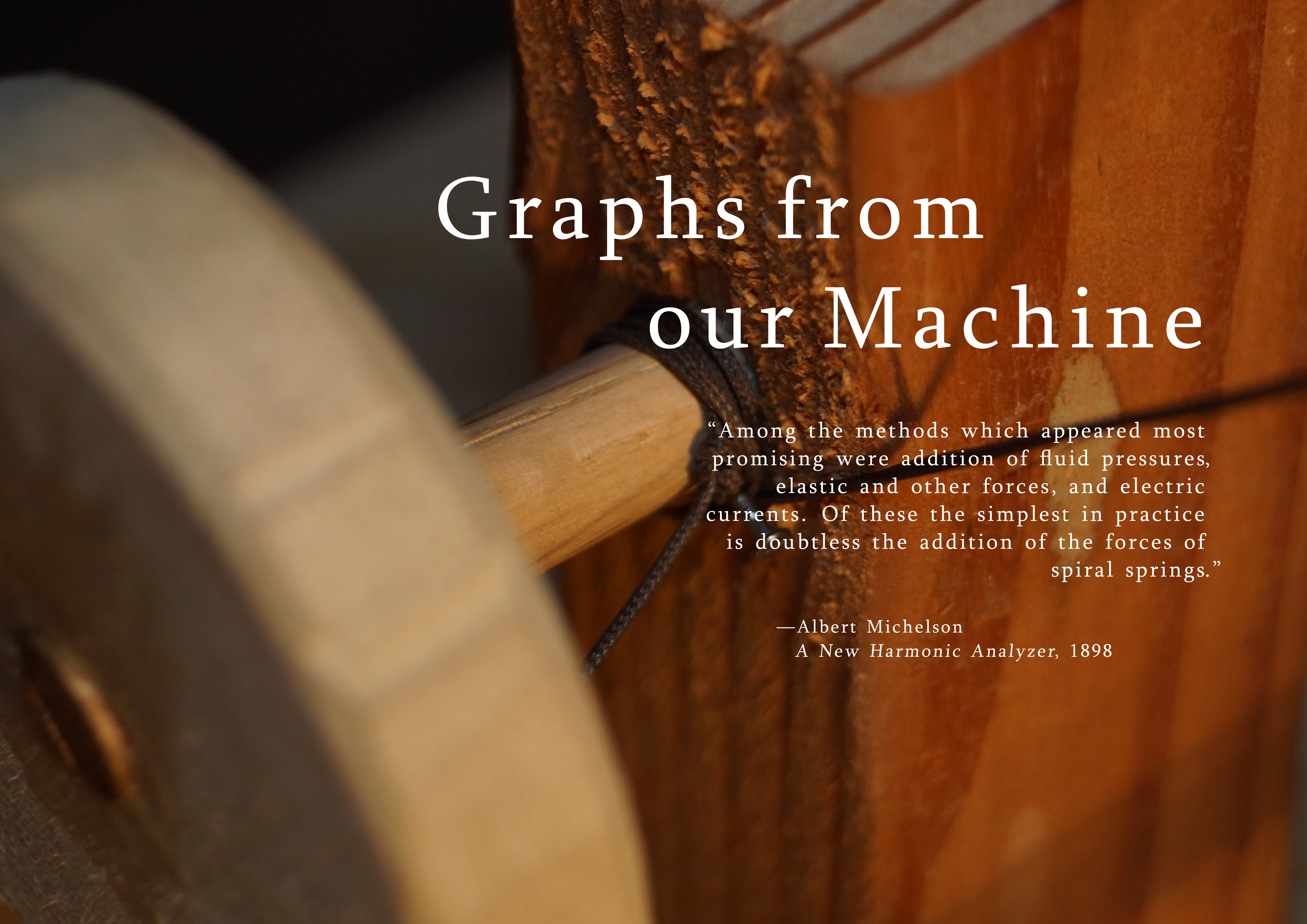
We initially imitated Michelson more closely and strung a wire from the end of the amplification bar to a point on the radius of the smaller of a pair of concentric wheels which serve to magnify the motion of the bar further. The springs we used were not strong enough to overcome the friction and rotate the gear, but luckily the motion at the end of the amplification bar was large enough without the augmentation.

Moving the pen closer to the summing lever has the effect of compressing the y-axis of the graph, and *vice-versa*.



A string is attached to a nail in the side of the axis rod of the fixed gears, and runs to a hook embedded in the opposite side of a board with paper nailed to the front.

