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Course #: **16720**
HW3

Q1.1
Answer:
(1)
the Lucas-Kanade algorithm assumes that a current estimate of **p** is known and then iteratively solves for increments to the parameters Δp; i.e. the following expression is (approximately) minimized:

$$\sum_x [I(W(x;p + \Delta p)) - T(x)]^2$$

The first order Taylor expansion of the above equation is:

$$\sum_x [I(W(x;p)) + \nabla I \frac{aW}{ap} \Delta p - T(x)]^2$$

The partial derivative of the expression with respect to Δp is:

$$\sum_x [\nabla I \frac{aW}{ap}]^T [I(W(x;p)) + \nabla I \frac{aW}{ap} \Delta p - T(x)]^2$$

So A = $\nabla I \frac{aW}{ap}$

The solution of Δp is:

$$\Delta p = H^{-1} \sum_x [\nabla I \frac{aW}{ap}]^T [T(x) - I(W(x;p))]^2$$

where is the n * n (Gauss-Newton approximation to the) Hessian matrix:

$$H = \sum_x [\nabla I \frac{aW}{ap}]^T [\nabla I \frac{aW}{ap}] = \sum_x A^T A$$

So $A^T A = [\nabla I \frac{aW}{ap}]^T [\nabla I \frac{aW}{ap}]$

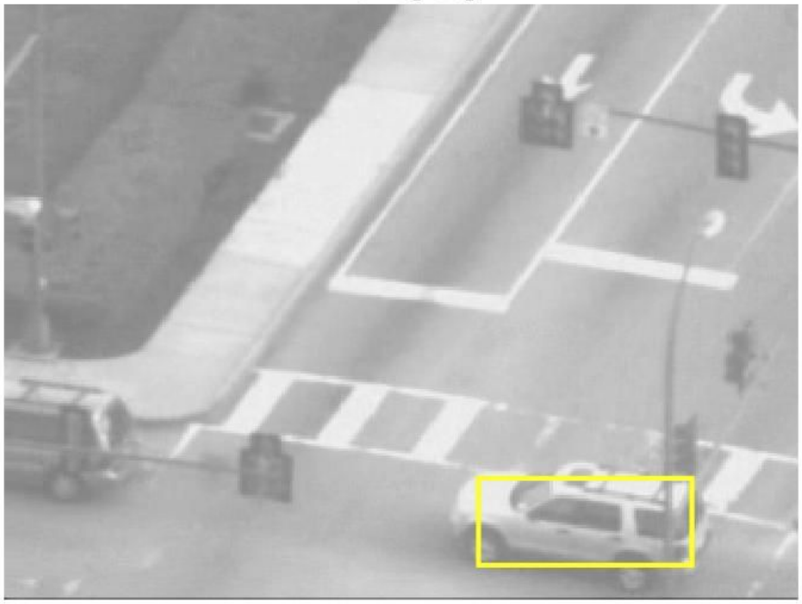
(2)

In order to get a solution of H, $A^T A$ should be invertible. In other word, $A^T A$ should not be singular.

Q1.3

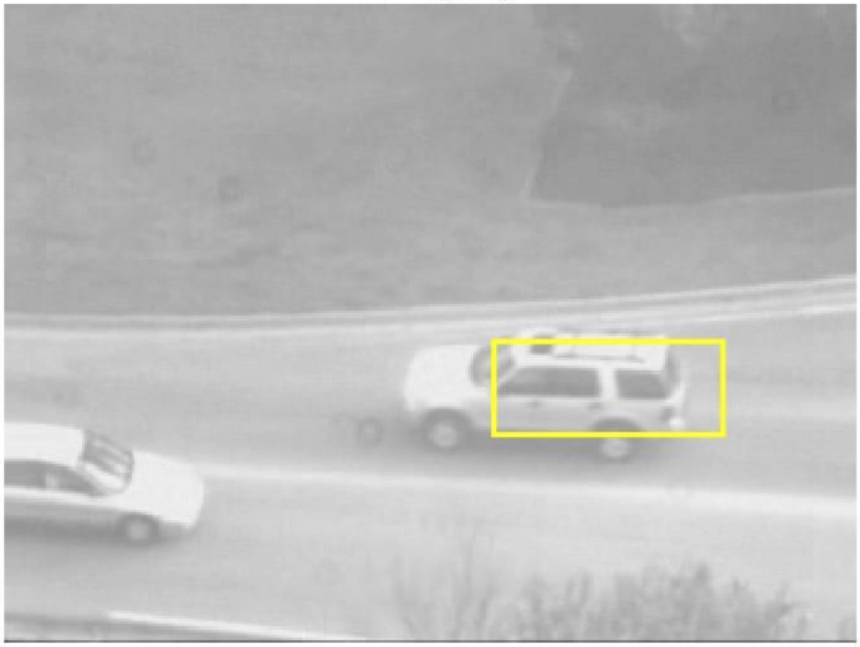
Tracking performance (image + bounding rectangle) at frames 2, 100, 200, 300 and 400 of the car:

