ECE 661: Homework 10

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Task 1

Take a pair of stereo images with the camera, with no particular constraints on how the second image is recorded vis-a-vis the first, as long as the two views are of the same scene. Using the two stereo images perform following tasks: image rectification, interest point detection and projective reconstruction.

Before implementing the 3 tasks, we can first calculate an initial fundamental matrix. If we can identify several pairs of corresponding points $p_i = (u_i, v_i, 1)$ and $p_i' = (u_i', v_i', 1)$ from two images, because of

$$p_i'^T F p_i = 0$$

We can reformulate the constraint as:

$$\begin{bmatrix} u_i u_i' & v_i u_i' & u_i' & u_i v_i' & v_i v_i' & v_i' & u_i & v_i & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

If we stack n such functions together, we can represent it as $\boldsymbol{W} f = 0$, solving this function and we can finally get F, usually the method is SVD. The solution f should be the singular vector associated with the least singular value of $\boldsymbol{W}$.

After we solve f out, we can easily get F, but since F should be singular, we need to add a constraint on it, which is achieved by setting the least singular value of F as 0. If the initial F can be decomposed by $F' = U \Sigma V^T$, the final fundamental matrix F is:

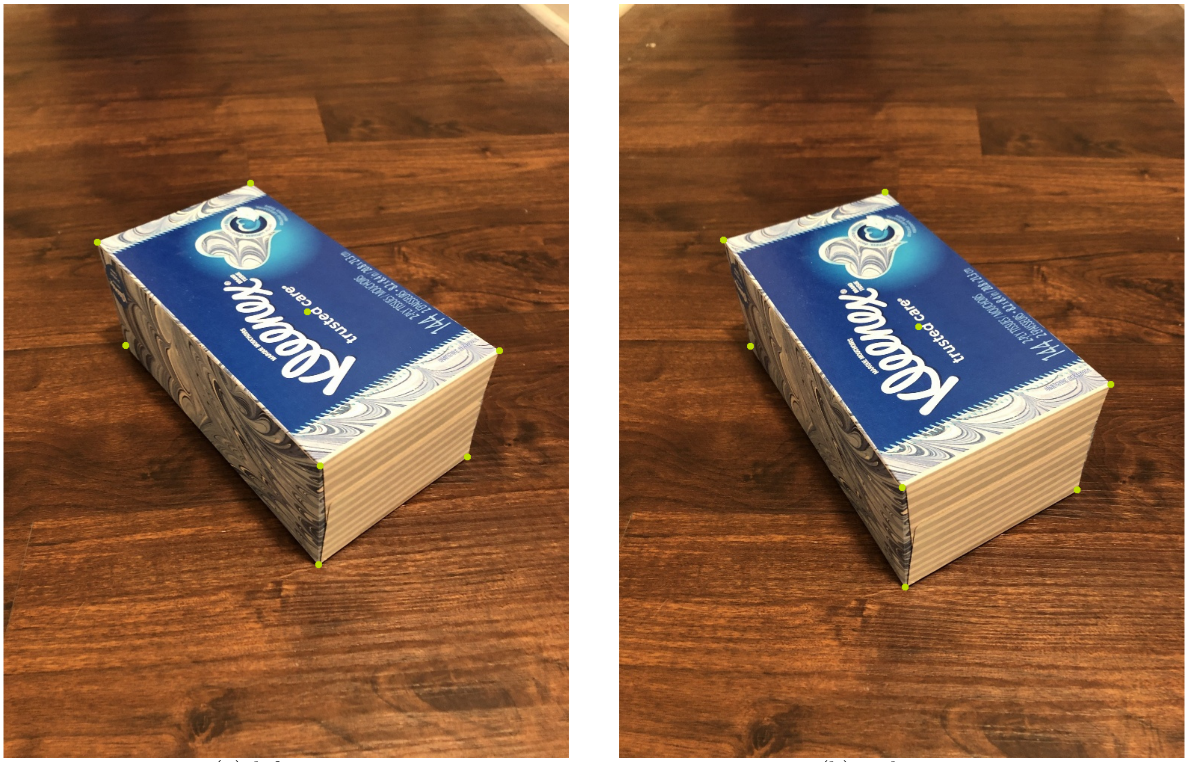
$$F = U \begin{bmatrix} \Sigma_{11} & 0 & 0 \\ 0 & \Sigma_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} V^T$$

8 point method for image rectification

In practice, since the fundamental matrix only has 7 DoF, 8 corresponding points is enough to solve it. The value of pixel coordinate is very large, normalized coordinates can bring much less error than the regular image coordinates. We normalize coordinates by translation and scaling, for translation, we move the image to make the center of 8 points (x_c, y_c) located at $(0, 0)$. For scaling, we multiply coordinates by a constant s to make the average distance from each point to the center is $\sqrt{2}$. The normalization can be done by matrix

$$\begin{bmatrix} s & 0 & -s * x_c \\ 0 & s & -s * y_c \\ 0 & 0 & 1 \end{bmatrix}$$

Assume the normalization matrices for two images are T_1 and T_2, the fundamental matrix we solved by using the normalized 8 points is F_p, the actually fundamental matrix is $F = T_2^T F_p T_1$



(a) left image (b) right image

Figure 1: A pair of stereo images, the manually selected interest points are marked by yellow

Image rectification is the process of making any two given images parallel. If we can make a pair of stereo images rectified, we can take use of the proterty of Essential matrix, and find the pairs of interest points much more easily. To achieve rectification, we need two homographical transformation matrix H_1 and H_2 for the pair of stereo images.

Since we do not know the camera matrix, we need to find a way to transform the epipoles of the stereo image pair to the point of infinity. We first find two epipoles e_1 and e_2, they are right and left null spaces of the fundamental matrix.

