

Chapter 3: Temperature Sensors and Thermal Actuators

1. Temperature scales and units

2. Temperature sensor types

I. Thermoresistive sensors and actuators:

- a) Resistance Temperature Detectors (RTD)
- b) Silicon resistive sensors
- c) Thermistors

II. Thermoelectric sensors:

- a) Thermocouples
- b) Semiconductor thermocouples
- c) Thermopiles
- d) Thermoelectric generators (Peltier Cell)

III. p-n junction temperature sensors

IV. Other temperature sensors

- a) Optical and acoustical sensors
- b) Thermomechanical sensors and actuators

A bit of history

- Temperature sensor is the *oldest sensors* (except magnetic compass)
- History of temperature measurements and thermometers:
 - **1600** – Introduction of thermometers (water expansion, mercury)
 - **1650** - first attempts at temperature scales (Boyle)
 - **1700** –Temperature scales (Magelotti, Renaldini, Newton) – didn't catch
 - **1708** - Fahrenheit scale (180 div between freezing and boiling points of water)
 - **1742** - Celsius scale (100 div between freezing and boiling points of water)
 - **1848** - Kelvin scale (based on Carnot's thermodynamic work)
 - **1927** - IPTS - International Practical Temperature Scale
 - **1821** - Seebeck effect (Thomas Johann Seebeck)
 - **1826** - first temperature sensor - a thermocouple - based on the Seebeck effect (Antoine Cesar Becquerel)
 - **1834** - Peltier effect (Charles Athanase Peltier).
 - *First Peltier cell built in 1960's (for cooling and heating for space applications)*
 - **1821** - discovery of temperature dependence of conductivity (Humphrey Davey)
 - *1871 - William Siemens builds the first resistive sensor made of platinum*

Temperature scales

Temperature scales:

- **Centigrade (Celsius (°C)) scale:** 0°C at freezing point of water (triple point of water is reference), 100°C at boiling point of water
- **Standard scale (K):** Kelvin scale (absolute zero is reference),
 - Absolute zero at -273.15 °C
 - Freezing point of water at 273.15 K,
 - Boiling point of water at 373.15 K
- **Fahrenheit scale (°F):**
 - Freezing point of water at 32 °F
 - Boiling point of water at 212 °F
 - Absolute zero at -459.67 °F
- **Conversion between scales:**

$$\text{From } ^\circ\text{C to K: } N [^\circ\text{C}] = (N + 273.15) [\text{K}]$$

$$\text{From } ^\circ\text{C to } ^\circ\text{F: } P [^\circ\text{C}] = (P \times 1.8 + 32) [^\circ\text{F}]$$

$$\text{From K to } ^\circ\text{C: } M [\text{K}] = (M - 273.15) [^\circ\text{C}]$$

$$\text{From } ^\circ\text{F to } ^\circ\text{C: } Q [^\circ\text{F}] = (Q - 32)/1.8 [^\circ\text{C}]$$

$$\text{From K to } ^\circ\text{F: } S [\text{K}] = (S - 273.15) \times 1.8 + 32 [^\circ\text{F}]$$

$$\text{From } ^\circ\text{F to K: } U [^\circ\text{F}] = (U - 32)/1.8 + 273.15 [\text{K}]$$

Heat and Heat Capacity

- **Heat** is a form of energy – units of joule [J]
 - $1 \text{ J} = 1 \text{ W.s}$, $1 \text{ kWh} = 3.6 \text{ MJ}$, $1 \text{ cal} = 0.239 \text{ J}$
 - (the kWh and the calorie (cal) are not SI units)
- **Thermal conductivity (k or λ)**: the ability of materials to conduct heat. Denoted as k or λ , units: (W/[m.K])
- **Heat capacity (C)**: Amount of heat (energy) necessary to change the temperature of a substance by a given amount. units: (J/K)
- **Molar heat capacity**: energy necessary to change the temperature of one mole of substance. Units: (J/[mol.K])
- **Specific heat capacity**: energy necessary to change the temperature of 1 kg of substance by 1 K . Units: (J/[kg.K])
- **Volumetric heat capacity**: energy necessary to change the temperature of 1 m^3 of substance (gas) by 1 K . Units: (J/[m³.K])
- Often specific heat capacity is given in units of (J/[g.K]) and that of volumetric heat capacity in units of (J/[cm³.K])