Frame[100]



Frame[200]

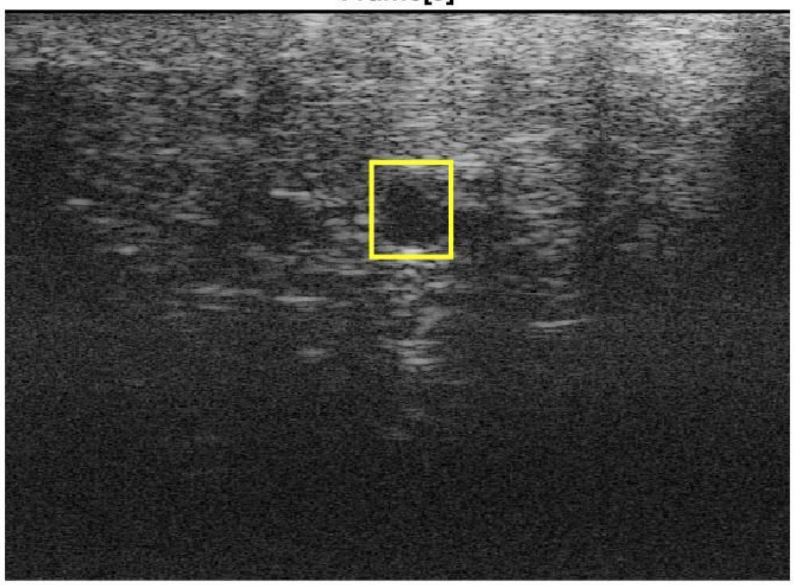
Frame[300]



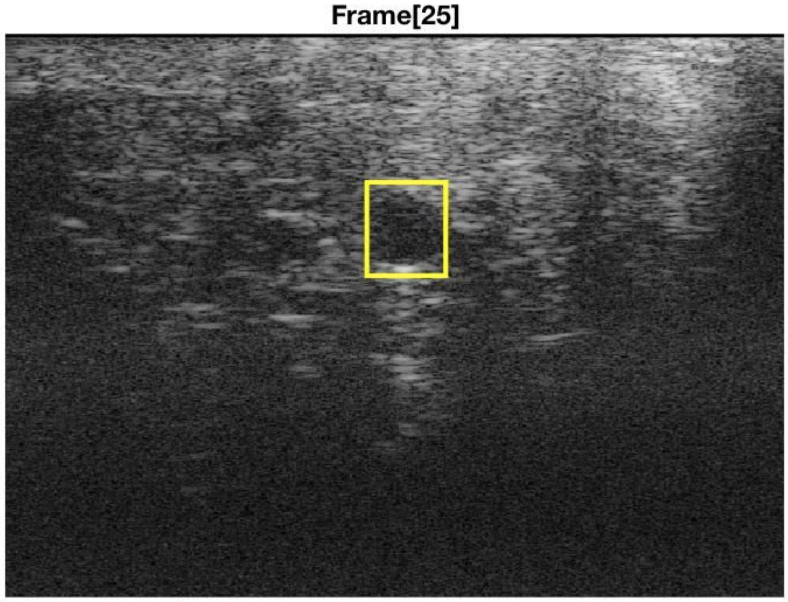
Frame[400]

Tracking performance (image + bounding rectangle) at frames 5, 25, 50, 75 and 100 of the beating vessel in ultrasound:

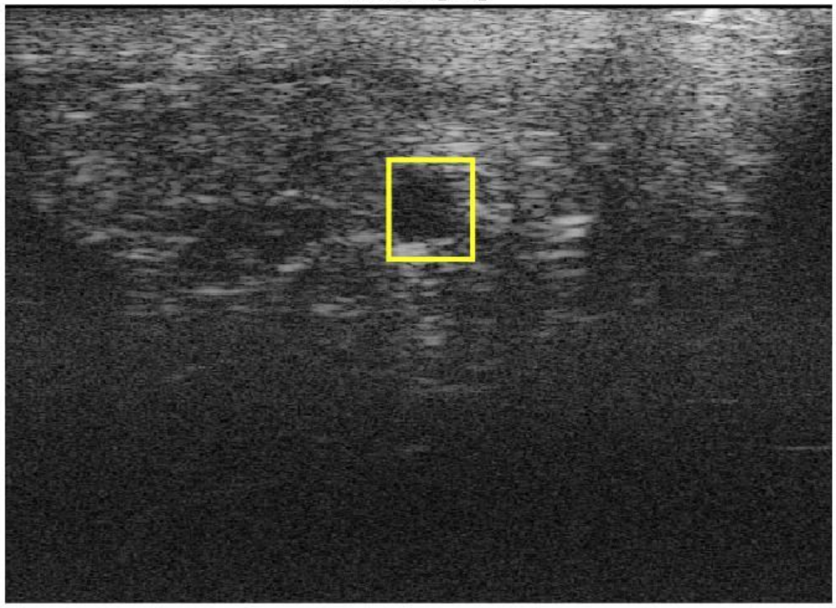
Frame[5]