We first solve H_2, which is a homographical transformation that brings e_2 to $(1, 0, 0)$. First we use a matrix T to set the coordinate of the image center as $(0, 0)$:

$$T = \begin{bmatrix} 1 & 0 & -\frac{\text{width}}{2} \\ 0 & 1 & -\frac{\text{height}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

After applying this, we use another rotation matrix R to send e_2 to the horizontal axis at a point $(f, 0, 1)$, where f is the distance from epipole to the image center. And after this, we just need to used G to send e_2 to $(1, 0, 0)$:

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{f} & 0 & 1 \end{bmatrix}$$

Finally, other points on the rectified image need to be transformed back to the original space, so the overall transformation is

$$H_2 = T^{-1}GRT$$

When solving H_1 we need to take H_2 into consideration. By solving

$$\underset{H_1}{argmin} \sum_i \left\| H_1 p_i - H_2 p_i' \right\|^2$$

We finally have

$$H_1 = H_A H_2 M$$

Where $M = [e_1]_\times F + e_1 v^T$, usually $v = [1, 1, 1]$ and

$$H_A = \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So if $\boldsymbol{a} = (a_1, a_2, a_3)$ is calculated, we will solve H_1, substitute $H_1 = H_A H_2 M$ into $\sum_i \left\| H_1 p_i - H_2 p_i' \right\|^2$, we can rewrite it as:

$$\underset{\boldsymbol{a}}{argmin} \sum_i (a_1 \hat{x}_i + a_2 \hat{y}_i + a_3 - \hat{x}_i')^2$$

This is a least-squares problem $W\boldsymbol{a} = b$ where

$$W = \begin{bmatrix} \hat{x}_1 & \hat{y}_1 & 1 \\ & \vdots & \\ \hat{x}_n & \hat{y}_n & 1 \end{bmatrix} \quad b = \begin{bmatrix} \hat{x}_1' \\ \vdots \\ \hat{x}_n' \end{bmatrix}$$

After projecting the original image to the rectified space respectively, because we need to translate the rectified space to a valid image space, we need to further align the height of two images to make sure the corresponding points are still at the same height. The final rectified images are:

(a) left image

(b) right image

Figure 2: A pair of stereo images after rectification

Interest points extraction and matching by Canny

We would like to extract more correspondence to refine the fundamental matrix, first we extract the interest points and then we find correspondence among them. The interest points are got by the Canny edge detector, which could extract a tremendous amount of points. After the edge detector, we got 6854 points in the left image and 7103 points on the right image, I used the SIFT to get their feature vectors. Since the images are rectified, the corresponding points are supposed to be in the similar height.

In this experiment, I only set the height tolerance to be ± 1 pixel. I scanned all candidates from top to bottom, I extracted points in 3 adjacent lines in both image and selected at most 3 pairs for each trial by their NCC scores. Finally I got 516 pairs and I randomly selected 100 pairs to show below.