Temperature sensors - general

- Temperature sensors can be deceptively simple, for example:
 - **Thermocouples:** any two dissimilar materials, welded together at one end and connected to a micro-voltmeter
 - **Peltier cell:** any thermocouple connected to a dc source (polarity defines heating or cooling)
 - **Resistive sensor:** a length of a conductor connected to an ohmmeter
- Some temperature sensors can act as actuators as well
- Can be used to measure other quantities that can affect temperature (electromagnetic radiation, air speed, flow, etc.)
- Many newer sensors are semiconductor based

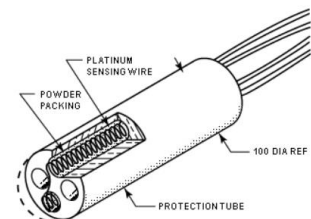
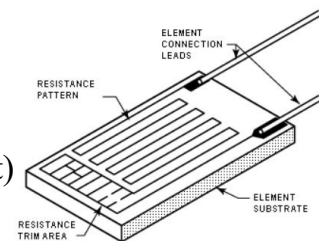
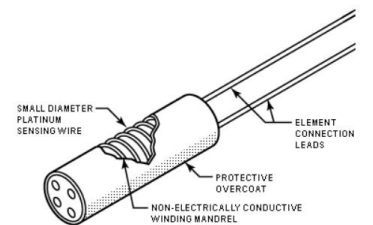
Temperature sensors - types

- **Thermoresistive sensors and actuators:**
 - Conductor based sensors and actuators (RTDs)
 - Semiconductor based sensors - thermistors
- **Thermoelectric sensors:**
 - Thermocouples and thermopiles
 - Peltier cells (used as actuators but can be used as sensors)
- **Semiconductor junction sensors**
- **Others**
 - Based on secondary effects (speed of sound, phase of light)
 - Indirect sensing (infrared thermometers – in chapter 4)
 - Expansion of metals, bimetals (actuators or sensors-actuators)

Thermoresistive sensors

Two basic types:

- Resistive Temperature Detector (RTD)
 - Metal wire
 - Thin film
 - Coiled Element RTD
 - Silicon based
- Thermistors (**Th**ermal **Res**istor)
 - Mainly NTC (Negative Temperature Coefficient)
 - PTC (Positive Temperature Coefficient)



Resistance temperature detectors

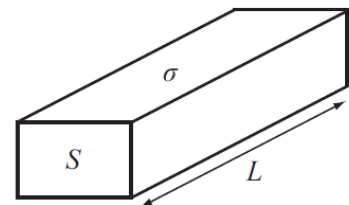
- Early sensors of this type were made of an appropriate metal such as *platinum*, *nickel*, or *copper*, depending on the application, temperature range, and often the cost.
- All RTDs are based on the change in **resistance**
- The resistance of a conductor of length L with constant cross-sectional area S and conductivity σ is:

$$R = \frac{L}{\sigma S} [\Omega]$$

- Conductivity is the measure of the ease at which an electric charge or heat can pass through a material and is temperature dependent.

$$\sigma = \frac{\sigma_0}{1 + \alpha[T - T_0]} \left[\frac{S}{m} \right]$$

Where α the temperature **coefficient of resistance (TCR)** [C^{-1}] of material, σ_0 is conductivity of conductor at reference temperature (T_0), usually 20°C



Resistance temperature detectors

- Resistance of the conductor as a function of temperature:

$$R = \frac{L}{\sigma_0 S} (1 + \alpha[T - T_0])[\Omega]$$

Or

$$R = R_0(1 + \alpha[T - T_0])[\Omega]$$

Where R_0 , is the resistance at the reference temperature, T_0 (base resistance)
 T and T_0 can be in degrees Celsius or kelvin (same division)

- Example:** Copper: $\sigma_0 = 5.8 \times 10^7 \text{ S/m}$, $\alpha = 0.0039/^\circ\text{C}$ at $T_0 = 20^\circ\text{C}$,
 Wire of cross-sectional area: $S = 0.1 \text{ mm}^2$, and length $L = 1 \text{ m}$

- Base resistance at 20°C :

$$R_0 = \frac{L}{\sigma_0 S} = \frac{1}{5.8 \times 10^7 \times 0.1 \times 10^{-6}} = 0.017 \Omega$$

- Change in resistance of:

$$\Delta R = R_0 \alpha \Delta T = 0.017 \times 0.0039 \times 1 = 6.63 \times 10^{-5} \Omega$$

Change of **0.38%** per $^\circ\text{C}$

Resistance temperature detectors

Conclusions from this example:

- For the sensor to be practical the conductor must be *long* and *thin* and/or *conductivity* must be *low* with *large temperature coefficient* to have easier signal processing.