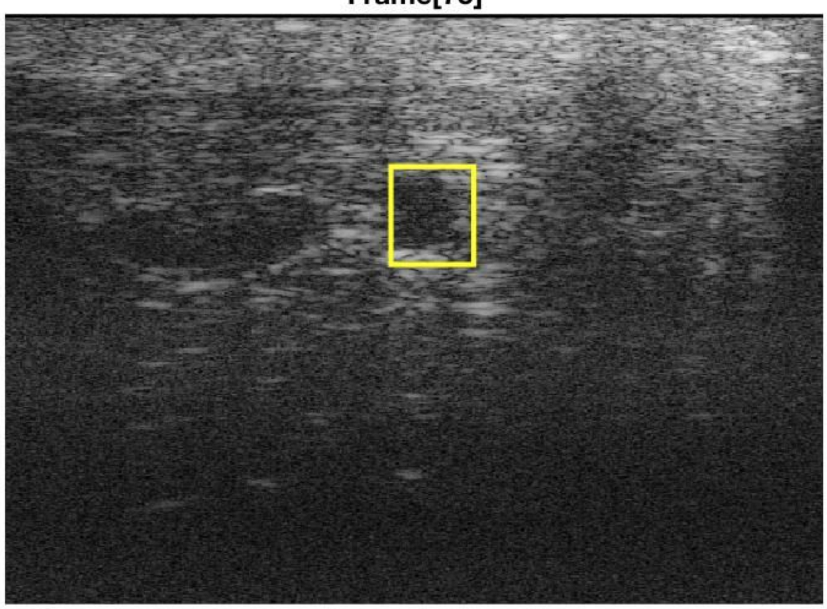
Frame[25]



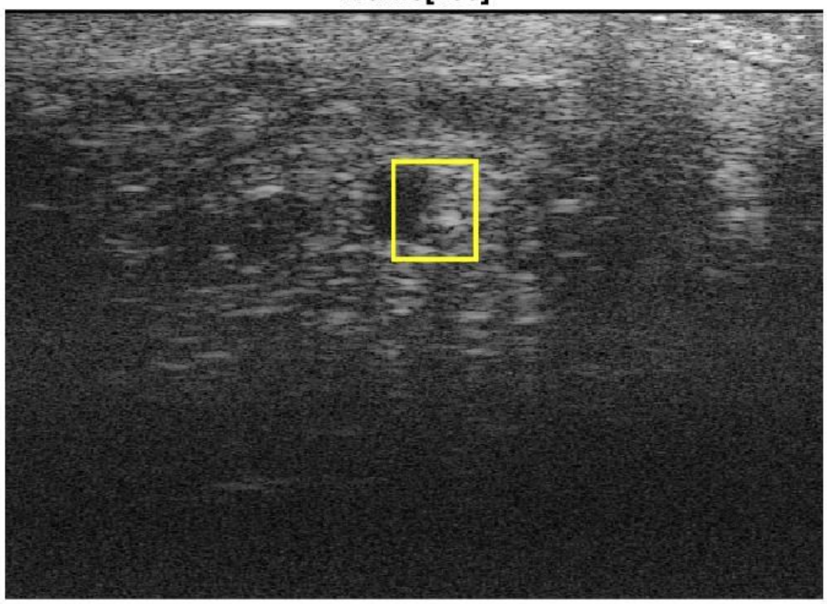
Frame[50]



Frame[75]



Frame[100]

