ECE 661: Homework 2

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Task 1

The projective transformation H from the source coordinates in homogeneous representational space $X = (x, y, 1)$ to target coordinates $Y = (u, v, 1)$ is:

$$Y = HX$$

H is a non-singular matrix and can be represented as:

$$H = \begin{bmatrix} a_{11} & a_{12} & t_1 \\ a_{21} & a_{22} & t_2 \\ v_1 & v_2 & 1 \end{bmatrix}$$

Thus, for each pair of X and Y, we have:

$$\begin{bmatrix} a_{11} & a_{12} & t_1 \\ a_{21} & a_{22} & t_2 \\ v_1 & v_2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

We can form 3 functions by this, they are:

$$\begin{aligned} a_{11}x + a_{12}y + t_1 &= u \\ a_{21}x + a_{22}y + t_2 &= v \\ v_1 x + v_2 y + 1 &= 1 \end{aligned}$$

Or,

$$\begin{aligned} a_{11}x + a_{12}y + t_1 &= v_1 xu + v_2 yu + u \\ a_{21}x + a_{22}y + t_2 &= v_1 xv + v_2 yv + v \end{aligned}$$

Then we have the equation that can be easily combined with equations from other pairs of points:

$$\begin{aligned} a_{11}x + a_{12}y + t_1 \qquad\qquad\qquad\qquad - v_1 xu - v_2 yu &= u \\ a_{21}x + a_{22}y + t_2 - v_1 xv - v_2 yv &= v \end{aligned}$$

It can be represented by matrix multiplication for coordinate pairs (x_i, y_i) and (u_i, v_i):

$$\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -x_i u_i & -y_i u_i \\ 0 & 0 & 0 & x_i & y_i & 1 & -x_i v_i & -y_i v_i \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ t_1 \\ a_{21} \\ a_{22} \\ t_2 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} u_i \\ v_i \end{bmatrix}$$

To solve the 8 unknown parameters, we need four pairs of associated points, the equations can be represented by:

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1u_1 & -y_1u_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1v_1 & -y_1v_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2u_2 & -y_2u_2 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2v_2 & -y_2v_2 \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3u_3 & -y_3u_3 \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3v_3 & -y_3v_3 \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4u_4 & -y_4u_4 \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4v_4 & -y_4v_4 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ t_1 \\ a_{21} \\ a_{22} \\ t_2 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix} \tag{1}$$

Define

$$T = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1u_1 & -y_1u_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1v_1 & -y_1v_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2u_2 & -y_2u_2 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2v_2 & -y_2v_2 \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3u_3 & -y_3u_3 \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3v_3 & -y_3v_3 \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4u_4 & -y_4u_4 \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4v_4 & -y_4v_4 \end{bmatrix} \quad P = \begin{bmatrix} a_{11} \\ a_{12} \\ t_1 \\ a_{21} \\ a_{22} \\ t_2 \\ v_1 \\ v_2 \end{bmatrix} \quad C = \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix}$$

We can solve P by

$$P = T^{-1}C \tag{2}$$

In **task 1**, the four associated coordinates from images 1a., 1b., 1c., 1d. are:

image	x_P	y_P	x_Q	y_Q	x_S	y_S	x_R	y_R
image 1a.	298	510	237	1605	1776	357	1684	1826
image 1b.	342	696	333	2328	1884	753	1881	2004
image 1c.	109	439	116	1366	1215	305	1096	1861
image 1d.	90	192	90	1118	1796	192	1796	1118

1.

First we substitute the 4 coordinates of image 1a and 1d into the equation (1), got matrix T and its inverse T^{-1}, then we can get the parameters of projective transformation matrix H from P by equation (2).
When we get H, we need find all associated pixel pairs in the two ranges from source image and target image. (The pixels in the given range of source image need to be replaced by the associated pixels from target image).
To achieve this, I used a mask image having the same size as the source image, whose pixels in the given area is white $(255, 255, 255)$ and others are black $(0, 0, 0)$. Iterating all pixels in the mask to find all coordinates $\{X_i\}$ that needs to be replaced by the pixels of $\{Y_i\}$ on target image, where $Y_i = HX_i$.
If the size of areas from two images is different, we'd better first resize them, or filter them and do interpolation. But since different areas in the task have similar size, we don't need to do that to make a good visual effect.

