

MSE 310 Assignment1

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Assignment #1

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Simon Fraser University
Mechatronic Systems Engineering

MSE 310 Sensors and Actuators

- 1- Nonlinear transfer function of a temperature sensor. A temperature sensor's transfer function gives the resistance of the sensor as a function of temperature as:

$$R(T) = R_0 e^{-\beta(1/T_0 - 1/T)}$$

Temperatures are in K. T_0 is a reference temperature, T is the temperature at which the resistance is measured and R_0 is the sensor's resistance at T_0 . β is a constant characteristic of the particular sensor, typically supplied by the manufacturer. A sensor is given with properties $R_0 = 100$ kΩ at $T_0 = 25^\circ\text{C}$ and $\beta = 3,560$. The resistance of the sensor is measured at 85°C as 13,100 W and at 25°C as 100 kΩ.

- (a) Assuming the measurements to be exact, what is the error in the transfer function at 85°C ?
 (b) What is the maximum error as a percentage when using the manufacturer's data in the range 0°C and 100°C if a new transfer function based on the measurements at 85°C and at 25°C is established? At what temperature does it occur?

- 2- The response of a temperature sensor is given as

$$R(T) = R_0 e^{-\beta(1/T - 1/T_0)} \Omega$$

where R_0 is the resistance of the sensor at temperature T_0 and β is a constant that depends on the material of the sensor. Temperatures T and T_0 are in K. Given: $R(T) = 1,000 \Omega$ at 25°C and $3,000 \Omega$ at 0°C . The sensor is intended for use between -45°C and 120°C . $T_0 = 20^\circ\text{C}$.

- a) Evaluate β for this sensor and plot the sensor transfer function for the intended span.
 b) Approximate the transfer function as a straight line connecting the end points and calculate the maximum error expected as a percentage of full scale.

1.a)

$$\begin{aligned} R(T) &= R_0 e^{-\beta(1/T_0 - 1/T)} \\ &= 100000 e^{-3560(\frac{1}{25} - \frac{1}{85})} \\ &= 13529.07 \end{aligned}$$

error = measured - calc
 $= 13100 - 13529.07$
 $= -429.07 \text{ W}$

$$\frac{-429.07}{13100} \times 100\% = 3.28\%$$

b)

$$\begin{aligned} R(0) &= 100000 e^{-3560(\frac{1}{25} + \frac{1}{100})} \\ &= 9072.71 \\ R(10) &= 100000 e^{-3560(\frac{1}{25} + \frac{1}{35})} \\ &= 218271.35 \\ R_{ss}(T) &= 13100 e^{-3560(\frac{1}{25} + \frac{1}{T})} \\ &= 8789.97 \end{aligned}$$

$$\frac{9072.71 - 8789.97}{9072.71} = 3.18\% \text{ at } 100^\circ\text{C}$$

2. a) $R(T) = R_0 e^{-\beta(\frac{1}{T} - \frac{1}{T_0})}$

$$\begin{aligned} 1000 &= R_0 e^{-\beta(\frac{1}{25} - \frac{1}{20})} \\ 3000 &= R_0 e^{-\beta(\frac{1}{120} - \frac{1}{20})} \end{aligned}$$

$$\beta = -3578.93, R_0 = 1227.14$$

b) $R(T) = R_0 e^{-\beta(\frac{1}{T} - \frac{1}{T_0})}$

$$\begin{aligned} R(-45) &= 39725.51 \\ R(120) &= 55.03 \end{aligned}$$

$$R'(T) = \frac{55.03 - 39725.51}{(120 - 45) - (-45 - 45)} = -240.43$$

$$y = mx + b$$

$$39725.51 = -240.43(120 - 45) + b, b = 94579.7$$

$$y = -240.43T + 94579.7$$

- 3- A strain gauge is a resistive sensor that changes its resistance according to the strain applied to it. Strain, denoted as ϵ , is the elongation (or contraction) in response to force. Two different sensors are used in an application by exposing both to exactly the same strain. The transfer functions of

$$R_1 = R_{01}(1 + 5.0\epsilon)$$

$$R_2 = R_{02}(1 + 2.0\epsilon)$$

R_{01} and R_{02} are the resistances of the two sensors when no strain is applied, ϵ is the strain and $g_1 = 5.0$ and $g_2 = 2.0$ are sensitivities of the two sensors. Calculate the sensitivity of the strain gauge made of the two strain gauges.