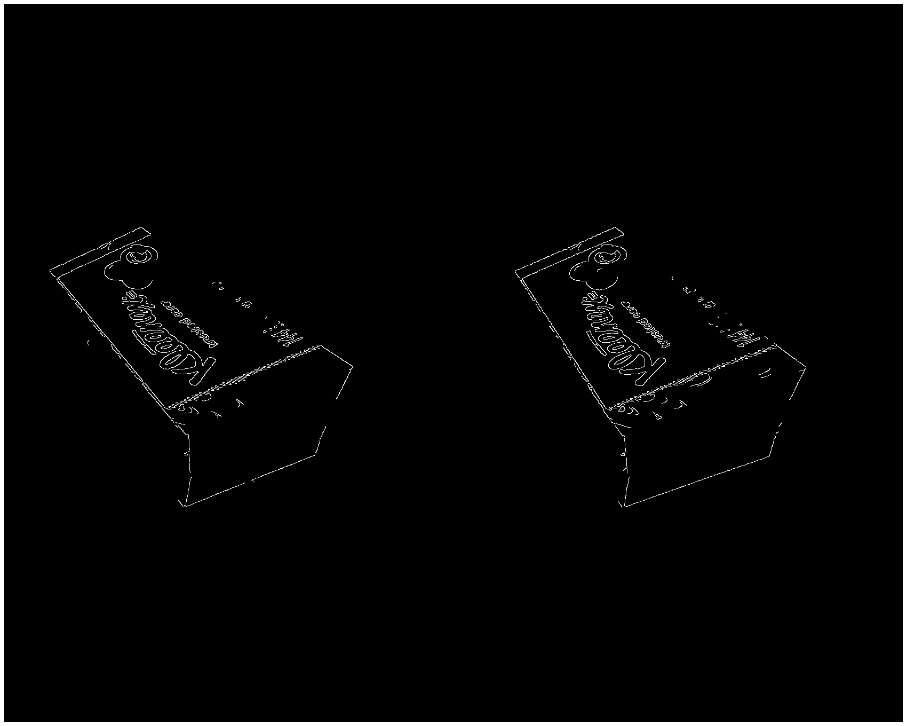


Figure 3: Edges detected by canny

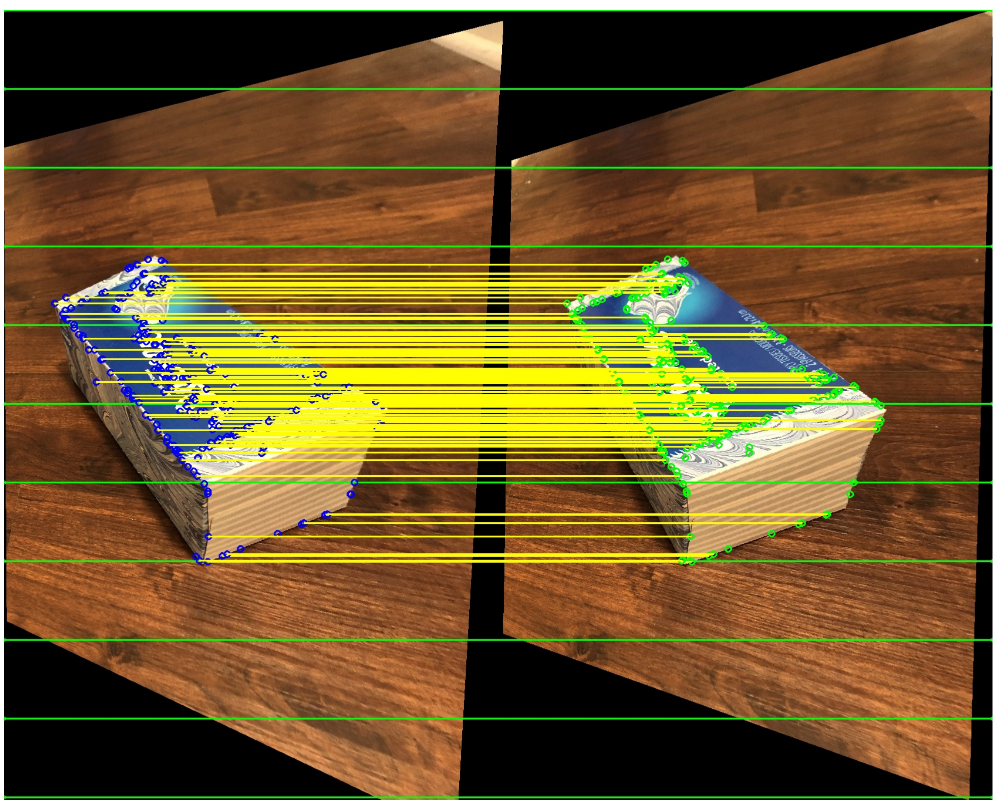


Figure 4: interest point correspondences

Projective reconstruction

We can find projective matrices that project the world coordinate of those points to two image spaces by: $P_1 = [I_{3\times3}|\vec{0}^T]$ and $P_2 = [[e_2]_\times F|e_2^T]$