The following figures are: images shown in Figs. 1d and 1a, (2) images shown in Figs. 1d and 1b, and (3) images shown in Figs. 1d and 1c.

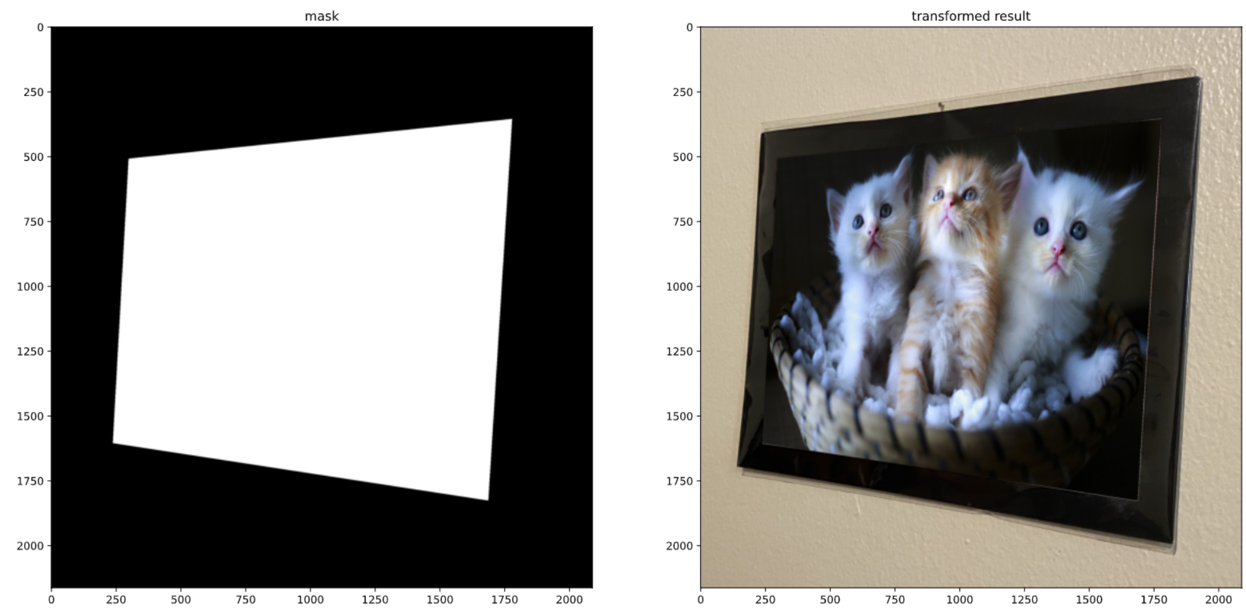

Figure 1: Mask in 1a and Fig. 1d in 1a

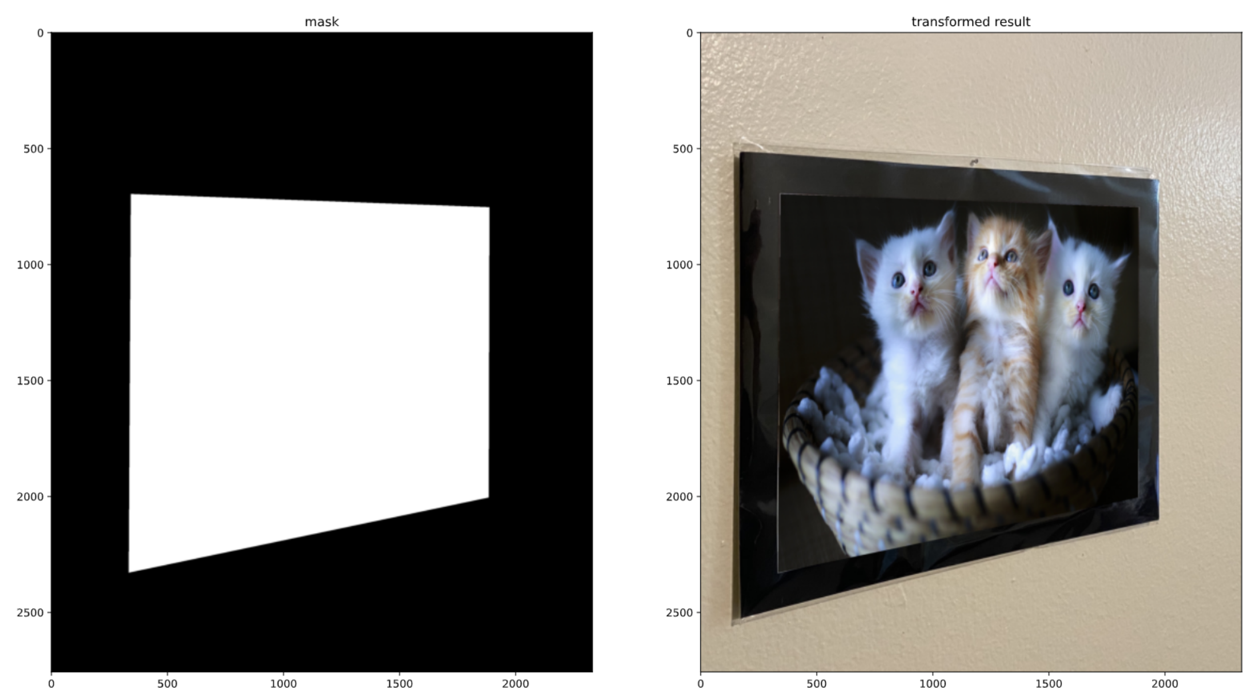

Figure 2: Mask in 1b and Fig. 1d in 1b

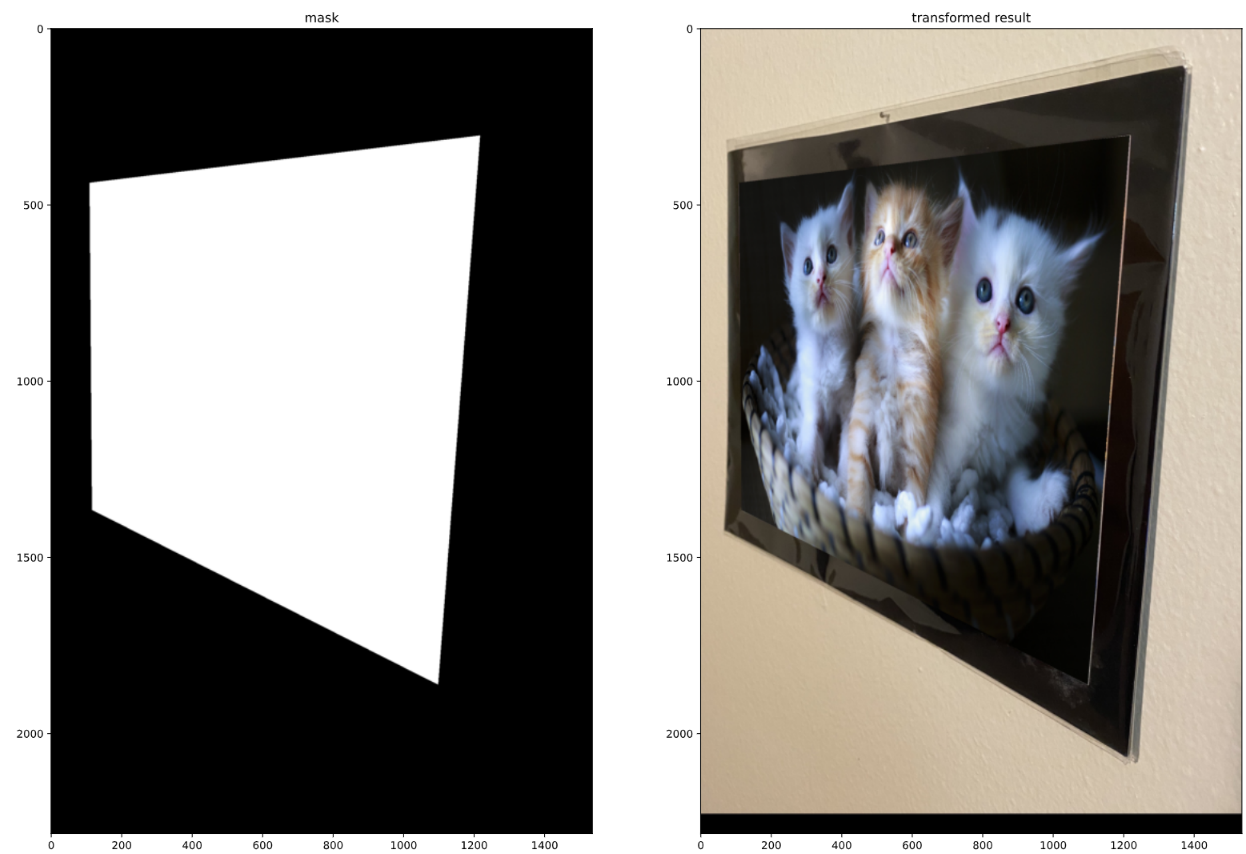

Figure 3: Mask in 1c and Fig. 1d in 1c

2.

Assume the homography between images shown in Figs. 1a and 1b are is H_{ab}, the homography between images shown in Figs. 1b and 1c are is H_{bc}, then