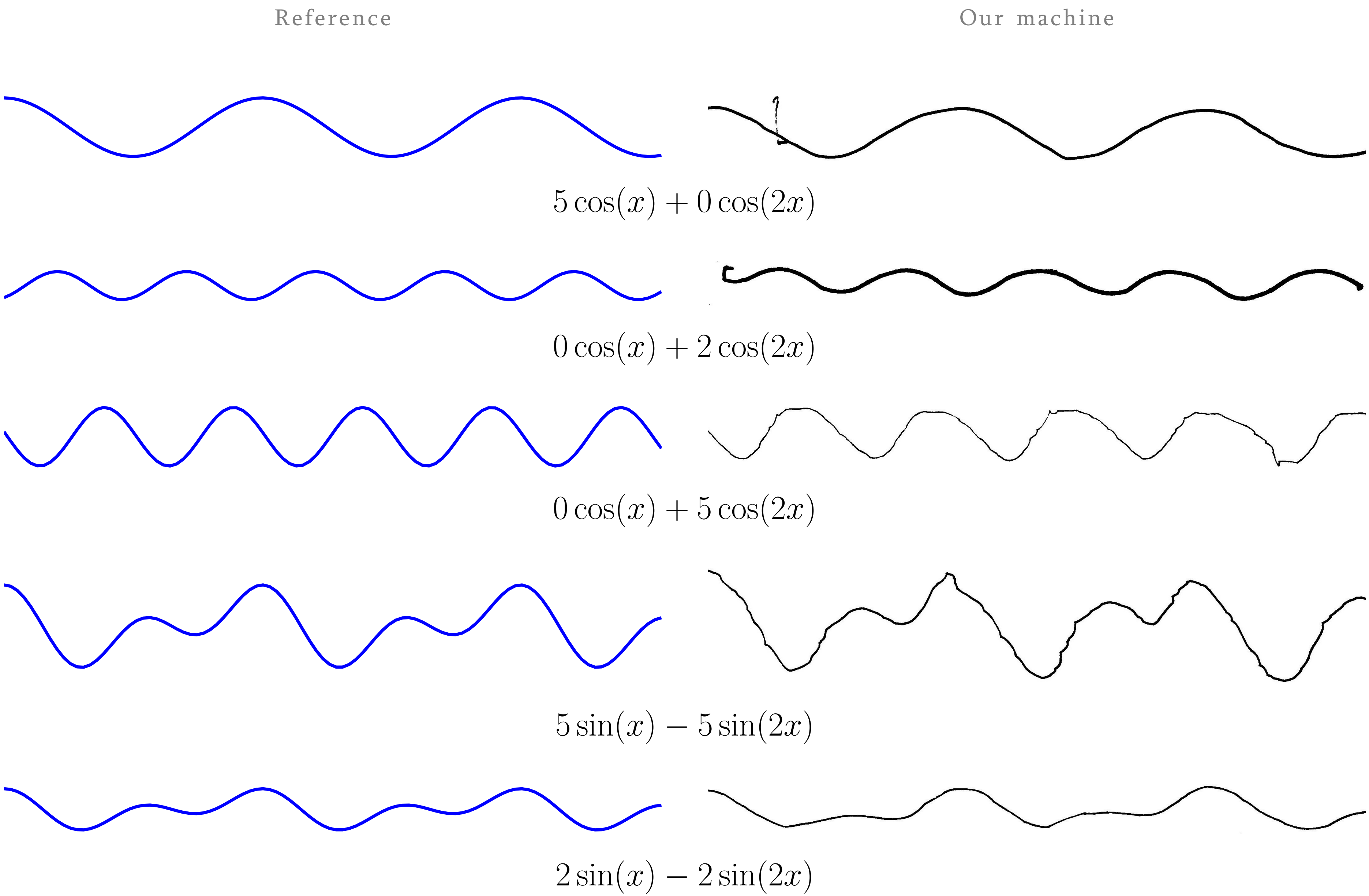
As the handle is turned, the string winds around the axis rod and pulls the paper away from the machine while the pen clamped to the amplification bar draws on it.