Material	Conductivity ¹ σ [S/m]	Temperature coefficient ³ of resistance [per °C]
Copper (Cu)	5.8×10^7	0.0039
Carbon (C)	3.0×10^5	-0.0005
Constantan (60% Cu, 40% Ni)	2.0×10^6	0.00001
Chromium (Cr)	5.6×10^6	0.0059
Germanium (Ge)	2.2	-0.05
Gold (Au)	4.1×10^7	0.0034
Iron (Fe)	1.0×10^7	0.0065
Mercury (Hg)	1.0×10^6	0.00089
Nichrome (NiCr)	1.0×10^6	0.0004
Nickel (Ni)	1.15×10^7	0.00672
Platinum (Pt) ²	9.4×10^6	0.003926 (at 0 °C)
Silicon (Si) (pure)	4.35×10^{-6}	-0.07
Silver (Ag)	6.1×10^7	0.0016
Titanium (Ti)	1.8×10^6	0.042
Tungsten (W)	1.8×10^7	0.0056
Zinc (Zn)	1.76×10^7	0.0059
Aluminum (Al)	3.6×10^7	0.0043

Resistance temperature detectors

- RTD range temperature is usually between -200°C to 600°C
- Transfer function can be obtained two ways
 - 1) the manufacturers follow the existing standards that specify the coefficient α that a sensor uses (part of the sensor's specifications).
 - Standard EN 60751, dealing with platinum RTDs, specifies $\alpha = 0.00385$ (this is sometimes called the “European curve”)
 - Other values are 0.003926 (“American curve”), 0.003916, and 0.003902, among others, and relate to grades of platinum
- This allows us to establish an approximate transfer function as follows:

$$R = R(0)(1 + \alpha T)[\Omega]$$

Where $R(0)$ is the resistance at 0°C (nominal resistance)

- This is an *approximate* value because α is itself **temperature dependent**
- However, for *small sensing spans* close to the nominal temperature, the temperature curve is nearly linear this transfer function is **sufficiently accurate**.

Resistance temperature detectors

2) For more accurate sensing, the Callendar-Van Dusen equation is used:

$$\text{For } T \geq 0^\circ\text{C} \quad R(T) = R(0)[1 + aT + bT^2] \quad [\Omega]$$

$$\text{For } T < 0^\circ\text{C} \quad R(T) = R(0)[1 + aT + bT^2 + c(T - 100)T^3] \quad [\Omega]$$

- Coefficients can be calculated from experiments.
- Also, they are available in standards' tables for various materials.
 - Standard EN 60751, $\alpha = 0.00385$ and the coefficients are

$$a = 3.9083 \times 10^{-3}, b = -5.775 \times 10^{-7} \quad T > 0$$

$$a = 3.9083 \times 10^{-3}, b = -5.775 \times 10^{-7}, c = -4.183 \times 10^{-12} \quad T < 0$$

- These equations are only needed for ***larger spans*** or if sensing is done at ***low*** or ***high*** temperatures

Resistance temperature detectors

Example: Wire-spool sensor

- A spool of magnet wire (copper wire insulated with a thin layer of polyurethane) contains 500 m of wire with a diameter of 0.2 mm . It is proposed to use the spool as a temperature sensor to sense the temperature in a freezer. The proposed range is between -45°C and $+10^\circ\text{C}$. A milliammeter is used to display the temperature by connecting the sensor directly to a 1.5 V battery and measuring the current through it.
- a) Calculate the resistance of the sensor and the corresponding currents at the minimum and maximum temperatures.
- b) Calculate the maximum power the sensor dissipates.

Note: from the given table, $\sigma_0 = 5.8 \times 10^7\text{ S/m}$, $\alpha = 0.0039/^\circ\text{C}$ at $T_0 = 20^\circ\text{C}$

Material	Conductivity ¹ σ [S/m]	Temperature coefficient ³ of resistance [per $^\circ\text{C}$]
Copper (Cu)	5.8×10^7	0.0039
Carbon (C)	3.0×10^5	-0.0005

Resistance temperature detectors

Solution:

$$R(T) = \frac{l}{\sigma_0 S} (1 + \alpha[T - 20^\circ]) \text{ } [\Omega]$$

$$R(-45^\circ) = \frac{500}{5.8 \times 10^7 \times \pi \times (0.0001)^2} (1 + 0.0039[-45 - 20^\circ]) = 204.84 \text{ } \Omega.$$

$$R(+10^\circ) = \frac{500}{5.8 \times 10^7 \times \pi \times (0.0001)^2} (1 + 0.0039[10 - 20^\circ]) = 263.7 \text{ } \Omega$$