the homography between images shown in Figs. 1a and 1c H_{ac} can be computed by $H_{ac} = H_{ab}H_{bc}$. The numerical solution of these three matrix are:

$$H_{ab} = \begin{bmatrix} 0.4634 & -0.0307 & 148.32 \\ -0.2046 & 0.6206 & 104.8 \\ -2.546 \times 10^{-4} & 3.363 \times 10^{-6} & 1 \end{bmatrix}$$

$$H_{bc} = \begin{bmatrix} 3.906 & -0.0693 & -21.18 \\ 1.219 & 1.7137 & -123.48 \\ -1.256 \times 10^{-3} & -9.701 \times 10^{-5} & 1 \end{bmatrix}$$

$$H_{ac} = \begin{bmatrix} 1.814 & -0.0966 & 143.11 \\ -0.0888 & 1.0675 & 32.49 \\ -2.656 \times 10^{-4} & -7.361 \times 10^{-5} & 1 \end{bmatrix}$$

Figure 4: Project area in Fig. 1a to 1c (left: projected 1a on 1c, right: original 1c)

Task 2

The four images are: Seleted areas on the four images are given by:

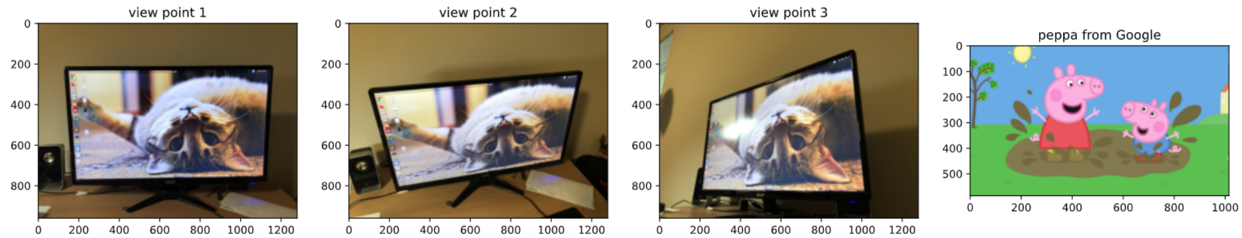

Figure 5: my monitor images from 3 viewpoints and a Peppa image

image	x_P	y_P	x_Q	y_Q	x_S	y_S	x_R	y_R
image 1a.	149	234	187	768	1135	228	1108	753
image 1b.	123	344	247	804	1131	254	1042	672
image 1c.	258	397	212	818	943	156	1011	831
image 1d.	24	21	24	517	906	21	906	517