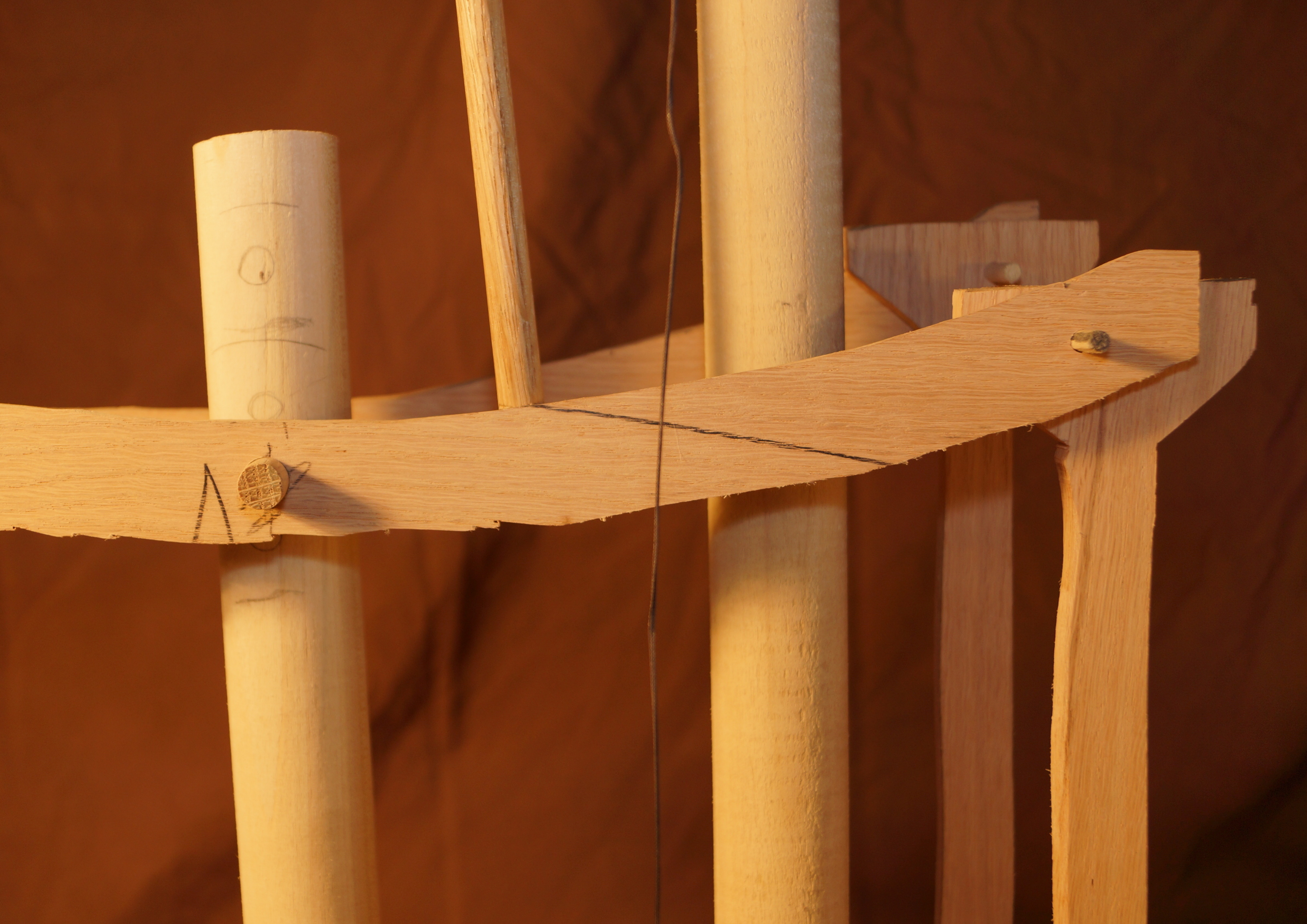
Graphs from our Machine

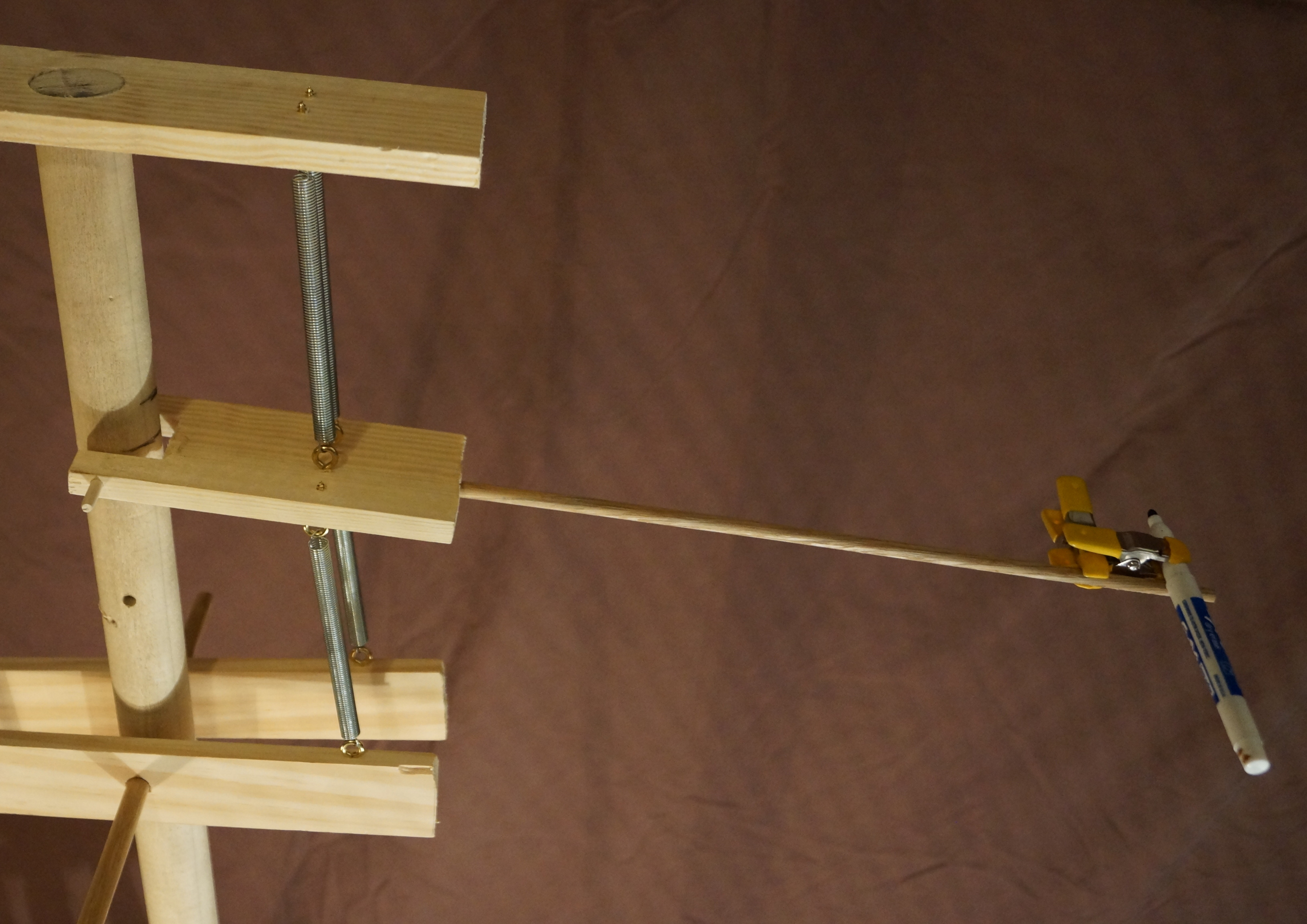
"Among the methods which appeared most promising were addition of fluid pressures, elastic and other forces, and electric currents. Of these the simplest in practice is doubtless the addition of the forces of spiral springs."