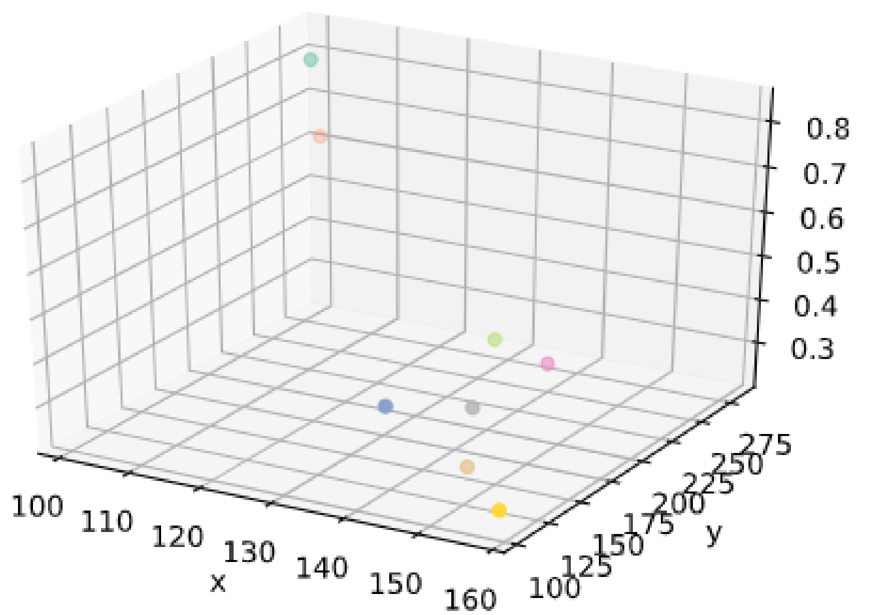


Figure 5: Reconstructed manually selected 8 points

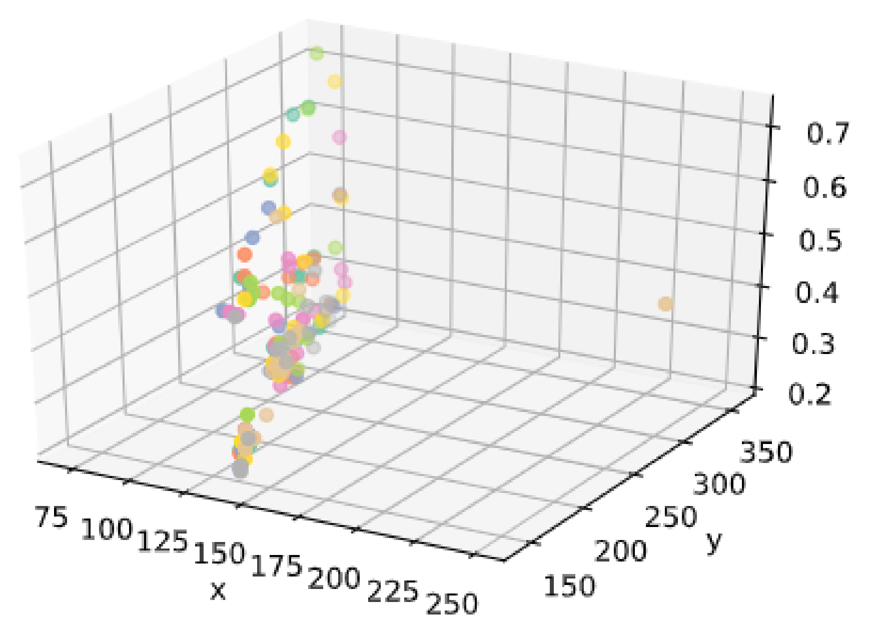


Figure 6: Reconstructed 100 points by interest points detector

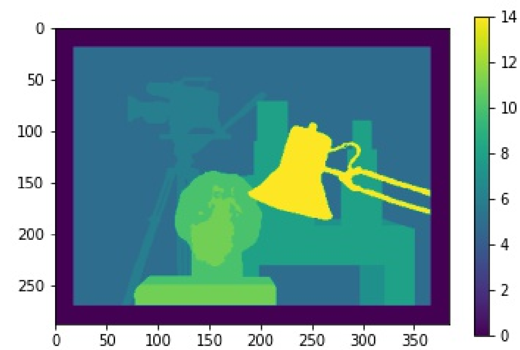
Task 2

Census transform is a transform in a local window, which can transform the relative pixel value in the window to a bit-vector. If the intensity of a pixel is greater than the central pixel, it is "1", otherwise it's "0". The bit vector got from census transform will serve to compare the local information with other windows. The census transform can remove the influence of absolute intensity and only focus on the local variation.
The error rate under different window size is shown below:

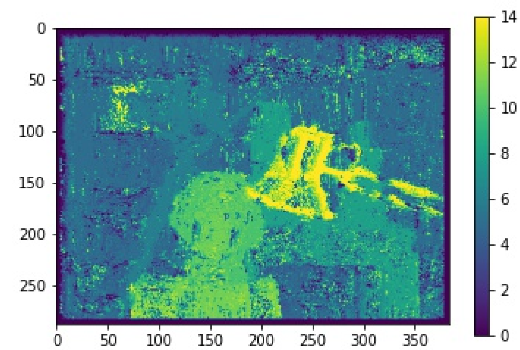
error rate (%)	$M = 9$	$M = 19$	$M = 29$
$\delta = 1$	66.24	79.03	86.70
$\delta = 2$	71.57	83.01	90.15

Table 1: percentile of pixels under error thresholds

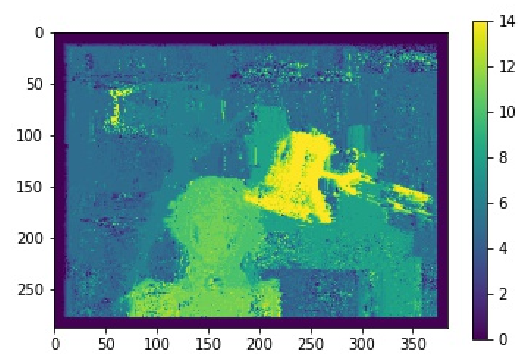
It's obvious that the accuracy of disparity map improved significantly when we enlarge the window size. The disparity maps and the error masks I got are shown below, where brighter parts means larger disparity:



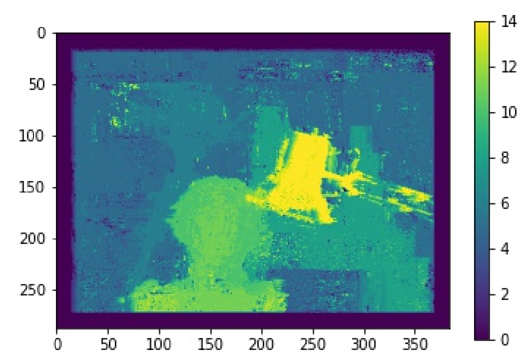
(a) Ground truth



(b) $M = 9$

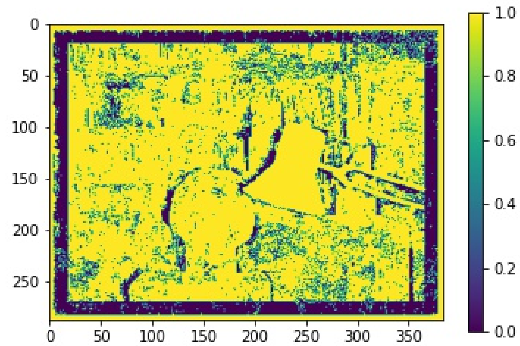


(c) $M = 19$

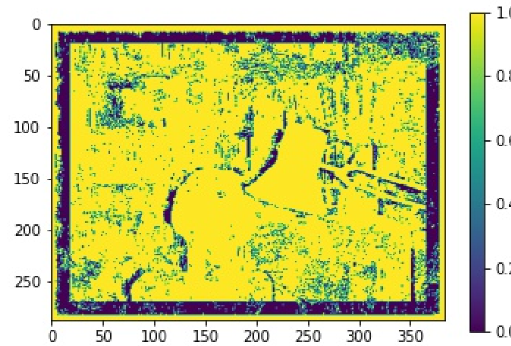


(d) $M = 29$

Figure 7: Disparity maps

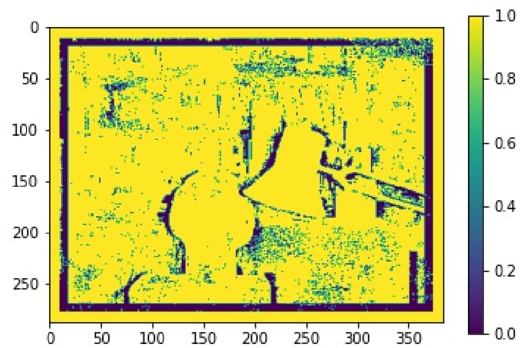


(a) $\delta = 1$

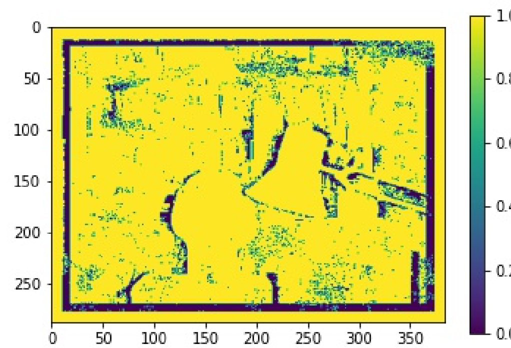


(b) $\delta = 2$

Figure 8: Error mask when $M = 9$

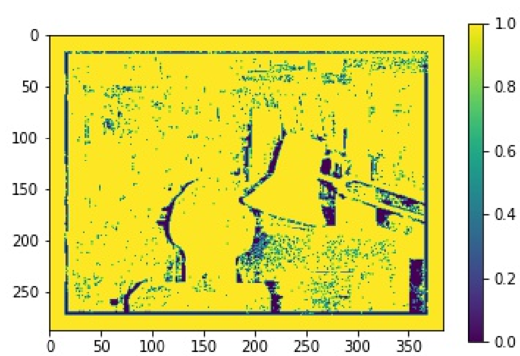


(a) $\delta = 1$

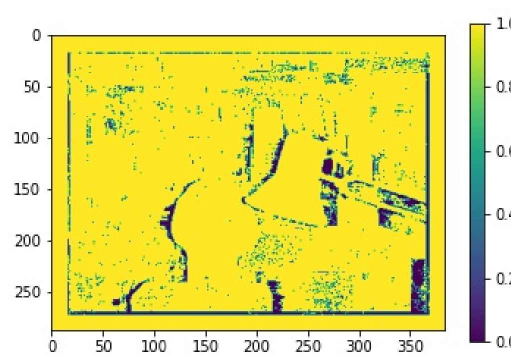


(b) $\delta = 2$

Figure 9: Error mask when $M = 19$



(a) $\delta = 1$



(b) $\delta = 2$

Figure 10: Error mask when $M = 29$

The brighter (yellow) color means correct.