Repeat the steps in task 1, we get the results:

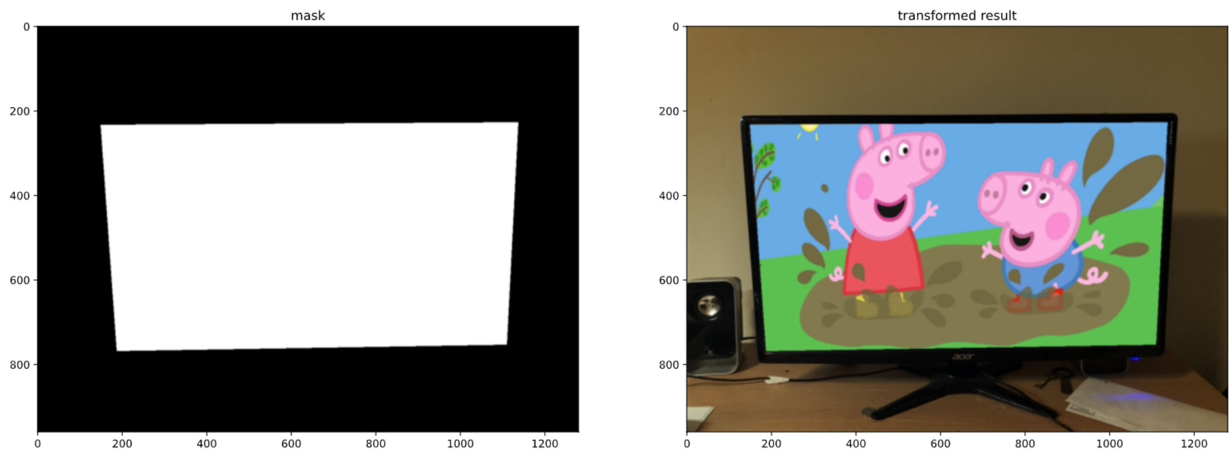

Figure 6: Mask in 1a and Fig. 1d in 1a

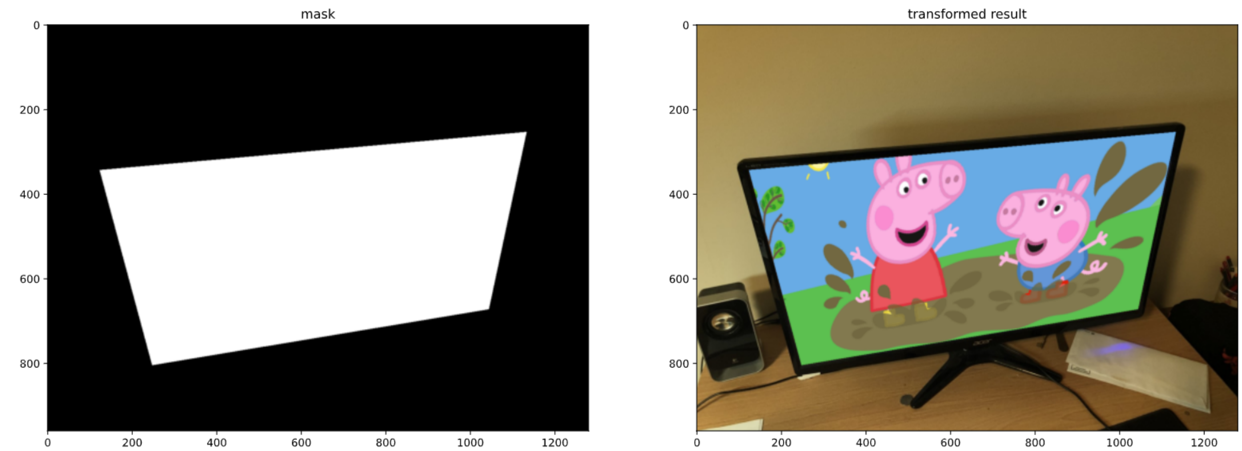

Figure 7: Mask in 1b and Fig. 1d in 1b