- a) If they are connected in series.
- b) If they are connected in parallel.
- c) From (a) and (b) show that sensitivity increases when the sensors are connected in series and decreases when connected in parallel. For simplicity use two identical sensors.

- 4- A new type of thermistor rated at $100 \text{ k}\Omega$ at 20°C is used to sense temperatures between -80°C and $+100^\circ\text{C}$. It is expected that the transfer function is a second-order polynomial. To evaluate its transfer function, the resistance of the thermistor is measured at -60°C as $320 \text{ k}\Omega$ and at $+80^\circ\text{C}$ as $20 \text{ k}\Omega$.
- a.) Find and plot the transfer function for the required span using a second-order polynomial.
 - b.) Calculate the resistance expected at 0°C .

- 5- Incandescent lightbulbs use a tungsten wire as the light-radiating filament by heating it to a temperature at which it is bright enough to produce light. The temperature of the wire can be estimated directly from the power rating and the resistance of the wire when it is cold. Given a 120 V , 100 W lightbulb with a resistance of 22Ω at room temperature (20°C):
- a. Calculate the temperature of the filament when the lightbulb is lit.
 - b. What are the possible sources of error in this type of indirect sensing? Explain.

3)

$$R_1 = R_{01}(1 + 5\epsilon) \quad g_1 = 5$$

$$R_2 = R_{02}(1 + 2\epsilon) \quad g_2 = 2$$

$$\text{a)} \quad R_T = R_1 + R_2$$

$$= R_{01} + 5\epsilon R_{01} + R_{02} + 2\epsilon R_{02}$$

$$= R_{01} + R_{02} + \epsilon(5R_{01} + 2R_{02})$$

$$\text{b)} \quad R_T = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{1}{R_{01} + 5\epsilon R_{01}} + \frac{1}{R_{02} + 2\epsilon R_{02}}$$

$$= \frac{R_{02} + 2\epsilon R_{02} + R_{01} + 5\epsilon R_{01}}{(R_{01} + 5\epsilon R_{01})(R_{02} + 2\epsilon R_{02})}$$

$$= \frac{R_{01} + R_{02} + \epsilon(5R_{01} + 2R_{02})}{R_{01}R_{02} + R_{01}R_{02}(7\epsilon R_{01}) + 10\epsilon^2 R_{01}R_{02}}$$