code

Task1

```
import cv2
import numpy as np
import os
import math
from scipy.optimize import least_squares
from scipy.linalg import null_space
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

path_root = '/home/xingguang/Documents/ECE661/hw10/Task2_Images'

def normlize(pt):
    center = np.average(pt, axis=0)
    npt = pt - center
    d = np.sqrt(np.sum(npt ** 2, axis=1))
    s = np.sqrt(2)/np.average(d)
    npt = s * npt
    T = np.array([[s, 0, -s*center[0]], [0, s, -s*center[1]], [0, 0, 1]])
    return npt, T


def findF(pts1, pts2, T1, T2):
    def A_unit(p1, p2):
        x, y = p1[0], p1[1]
        x_, y_ = p2[0], p2[1]
        return np.asarray([[x*x_, y*x_, x_, x*y_, y*y_, y_, x, y, 1]])
    A_list = [A_unit(pts1[i], pts2[i]) for i in range(pts1.shape[0])]
    A = np.concatenate(A_list, axis=0)
    u, d, vt = np.linalg.svd(A)
    F = vt[-1].reshape((3, 3))
    U, D, VT = np.linalg.svd(F)
    DD = np.diag([D[0], D[1], 0])
    nF = U.dot(np.dot(DD, VT))
    denormalized_F = np.dot(T2.T, np.dot(nF, T1))
    return denormalized_F


def skew(x):
    return np.array([[0, -x[2], x[1]],
                     [x[2], 0, -x[0]],
                     [-x[1], x[0], 0]])


def findP(F, e2):
    P1 = np.array([[1,0,0,0], [0,1,0,0], [0,0,1,0]])
    P2 = np.hstack([np.dot(skew(e2), F), e2.reshape((-1,1))])
    return P1, P2

def definePQRS(width, height):
    P = [0, 0]
    Q = [0, height]
    S = [width, 0]
    R = [width, height]
    return np.asarray([P, Q, S, R])
```

```python
def projTransform(H, source):
    nps = source.shape[0]
    source_rep = np.concatenate((source, np.ones((nps,1))), axis=1)
    t_homo = np.matmul(H, source_rep.T).T
    t_norm = t_homo / t_homo[:,2].reshape((nps,1))
    return t_norm[:,:2]


def projResizeRange(source, H):
    corners = definePQRS(source.shape[1], source.shape[0])
    expanded = projTransform(H, corners)
    l, r = np.amin(expanded[:, 0]), np.amax(expanded[:, 0])
    t, b = np.amin(expanded[:, 1]), np.amax(expanded[:, 1])

    height, width = (b-t), (r-l)
    H = np.linalg.inv(H)
    out = np.zeros((int(b-t), int(r-l), 3), dtype=source.dtype)
    print("Size of rectified image is:", out.shape)
    for i in range(out.shape[0]):
        for j in range(out.shape[1]):
            coor_source = np.asarray([[j+l, i+t]])
            coor_target = projTransform(H, coor_source).squeeze()
            h, w = int(coor_target[1]), int(coor_target[0])
            if 0 <= h < source.shape[0] and 0 <= w <source.shape[1]:
                out[i,j,:] = source[h, w,:]
    translation = np.array([[1, 0, -l], [0, 1, -t], [0, 0, 1]])
    return out, translation


def filter_corner(edges, pts):
    l, r = np.amin(pts[:, 0]), np.amax(pts[:, 0])
    t, b = np.amin(pts[:, 1]), np.amax(pts[:, 1])
    for i in range(edges.shape[0]):
        for j in range(edges.shape[1]):
            if j < l or j > r or i < t or i > b:
                edges[i,j] = 0
    return edges


def drawPoints(img, p, c=(0, 0, 255)):
    out = img.copy()
    for i in range(len(p)):
        point = (int(p[i][0]), int(p[i][1]))
        out = cv2.circle(out, point, radius=10, color=c, thickness=-1)
    return out


def drawLines(img, p_start, p_end, color=(0,255,0)):
    image = np.copy(img)
    for i in range(len(p_start)):
        image = cv2.circle(image, p_start[i], 3, color, 1)
        image = cv2.circle(image, p_end[i], 3, color, 1)
        image = cv2.line(image, p_start[i], p_end[i], color, 2)
    return image


def stack_img(im1, im2):
```

```python
    h1, w1 = im1.shape[0], im1.shape[1]
    h2, w2 = im2.shape[0], im2.shape[1]
    nh = min(h1, h2)
    h1_t = 0
    h1_b = h1_t + nh
    h2_t = 0
    h2_b = h2_t + nh
    w = w1 + w2
    lps = [(0, i) for i in range(0, nh, int(nh/10))]
    rps = [(w-1, i) for i in range(0, nh, int(nh/10))]
    if len(im1.shape) == 3:
        stacked_im = np.zeros((nh, w, 3)).astype(np.uint8)
        stacked_im[:, :w1, :] = im1[h1_t:h1_b, :, :]
        stacked_im[:, w1:w, :] = im2[h2_t:h2_b, :, :]
        stacked_im = drawLines(stacked_im, lps, rps)
    else:
        stacked_im = np.zeros((nh, w)).astype(np.uint8)
        stacked_im[:, :w1] = im1[h1_t:h1_b, :]
        stacked_im[:, w1:w] = im2[h1_t:h1_b, :]
        # stacked_im = drawLines(stacked_im, lps, rps, color=255)
    return stacked_im


def rectify(img1, img2, pts1, pts2, F):
    height = img1.shape[0]
    width = img1.shape[1]
    num_pts = len(pts1)
    hp1 = np.hstack((pts1, np.ones((num_pts, 1))))
    hp2 = np.hstack((pts2, np.ones((num_pts, 1))))
    e1 = null_space(F).flatten()
    e1 /= e1[2]
    e2 = null_space(F.T).flatten()
    e2 /= e2[2]

    theta = np.arctan(-1 * (e2[1] - height/2) / (e2[0] - width/2))
    f = np.cos(theta) * (e2[0] - width/2) - np.sin(theta) * (e2[1] - height/2)

    R = np.asarray([[np.cos(theta), -np.sin(theta), 0],
                    [np.sin(theta), np.cos(theta), 0],
                    [0, 0, 1]])
    T = np.asarray([[1, 0, -width/2],
                [0, 1, -height/2],
                [0, 0, 1]])
    G = np.asarray([[1, 0, 0],[0, 1, 0],[-1/f, 0, 1]])
    H2 = np.dot(np.dot(np.dot(np.linalg.inv(T), G), R), T)

    M = np.dot(skew(e2), F) + np.dot(e2.reshape((-1, 1)), np.asarray([[1, 1, 1]]))
    new_hp1 = np.dot(np.dot(H2, M), hp1.T).T
    new_hp1 = new_hp1 / new_hp1[:, -1].reshape((-1, 1))
    new_hp2 = np.dot(H2, hp1.T).T
    new_hp2 = new_hp2 / new_hp2[:, -1].reshape((-1, 1))

    W = new_hp1.copy()
    b = new_hp2[:, 0].copy()
    a = np.dot(np.linalg.pinv(W), b)
    H_a = np.asarray([[a[0], a[1], a[2]], [0, 1, 0], [0, 0, 1]])
    H1 = np.dot(np.dot(H_a, H2), M)
    H2 = H2 / H2[-1, -1]
    H1 = H1 / H1[-1, -1]
```

```
    return H1, H2, e1, e2


def align_imgs(img1, img2, H1, H2):
    new_img1, translation1 = projResizeRange(img1, H1)
    new_img2, translation2 = projResizeRange(img2, H2)
    hoffset1 = translation1[1, 2]
    hoffset2 = translation2[1, 2]
    mismatch = abs(hoffset1-hoffset2)
    nT = np.array([[1,0,0], [0,1,mismatch], [0,0,1]])
    mismatch = int(mismatch)
    nH1 = np.dot(translation1, H1)
    nH2 = np.dot(translation2, H2)
    if hoffset1 < hoffset2:
        rct1 = np.zeros((new_img1.shape[0]+mismatch, new_img1.shape[1], 3), dtype=
   np.uint8)
        rct1[mismatch:, :, :] = new_img1
        rct2 = new_img2