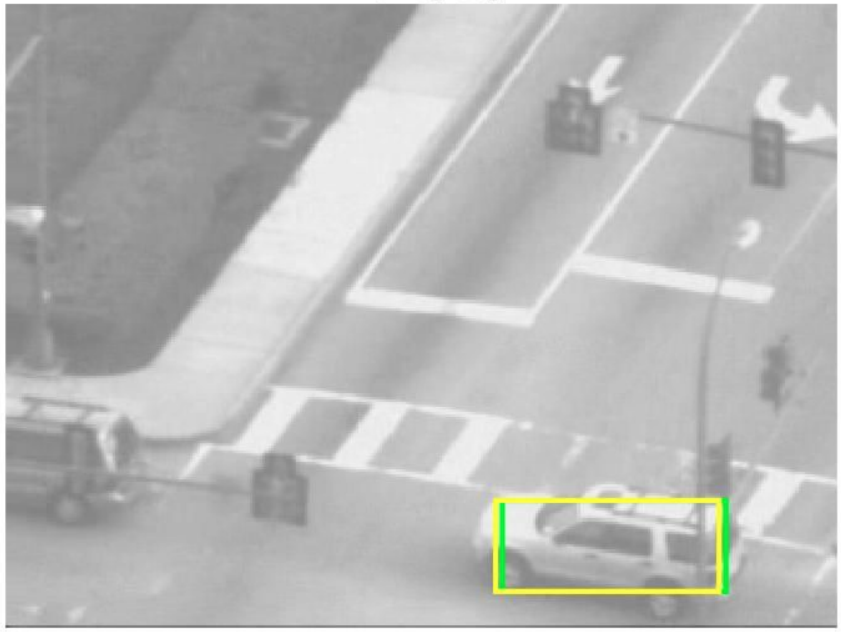


Q 1.4 (Bonus):

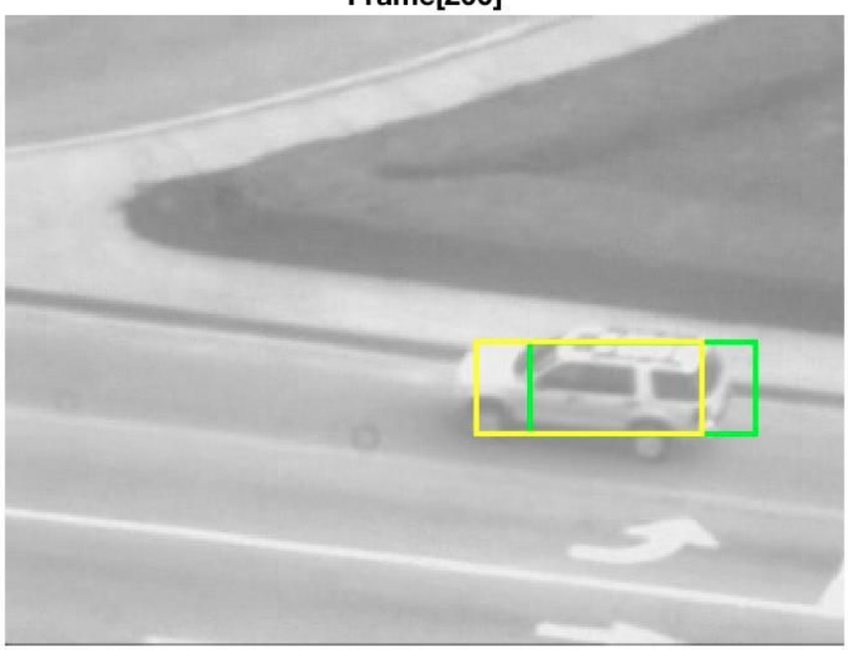
Tracking performance (image + bounding rectangle) at frames 2, 100, 200, 300 and 400 of the car:

Frame[2]

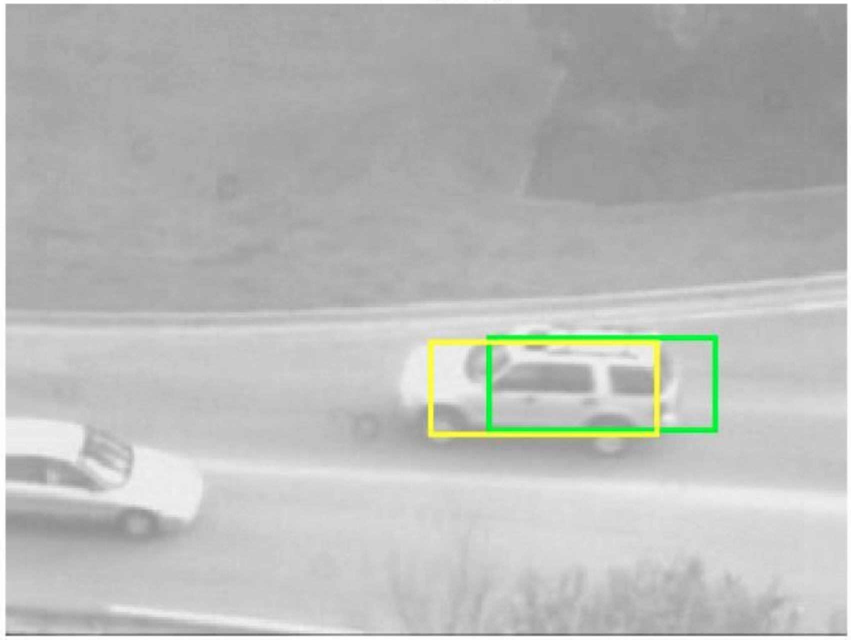
Frame[100]