c) plugging in numbers in a) sensitivity ↑

b) sensitivity ↓

$$\text{4) Secondary} \Rightarrow R = aT^2 + bT + c$$

$$\text{a)} \quad R(20) = 100000 = a(20)^2 + b(20) + c = 400a + 20b + c$$

$$R(80) = 20000 = a(80)^2 + b(80) + c = 6400a + 80b + c$$

$$R(-60) = 320000 = a(-60)^2 + b(-60) + c = 3600a - 60b + c$$

plugging into equation: $a = 10.12 \quad b = -2345.24 \quad c = 142557.19$

$$R = aT^2 + bT + c$$

$$= 10.12T^2 + (-2345.24)T + 142557.19$$

$$\text{b) } @ T=0, \quad R + C = 142557.19 \Omega$$

$$5) \quad T_0 = 20^\circ\text{C} \quad V = 120\text{V}$$

$$\text{a)} \quad R_0 = 22\Omega, \quad P = 100\text{W}$$

tungsten $\alpha = 0.0056$

b) It might be the wire

↳ Indirect contact

↳ Slow reaction

$$P = \sqrt{VR}$$

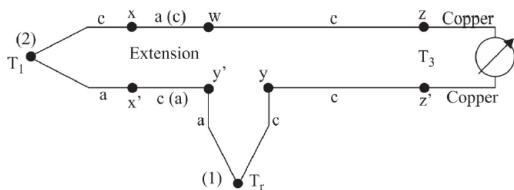
$$100 = 120^2 R, \quad R = 144 \Omega$$

$$R = R_0(1 + \alpha(T - T_0))$$

$$144 = 22(1 + 0.0056(T - (20 + 273.15)))$$

$$T = 1293.91 \text{ K}$$

- 6- An RTD can be built relatively easily. Consider a copper RTD made of magnet wire (copper wire insulated with a polymer). The wire is 0.1 mm thick and the nominal resistance required is 120Ω at 20°C . Neglect the thickness of the insulating polymer.
- How long must the wire be?
 - Assuming we wish to wind the copper wire into a single spiral winding 6 mm in diameter so that it can be enclosed in a stainless-steel tube, what is the minimum length of the RTD?
 - Calculate the range of resistance of the RTD for use between -45°C and 120°C .
- 7- A thermistor with a nominal resistance of $15 \text{k}\Omega$ at 25°C carries a current of 5mA . At an ambient temperature of 30°C (measured with a thermocouple), the resistance of the thermistor is $12.5 \text{k}\Omega$. The current is now removed and the resistance of the thermistor drops to $12.35 \text{k}\Omega$. Calculate the error due to self-heat of the thermistor in $^\circ\text{C}$ per milliwatt ($^\circ\text{C}/\text{mW}$).
- 8- A K-type thermocouple measures temperature $T_1 = 100^\circ\text{C}$ and has reference temperature $T_r = 0^\circ\text{C}$. In a particular application it becomes necessary to extend the length of the wires leading to the sensing junction as shown in Figure 3.33. The $y-y'$ and $z-z'$ junctions are each held in a 25°C temperature zone. Calculate the reading of the voltmeter under the following conditions ($c = \text{chromel}$, $a = \text{alumel}$):
- The extension section is absent. (i.e., x and w are the same point and x' and y' are the same point).
 - The extension is made of a pair of copper wires. The $x-x'$, $y-y'$, $z-z'$ and w junctions are held in a temperature zone at 25°C .
 - To improve accuracy, the extension section is made of the same wires as the junction, with the alumel wire on top and the chromel wire on the bottom. The junctions $x-x'$, $y-y'$, $z-z'$, are in a temperature zones, at 25°C . The junction w is at 20°C . Calculate the error in the reading of temperature T_1 .
 - In a further attempt to reduce the error, the extension is flipped so that now the chromel wire is on top and the alumel wire is on the bottom while the junctions conditions are the same as in (c). Does that resolve the issue? Explain



6. a) $R = \frac{\rho L}{A}$
 $L = \frac{R(0.25\pi \times D^2)}{\rho}$
 $= \frac{120 \times 0.25\pi \times (0.1 \times 10^{-3})^2}{1.68 \times 10^{-8}}$
 $= 56.1 \text{m}$

b) $d = 6 \text{mm}$
 \dots
 How to find min length???

c) $R = R_0(1 + \alpha(T_0 - T))$ $\alpha_{\text{copper}} = 0.00393$
 $= 120(1 + 0.00393(-45 - 20))$
 $= 89.5 \text{k}\Omega$

$R_{\text{max}} \leq R \leq R_{\text{min}}$
 $R_{\text{max}} = 120(1 + 0.00393(100 - 20))$
 $= 167.6 \text{k}\Omega$

7. $R(25^\circ\text{C}) = 15 \text{k}\Omega$
 $\alpha(15^\circ\text{C}) = 5 \text{m}\Omega/\text{K}$
 $R(20^\circ\text{C}) = 12.5 \text{k}\Omega$
 $R(30^\circ\text{C}) = 12.35 \text{k}\Omega$

$12500 = 15000 e^{\frac{\beta(1-\frac{1}{20+25})}{30+25}}$
 $\beta = 3295.8$

$12350 = 15000 e^{3295.8(\frac{1}{T} - \frac{1}{20+25})}$
 $T = 303.481$

$e = \frac{\Delta T}{\Delta P} = \frac{303.481 - 298.15}{5 \text{m}\Omega \times 150}$
 $= 0.001483 \text{ }^\circ\text{C/mA}$

8. a) $V = \alpha(T_1 - T_r)$
 $\alpha = 41 \text{ mV}/\text{K}$
 $V = 41 \times 10^{-6} \times 100$
 $= 4.1 \times 10^{-3} \text{ V}$

b) $V = \alpha(T_1 - T_r)$
 $= 41 \times 10^{-6} \times (75^\circ\text{C})$
 $= 3.075 \times 10^{-3} \text{ V}$

c) $41 \times 10^{-6} [L(80^\circ\text{C}) - 2.28 \times 10^{-3} \text{ V}]$

d) No, still not same material / at same temperature.

$$\frac{3.28 - 3.075}{3.28} = 6.25\%$$