        nH1 = np.dot(nT, nH1)
    elif hoffset1 > hoffset2:
        rct2 = np.zeros((new_img2.shape[0]+mismatch, new_img2.shape[1], 3), dtype=
   np.uint8)
        rct2[mismatch:, :, :] = new_img2
        rct1 = new_img1
        nH2 = np.dot(nT, nH2)
    return rct1, rct2, nH1, nH2


def findKPs(img, edges, max_num = 500):
    cy_x = np.argwhere(edges > 100)
    edges_coord =np.hstack([cy_x[:,1, np.newaxis], cy_x[:,0, np.newaxis]])
    num_p = len(edges_coord)
    print(num_p)
    descriptor = cv2.xfeatures2d.SIFT_create()
    kp = [cv2.KeyPoint(p[0],p[1],20) for p in edges_coord]
    pos, feature = descriptor.compute(img, kp)
    coors =[[p.pt[0],p.pt[1]] for p in pos]
    pos = np.array(coors, dtype=np.int)
    return pos,feature


def NCC(feat1, feat2):
    N,M=feat1.shape[0], feat2.shape[0]

    window_m1 = np.mean(feat1, axis=-1,keepdims=True)
    window_m2 = np.mean(feat2, axis=-1,keepdims=True)

    f1 = feat1 - window_m1
    f2 = feat2 - window_m2

    f1_tile = np.tile(f1[:,None,:],(1, M, 1))
    f2_tile = np.tile(f2[None,:,:],(N, 1, 1))

    nominator = (f1_tile * f2_tile).sum(axis=-1)
    denominator = np.maximum(((f1_tile**2).sum(axis=-1))*((f2_tile**2).sum(axis
   =-1)),1e-8)
    return -nominator/np.sqrt(denominator)
```

```python
def matching(f1, f2, top_match):

    matching=[]
    for i in range(f1.shape[0]):
        matrix = NCC(f1[i:i+1],f2)
        min_idx = matrix[0].argmin(axis=-1)
        min_dis = matrix[0][min_idx]
        matching.append([min_dis, i, min_idx])

    matching = sorted(matching, key=lambda x: x[0])
    origin_idx = [ matching[i][1] for i in range(min(top_match, len(f1)))]
    target_idx = [ matching[i][2] for i in range(min(top_match, len(f1)))]
    return origin_idx,target_idx


def correspondence(kp1, kp2, f1, f2):

    left_kp_matched, right_kp_matched = [],[]
    for p, f in zip(kp1,f1):
        mask = (kp2[:,1] < (p[1] + 1)) & (kp2[:,1] > (p[1] - 1))
        candidates_p = kp2[mask]
        candidates_f = f2[mask]
        if len(candidates_p)>0:
            left_idx, right_idx = matching(f[None,:],candidates_f,1)
            left_kp_matched.append(p)
            right_kp_matched.append(candidates_p[right_idx[0]])
    return np.array(left_kp_matched), np.array(right_kp_matched)


def triangular3D(p1, p2, P1, P2):
    p1_cam1, p2_cam1, p3_cam1 = P1[0], P1[1], P1[2]
    p1_cam2, p2_cam2, p3_cam2 = P2[0], P2[1], P2[2]
    A = []
    for i in range(p1.shape[0]):
        x1, y1, x2, y2 = p1[i,0], p1[i,1], p2[i,0], p2[i,1]
        sub_A = []
        sub_A.append(x1*p3_cam1 - p1_cam1)
        sub_A.append(y1*p3_cam1 - p2_cam1)
        sub_A.append(x2*p3_cam2 - p1_cam2)
        sub_A.append(y2*p3_cam2 - p2_cam2)
        sub_A = np.array(sub_A)
        A.append(sub_A)

    A = np.array(A)
    ATA = np.einsum('ijk,ikm->ijm', np.transpose(A,(0,2,1)),A)
    U, d, Vh = np.linalg.svd(ATA)
    X = Vh[:,-1,:]
    X = X / X[:,-1, np.newaxis]
    return X


def errfunc(F, pts1, pts2):
    #print('F in lm= {}'.format(F))
    F = np.append(F,1).reshape(3,3)
    U, D, VT = np.linalg.svd(F)
    DD = np.diag([D[0], D[1], 0])
    F = U.dot(np.dot(DD, VT))
    e2 = null_space(F.T).flatten()
    e2 /= e2[-1]
```

```
    P1, P2 = findP(F, e2)

    world_pts=triangular3D(pts1, pts2, P1, P2)

    proj1 = np.dot(P1, world_pts.T)
    proj1 /= proj1[2]
    proj2 = np.dot(P2,world_pts.T)
    proj2 /= proj2[2]
    err = np.append(proj1[:2]-pts1[:2], proj2[:2]-pts2[:2])
    return err

def lm(pts1, pts2, F):
    pts1=np.array(pts1).T
    pts2=np.array(pts2).T

    F = F.flatten()
    F = F[:-1].reshape(8,1)

    ans=optimize.leastsq(errfunc,F,args=(pts1,pts2))
    F = F.flatten()
    print('change = ', ans[0] - F)

    F=np.append(ans[0],1).reshape(3,3)
    e2 = null_space(F.T).flatten()
    e2 /= e2[-1]
    _, P2 = findP(F, e2)
    return F, P2


img1 = cv2.imread("i1.jpg")
img2 = cv2.imread("i2.jpg")
pts1 = np.array([[125,
   318],[163,456],[330,239],[423,617],[421,749],[663,463],[620,605],[406, 411]])
pts2 = np.array([[139,
   315],[175,457],[355,251],[378,646],[382,779],[657,508],[612,649],[400, 431]])
npt1, T1 = normlize(pts1)
npt2, T2 = normlize(pts2)

F = findF(npt1, npt2, T1, T2)
H1, H2, e1, e2 = rectify(img1, img2, pts1, pts2, np.copy(F))
rct1, rct2, nH1, nH2 = align_imgs(img1, img2, H1, H2)
plt.imshow(rct1)
cv2.imwrite("rct1.jpg", rct1)
new_pts1 = projTransform(nH1, pts1)
i1 = drawPoints(rct1, new_pts1)
plt.imshow(i1)
cv2.imwrite("rct2.jpg", rct2)

new_pts1 = projTransform(nH1, pts1)
new_pts2 = projTransform(nH2, pts2)
g1 = cv2.cvtColor(rct1, cv2.COLOR_BGR2GRAY)
g1 = cv2.GaussianBlur(g1, (3,3), 3)
edges1 = cv2.Canny(g1, 255*1.5, 255)
edges1 = filter_corner(edges1, new_pts1)

g2 = cv2.cvtColor(rct2, cv2.COLOR_BGR2GRAY)
g2 = cv2.GaussianBlur(g2, (3,3), 3)
edges2 = cv2.Canny(g2, 255*1.5, 255)
edges2 = filter_corner(edges2, new_pts2)
```

```
stacked_edges = stack_img(edges1, edges2)
cv2.imwrite("canny.jpg", stacked_edges)
plt.imshow(stacked_edges)

kp1, feature1 = findKPs(rct1, edges1)
kp2, feature2 = findKPs(rct2, edges2)
matched1, matched2 = correspondence(kp1, kp2, feature1, feature2)

stacked_img = stack_img(rct1, rct2)
plt.figure(figsize=(15,25))
plt.imshow(stacked_img)

sel = np.random.choice(len(matched1), size=(128,), replace=False)
selected1 = matched1[sel]
selected2 = matched2[sel]
for (x1,y1), (x2,y2) in zip(selected1, selected2):
    cv2.circle(stacked_img, (int(x1),int(y1)),5, (255,0,0),thickness=2)
    cv2.circle(stacked_img, (int(x2)+rct1.shape[1],int(y2)),5, (0,255,0),thickness
    =2)
    cv2.line(stacked_img, (int(x1),int(y1)),(int(x2)+rct1.shape[1],int(y2)),
    (0,255,255),thickness=2)

plt.figure(figsize=(15,25))
plt.imshow(stacked_img)
cv2.imwrite('corresp.jpg', stacked_img)

P1, P2 = findP(F, e2)
X = triangular3D(pts1, p2, P1, P2)
fig = plt.figure()
ax = fig.add_subplot(111,projection='3d')
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
ax.scatter(X[:,0],X[:,1],X[:,2], cmap=plt.cm.get_cmap('Set2', len(X)), c=np.arange
    (len(X)))

X1 = triangularization(selected1, selected2, P1, P2)
fig = plt.figure()
ax = fig.add_subplot(111,projection='3d')
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
ax.scatter(X1[:,0],X1[:,1],X1[:,2], cmap=plt.cm.get_cmap('Set2', len(X1)), c=np.
    arange(len(X1)))
```