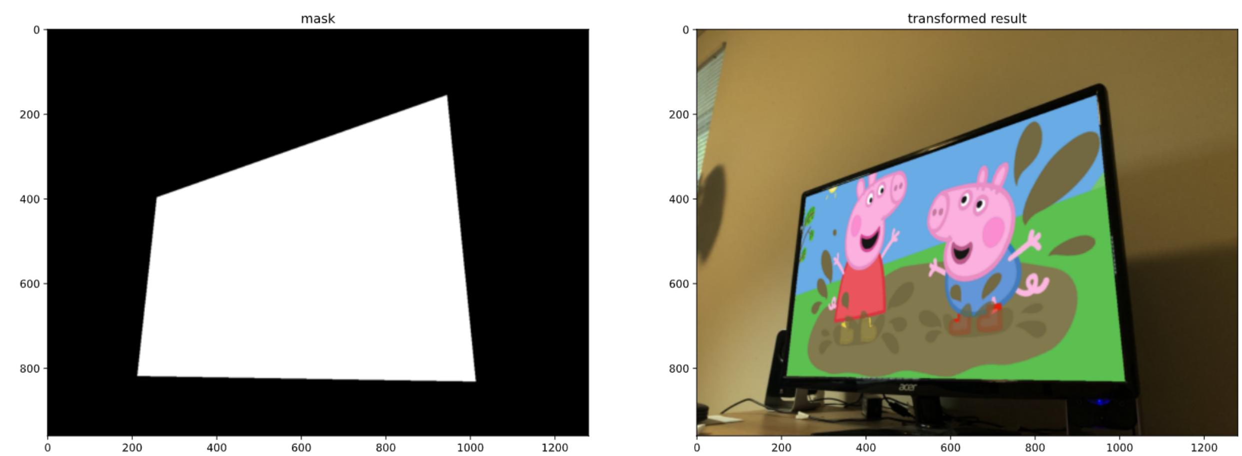

Figure 8: Mask in 1c and Fig. 1d in 1c

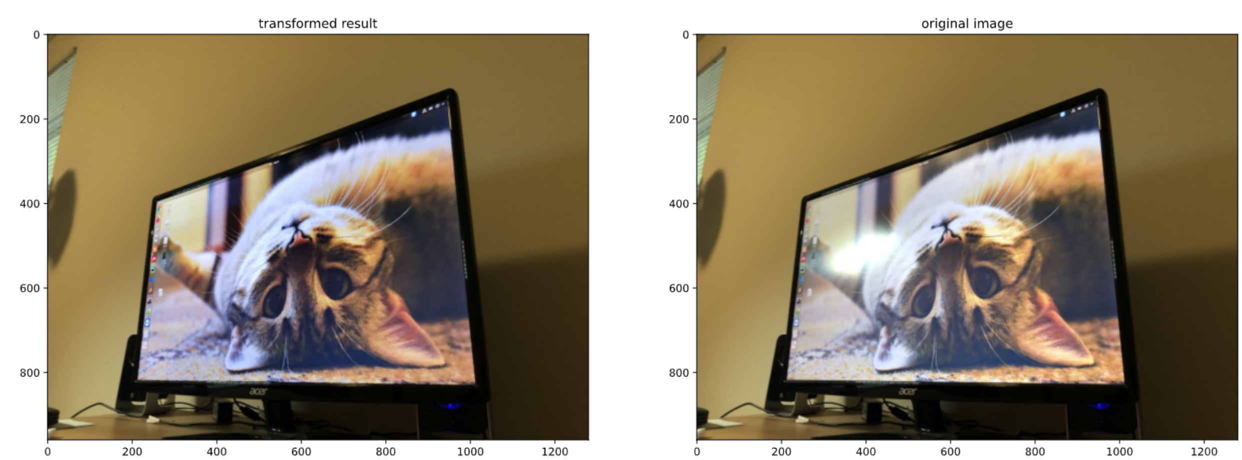

Figure 9: Project area in Fig. 1a to 1c (left: projected 1a on 1c, right: original 1c)