Frame[200]



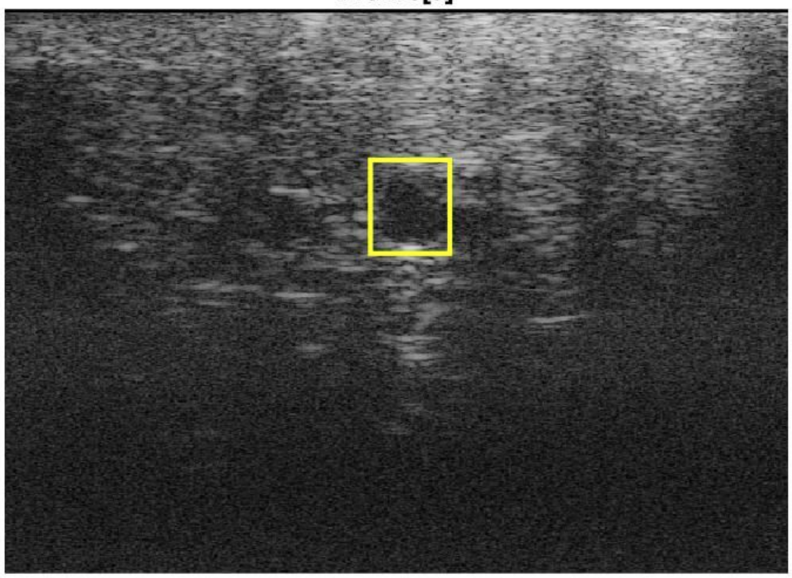
Frame[300]



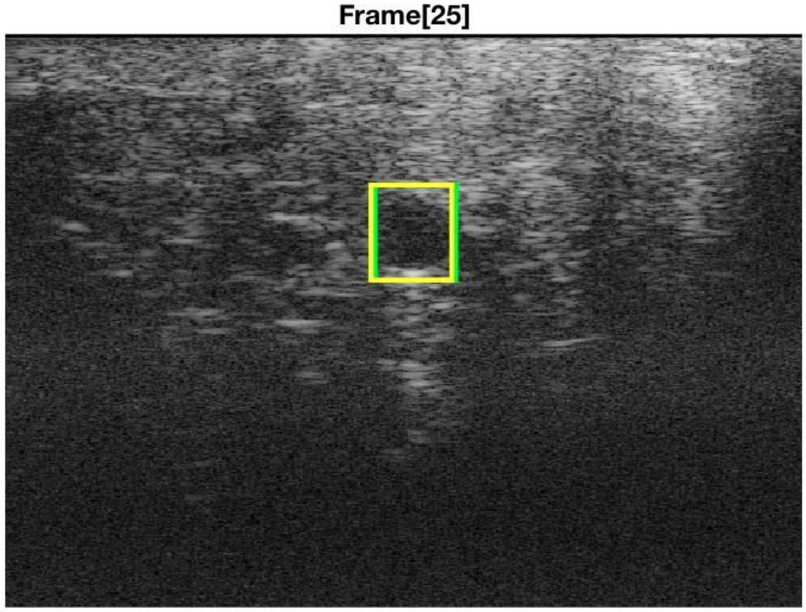
Frame[400]

Tracking performance (image + bounding rectangle) at frames 5, 25, 50, 75 and 100 of the beating vessel in ultrasound:

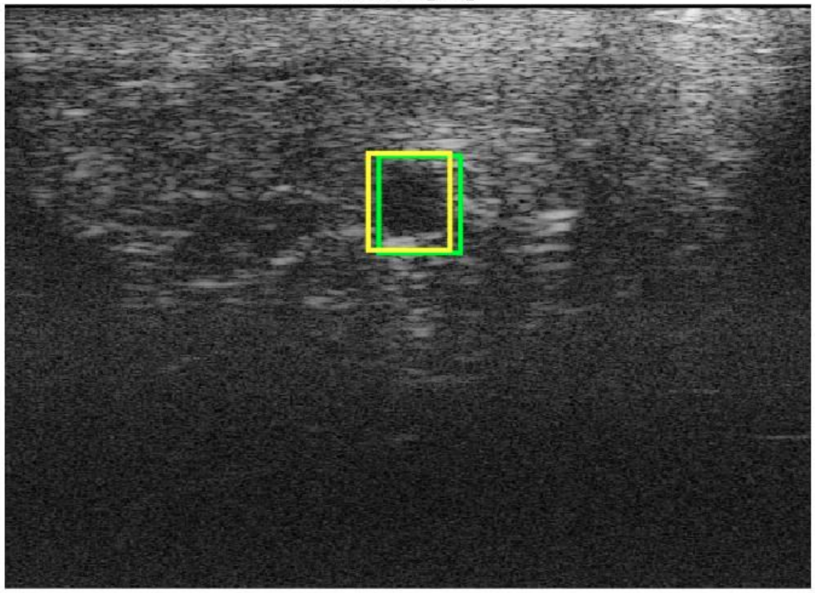
Frame[5]