- The resistance changes from 204.84 Ω at -45 $^\circ\text{C}$ to 263.7 Ω at 10 $^\circ\text{C}$.
- The currents are

$$I(-45^\circ) = \frac{1.5}{204.84} = 7.323 \text{ mA} \quad I(+10^\circ) = \frac{1.5}{263.7} = 5.688 \text{ mA}$$

- The power dissipated is

$$P(+10^\circ) = I^2 R = (5.688 \times 10^{-3})^2 \times 263.7 = 8.53 \text{ mW}$$

$$P(-45^\circ) = I^2 R = (7.323 \times 10^{-3})^2 \times 204.84 = 10.98 \text{ mW}$$

- The power is **low** which is ideal for temperature sensors since power dissipated in the sensor can lead to *errors* due to self heating if it is high.

Resistance temperature detectors

Example2: Wire RTD resistance and sensitivity

- A wire-wound RTD sensor is made of pure platinum wire, 0.1 mm in diameter, to have a resistance of 25 Ω at 0 $^{\circ}\text{C}$. Assume here that the TCR is constant with temperature.
- a) Find the necessary length for the wire.
- b) Find the resistance of the RTD at 100
- c) Find the sensitivity of the sensor in ohms/degree Celsius

Solution:

From the table, $\sigma_0 = 9.4 \times 10^6 \text{ S/m}$ at $T_0 = 20^{\circ}\text{C}$, and $\alpha = 0.003926/^{\circ}\text{C}$ at $T_0 = 0^{\circ}\text{C}$

$$a) \quad R(T) = \frac{l}{\sigma_0 S} (1 + \alpha[T - 20^{\circ}]) \text{ } [\Omega]$$

$$25 = \frac{l}{9.4 \times 10^6 \times \pi \times (0.05 \times 10^{-3})^2} (1 + 0.003926[0 - 20]) = 12.48154l \text{ } [\Omega]$$

$$l = \frac{25}{12.48154} = 2.003 \text{ m}$$

Resistance temperature detectors

Solution:

b)

$$R(100\text{ }^{\circ}\text{C}) = \frac{2.003}{9.4 \times 10^6 \times \pi \times (0.05 \times 10^{-3})^2} (1 + 0.003926[100 - 20]) = 35.652\ \Omega$$

c)

Sensitivity is the slope of transfer function which means the amount of change in resistance by increasing the temperature by 1 °C

$$R(T + 1) - R(T) = \frac{l}{\sigma_0 S} (1 + \alpha[T + 1 - 20^{\circ}]) - \frac{l}{\sigma_0 S} (1 + \alpha[T - 20^{\circ}]) = \frac{l\alpha}{\sigma_0 S} [\Omega]$$

$$\Delta R = \frac{l\alpha}{\sigma_0 S} = \frac{2.003 \times 0.003926}{9.4 \times 10^6 \times \pi \times (0.05 \times 10^{-3})^2} = 0.1065\ \Omega$$

- The sensitivity is therefore 0.1065 Ω /°C
- Check: Since the resistance is linear with temperature, the sensitivity is the same everywhere and thus we can write the resistance at 100 °C as

$$R(100\text{ }^{\circ}\text{C}) = R(0\text{ }^{\circ}\text{C}) + 100 \times \Delta R = 25 + 100 \times 0.1065 = 35.65\ \Omega$$

Resistance temperature detectors

Example 3: RTD representation and accuracy

- A platinum RTD with nominal resistance of $100\ \Omega$ at $0\ ^\circ\text{C}$ is specified for the range from $-200\ ^\circ\text{C}$ to $+600\ ^\circ\text{C}$. The engineer has the option of using the approximate transfer function (linear) or the exact transfer function in (polynomial). Assume $\alpha = 0.00385/^\circ\text{C}$
- a) Calculate the error incurred by using the approximate transfer function at the extremes of the range.
- b) What are the errors if the range used is from $-50\ ^\circ\text{C}$ to $+100\ ^\circ\text{C}$

From standard EN 60751, $\alpha = 0.00385$ and the coefficients are

$$a = 3.9083 \times 10^{-3}, b = -5.775 \times 10^{-7} \quad T > 0$$

$$a = 3.9083 \times 10^{-3}, b = -5.775 \times 10^{-7}, c = -4.183 \times 10^{-12} \quad T < 0$$