Task2

```
import numpy as np
import cv2
from matplotlib import pyplot as plt

def ConsusTrans(img, M):
    s = (M-1)//2
    x_min, x_max = s, img.shape[1] - 1 - s
    y_min, y_max = s, img.shape[0] - 1 - s
    c = np.transpose([np.tile(np.arange(x_min, x_max,1), y_max-y_min), \
```

```
            np.repeat(np.arange(y_min, y_max, 1), x_max-x_min)])

    pc = img[c[:,1], c[:,0]]
    codes = []
    for x_shift in range(-s, s+1):
        for y_shift in range(-s, s+1):
            pixels = img[c[:,1]+y_shift, c[:,0]+x_shift]
            codes.append(pc < pixels)

    codes = np.array(codes)
    img_bits = np.zeros((img.shape[0], img.shape[1],M*M),dtype=np.bool)
    img_bits[c[:,1], c[:,0]] = codes.T
    return img_bits


path_root = '/home/xingguang/Documents/ECE661/hw10/Task2_Images'
img1 = cv2.imread(os.path.join(path_root, "Left.ppm"), cv2.IMREAD_GRAYSCALE)
img2 = cv2.imread(os.path.join(path_root, "Right.ppm"), cv2.IMREAD_GRAYSCALE)
img1_gt = cv2.imread(os.path.join(path_root, "left_truedisp.pgm"),cv2.
    IMREAD_GRAYSCALE)
img1_gt = (img1_gt.astype(np.float32)/16).astype(np.int)
npixel = img1_gt.shape[0] * img1_gt.shape[1]
plt.figure()
plt.imshow(img1_gt)
plt.colorbar()
plt.savefig("./Task2_output/GT.jpg")

M  =  29
bit_im1 = ConsusTrans(img1, M)
bit_im2 = ConsusTrans(img2, M)

s = (M-1)//2
x_min, x_max = s, bit_im1.shape[1] - 1 - s
y_min, y_max = s, bit_im1.shape[0] - 1 - s
c = np.transpose([np.tile(np.arange(x_min, x_max,1), y_max-y_min), \
                    np.repeat(np.arange(y_min, y_max, 1), x_max-x_min)])

ddis = np.zeros((img1.shape[0], img1.shape[1]))
census_map = -np.inf*np.ones((img1.shape[0], img1.shape[1]))
for sh in range(15):
    # left image, and the right image within a certain shifted ROI
    mask = c[:,1]-sh >=s
    roi1 = c[mask]
    roi2 = c[mask] - np.array([[sh,0]])

    roi1_bits = bit_im1[roi1[:,1], roi1[:,0],:]
    roi2_bits = bit_im2[roi2[:,1], roi2[:,0],:]
    census_ROI = np.sum(roi1_bits==roi2_bits, axis=-1)

    census_mask = census_map[roi1[:,1], roi1[:,0]] < census_ROI
    roi2d = roi1[census_mask]
    ddis[roi2d[:,1], roi2d[:,0]] = sh
    census_map[roi2d[:,1], roi2d[:,0]] = census_ROI[census_mask]

plt.figure()
plt.imshow(ddis)
plt.colorbar()
plt.savefig(f"./Task2_output/disparity_{M}.jpg")
```

```python
for delta in [1, 2]:
    mask = (disparity_dis-img1_gt)<=delta
    error = np.sum(np.abs((disparity_dis-img1_gt))<=delta)
    print(f"percent delta={delta}," , 100*error/npixel)

    plt.figure()
    plt.imshow(mask)
    plt.colorbar()
    plt.savefig(f"./Task2_output/mask_{delta}_{M}.jpg")
```