Analytic Number Theory Fall 2025 - Homework
Homework 5 does not need to be submitted **Name:**

Directions

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Homework 5

(1) Prove that for $x > 1$ and $c > 0$ we have
$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{x^s}{s}\mathrm{d}s = 1.$$

(2) Assume that $u > 0$, express
$$\frac{1}{2\pi i}\int_{2-i\infty}^{2+i\infty}\frac{u^s}{s^2}\left(-\frac{\zeta'(s)}{\zeta(s)}\right)\mathrm{d}s$$
as a finite sum involving $\Lambda(n)$.

(3) Derive an asymptotic formula for the logarithm of the least common multiple of the first n positive integers $\log \operatorname{lcm}[1, 2, \cdots, n]$ as $n \to \infty$.

(4) We say x is an n-bit integer if x is an integer such that $2^{n-1} \leqslant x < 2^n$, prove that there are at least $\dfrac{2^{n-1}}{2n}$ prime n-bit integers.

(5) Prove that
$$\sum_{p\leqslant x}\frac{1}{p} = \log\log x + O(1)$$

(6) Prove the estimate
$$\pi(x) = O(\frac{x}{\log\log x})$$
by sieve method.