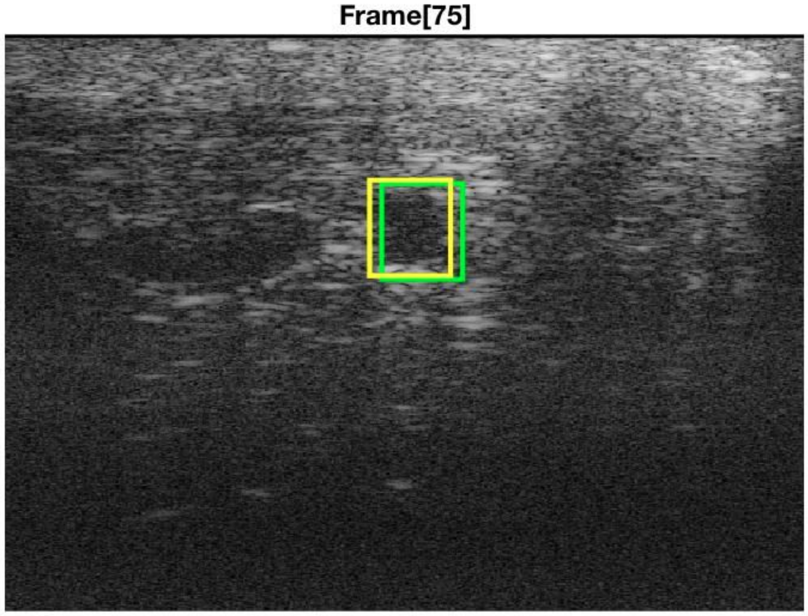
Frame[25]

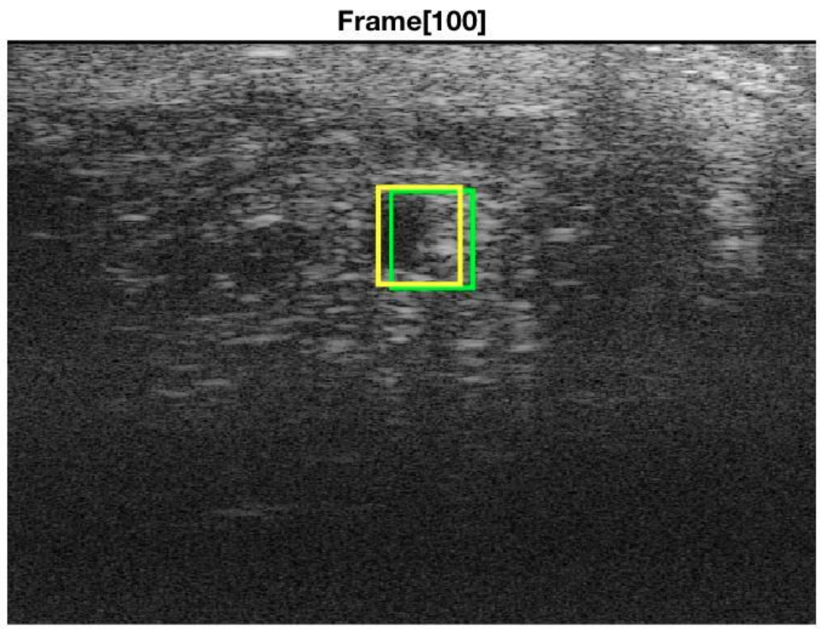


Frame[50]



Frame[75]





Q 2.1:

From

$$I_{t+1} = I_t + \sum_{c=1}^{k} w_c B_c$$

We can get

$$\sum_{c=1}^{k} w_c B_c = I_{t+1} - I_t$$

$$B_c \sum_{c=1}^{k} w_c B_c = B_c(I_{t+1} - I_t)$$

Because B s are orthogonal to each other, $B_c \times B_k = 0$, if c != k.