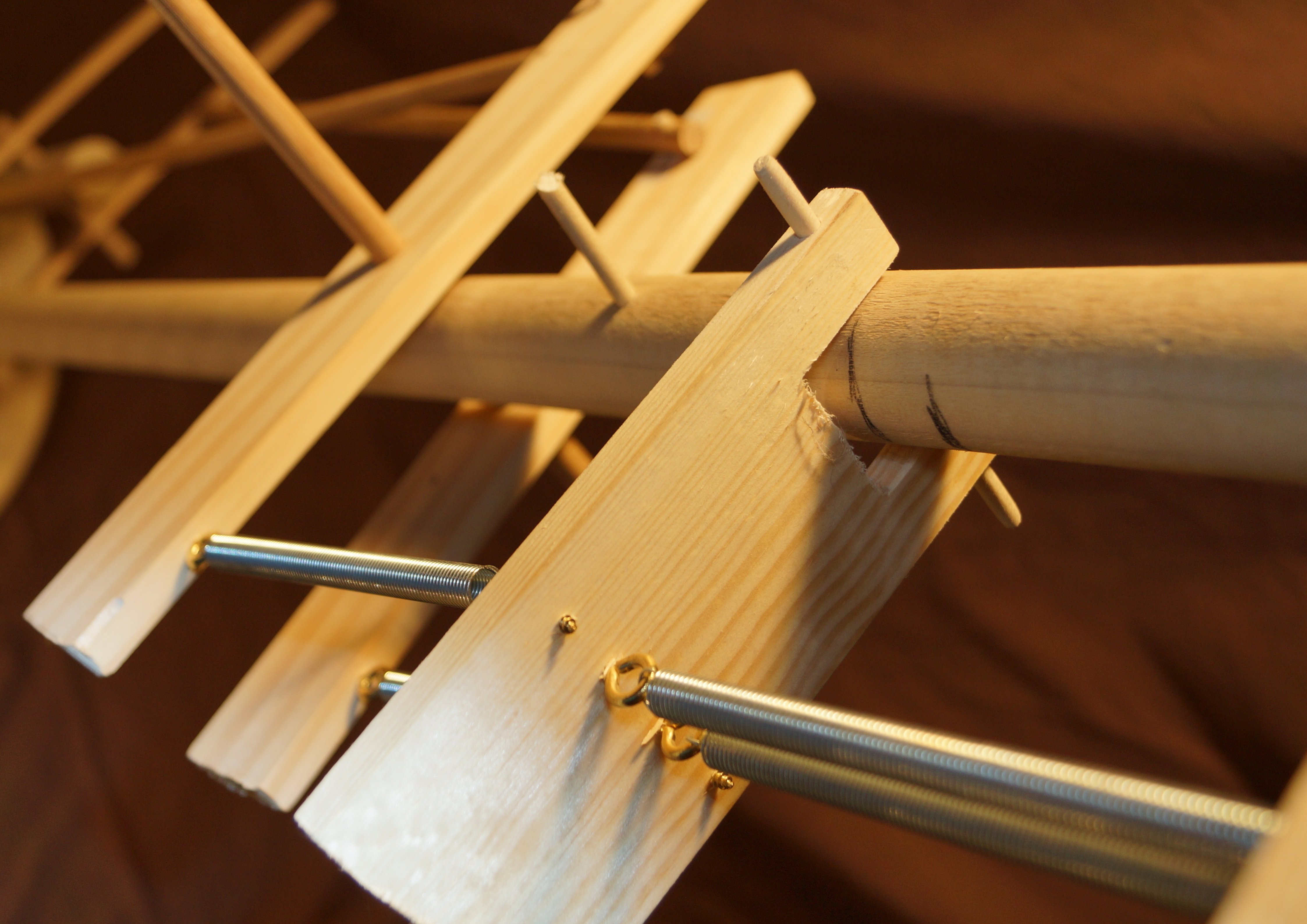
—Albert Michelson
A New Harmonic Analyzer, 1898


Reference
Our machine
$5\cos(x) + 0\cos(2x)$
$0\cos(x) + 2\cos(2x)$
$0\cos(x) + 5\cos(2x)$
$5\sin(x) - 5\sin(2x)$
$2\sin(x) - 2\sin(2x)$