Resistance temperature detectors

Solution:

a) linear TF $R(600\text{ }^{\circ}\text{C}) = R(0)[1 + \alpha T] = 100[1 + 0.00385 \times 600] = 331\ \Omega$

$$R(-200\text{ }^{\circ}\text{C}) = 100[1 - 0.00385 \times 200] = 23\ \Omega$$

nonlinear TF

$$R(600\text{ }^{\circ}\text{C}) = R(0)[1 + aT + bT^2]$$

$$100[1 + 3.9083 \times 10^{-3} \times 600 - 5.775 \times 10^{-7} \times 600^2] = 313.708\ \Omega$$

$$R(-200\text{ }^{\circ}\text{C}) = R(0)[1 + aT + bT^2 + c(T - 100)T^3] \quad [\Omega]$$

$$100[1 + 3.9083 \times 10^{-3} \times (-200) - 5.775 \times 10^{-7} \times 200^2$$

$$-4.183 \times 10^{-12} \times (-300) \times (-200)^3] = 18.52\ \Omega$$

- The resistance calculated with the approximate formula is higher by 5.51% at 600°C and higher by 24.19% at -200 °C.
- These deviations ***are not acceptable*** and use of the Callendar–Van Dusen relations is *essential*.

Resistance temperature detectors

Solution:

b) linear TF $R(100\text{ }^{\circ}\text{C}) = R(0)[1 + \alpha T] = 100[1 + 0.00385 \times 100] = 138.5\ \Omega$
 $R(-50\text{ }^{\circ}\text{C}) = 100[1 - 0.00385 \times 50] = 80.75\ \Omega$

nonlinear TF

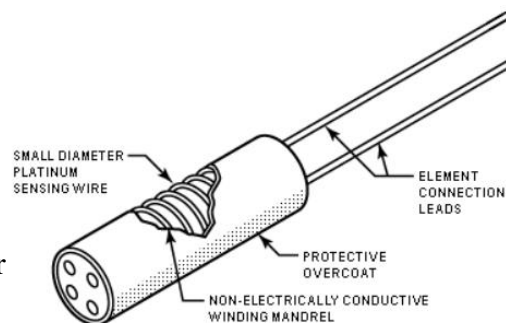
$$R(100\text{ }^{\circ}\text{C}) = 100[1 + 3.9083 \times 10^{-3} \times 100 - 5.775 \times 10^{-7} \times 100^2] = 138.5055\ \Omega$$

$$R(-50\text{ }^{\circ}\text{C}) = 100[1 + 3.9083 \times 10^{-3} \times (-50) - 5.775 \times 10^{-7} \times 50^2 - 4.183 \times 10^{-12} \times (-150) \times (-50)^3] = 80.3063\ \Omega$$

- The resistance calculated with the approximate formula is only 0.0397% lower at 100°C and lower by 0.552% at -50 °C.
- These deviations are ***acceptable*** and the *approximate formula can be* used.

Construction - Wire RTD

- Careful to minimize the effect of tension on wires in designing RTDs which has the same effect on resistance as a change in temperature
- A characteristic property of wire RTDs is their *relatively low resistance*
 - High resistances would require very long wires or very thin wires.
 - Another consideration is cost.
- Satisfactory resistance for wire sensors is from a *few ohms* to a *few tens of ohms*
- Wire is made of a fairly *thin, uniform* wire wound in a small diameter coil
- Coil is supported on a suitable support such as *mica* or *glass*.
- Wire and coil are enclosed in a *glass, ceramic* or *highly conductive metal*
 - Allow better heat transfer to the sensing wire and so, faster response of the sensor



Material - Wire RTD

- RTDs are typically made out of *Platinum* wire.
- *Nickel & Copper* have been used, but for RTDs, Platinum is superior to the other metals.

Platinum - used for precision applications

- Chemically stable at high temperatures
- Resists oxidation
- Can be made into thin wires of high chemical purity
- Resists corrosion
- Can withstand severe environmental conditions.
- Useful to about 800 °C and down to below –250 °C.
- Very sensitive to strain
- Sensitive to chemical contaminants

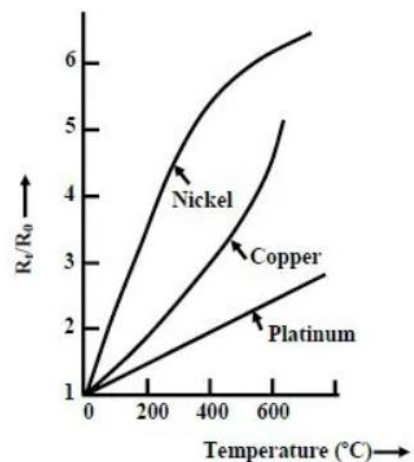
Material - Wire RTD

- **Nickel**

- Less-demanding applications and cheaper
- Can be used from about -100 °C to about 500 °C, but its R–T curve becomes non-linear at temperatures above 300 °C