Thus:

$$B_c \sum_{c=1}^{k} w_c B_c = \left|Bc\right|^2 w_c = B_c(I_{t+1} - I_t)$$

$$w_c = \frac{B_c}{\left|Bc\right|^2} (I_{t+1} - I_t)$$

Q2.3

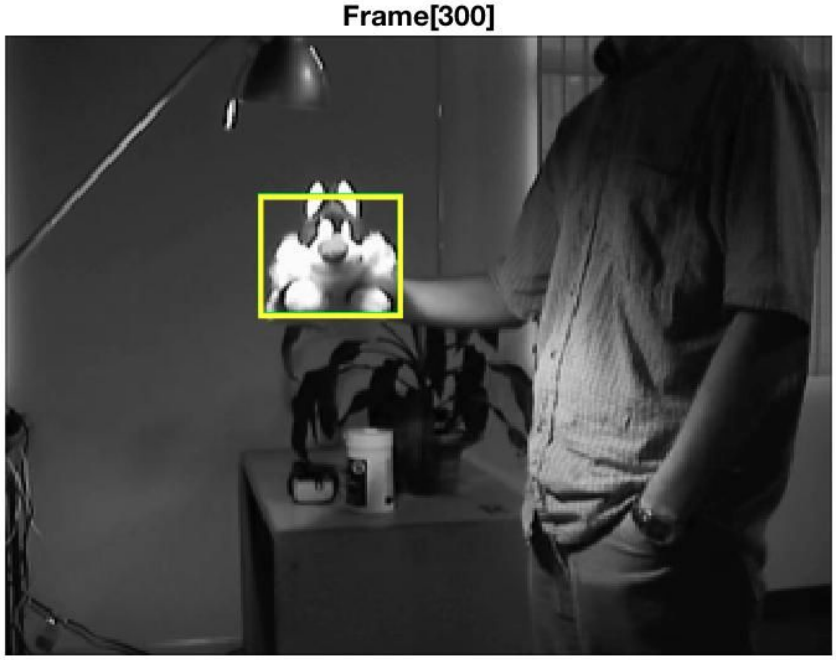
Tracking performance (image + bounding rectangle) at frames 2, 200, 300, 350 and 400 of the toy:

Note: I can't see the huge difference of LucasKanadeInverseCompositional and LucasKanadeBasis in terms of this case. Therefore, in the report picture, you can't see the green rectangle because it is overlapped by the yellow rectangle.

Frame[2]

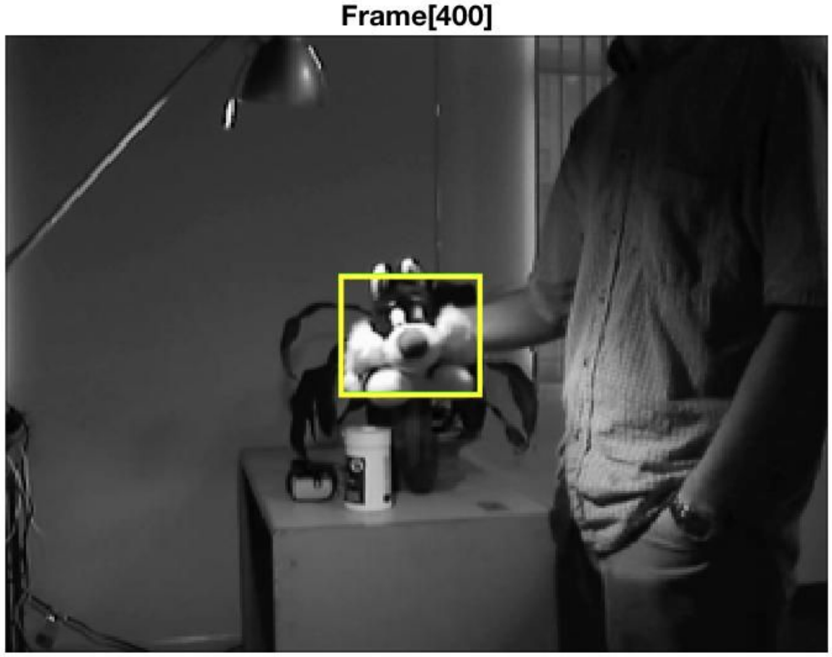
Frame[200]

Frame[300]



Frame[350]