Photographs





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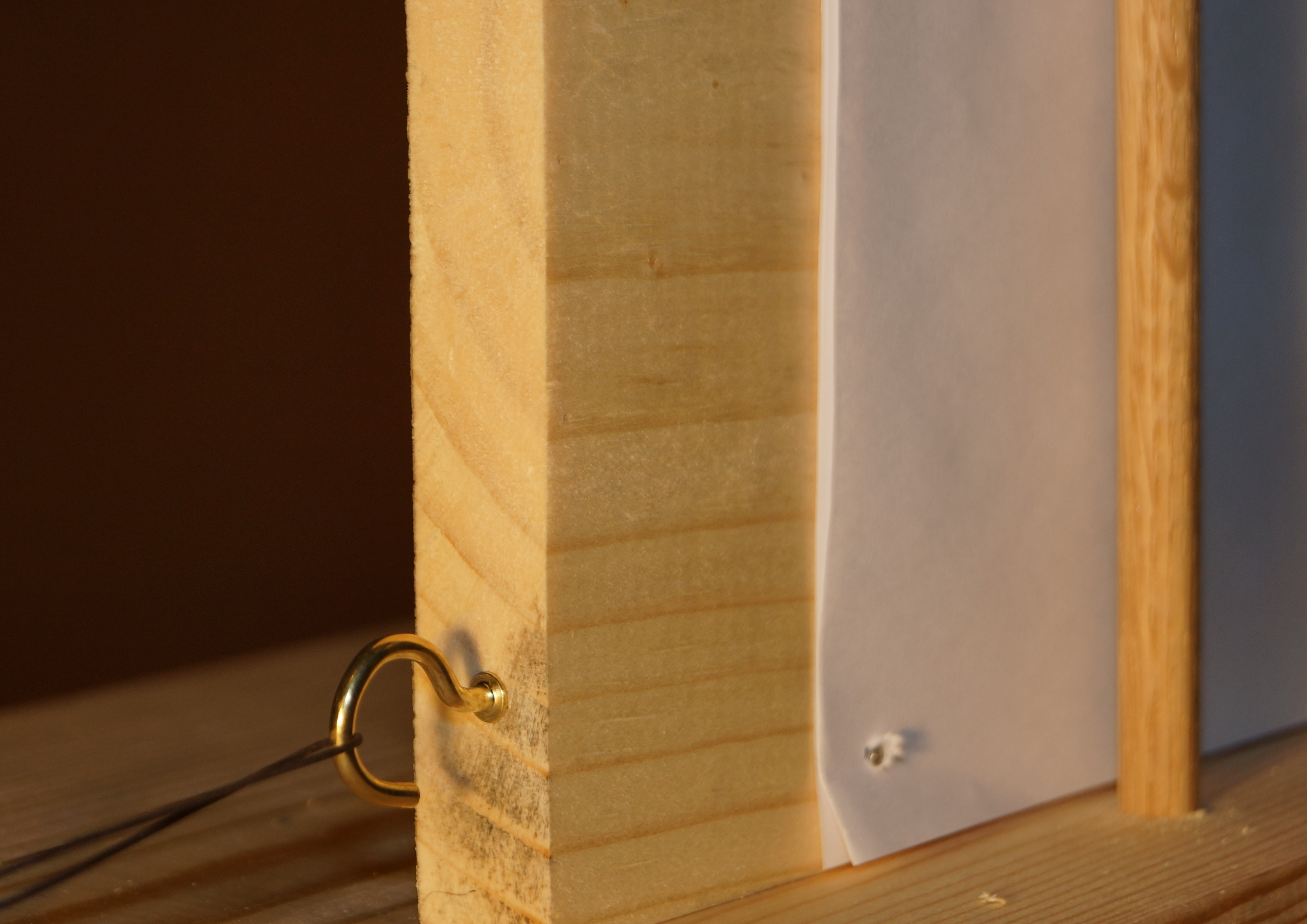

Final Words

Michelson's machine is obviously far more sophisticated than our reduced version. In addition to a lower number of gears, ours provides almost no opportunities for customization; this contrasts with the original harmonic analyzer which offered the ability to change the speed of the platen's movement, easily switch between sines and cosines, and more.

Wood is inherently less precise than metal for making a machine such as this, and therefore our main design goals were simplicity and fault-tolerance. These requirements motivated us to part from Michelson's design in several places, most notably in using pulleys instead of contact gears and the design of the uppermost level of the machine. Even so, the graphs produced by our machine are not of high enough quality to be useful. They are closer to abstract art than mathematics.

Our machine also does not have enough gears to do interesting harmonic analysis and can't put much of the *analyzer* in *harmonic analyzer*. However, we expect that going far beyond two would exacerbate the inaccuracies caused by wood construction and defeat the purpose of supplying more gears.

Undertaking a project like this, no matter how much it ends up deviating from its inspiration, gives us an appreciation for the tremendous labor that Michelson put into his own work. We hope that our small efforts will encourage others to explore the world of analog computers, even if for no other reason than to correct our mistakes. We have greatly enjoyed the time we spent on this project, and we owe our appreciation to Bill Hammack, who brought to light the timeless charm and wit of Albert Michelson's Harmonic Analyzer.

Supplements

"Albert Michelson's Harmonic Analyzer" by Bill Hammack, Steve Kranz and Bruce Carpenter, PDF:
http://www.engineerguy.com/fourier/pdfs/albert-michelsons-harmonic-analyzer.pdf

Engineer Guy's explanation of Albert Michelson's Harmonic Analyzer, Video 1/4:
https://www.youtube.com/watch?v=NAsM30MAHLg

A video of our version of the harmonic analyzer in action:
https://www.youtube.com/watch?v=x4R5zV_sqMQ