- **Copper**

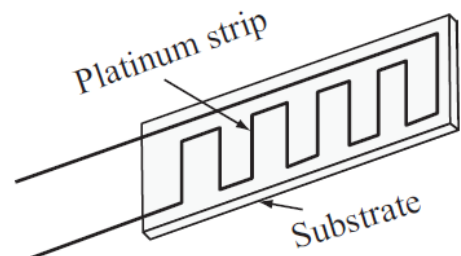
- Less-demanding applications and cheaper
- Wire length needed is long (high conductivity)
- Reduced temperature range (copper only works up to about 300 °C)
- Oxidizes and it cannot be used over 150 °C
- Not suitable for corrosive environments (unless properly protected)



Thin Film RTD

Thin film sensors:

- Produced by depositing a thin layer of a suitable material (platinum or its alloys) on electrically non-conducting, and thermally conducting ceramic
- Etched to form a long strip and potted in epoxy or glass to protect it.
- Small and relatively inexpensive (some are only a few mm²)
- Often the choice in modern sensors especially when the very high precision of Platinum wire sensors is not needed.



Thin Film RTD

Advantages of wire-wound resistors:

- High accuracy along the entire temperature range, required in certain laboratory settings or for applications like custody transfer

Disadvantages of wire-wound resistors

- Not vibration resistant
- Larger footprint (around 25mm, or 1 inch) in length
- As much as 10 times more expensive than a thin-film resistor

Advantages of thin-film resistors

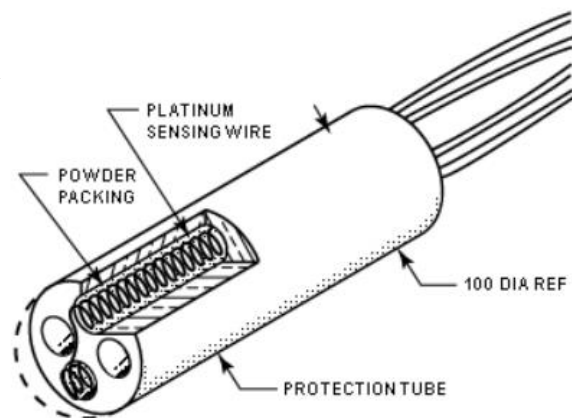
- Compact size (around 3mm, or 1/8-inch) in length
- Vibration resistant, thanks to their smaller mass and no moving parts, making them ideal for use in compressors and bearing applications
- About 1/10th of the cost of a wire-wound resistor

Disadvantages of thin-film resistors

- Loss of accuracy at low and high temperatures

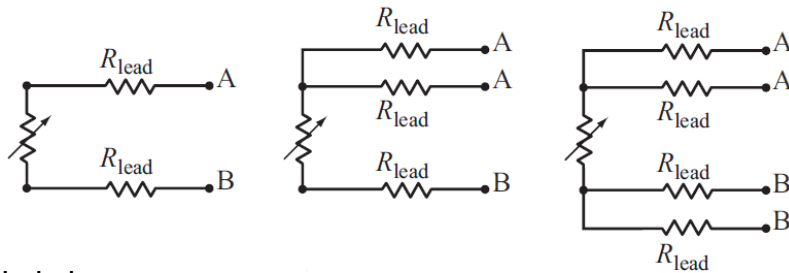
Coiled Element RTD

- This “strain-free” design allows the sensing wire to expand and contract freely.
- The sensing element is a small coil of platinum wire that resembles a filament in light bulb.
- The coil is inserted into bores of the mandrel and packed with a very finely ground ceramic powder.
- This permits the sensing wire to move, while still remaining in good thermal contact with the process.