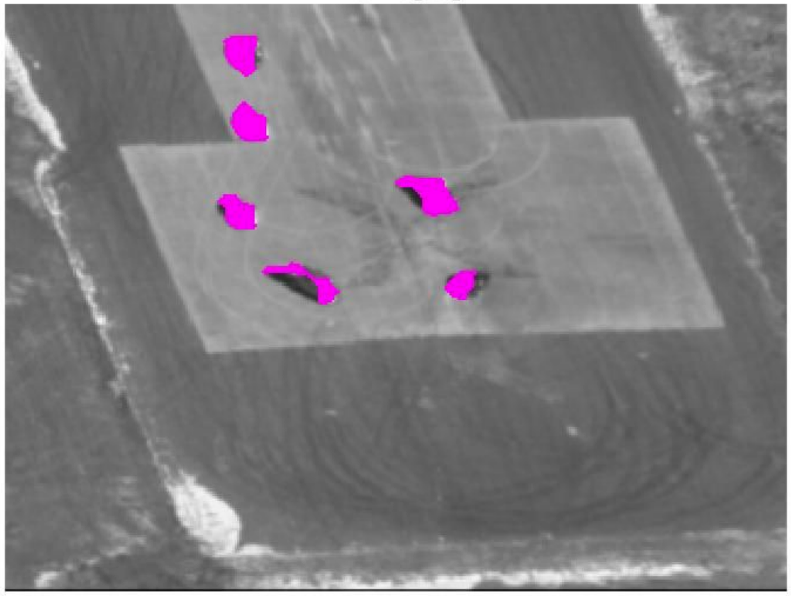
Frame[400]



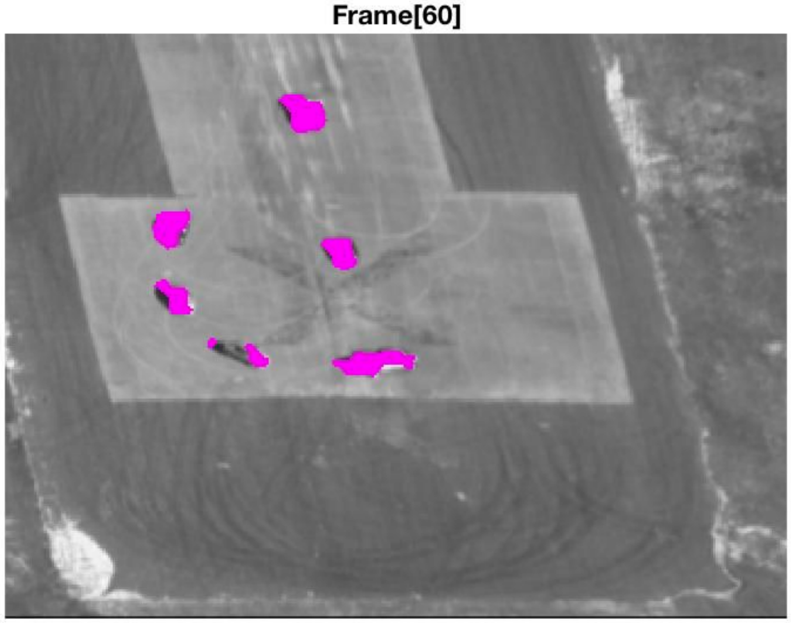
Q3.3

Tracking performance at frames 30, 60, 90 and 120:

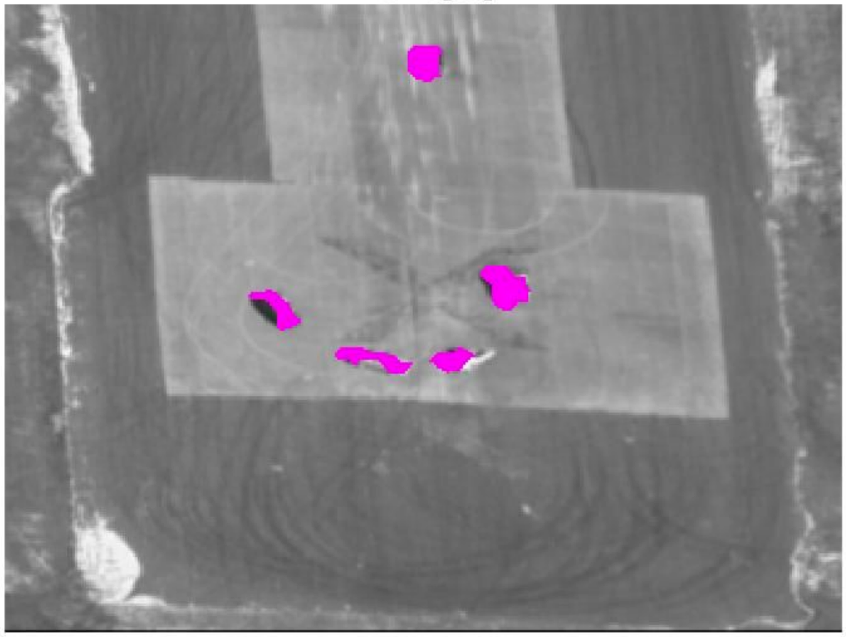
Frame[30]



Frame[60]



Frame[90]



Frame[120]