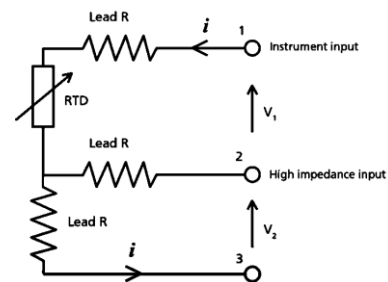
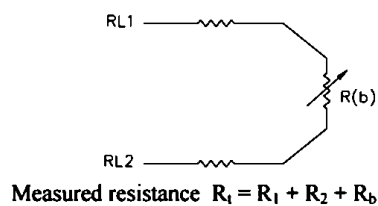
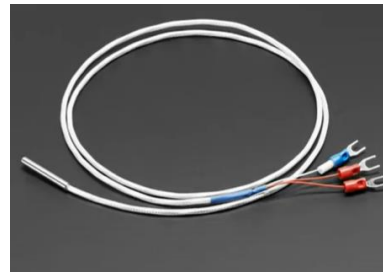
Connection of RTD

- RTD measurements are subject to errors due to the connecting wires resistance and temperature variations along these wires
- In *low resistance RTDs*, the resistance of the lead wires also changes with temperature, and can add to errors in the sensing circuit as these resistances are **not negligible** (except in *high-resistance thin-film RTDs*).
- Because of this, some commercial sensors come in **two-, three-, or four-wire** configurations to facilitate compensation for the lead wires
 - Two-wire configurations **cannot** be compensated
 - Three- and four-wire sensors **allow** compensation of the lead resistance and should be used when *high precision* is essential.



SFU

Connection of RTD



$$V_1 = i(RTD + 2 \times [\text{lead wire}\Omega])$$

$$V_2 = i(2 \times \text{lead wire}\Omega)$$

$$V_2 - V_1 = i(RTD)$$

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Self heat in RTDs

- RTDs are subject to errors due to *increases* in their own temperature produced by the heat generated in them by the *current* used to measure their resistance.
- Applicable to wire-wound or thin film sensors
- Power dissipated: $P_d = I^2 R$ (I is current and R the sensor's resistance)
- **Self heat** depends on size, construction and environment
 - Lower in large elements, higher in small elements
 - Environmental factors (moving air or standing air, etc.) affect significantly
 - Important to lower the current as much as possible
- Given as temperature rise per unit power ($^{\circ}\text{C}/\text{mW}$) or power needed to raise temperature ($\text{mW}/^{\circ}\text{C}$) by manufacturer
 - Self-heat errors are of the order of $0.01^{\circ}\text{C}/\text{mW}$ to $0.2^{\circ}\text{C}/\text{mW}$

Self heat in RTDs

Example: Self heat in RTDs

- A 100 Ω (at 0 $^{\circ}\text{C}$) platinum RTD operates between -200 $^{\circ}\text{C}$ to +850 $^{\circ}\text{C}$. Its self-heat is provided in its data sheet as 0.08 $^{\circ}\text{C}/\text{mW}$ in air (typically, this value is given at a low airspeed of 1 m/s). Calculate the maximum error expected due to self-heat if
 - Resistance is measured by applying a constant voltage of 1 V across the sensor.
 - Resistance is measured by applying a constant current of 10 mA through the sensor.

$$a = 3.9083 \times 10^{-3}, b = -5.775 \times 10^{-7} \quad T > 0$$

$$a = 3.9083 \times 10^{-3}, b = -5.775 \times 10^{-7}, c = -4.183 \times 10^{-12} \quad T < 0$$

Solution:

a)

$$\begin{aligned} R(-200^{\circ}\text{C}) &= R(0)[1 + aT + bT^2 + c(T - 100)T^3] \\ &= 100[1 + 3.9083 \times 10^{-3} \times (-200) - 5.775 \times 10^{-7} \times 200^2 \\ &\quad - 4.183 \times 10^{-12} \times (-200 - 100) \times (-200)^3] = 18.52 \, \Omega \\ R(850^{\circ}\text{C}) &= R(0)[1 + aT + bT^2] \\ &= 100[1 + 3.9083 \times 10^{-3} \times 850 - 5.775 \times 10^{-7} \times 850^2] \\ &= 390.48 \, \Omega. \end{aligned}$$

$$P(-200^{\circ}\text{C}) = \frac{V^2}{R} = \frac{1}{18.52} = 54 \, \text{mW}$$

$$P(850^{\circ}\text{C}) = \frac{V^2}{R} = \frac{1}{390.48} = 2.56 \, \text{mW}$$

SFU

Self heat in RTDs

a)

error at $-200\text{ }^{\circ}\text{C}$ is $54 \times 0.08 = 4.32\text{ }^{\circ}\text{C}$

error at $850\text{ }^{\circ}\text{C}$ is $2.56 \times 0.08 = 0.205\text{ }^{\circ}\text{C}$

- The maximum error occurs at **$-200\text{ }^{\circ}\text{C}$** and equals **$4.3\text{ }^{\circ}\text{C}$** or **$2.15\%$** .
- At the high end of the span the error is only **$0.2\text{ }^{\circ}\text{C}$** .

b)

$$P(-200^{\circ}\text{C}) = I^2 R = (10 \times 10^{-3})^2 \times 18.52 = 1.85\text{ mW}$$

$$P(850^{\circ}\text{C}) = I^2 R = (10 \times 10^{-3})^2 \times 390.48 = 39\text{ mW}$$

error at $-200\text{ }^{\circ}\text{C}$ is $1.85 \times 0.08 = 0.148\text{ }^{\circ}\text{C}$.

error at $850\text{ }^{\circ}\text{C}$ is $39 \times 0.08 = 3.12\text{ }^{\circ}\text{C}$

- The maximum error occurs at **$850\text{ }^{\circ}\text{C}$** and equals **$3.12\text{ }^{\circ}\text{C}$** or **$0.37\%$** .
- Both methods are used and in both the errors vary with temperature.
- Use of a ***current*** source reduces the errors throughout the temperature range.
- To reduce the error, the current can be reduced, but it cannot be too small, because of difficulties in measurements as well as noise may be encountered.

Response Time in RTDs

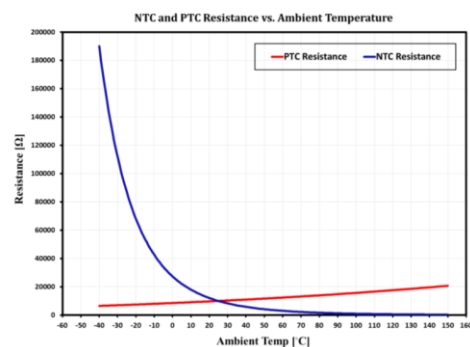
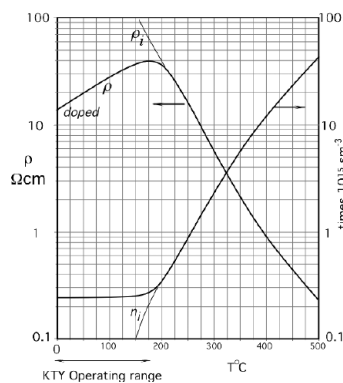
- Response of most *temperature sensors* is slow, especially if they are physically large.
- Provided as part of data sheet
- Given in air or in water or both, moving or stationary
- Given as time to reach 90%, 50% (or other) of steady state
 - 50% of steady state means the sensor has reached a temperature equal to its initial value plus 50% of the step.
- Generally slow
- Wire RTDs are slower (because of their physical size)
- Typical values 0.5 sec in water to 100 sec in moving air

Silicon Resistive Sensors

- **Valence electrons** are those bound to atoms and hence not free to move
- **Conduction electrons** are free to move and affect the current through the semiconductor.
- For an electron to move into the *conduction band*, it must acquire ***additional energy*** which is called ***band gap energy*** and is material dependent
 - This additional energy comes from ***heat, light, nuclear, electromagnetic***
- The ***higher*** the temperature, the ***higher*** the number of electrons available and hence the ***higher*** the current that can flow through the device (i.e., the ***lower*** its resistance).
 - As temperature increases, the resistance decreases
 - Pure semiconductors such as silicon typically have NTC characteristics

Silicon Resistive Sensors

- Semiconductors are *rarely* used as pure (intrinsic) materials.
- Impurities are introduced into the intrinsic material in a process called *doping*.
- If silicon doped with an n-type impurity such as arsenic (As) or antimony (Sb), the reverse effect (PTC) is observed below a certain temperature.
 - For n-type silicon, a PTC is observed below about 200°C (NTC above)
 - PTC silicon resistive (Cylister), are mainly used for fuses
 - while NTC sensors are just used for temperature sensors



Silicon Resistive Sensors

- PTC Silicon resistive sensors can operate in a *limited range* of temperatures like most semiconductors devices based on silicon
- Maximum range is between -55°C to $+150^{\circ}\text{C}$.
- Typical range: -45°C to $+85^{\circ}\text{C}$ or 0°C to $+80^{\circ}\text{C}$
- Resistance: typically $1\text{k}\Omega$ at 25°C .
- Made as a small chip with two electrodes and encapsulated in epoxy, etc
- Resistance is calculated using the Callendar-Van Dusen Equation

$$R(T) = R(0) \left[1 + a(T - T_{\text{ref}}) + b(T - T_{\text{ref}})^2 + HOT \right] \quad [\Omega]$$

HOT stands for “higher order terms” indicating that additional coefficients may be used to improve representation

